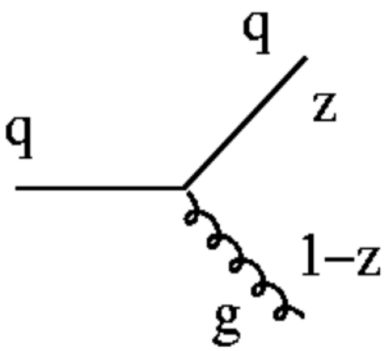
Monte Carlo School 2015
13-17 April 2015
DESY, Hamburg

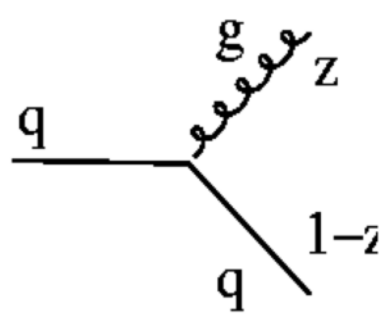
Exercise 2 - Introduction

- Schedule:
  - Tuesday - Exercise 1:
    - Random numbers
    - MC method
    - MC integration
  - Wednesday - Exercise 2:
    - Sudakov form factor
    - MC solution of evolution equation
  - Thursday - Exercise 3
    - Calculation & simulation of Higgs production
    - Using MC solution of evolution equation → calculation of pt spectrum of Higgs at LHC

Splitting functions in lowest order

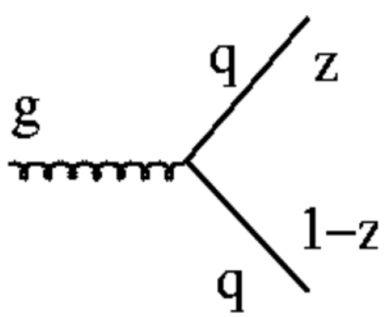


$$P_{qq} = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$

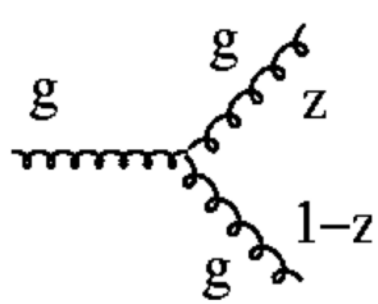


$$P_{gq} = \frac{4}{3}\left(\frac{1+(1-z)^2}{z}\right)$$

similarity to EPA...



$$P_{qg} = \frac{1}{2}\left(z^2+(1-z)^2\right)$$



$$P_{gg} = 6\left(\frac{1-z}{z}+\frac{z}{1-z}+z(1-z)\right)$$

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13-17 April 2015
DESY, Hamburg

Collinear factorization: DGLAP

- introduce new scale $\mu^2 \gg \chi^2$ and include soft, non-perturbative physics into renormalized parton density:
- $$q_i(x,\mu^2) = q_i^0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i^0(\xi)P_{qq}\left(\frac{x}{\xi}\right) + g^0(\xi)P_{qg}\left(\frac{x}{\xi}\right)\right]\log\left(\frac{\mu^2}{\chi^2}\right)$$

- Dokshitzer Gribov Lipatov Altarelli Parisi equation:

V.V. Gribov and L.N. Lipatov Sov. J. Nucl. Phys. 438 and 675 (1972) 15, L.N. Lipatov Sov. J. Nucl. Phys 94 (1975) 20, G. Altarelli and G. Parisi Nucl.Phys.B 298 (1977) 126, Y.L. Dokshitser Sov. Phys. JETP 641 (1977) 46

$$\frac{dq_i(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i(\xi,\mu^2)P_{qq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{qg}\left(\frac{x}{\xi}\right)\right]$$

- BUT there are also gluons....

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[\sum_i q_i(\xi,\mu^2)P_{gq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{gg}\left(\frac{x}{\xi}\right)\right]$$

- DGLAP is the analogue to the beta function for running of the coupling

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13-17 April 2015
DESY, Hamburg

What is happening at small x ?

- For $x \to 0$ only gluon splitting function matters:

$$P_{gg} = 6\left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right) = 6\left(\frac{1}{z} - 2 + z(1-z) + \frac{1}{1-z}\right)$$

$$P_{gg} \sim 6\frac{1}{z} \text{ for } z \to 0$$

- evolution equation is then:

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi} g(\xi,\mu^2) P_{gg}\left(\frac{x}{\xi}\right)$$

$$xg(x,t) = xg(x,\mu_0^2) + \frac{3\alpha_s}{\pi}\int_{t_0}^{t} d\log t' \int_x^1 \frac{d\xi}{\xi}\xi g(\xi,t')$$ with $t = \mu^2$

Solving integral equations

Weisstein, Eric W. "Integral Equation Neumann Series." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/IntegralEquationNeumannSeries.html

- Integral equation of Fredholm type:

$$\phi(x) = f(x) + \lambda \int_a^b K(x,y)\phi(y)dy$$

- solve it by iteration (Neumann series):

$$\begin{aligned}
\phi_0(x) &= f(x) \\
\phi_1(x) &= f(x) + \lambda \int_a^b K(x,y) f(y) dy \\
\phi_2(x) &= f(x) + \lambda \int_a^b K(x,y_1) f(y_1) dy_1 + \lambda^2 \int_a^b \int_a^b K(x,y_1) K(y_1,y_2) f(y_2) dy_2 dy_1 \\
\phi_n(x) &= \sum_{i=0}^{n} \lambda^i u_i(x) \\
u_0(x) &= f(x) \\
u_1(x) &= \int_a^b K(x,y) f(y) dy \\
u_n(x) &= \int_a^b \cdots \int_a^b K(x,y_1) K(y_1,y_2) \cdots K(y_{n-1},y_n) f(y_n) dy_1 \cdots dy_n
\end{aligned}$$

with the solution:

$$\phi(x) = \lim_{n\to\infty} q_n(x) = \lim_{n\to\infty} \sum_{i=0}^{n} \lambda^i u_i(x)$$

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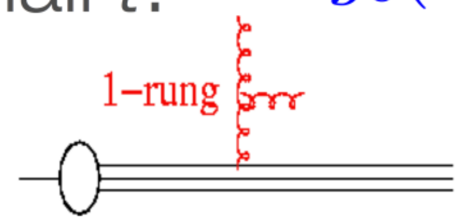
13-17 April 2015
DESY, Hamburg

Estimates at small x: DLL

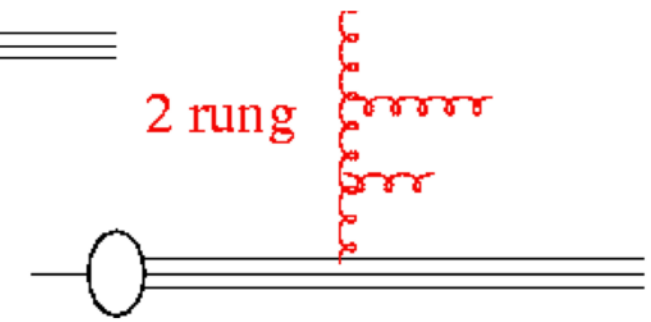
$$xg(x,t) = xg(x,\mu_0^2) + \frac{3\alpha_s}{\pi}\int_{t_0}^{t} d\log t' \int_x^1 \frac{d\xi}{\xi}\xi g(\xi,t') \quad \text{with} \quad t=\mu^2$$

- use constant starting distribution at small t: $xg_0(x) = C$

$$xg_1(x,t) = C + \frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}C$$

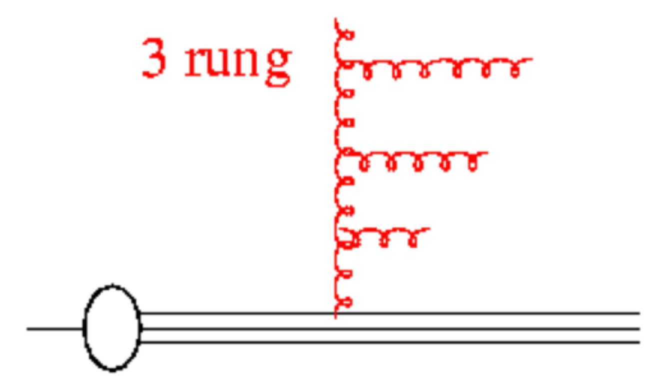


$$xg_2(x,t) = C + \frac{1}{2}\frac{1}{2}\left(\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}\right)^2 C$$



$$\vdots$$

$$xg_n(x,t) = C + \frac{1}{n!}\frac{1}{n!}\left(\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}\right)^n C$$



$$xg(x,t) = \sum_n \frac{1}{n!}\frac{1}{n!}\left(\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}\right)^n C$$

$$xg(x,t) \sim C\exp\left(2\sqrt{\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}}\right)$$

double leading log approximation (DLL)

Divergencies again...

- collinear divergencies factored into renormalized parton distributions
- what about soft divergencies ?

treated with "plus" prescription

$$\frac{1}{1-z} \rightarrow \frac{1}{1-z}_+ \qquad \textbf{with} \qquad \int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{(1-z)}$$

- soft divergency treated with Sudakov form factor:

$$\Delta(t) = \exp\left[- \int_{t_0}^{t} \frac{dt'}{t'} \int^{z_{max}} dz \frac{\alpha_s}{2\pi} \tilde{P}(z) \right]$$

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13-17 April 2015
DESY, Hamburg

DGLAP evolution again....

- differential form: $$t\frac{\partial}{\partial t} f(x,t) = \int \frac{dz}{z}\frac{\alpha_s}{2\pi} P_+(z)\, f\left(\frac{x}{z},t\right)$$

$$\Delta_s(t) = \exp\left(-\int_x^{z_{max}} dz \int_{t_0}^{t} \frac{\alpha_s}{2\pi}\frac{dt'}{t'}\tilde{P}(z)\right)$$

- differential form using f/Δ_s with

$$t\frac{\partial}{\partial t}\frac{f(x,t)}{\Delta_s(t)} = \int \frac{dz}{z}\frac{\alpha_s}{2\pi}\frac{\tilde{P}(z)}{\Delta_s(t)} f\left(\frac{x}{z},t\right)$$

- integral form

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z}\int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')}\tilde{P}(z) f\left(\frac{x}{z},t'\right)$$

no – branching probability from t_0 **to** t

Sudakov ff: all loop resum...

$g \to gg$ Splitting Fct $\tilde{P}(z) = \frac{\bar{\alpha}_s}{1-z} + \frac{\bar{\alpha}_s}{z} + \ldots$

- Sudakov form factor .... all loop resummation

$$\Delta_S = \exp\left(- \int dz \int \frac{dq'}{q'} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)$$

$$\Delta_S = 1 + \left(- \int dz \int \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)^1 + \frac{1}{2!} \left(- \int dz \int \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)^2 \ldots$$

$$\tilde{P}(z) \left[1 - \int \int dz \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) + \frac{1}{2!} \left(- \int \int dz \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)^2 - \ldots - \right]$$

DGLAP re-sums leading logs...

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via iteration:

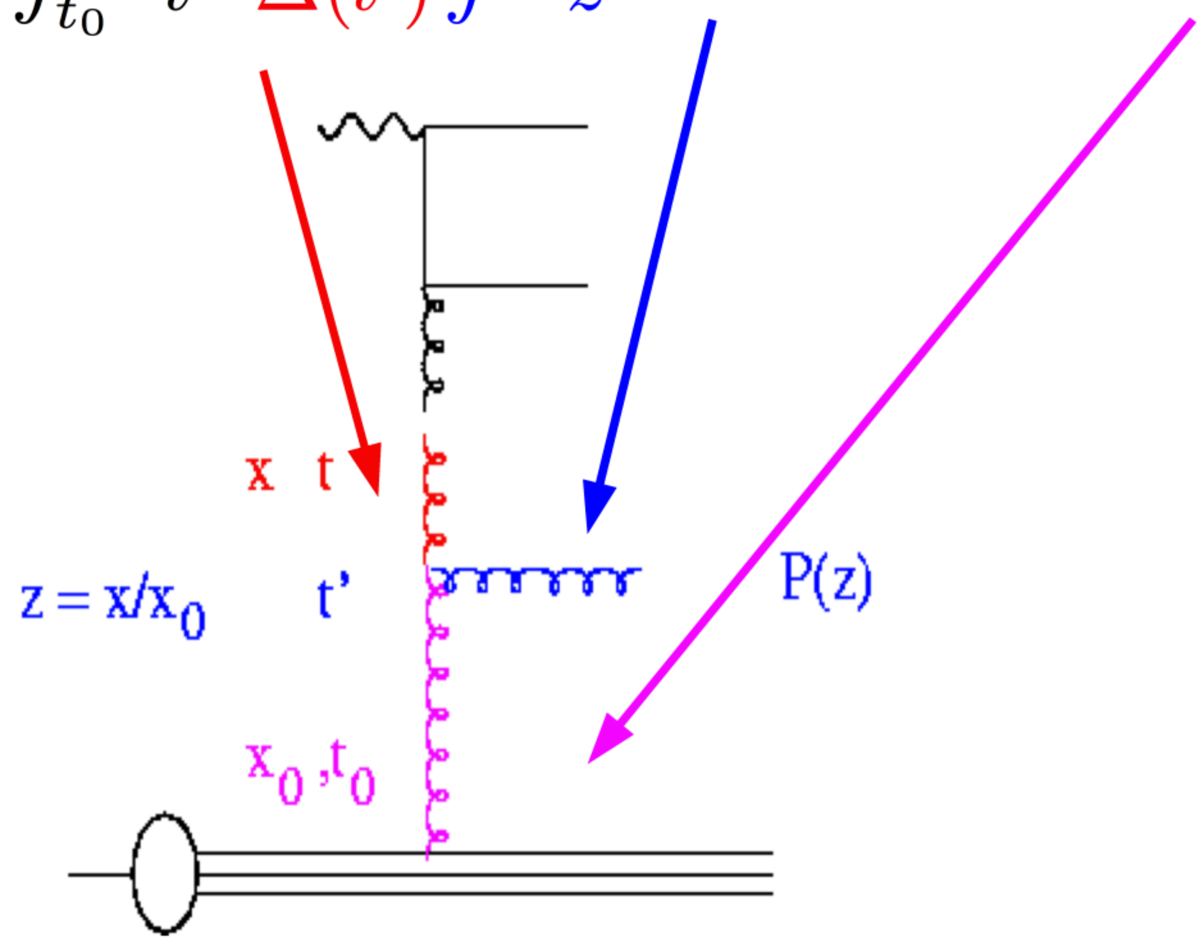
from t' to t w/o branching

branching at t'

from t_0 to t' w/o branching

$$f_0(x,t) = f(x,t_0)\Delta(t)$$

$$f_1(x,t) = f(x,t_0)\Delta(t) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z,t_0)\Delta(t')$$



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13-17 April 2015
DESY, Hamburg

DGLAP re-sums leading logs...

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z}\int \frac{dt'}{t'}\cdot\frac{\Delta_s(t)}{\Delta_s(t')}\tilde{P}(z) f\left(\frac{x}{z},t'\right)$$

- solve integral equation via iteration:

from t' to t w/o branching

branching at t'

from t_0 to t' w/o branching

$$f_0(x,t) = f(x,t_0)\Delta(t)$$

$$f_1(x,t) = f(x,t_0)\Delta(t) + \int_{t_0}^{t}\frac{dt'}{t'}\frac{\Delta(t)}{\Delta(t')}\int\frac{dz}{z}\tilde{P}(z) f(x/z,t_0)\Delta(t')$$

$$= f(x,t_0)\Delta(t) + \log\frac{t}{t_0} A\otimes\Delta(t) f(x/z,t_0)$$

$$f_2(x,t) = f(x,t_0)\Delta(t) + \log\frac{t}{t_0} A\otimes\Delta(t) f(x/z,t_0) + \frac{1}{2}\log^2\frac{t}{t_0} A\otimes A\otimes\Delta(t) f(x/z,t_0)$$

$$f(x,t) = \lim_{n\to\infty} f_n(x,t) = \lim_{n\to\infty}\sum_n \frac{1}{n!}\log^n\left(\frac{t}{t_0}\right) A^n\otimes\Delta(t) f(x/z,t_0)$$

DGLAP re-sums $\log t$ to all orders !!!!!!!!!!!!!!!!

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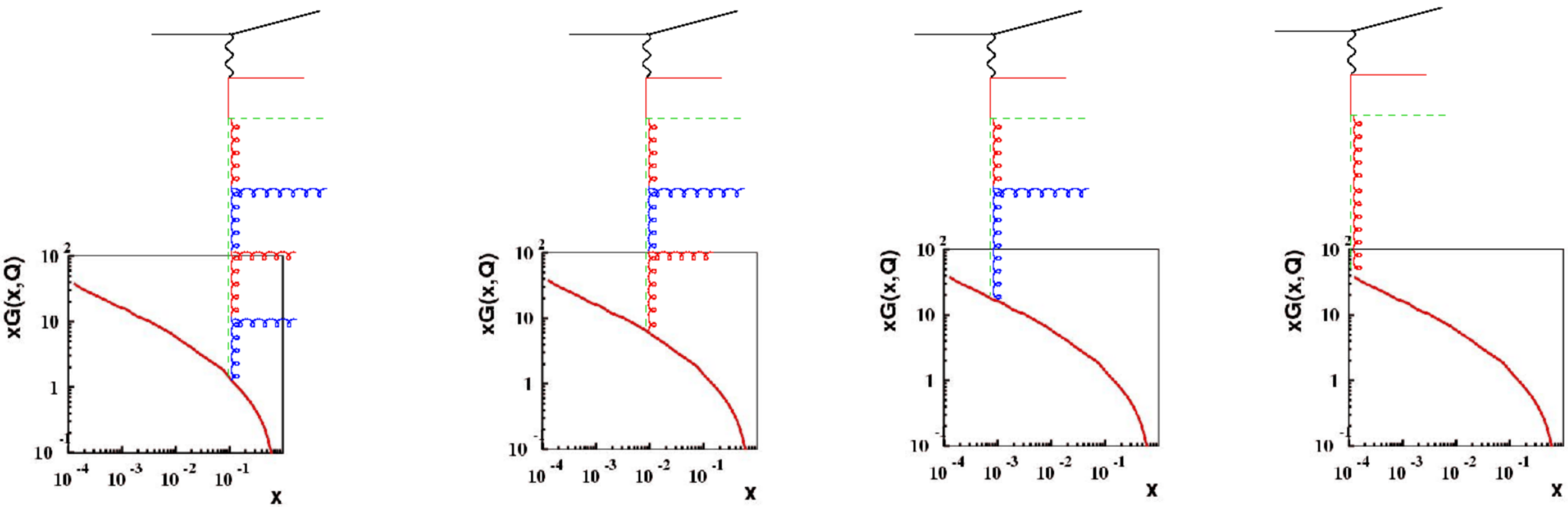
DGLAP evolution equation... again...

- for fixed x and Q^2 chains with different branchings contribute
- iterative procedure, spacelike parton showering



$$f(x,t) = f_0(x,t_0)\Delta_s(t) + \sum_{k=1}^{\infty} f_k(x_k,t_k)$$

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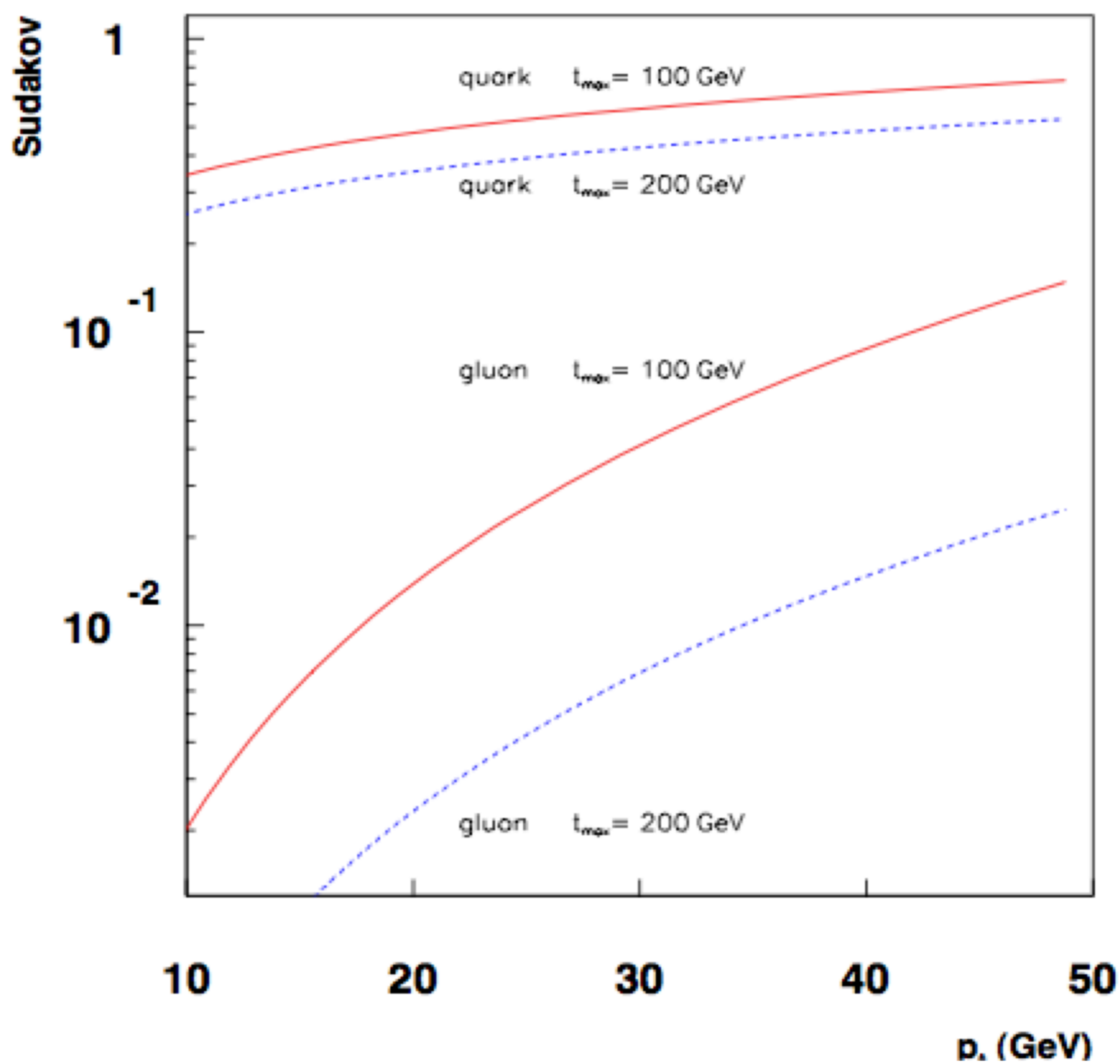
13-17 April 2015
DESY, Hamburg

Sudakov form factor

- what is the limit on z- integration ?
  - resolvable branching ?
  - $z < 1 - \frac{Q_0^2}{Q_b^2}$ with Q_0 a soft cutoff
- probability of no -radiation between Q_a and Q_b
  - $$\Pi(Q_a^2, Q_b^2) = \frac{\Delta(Q_a^2)}{\Delta(Q_b^2)} = \exp\left[- \int_{Q_b^2}^{Q_a^2} \frac{dq^2}{q^2} \int_0^{z_{cut}} dz \frac{\alpha_s}{2\pi} P(z) \right]$$

Sudakov form factors



Solving evolution equation with Monte Carlo

Evolution equation and Monte Carlo

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \frac{\alpha_s}{2\pi} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

$$= f(x,t_0)\Delta_s(t) + \int dz \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \frac{\alpha_s}{2\pi} \tilde{P}(z) f(x',t')\,\delta(x - zx')dx'$$

- $f(x', t_0)\Delta_s(t', t_0)$ **is probability from previous branching**
- use Sudakov: $$\Delta(t) = \exp\left[-\int_{t_0}^{t} \frac{dt'}{t'} \int^{z_{max}} dz \frac{\alpha_s}{2\pi} \tilde{P}(z)\right]$$
- generate *t* according to Sudakov

$$\frac{\partial}{\partial t'} \frac{\Delta_s(t)}{\Delta_s(t')} = \frac{\Delta_s(t)}{\Delta_s(t')} \left[\frac{1}{t'}\right] \int^{z_{max}} dz \frac{\alpha_s}{2\pi} \tilde{P}(z)$$

➔solve it for *t:* $\log \Delta_s(t, t') = \log R$

- generate z according to $$\int_{\epsilon}^{z} dz \frac{\alpha_s}{2\pi} P(z) = R \int_{\epsilon}^{1-\epsilon} dz \frac{\alpha_s}{2\pi} P(z)$$

Evolution equation and MC

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z}\int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')}\tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via iteration:

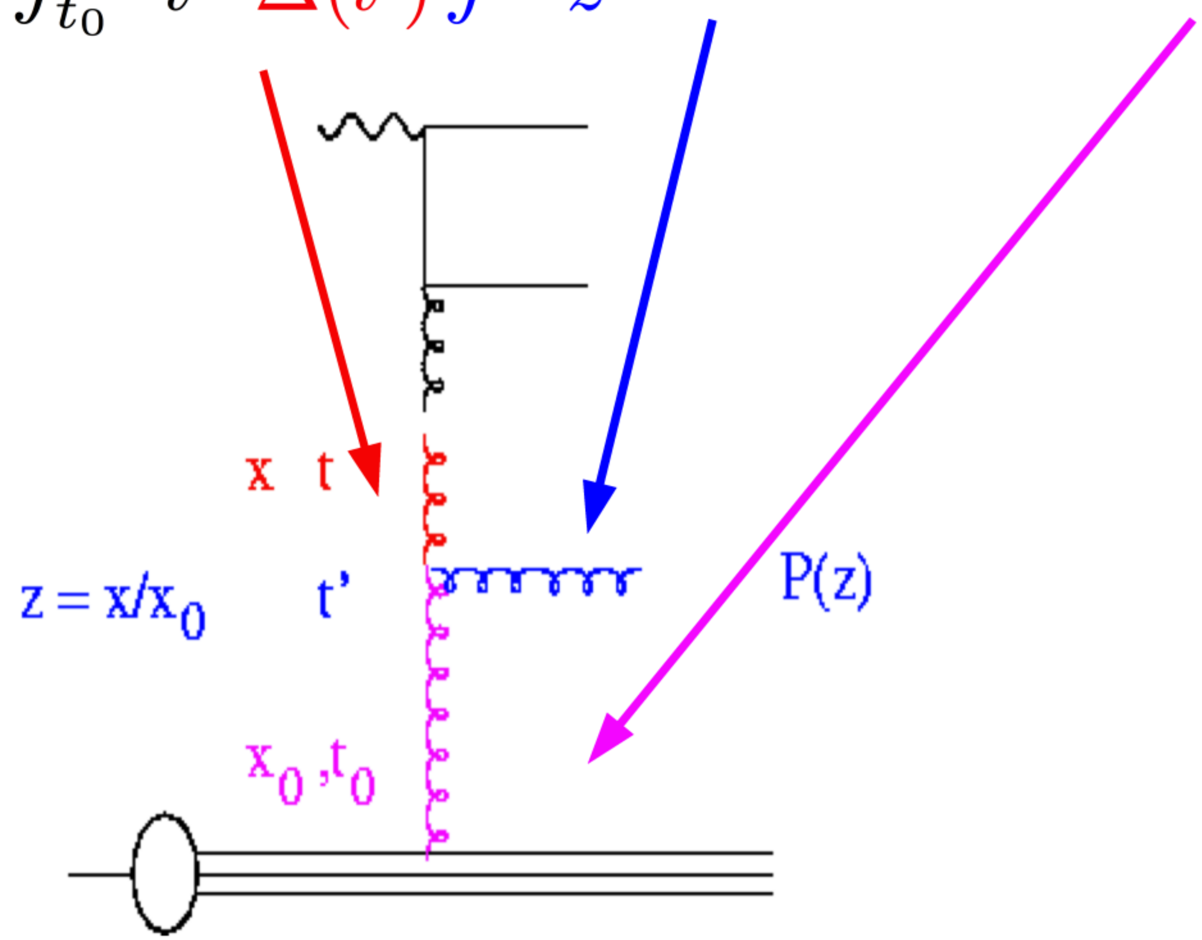
from t' to t w/o branching

branching at t'

from t_0 to t' w/o branching

$$f_0(x,t) = f(x,t_0)\Delta(t)$$

$$f_1(x,t) = f(x,t_0)\Delta(t) + \int_{t_0}^{t} \frac{dt'}{t'}\frac{\Delta(t)}{\Delta(t')}\int \frac{dz}{z}\tilde{P}(z) f(x/z, t_0)\Delta(t')$$



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13-17 April 2015
DESY, Hamburg

Evolution equation and MC

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via iteration:

from t' to t w/o branching

branching at t'

from t_0 to t' w/o branching

$$\begin{aligned}
f_0(x,t) &= f(x,t_0)\Delta(t) \\
f_1(x,t) &= f(x,t_0)\Delta(t) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z,t_0)\Delta(t') \\
&= f(x,t_0)\Delta(t) + \log\frac{t}{t_0} A \otimes \Delta(t) f(x/z,t_0) \\
f_2(x,t) &= f(x,t_0)\Delta(t) + \log\frac{t}{t_0} A \otimes \Delta(t) f(x/z,t_0) + \\
&\quad \frac{1}{2}\log^2\frac{t}{t_0} A \otimes A \otimes \Delta(t) f(x/z,t_0) \\
f(x,t) &= \lim_{n\to\infty} f_n(x,t) = \lim_{n\to\infty} \sum_n \frac{1}{n!} \log^n\left(\frac{t}{t_0}\right) A^n \otimes \Delta(t) f(x/z,t_0)
\end{aligned}$$

summing up all contribution up to t … advantage of importance sampling….

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Monte Carlo solution of evolution

updfevolv is hosted by Hepforge, IPPP Durham

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uPDFevolv 1.0.0

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uPDFevolv manual

uPDFevolv is an evolution code for TMD parton densities using the CCFM evolution equation.

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Version

1.0.0

Date

2014

Table of Contents

Exercise 2

Exercises 2 (15. April 2015)

6. Calculate the Sudakov form factor for the scales $t_2 = 10, 100, 500$ GeV^2 as a function of t_1 and plot it as a function of t_1. Use q as the argument for α_s, and check the differences. For the z integral use $z_{min} = 0.01$ and $z_{max} = 0.99$.

$$\log \Delta_S = - \int_{t_1}^{t_2} \frac{dt}{t} \int_{z_{min}}^{z_{max}} dz \frac{\alpha_s}{2\pi} P(z)$$

   Use the gluon and also the quark splitting functions :

$$P_{gg} = 6 \left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z) \right)$$

   and

$$P_{qq} = \frac{4}{3} \frac{1+z^2}{1-z}$$

7. write a program to evolve a parton density $g(x) = 3(1-x)^5/x$ from a starting scale $t_0 = 1$ GeV^2 to and higher scale $t = 100$ GeV^2. Do the evolution only with fixed $\alpha_s = 0.1$ and an approximate gluon splitting function $P_{gg} = 6(\frac{1}{z} + \frac{1}{1-z})$. To avoid the divergent regions use $z_{min} = \epsilon$ and $z_{max} = 1 - \epsilon$ with $\epsilon = 0.1$. Calculate the Sudakov form factor for evolving from t_1 to t_2 using only the $\frac{1}{(1-z)}$ part of the splitting function. Generate z according to P_{gg}. Repeat the branching until you reach the scale t. Plot the $xg(x)$ as a function of x for the starting distribution and for the evolved distribution. Repeat the same exercise but with $P_{qq} = \frac{4}{3}\frac{1+z^2}{1-z}$.

   Calculate and plot the transverse momentum of the parton after the evolution. At the starting scale the partons can have a intrinsic k_t, which is generated by a gauss distribution with $\mu = 0$ and $\sigma = 0.7$ (use generating a gauss distribution from Exercise 1).

   Compare the k_t distribution using P_{gg} and P_{qq}. What is different ?

How to get started

- utilities:

```
courselib.h: include headers
ranlxd.h, ranlxd.cc: random number generator ranlux
```

- initialize ROOT (needed for plotting)

```
module avail
module load root/5.34
```

- copy all the templates (be careful, do not to overwrite …)

  cp -rp /afs/desy.de/user/s/school30/public/Exercises .

- compiling and running:

```
cd exercise-1
make -f makefile-example-1
./example-1
```

- templates are provided which include the general structure – you only have to fill the interesting – important parts … good luck

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Computing setup

- Connect either to eduroam or to the school network:

  Name: terascale

  WPA/WPA2-PSK: XxPWjNH7

- All will get school accounts for naf:
  - for example: ssh -X school30@naf-school01.desy.de
  - create folder:

    `cd public`
  - copy all templates:

    cp -rp /afs/desy.de/user/s/school30/public/Exercises .

- Writeup, Exercise sheets, templates and solutions at:

  `http://www.desy.de/~jung/mcschool2015/`