Theoretical uncertainties

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Assessing and improving accuracy

Theoretical uncertainties: fixed order

For fixed-order calculations we have two natural handles to evaluate theoretical uncertainties, the renormalisation and factorisation scales μ_R and μ_F

$$\alpha_s^n(x\mu_R) = \alpha_s^n + (n\,\beta_0 \ln x)\,\alpha_s^{n+1}(\mu_R)$$

By varying these scales we generate a higher-order contribution

The relevant questions here are

- What are the default choices of μ_R and μ_F?
- What is the range over which we should vary these scales?
- How should we add uncertainties?

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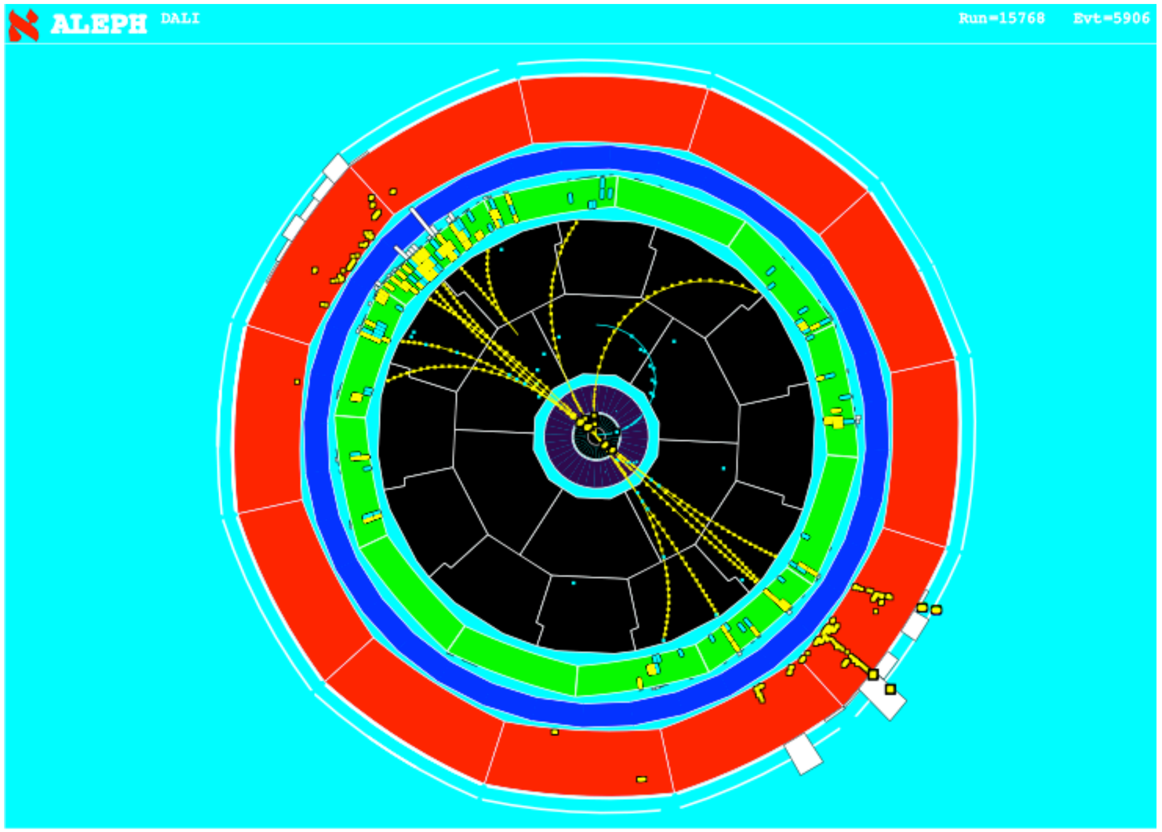
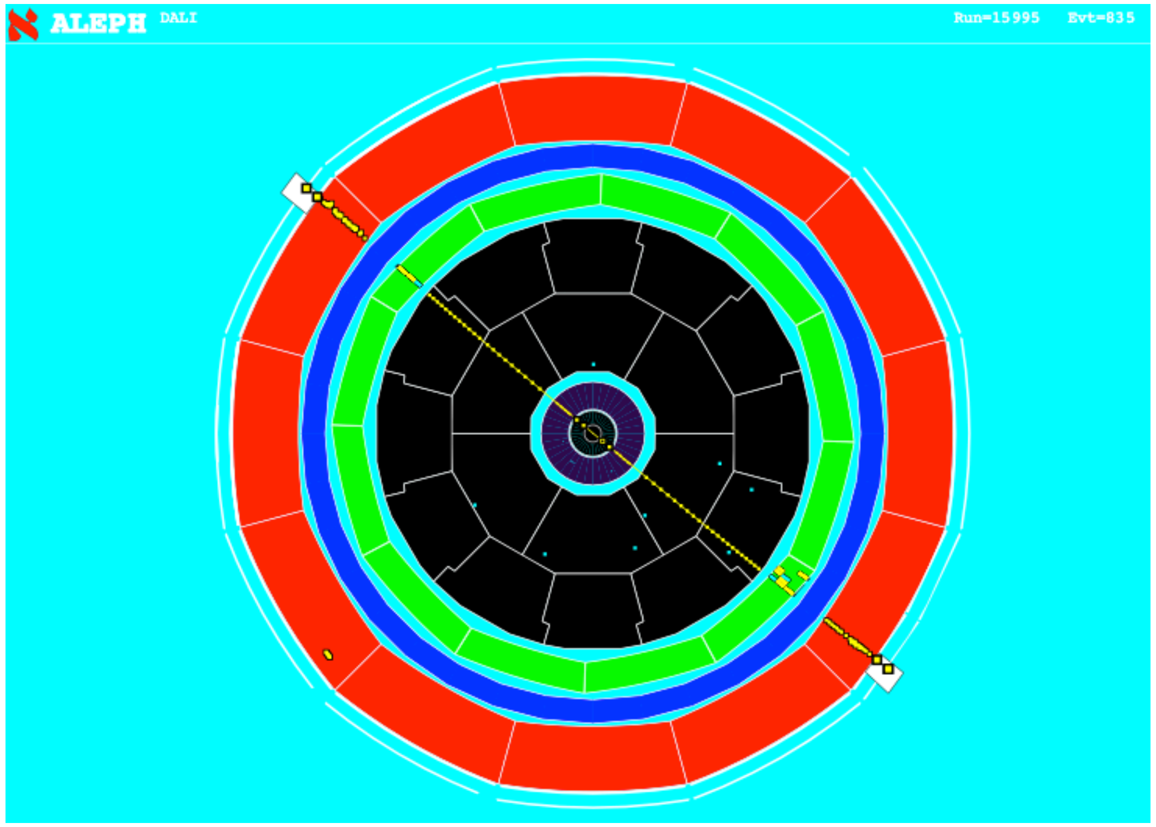
Unfortunately, there is no theoretically sound answer to any of these questions

Short-distance observables

Consider a simple counting observable in e^+e^- annihilation

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$





Renormalisation group analysis

Since the ratio R is IRC and collinear safe, it admits a massless limit

$$R\left(\alpha_s(\mu_R^2), \frac{Q^2}{\mu_R^2}, \frac{m_q^2}{\mu_R^2}\right) = \hat{R}\left(\alpha_s(\mu_R^2), \frac{Q^2}{\mu_R^2}\right) + \mathcal{O}\left(\left(\frac{m_q^2}{Q^2}\right)^p\right)$$

The massless limit $\hat{R}$ does not depend on μ_R $\Rightarrow$ renormalisation group

$$\left(\mu_R^2 \frac{\partial}{\partial \mu_R^2} + \beta(\alpha_s)\frac{\partial}{\partial \alpha_s}\right)\hat{R}\left(\alpha_s(\mu_R^2), \frac{Q^2}{\mu_R^2}\right) = 0 \quad \Rightarrow \quad Q^2 \frac{\partial \hat{R}}{\partial Q^2} = \beta(\alpha_s)\frac{\partial \hat{R}}{\partial \alpha_s}$$

The formal solution of this equation is

$$\hat{R}\left(\alpha_s(\mu_R^2), \frac{Q^2}{\mu_R^2}\right) = \hat{R}\left(\alpha_s(Q^2), 1\right)$$

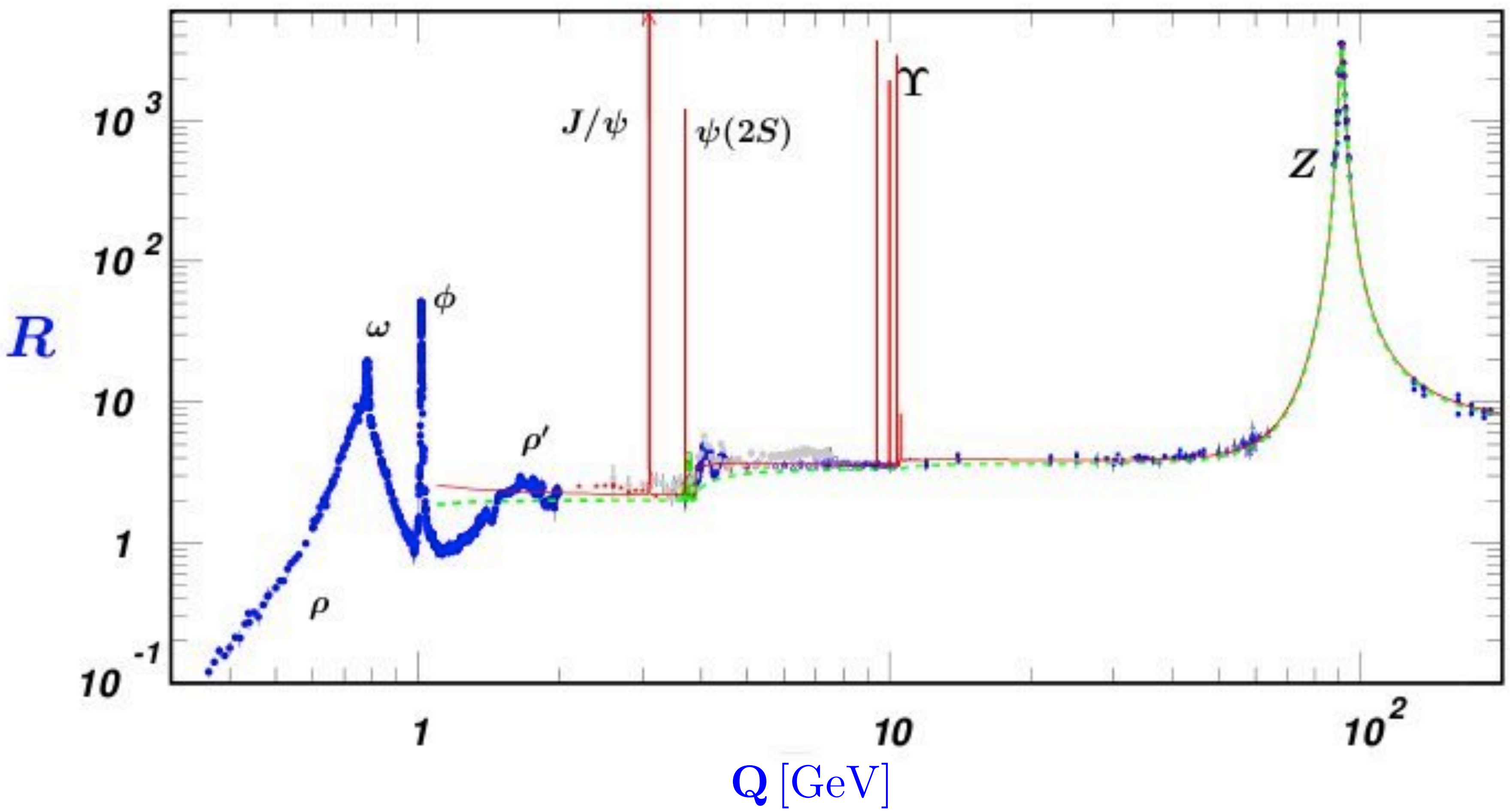
Renormalisation group, and the fact that $\hat{R}$ depends on a single hard scale provide enough condition to determine the default value of μ_R

Theoretical uncertainties: central value

For an observable characterised by a single scale, the dependence on the renormalisation scale appears in virtual corrections as follows

$$\hat{R}\left(\alpha_s(\mu_R^2), \frac{Q^2}{\mu_R^2}\right) = 1 + R_1\alpha_s(\mu_R^2) + \left(R_1\beta_0 \ln\frac{Q^2}{\mu_R^2} + R_2\right)\alpha_s^2(\mu_R^2) + \mathcal{O}(\alpha_s^3)$$

Choosing $\mu_R^2 = Q^2$ resums all terms $\ln(\mu_R^2/Q^2)$ in $\alpha_s(Q^2)$



Theoretical uncertainties: central value

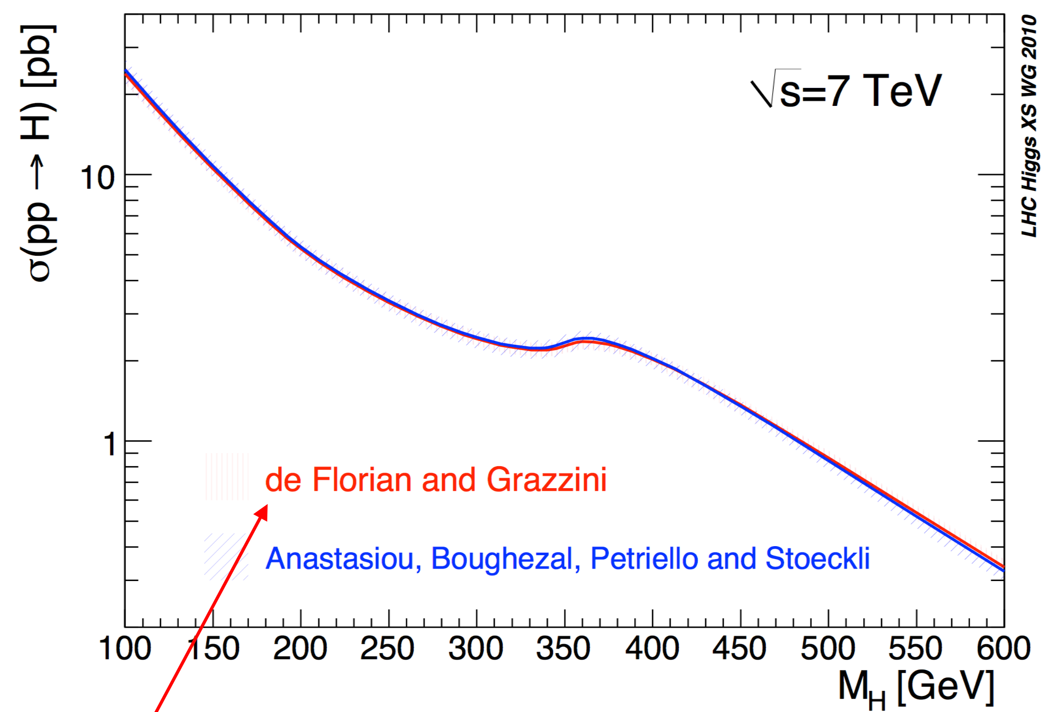
For an observable characterised by multiple scales at leading order

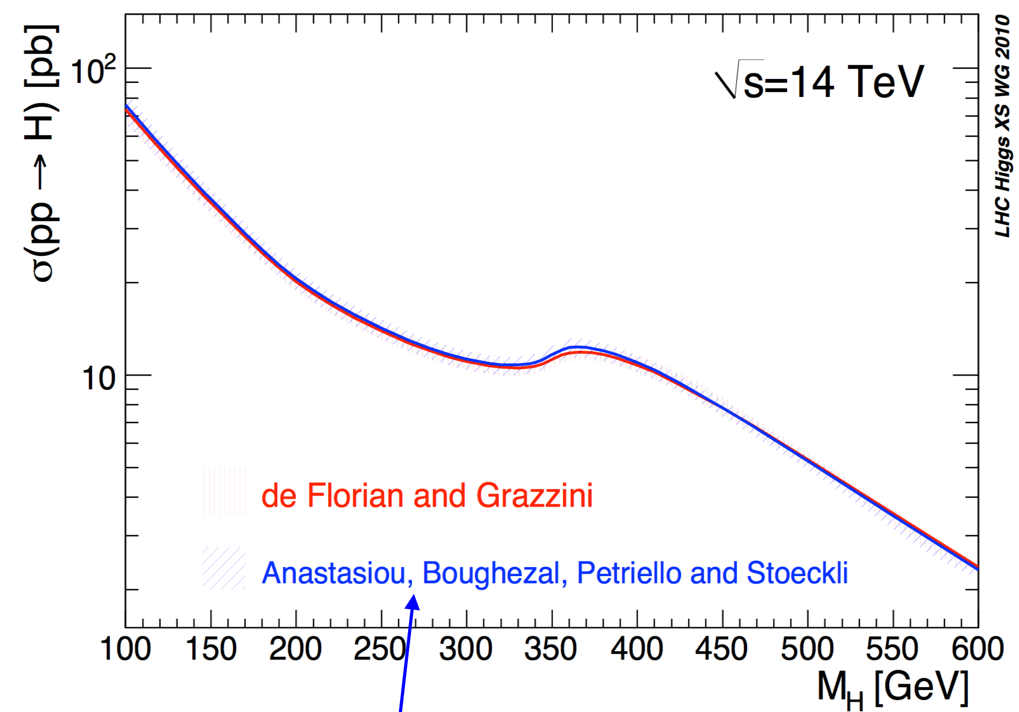
$$\sigma\left(\alpha_s(\mu^2), s_1, \ldots, s_n, \mu_R^2\right) \sim \sigma_0 \alpha_s^n + \left(\beta_0 \ln \frac{s_1 \ldots s_n}{\mu_R^{2n}} \sigma_0 + \sigma_1\right) \alpha_s^{n+1} + \mathcal{O}(\alpha_s^{n+2})$$

1. The choices that cancels the logarithm is $\mu_R^2 = (s_1\, s_2 \ldots s_n)^{1/n}$
2. It is not guaranteed that choosing that scale leads to a series that behaves better perturbatively. There might be for instance further scales coming from jet resolution parameters, kinematical cuts, etc.
3. The similarity is however deceiving: the extra power of the coupling accounts for the emission of an extra gluon, which might have nothing to do with the physics responsible for the tree-level process

Theoretical uncertainties: central scale

Since for one emission $\alpha_s = \alpha_s(k_t)$, a good practice is to try to estimate the typical scale for gluon radiation: this might depend on the observable





NNLL threshold resummation
central scale m_H

NNLO fixed order
central scale $m_H/2$

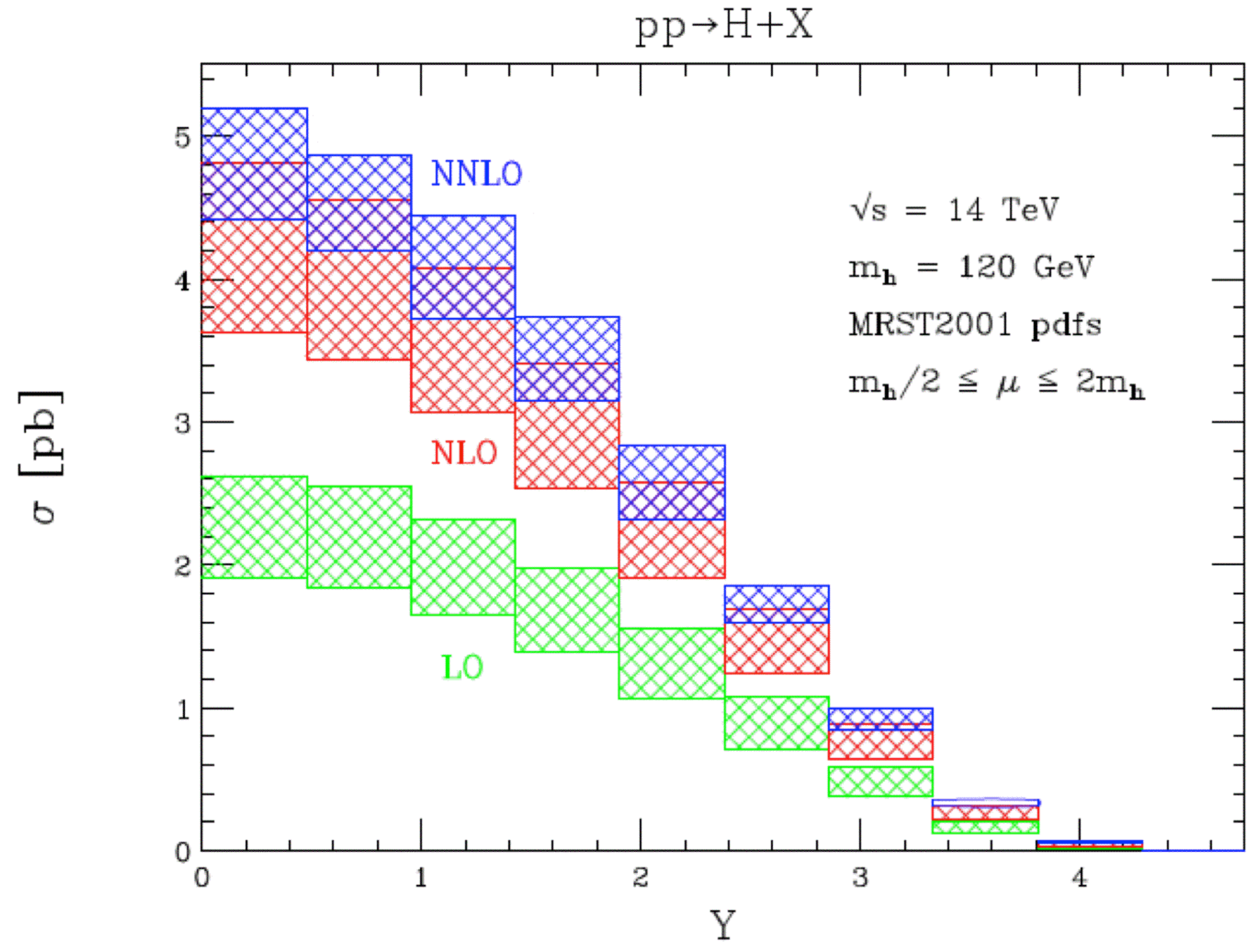
One can find an "optimal" scale for the fixed order by requiring that K-factors are minimised, this gives the choice $m_H/2$

A more systematic approach is given by the MiNLO procedure
[Hamilton Nason Oleari Zanderighi]

Theoretical uncertainties: scale variation

Only after one has identified a central scale does it make sense to take variations of factors of two, so as not to generate large logarithms

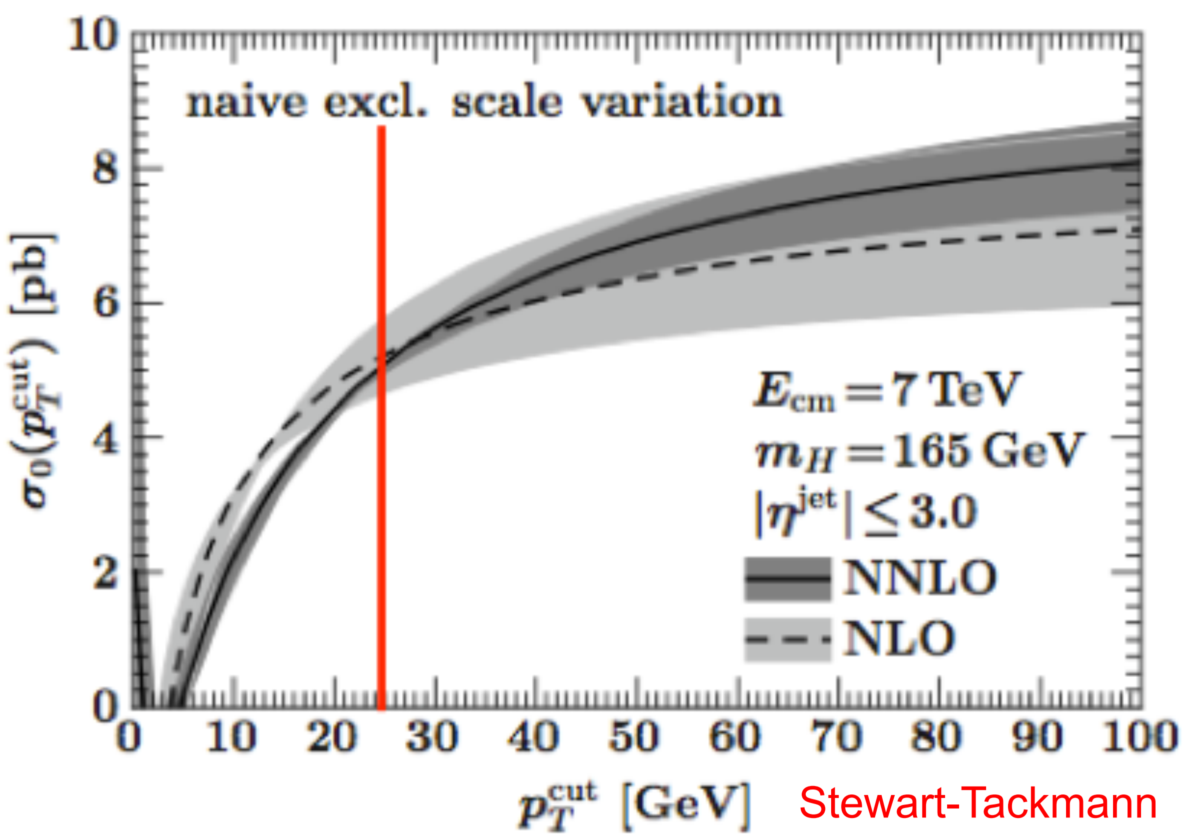
$$\alpha_s^n(x\mu_R) = \alpha_s^n + (n\,\beta_0 \ln x)\,\alpha_s^{n+1}(\mu_R)$$



This works as soon as you reach the first non-trivial order in α_s

Theoretical uncertainties: scale variations

Scale variations are able to highlight pathological behaviours of cross sections, for instance infrared sensitivity



Stewart-Tackmann

$$\sigma_{0-\text{jet}} = \sigma_{\text{tot}} - \sigma_{\geq 1-\text{jet}}$$

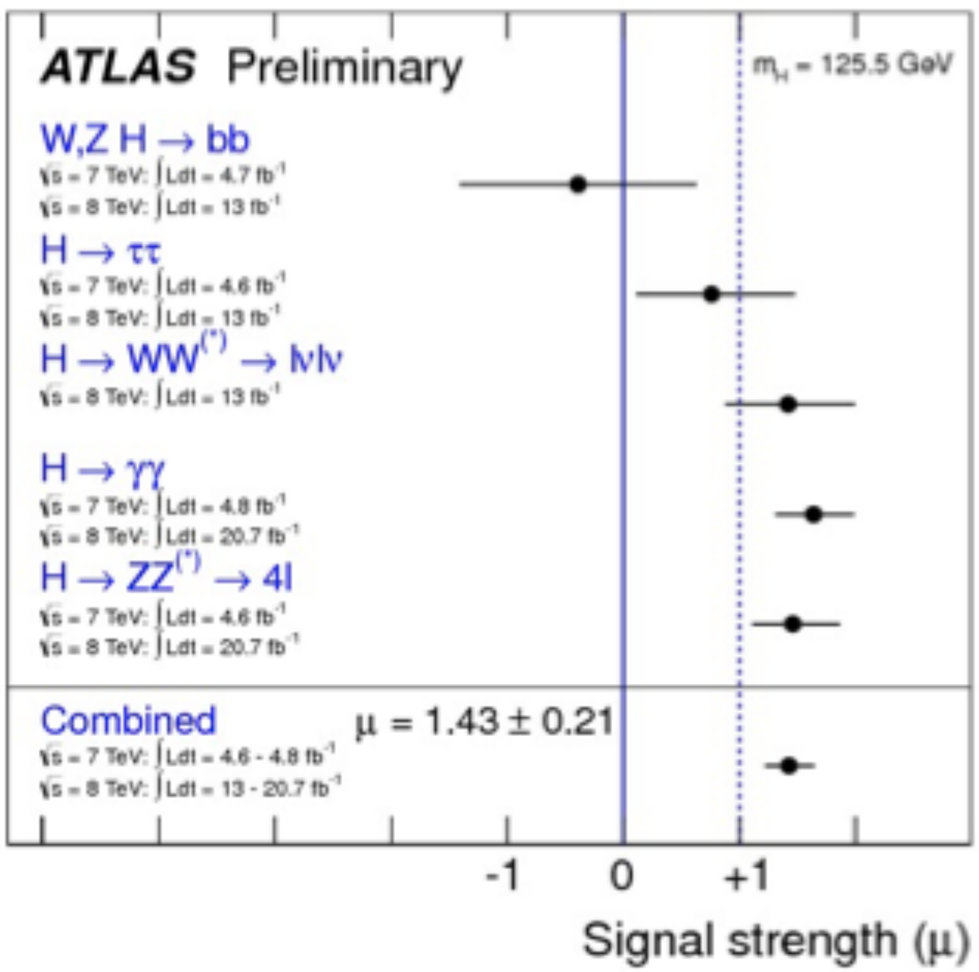
large K-factor   large logarithms

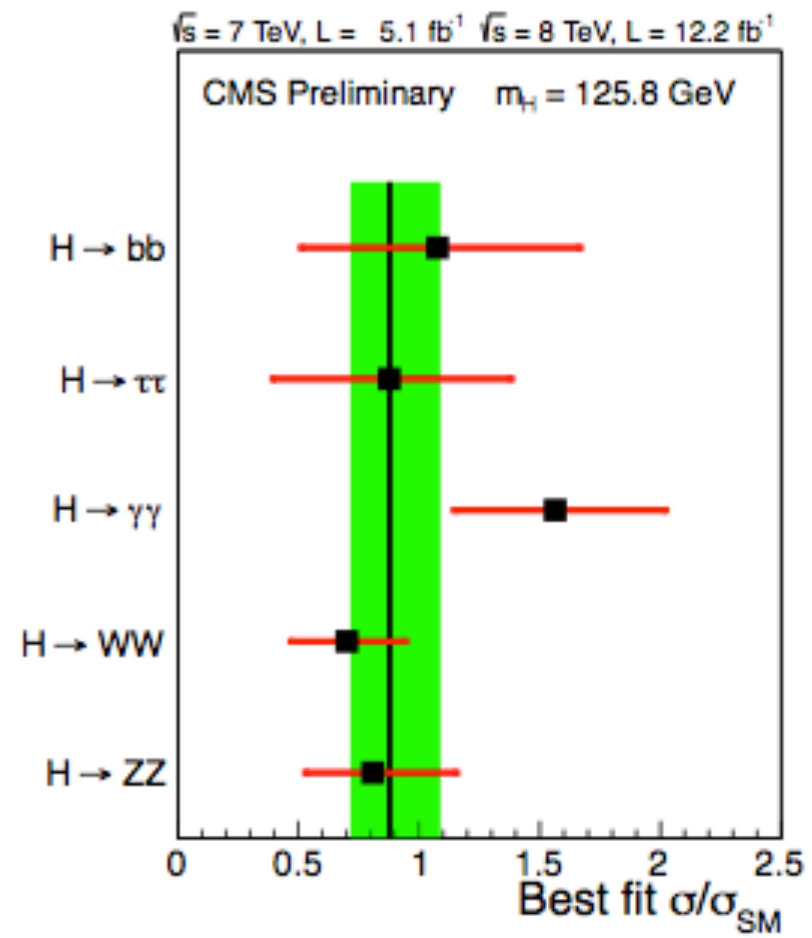
The cancellation of two large effects gives a spurious vanishing of scale uncertainties at low values of the jet-veto resolution $p_T^{\rm cut}$

A vanishing scale uncertainty is clearly not a good estimate of missing higher orders…

Higgs production with a jet-veto

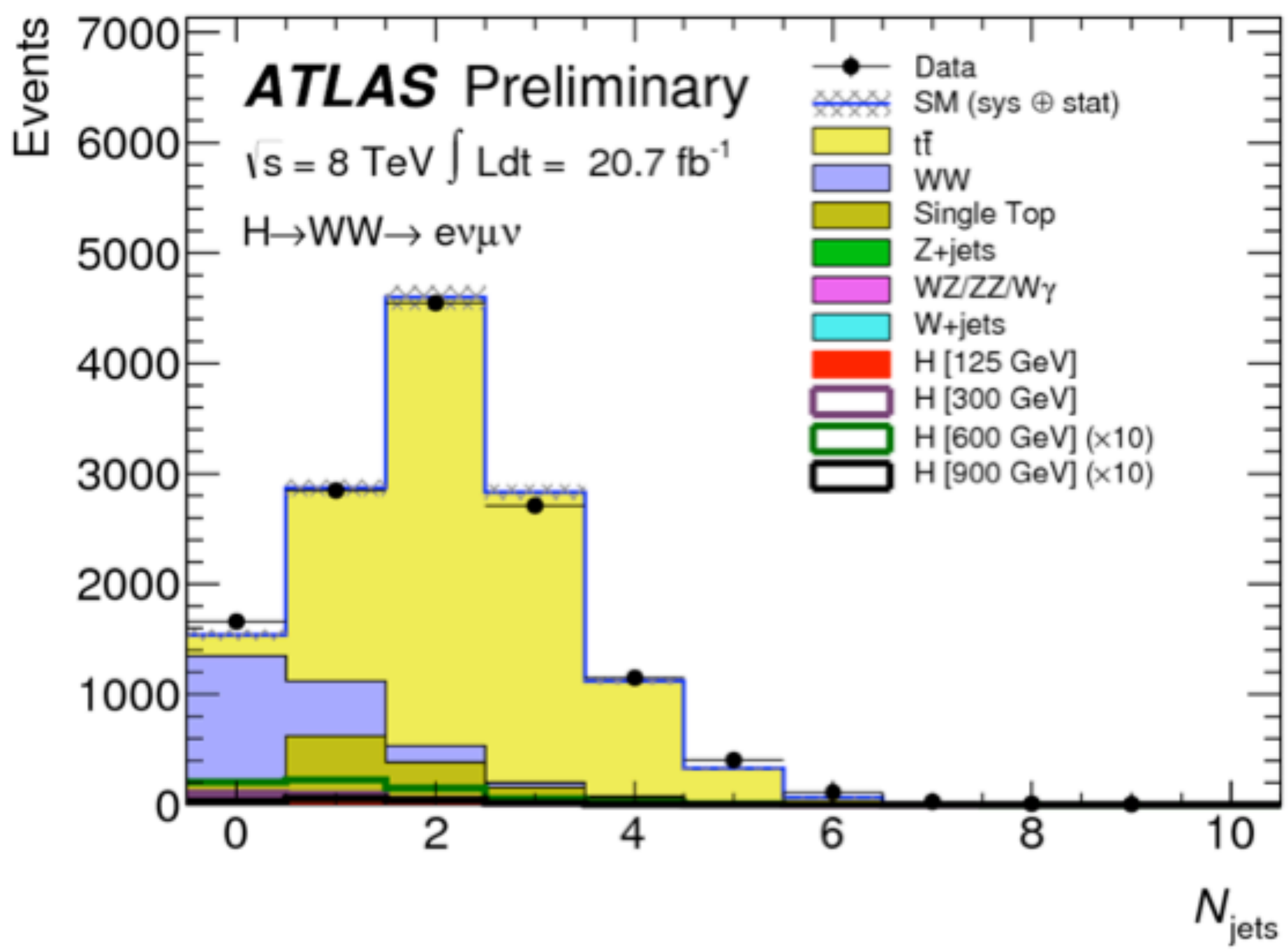
The main interest in jet-veto cross sections is to establish whether the boson found at the LHC is compatible with the Standard Model Higgs





Higgs production with a jet-veto

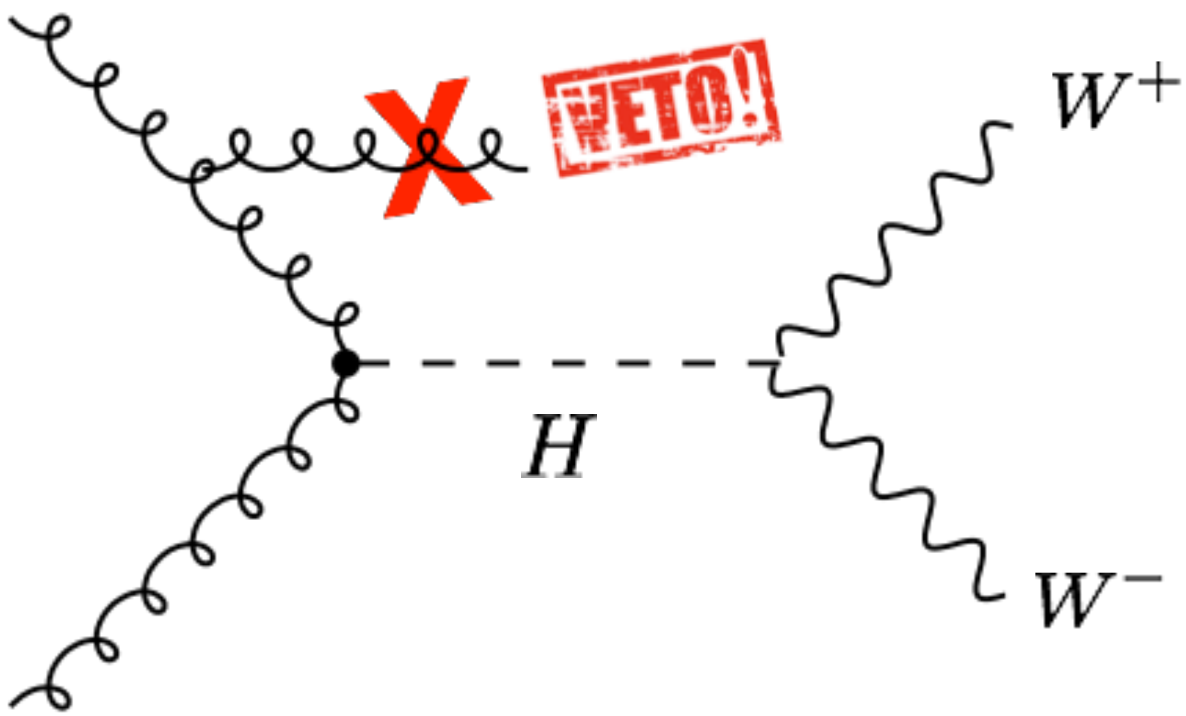
In order to suppress the large top-antitop background to $H \to WW$ we require that all jets have a transverse momentum below a threshold $p_{t,\mathrm{veto}}$



This works: the zero-jet cross section $\sigma_{0-\mathrm{jet}}$ is least contaminated by the huge (yellow) top-antitop background

Jet-veto as a two-scale problem

The zero-jet cross section is characterised by two scales, the Higgs mass m_H and the jet resolution $p_{\mathrm{t,veto}}$

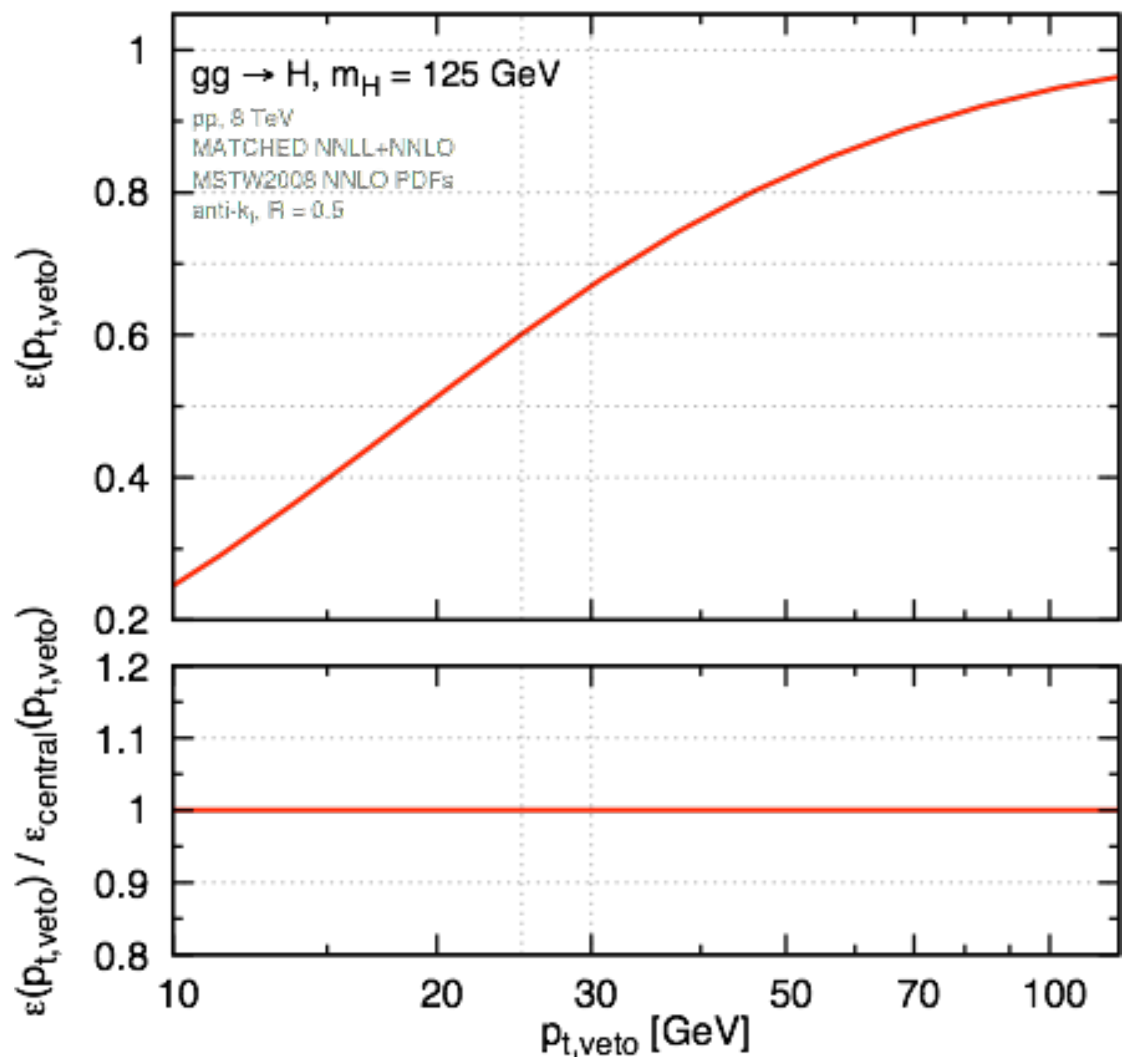


The jet-veto condition restricts the phase space available to gluons, so we expect logarithmically enhanced contributions $\ln(m_H/p_{\mathrm{t,veto}})$ at all orders

Does a resummation of large logarithms help solve the problem of the weird behaviour of scale uncertainties?

Resummation uncertainties

Resummation has more handles to assess theoretical uncertainties

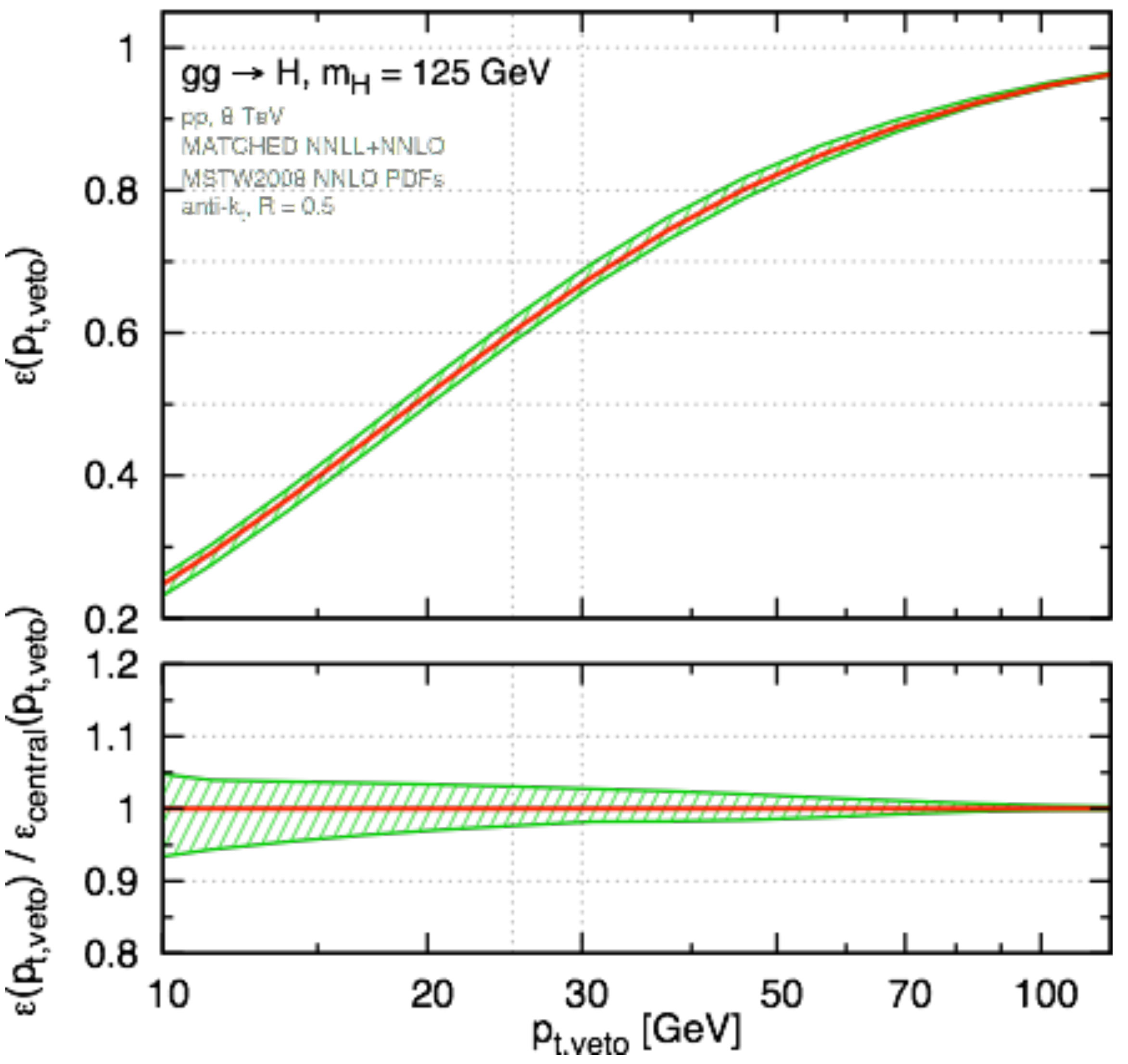


Resummation uncertainties

Resummation has more handles to assess theoretical uncertainties

1. "Traditional" variation of renormalisation and factorisation scales in the range

$$\frac{m_H}{4} \leq \mu_R, \mu_F \leq m_H \qquad \frac{1}{2} \leq \frac{\mu_F}{\mu_R} \leq 2$$



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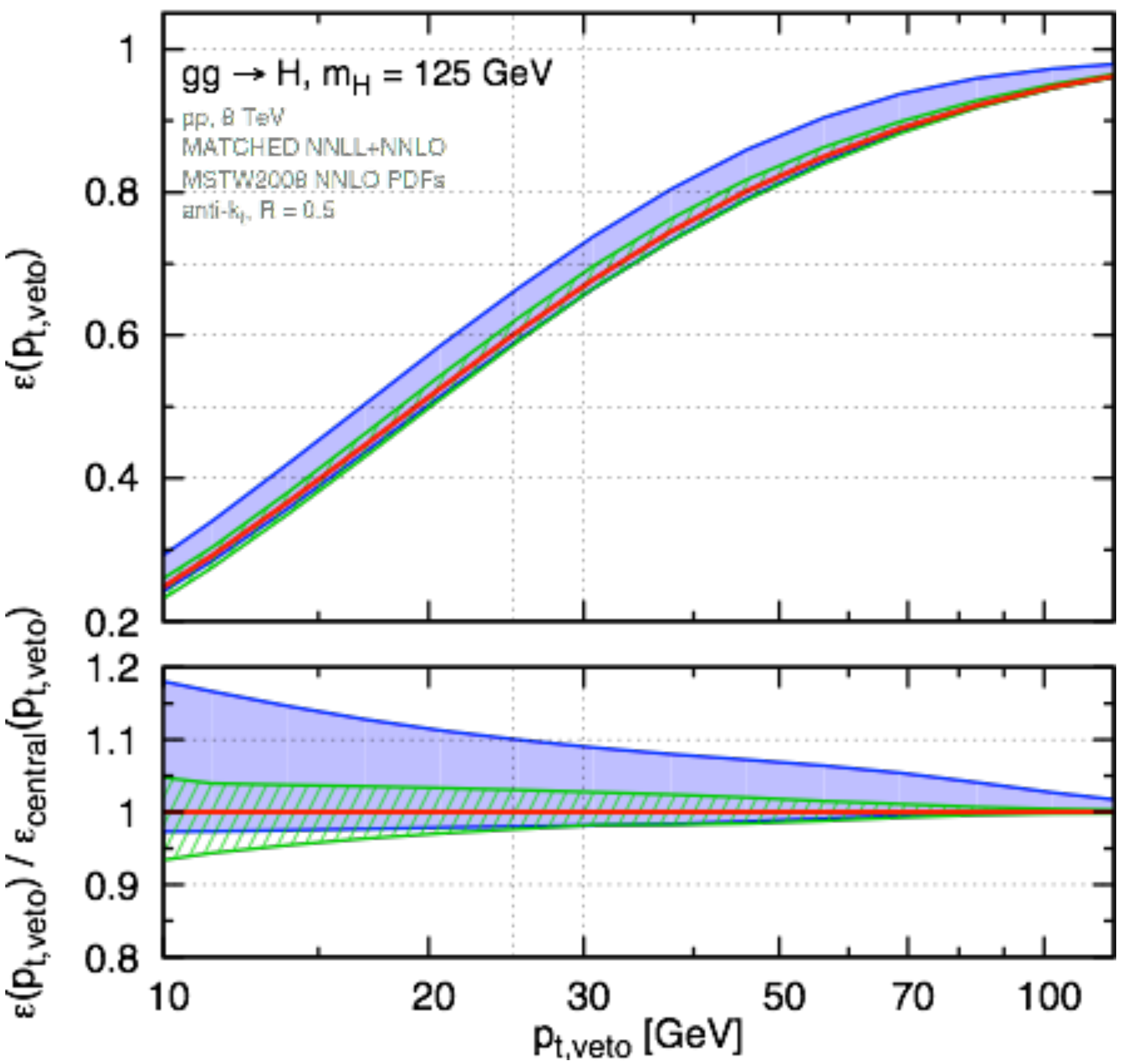
2. Resummation scale: change in the logs to be resummed, giving an idea of higher logarithmic corrections
$$\ln\left(\frac{m_H}{p_{t,\mathrm{veto}}}\right) \to \ln\left(\frac{Q}{p_{t,\mathrm{veto}}}\right)$$



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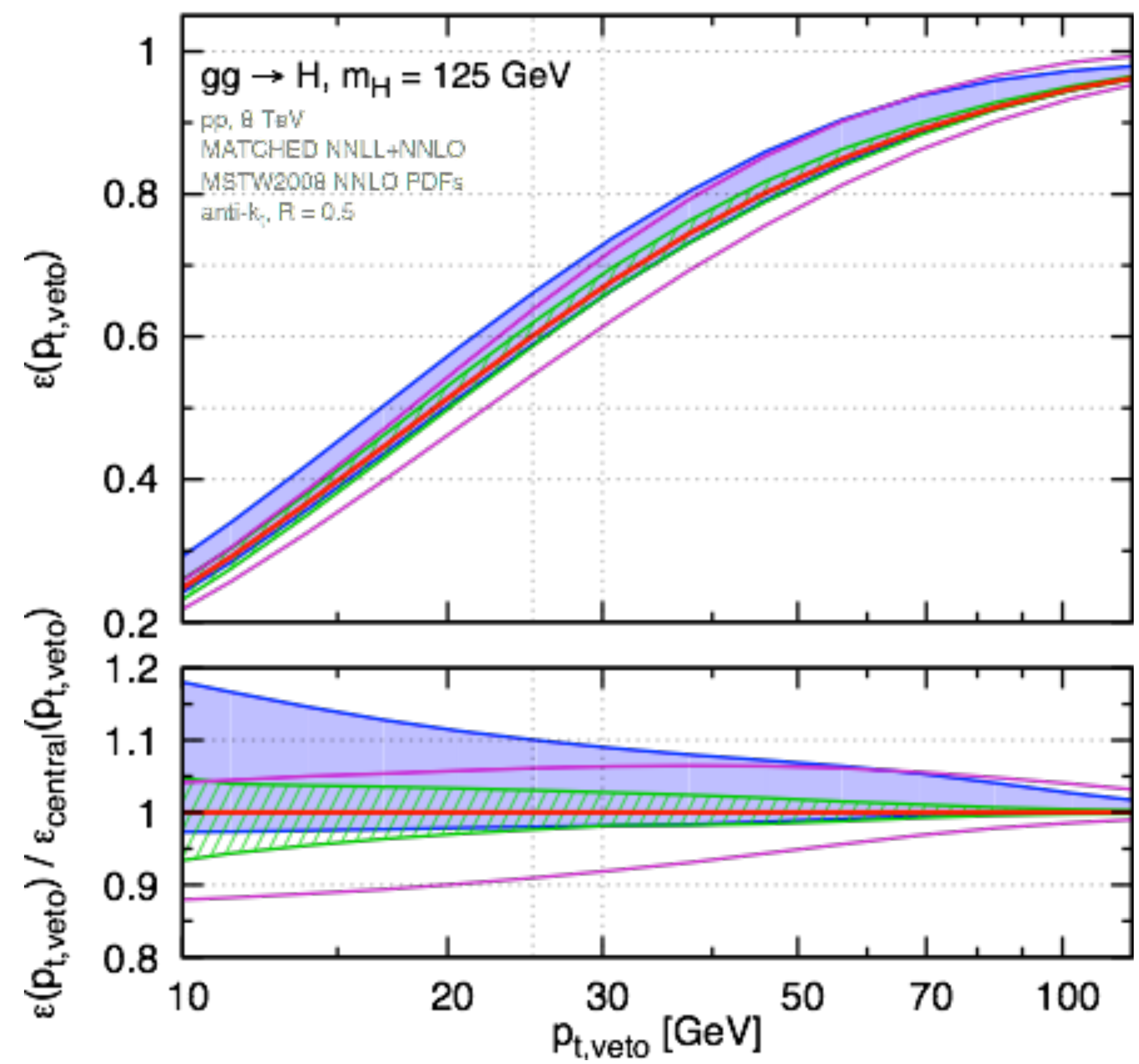
3. Variation of the scheme with which resummation is matched to exact fixed order



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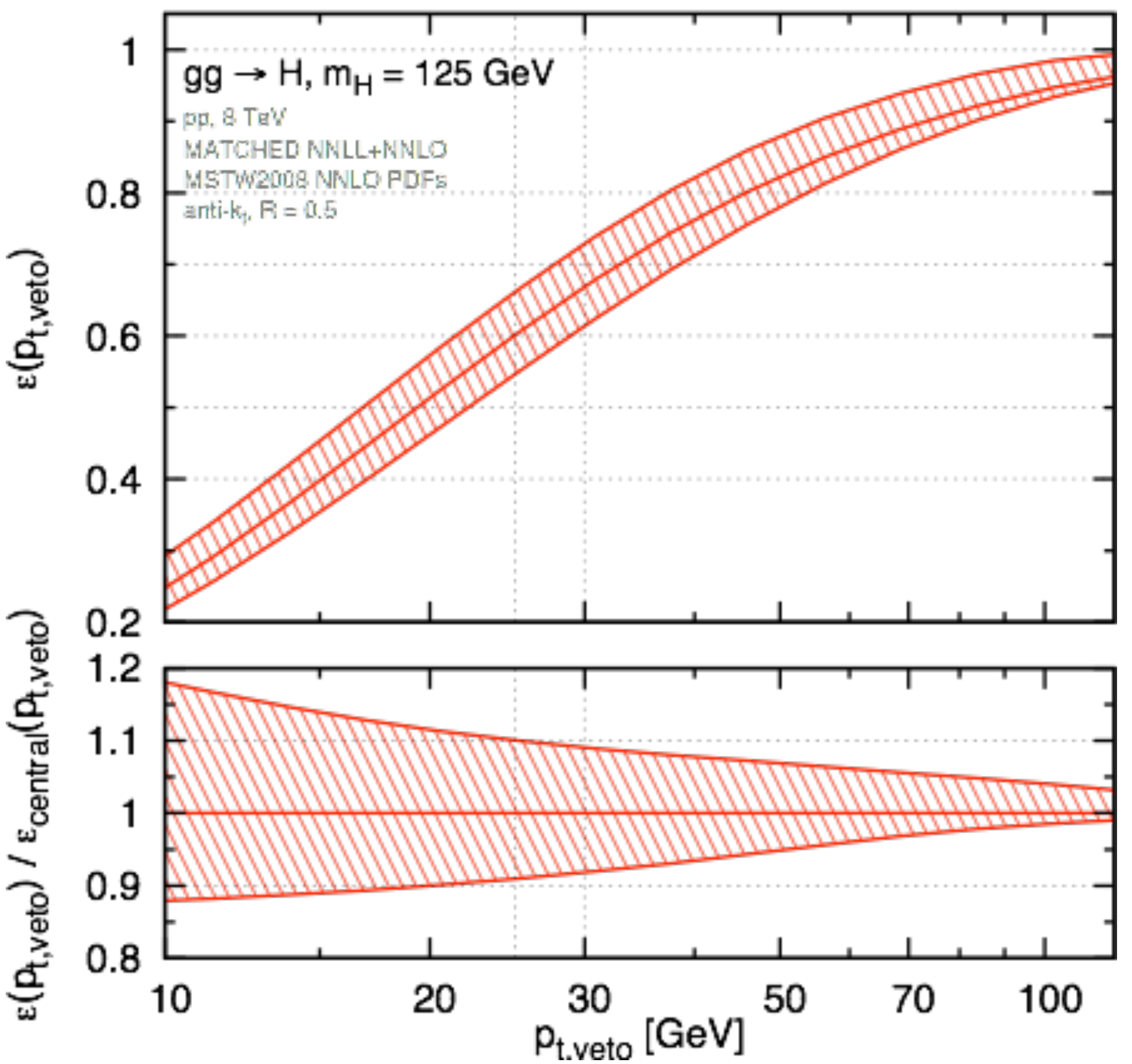
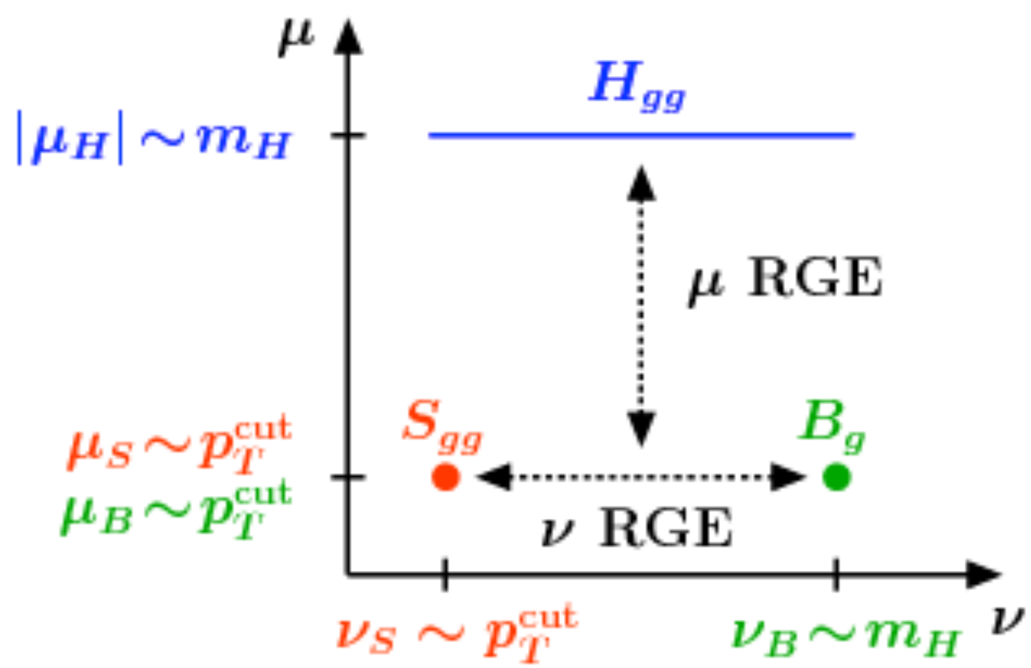
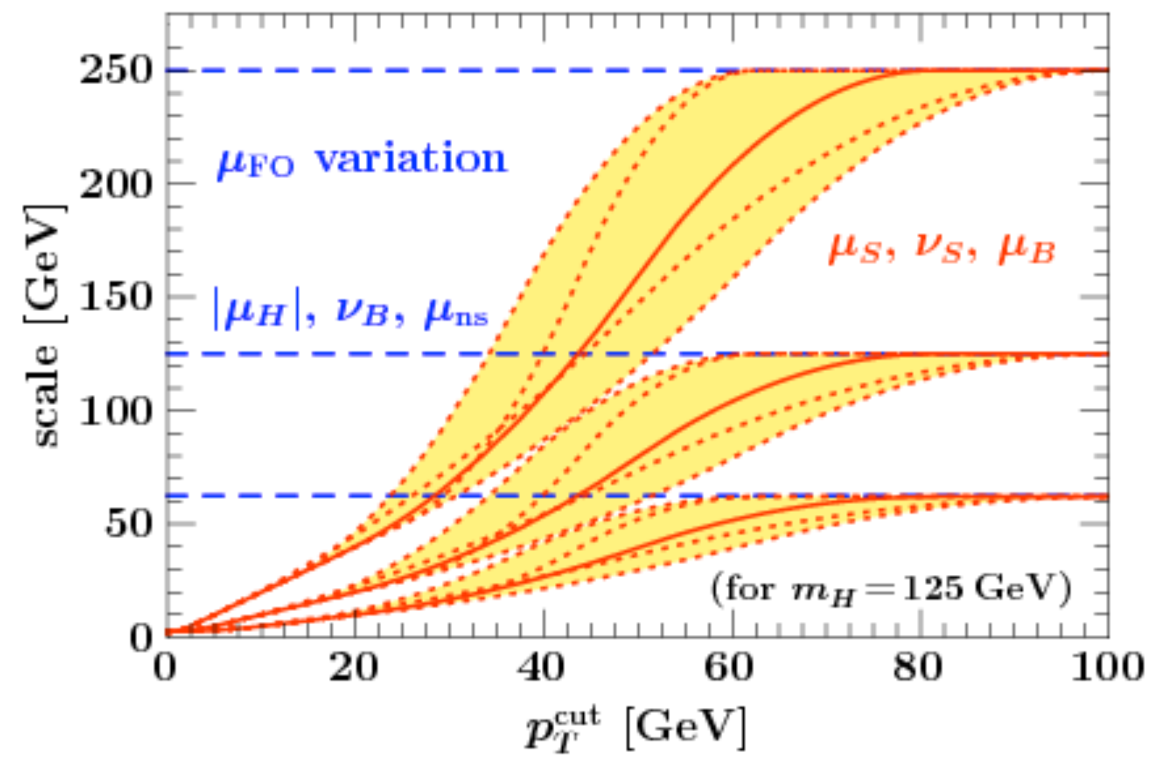


Total uncertainty: envelope of all these curves

Resummation uncertainties

Other resummed predictions have different central scales, a wider range of resummation scales, and the range of scale variation is a function of $p_{\mathrm{t,veto}}$





Stewart Tackmann Walsh Zuberi

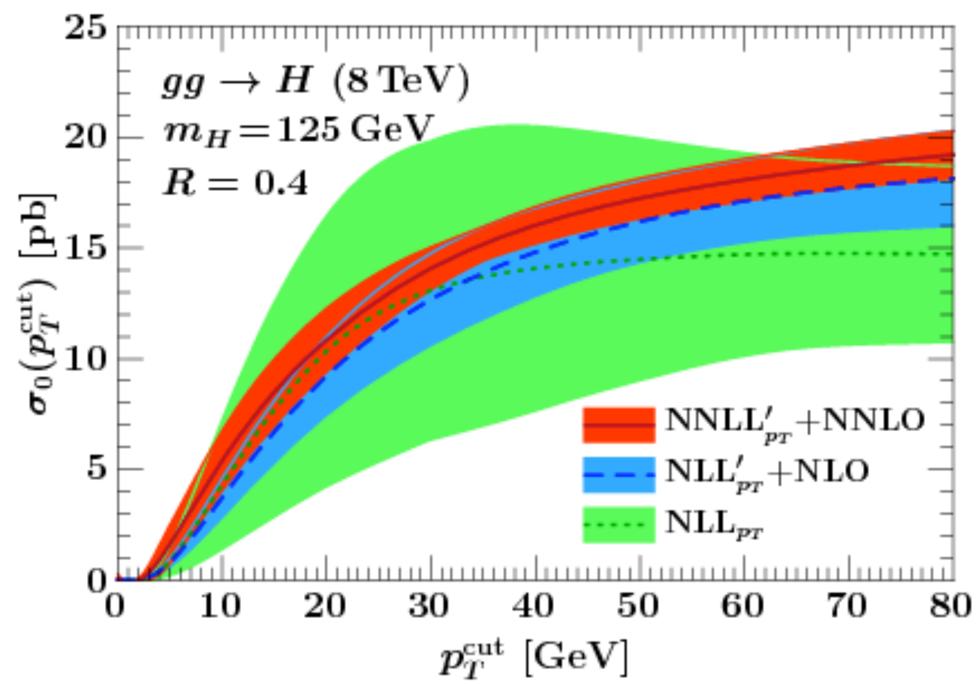
Each scale corresponds to a different tower of logarithms to be resummed

Scales are small when the resummation is important, and large where the fixed-order is OK $\Rightarrow$ smooth matching between resummation and NNLO

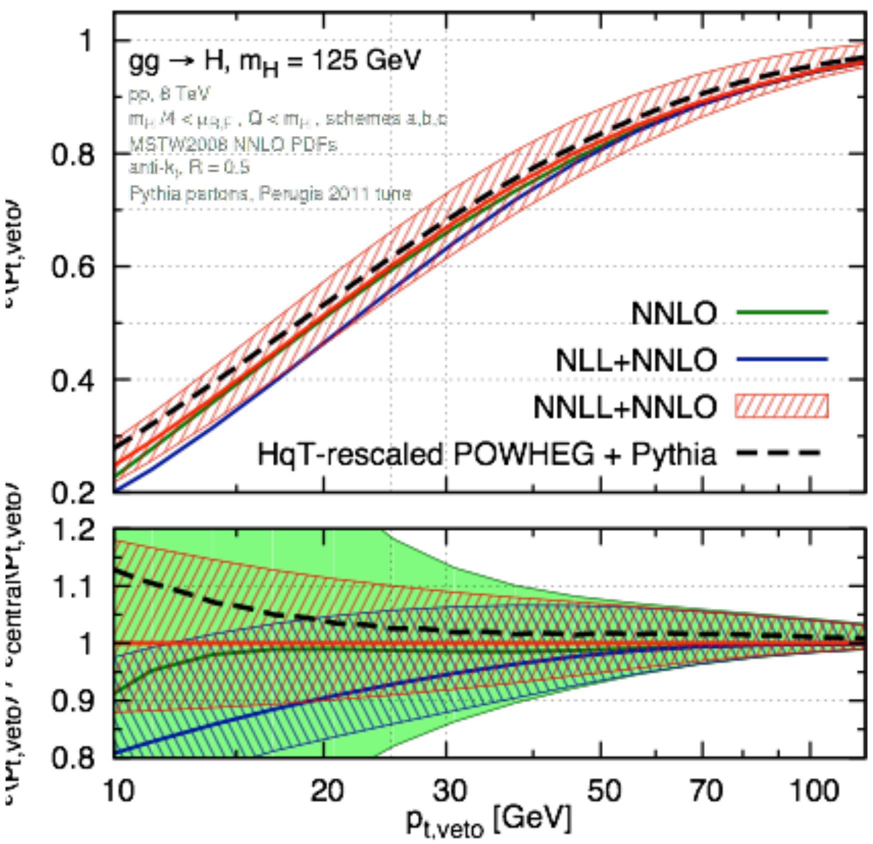
Resummation uncertainties

In all resummed calculations for the zero-jet cross section, uncertainties reduce consistently when increasing the resummation accuracy

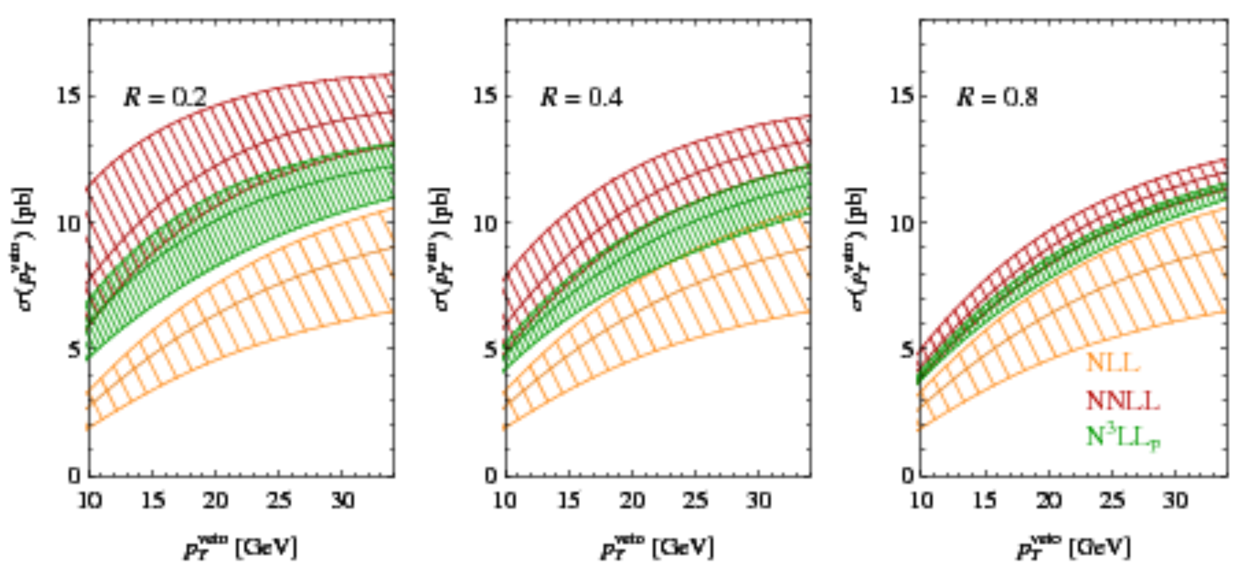
Stewart Tackmann Walsh Zuberi



Banfi Monni Salam Zanderighi

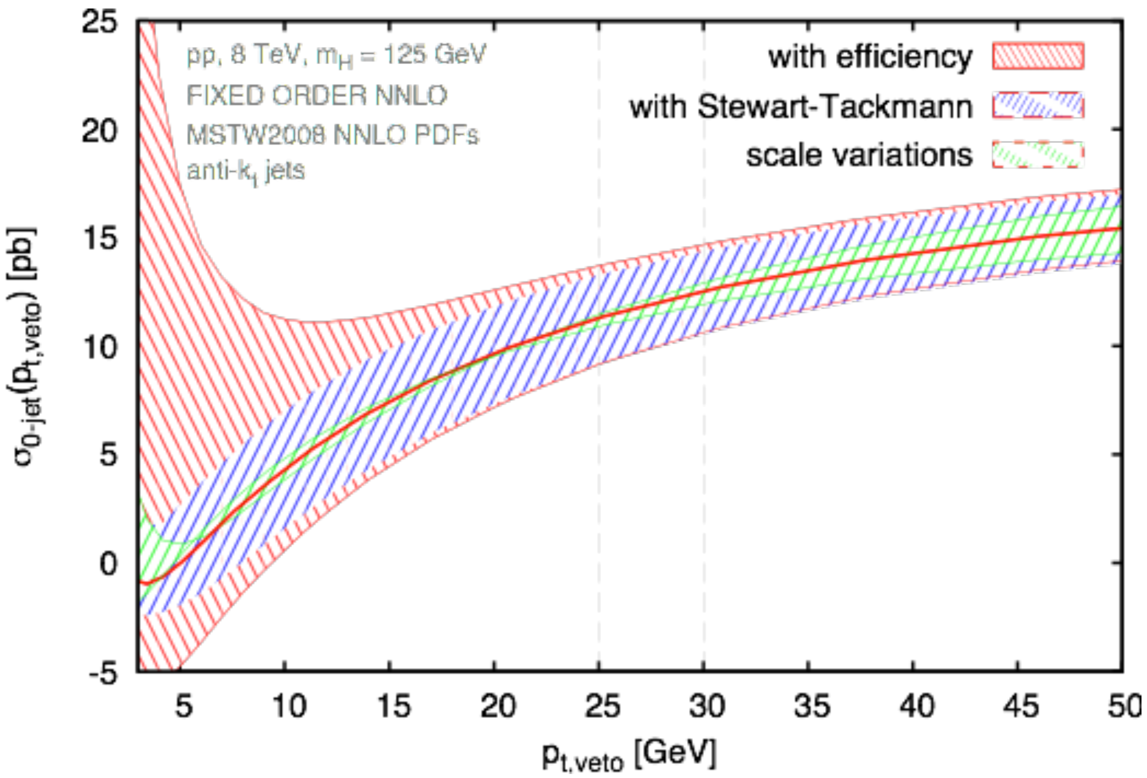


Becher Neubert Rothen



Resummation vs fixed-order uncertainties

At fixed-order, due to infrared sensitivity, different methods to assess uncertainties, all compatible within perturbative accuracy, give different results



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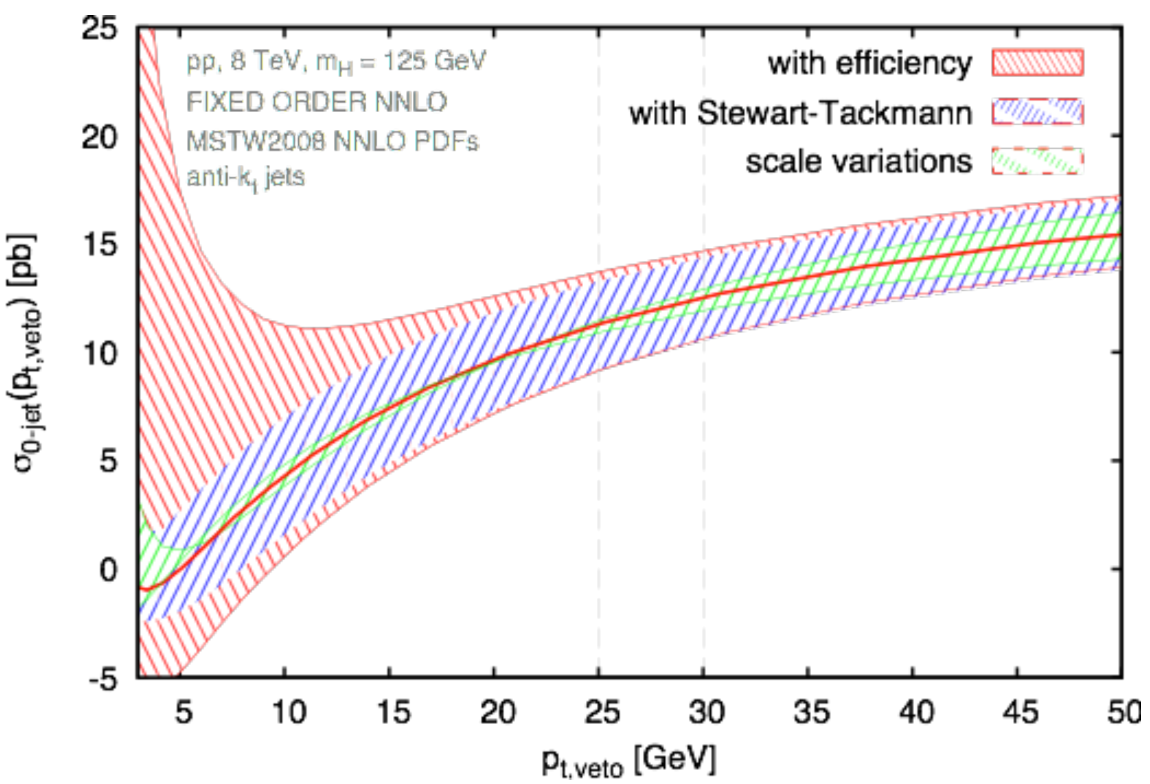

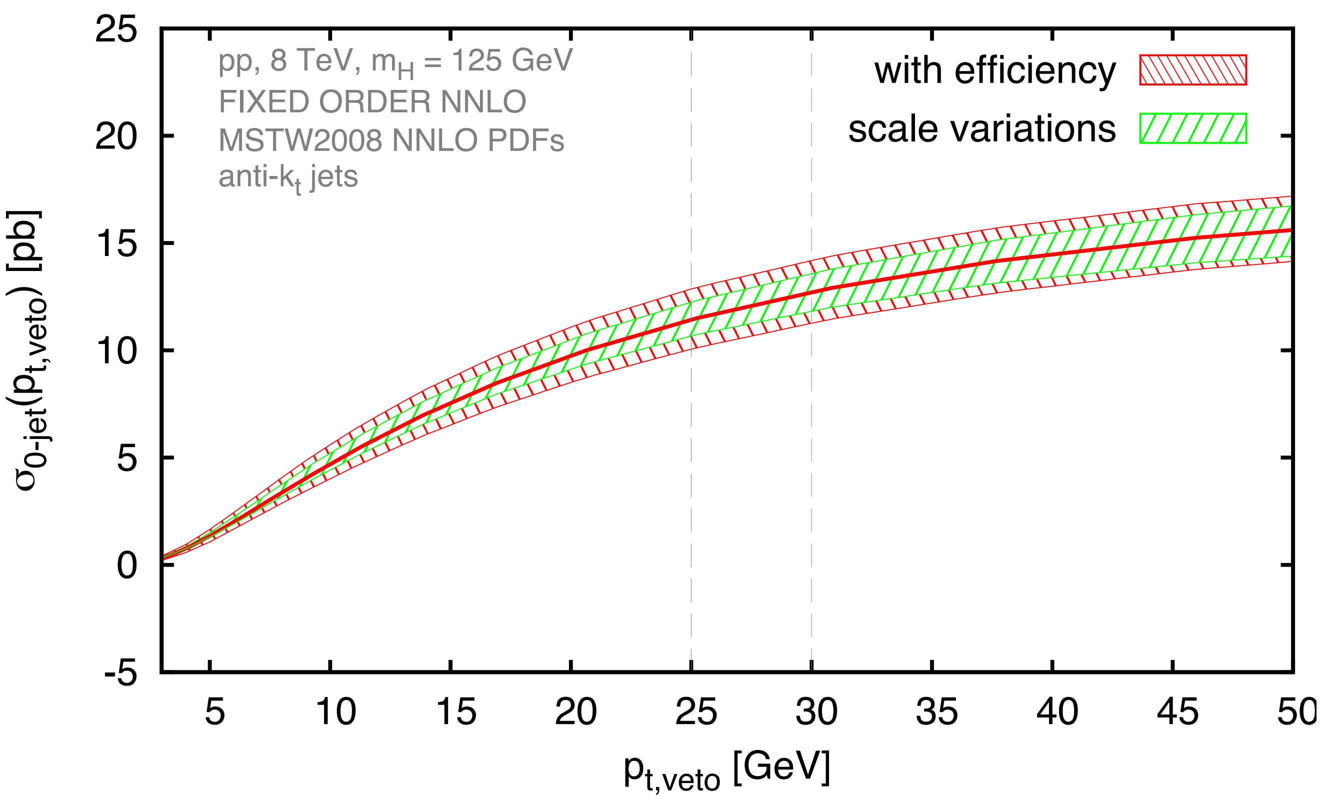

After resummation of large logarithms, also naive scale variations are a sensible way to estimate theoretical uncertainties, at NNLL around 10-12%

The main message is: if you feel you have to resum logs, just do it!

Summary

In this lecture we have learnt

1. variation of renormalisation and factorisation scales is a theoretically sound procedure for sufficiently inclusive observables

2. for less inclusive observables, problems in scale variations might give an indication of their infrared sensitivity

3. methods to assess uncertainties for resummed predictions