Lecture on combining fixed-order calculations and parton showers

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Outline

Introduction

- ◇ Short parton shower introduction
- ◇ Why do we need matching and merging?

ME+PS calculations (on the blackboard)

- ◇ Towards differential event descriptions.
- ◇ Matching (at NLO) and merging (at LO).

Resources:

`montecarlonet.org` and `http://users.phys.psu.edu/~cteq`

...let's see how far we get!

Parton showers

Factorisation: Divide and conquer

Every cross section containing an additional collinear gluon can be factorised as

$$d\sigma(\mathrm{pp} \to Y + \mathrm{g} + X) = d\sigma(\mathrm{pp} \to Y + X) \int \frac{dp_\perp^2}{p_\perp^2} \frac{dz}{z} \frac{\alpha_s}{2\pi} \frac{f(\frac{x_a}{z}, t)}{f_a(x_a, t)} P(z)$$

with the splitting kernels $P(z)$. This is independent of the process $\mathrm{pp} \to Y + X$.

Multi-parton cross sections can be approximated by "dressing up" low-multiplicity results with many collinear partons.

The splitting kernels have a probabilistic interpretation:

$$\int_{p^2_{\perp min}}^{p^2_{\perp max}} \frac{dp_\perp^2}{p_\perp^2} \int_{z_{min}}^{z_{max}} dz \frac{\alpha_s}{2\pi} P(z) \equiv$$

Probability of emitting a gluon with momentum fraction $1 - z \in [z_{min}, z_{max}]$ and transverse momentum $p_\perp \in [p_{\perp min}, p_{\perp max}]$.

Logarithms

Integrating the splitting probability, we get

$$\begin{aligned}\int_{p^2_{\perp min}}^{p^2_{\perp max}} \frac{dp_\perp^2}{p_\perp^2} \int_{z_{min}}^{z_{max}} dz \frac{\alpha_s}{2\pi} P(z) &\approx \int_{p^2_{\perp min}}^{p^2_{\perp max}} \frac{dp_\perp^2}{p_\perp^2} \int_{z_{min}}^{z_{max}} dz \frac{\alpha_s}{2\pi} \frac{2C_{F/A}}{(1-z)} \\ &\approx \alpha_s \ln\left(\frac{p^2_{\perp max}}{p^2_{\perp min}}\right) \ln\left(\frac{z_{max}}{z_{min}}\right)\end{aligned}$$

More generally, we find

$$d\sigma(\mathrm{pp} \to Y + ng) = d\sigma(\mathrm{pp} \to Y) \otimes \alpha_s^n \left(c_{2n} L^{2n} + c_{2n-1} L^{2n-1} + \cdots + c_0\right)$$

with $L = \ln\left(Q^2/p^2_{\perp min}\right)$, $Q^2 = \mathcal{O}(p^2_{\perp max})$, $p^2_{\perp min} = \mathcal{O}(\Lambda_{\mathrm{QCD}})$.

$\Rightarrow$ Multi-parton cross sections can be approximated by leading (double) log. But logs diverge and we need to do something about that!
$\Rightarrow$ Need something to tame this.
$\Rightarrow$ Resummation of large logarithms!

Generating resummed form factors numerically

Think about nuclear decay. With the "decay constant" $P(z)$, the "naive" probability of a decay between two evolution times $[t, t+\delta t]$

$$\delta t \int_0^1 dz P(z)$$

Probability of no decay between two times

$$1 - \delta t \int dz P(z)$$

Probability of not having any decays in n intervals of step size $\delta t/n$:

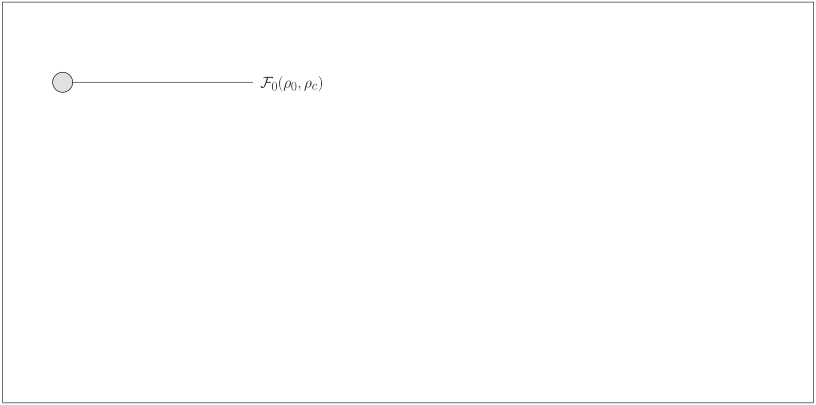
$$\left[1 - \frac{\delta t}{n} \int dz P(z)\right]^n$$

which, for continuous intervals $\frac{\delta t}{n} \to dt$ becomes

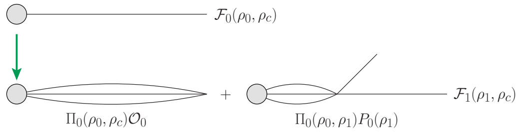
$$\exp\left\{-\int_t^{t+\delta t} dt \int dz P(z)\right\} \Rightarrow \Pi(t+\delta t, t) = \exp\left\{-\int_t^{t+\delta t} dt \left[A \ln(t) + B\right]\right\}$$

Parton shower resummation

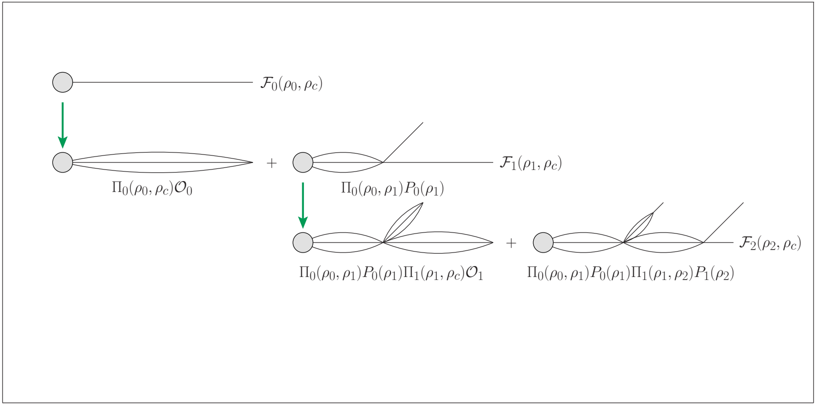
Parton showers attempt to resum, starting from an input state:



Parton shower can produce no hard splitting, or a hardest splitting



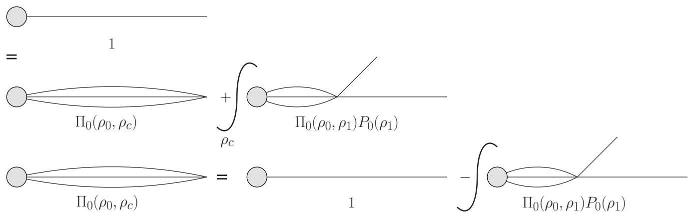
...and then no further splitting, or a second hardest splitting



There is only one hardest splitting, only one 2nd hardest splitting...
$\Rightarrow$ PS results never overlap because of no-emission probabilities.

Parton shower resummation

Parton showers are **unitary**: The probabilities add one.



No-emission probability = 1 – Emission probability = Sudakov

Showering will never change inclusive cross sections.

Parton showers vs. fixed order

We know two ways how to calculate multi-emission states:

- Fixed-order perturbation theory:

  + Contains all terms at one order.
  + Good for high relative $p_\perp$.
  − Only feasible for few emissions.

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  - $+$ Good for any number of emissions.
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Experiments at the LHC measure both low-$p_\perp$ and high-$p_\perp$ phenomena (e.g. in $p_{\perp,Z}$ or H_T).

To describe data, we need to combine the strengths of showers and matrix elements.

Where do we start?

1. Where do we trust the PS, where do we trust the ME?

2. How can we add fixed-order calculations?

3. What are the requirements on a PS?

4. How can we replace approximate PS terms with full fixed order?

5. How do we minimise our footprint?

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5. How do we minimise our footprint?

   Keep previous improvements. Control technical parameters!

Improvement schemes

- Matrix element corrections. Old, but still very good.
- Matrix element matching.
  - Typically combined with NLO corrections.
  - Very hard to iterate.
- Matrix element merging.
  - Combine many MEs. Historically: Slice phase space in two, use ME for hard jets, PS for soft jets. Introduces resolution criterion.
  - Very easy to iterate.

We will concentrate on matching and merging in this school.

...blackboard from here on!

Shorthands:

$\mathcal{O}_n$: Observable evaluated on an n-parton configuration

B_n: Differential n-parton cross section

$$\sum_{a,b} dx_b x_a f_{a/A}(x_a, \mu_F) f_{b/B}(x_b, \mu_F) \frac{1}{4\sqrt{\left(p_a p_b\right)^2 - M_a^2 M_b^2}} \left|\mathcal{M}\left(a+b \to c_1 + \cdots + c_n\right)\right|^2 \frac{d^3 p_1}{(2\pi)^3 2E_{p_1}} \cdots \frac{d^3 p_n}{(2\pi)^3 2E_{p_n}} (2\pi)^4 \delta^{(4)}\Big(p_a + p$$

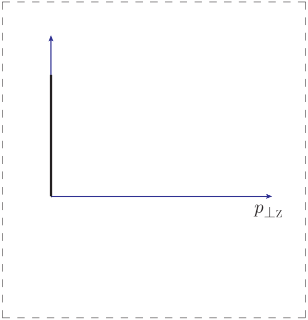
$\int_n$: Integrate over the n-parton phase space.

$\Pi(t_0, t_1)$: Probability of not having an emission between two evolution times (i.e. resolution scales). We will frequently drop the arguments, meaning that these scales are set as in a parton shower.

Problem: Getting a few distributions right - rapidity and transverse momentum

Start with the simplest case: $pp \to e^+e^-$.

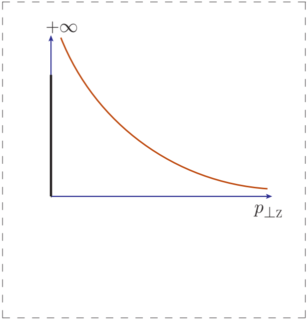
Now we can describe the rapidity of a e^+e^- pair with zero $p_\perp$.



$\mathrm{B}_0\mathcal{O}_0$

Non-zero $p_\perp$s are generated by $pp \to e^+e^- +$ parton.

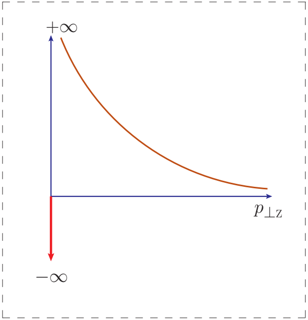
But now the rapidity diverges because $\mathrm{B}_1 \to \infty$ for $p_\perp \to 0$.



$$\mathrm{B}_0 \mathcal{O}_0 \qquad + \int_1 \mathrm{B}_1 \mathcal{O}_1$$

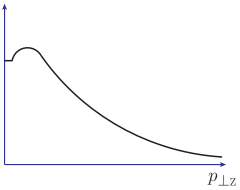
As a fix, we can subtract what we have added (up to finite terms, this is an NLO calculation).

But this does not give a very physical prediction.



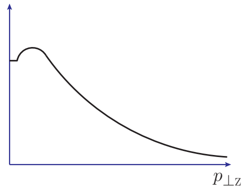
$$\mathrm{B}_0\mathcal{O}_0 \;-\; \int_1 \mathrm{B}_1\mathcal{O}_0 \quad + \; \int_1 \mathrm{B}_1\mathcal{O}_1$$

The divergence in B_1 can be regularized by a Sudakov factor.
Don't forget to subtract what we have added!



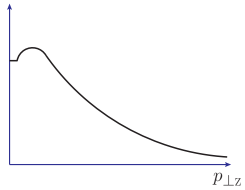
$$\mathrm{B}_0\mathcal{O}_0 \; - \; \int_1 \mathrm{B}_1\mathcal{O}_0\Pi_0 \; + \; \int_1 \mathrm{B}_1\mathcal{O}_1\Pi_0$$

Now using $\Pi_0 = \exp\left\{-\int_1 \mathrm{B}_1/\mathrm{B}_0\right\}$ we can massage this a bit.
The last term is just what showering B_0 gives.



$$\begin{aligned}
&\mathrm{B}_0\mathcal{O}_0 \;-\; \int_1 \mathrm{B}_1\mathcal{O}_0\Pi_0 \;+\; \int_1 \mathrm{B}_1\mathcal{O}_1\Pi_0 \\
=\;& \mathrm{B}_0\mathcal{O}_0\left[\,1 \;-\; \int_1 \frac{\mathrm{B}_1}{\mathrm{B}_0}\mathcal{O}_0\Pi_0\,\right] \;+\; \int_1 \mathrm{B}_1\mathcal{O}_1\Pi_0 \\
=\;& \mathrm{B}_0\mathcal{O}_0\Pi_0 \;+\; \int_1 \mathrm{B}_1\mathcal{O}_1\Pi_0 \;=\; \mathrm{B}_0\left[\mathcal{O}_0\Pi_0 \;+\; \int_1 \frac{\mathrm{B}_1}{\mathrm{B}_0}\mathcal{O}_1\Pi_0\right]
\end{aligned}$$

Note that there will always be a kink at ≈ 1 GeV, since some events just won't branch. This is smoothed out by primordial $k_\perp$ (non-perturbative effect)



$$\mathrm{B}_0 \left[\mathcal{O}_0 \Pi_0 \; + \; \int_1 \frac{\mathrm{B}_1}{\mathrm{B}_0} \mathcal{O}_1 \Pi_0 \right]$$

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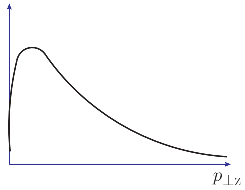
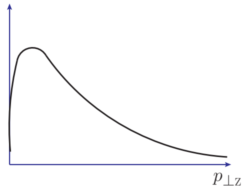


$$\mathrm{B}_0 \left[\mathcal{O}_0 \Pi_0 \;+\; \int_1 \frac{\mathrm{B}_1}{\mathrm{B}_0} \mathcal{O}_1 \Pi_0 \right]$$

By assumption, the $p_\perp$ spectrum is given by B_1 + higher orders.
The rapidity spectrum is given by B_0.
We can promote this to NLO accuracy by replacing the prefactor $\mathrm{B}_0 \rightarrow$ full NLO result.

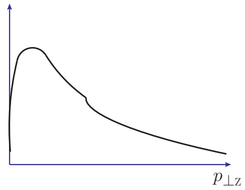


$$\mathrm{B}_0\mathcal{O}_0 \; - \; \int_1 \mathrm{B}_1\mathcal{O}_0\Pi_0 \; + \; \int_1 \mathrm{B}_1\mathcal{O}_1\Pi_0$$

$$= \; \mathrm{B}_0\mathcal{O}_0\left[\, 1 \; - \; \int_1 \frac{\mathrm{B}_1}{\mathrm{B}_0}\mathcal{O}_0\Pi_0 \right] \; + \; \int_1 \mathrm{B}_1\mathcal{O}_1\Pi_0$$

$$= \; \mathrm{B}_0\mathcal{O}_0\Pi_0 \; + \; \int_1 \mathrm{B}_1\mathcal{O}_1\Pi_0 \; = \; \overline{\mathrm{B}}_0\left[\mathcal{O}_0\Pi_0 \; + \; \int_1 \frac{\mathrm{B}_1}{\mathrm{B}_0}\mathcal{O}_1\Pi_0\right]$$

Many such NLO + PS matchings exist, as we can "freely" choose the higher orders. For example, an different B_1' can be accommodated by a subtraction.



$$
\begin{aligned}
& \overline{\mathrm{B}}_0 \left[\mathcal{O}_0 \Pi_0 \; + \; \int_1 \frac{\mathrm{B}_1}{\mathrm{B}_0} \mathcal{O}_1 \Pi_0 \right] \\
= \;\; & \overline{\mathrm{B}}_0' \left[\mathcal{O}_0 \Pi_0' \; + \; \int_1 \frac{\mathrm{B}_1'}{\mathrm{B}_0} \mathcal{O}_1 \Pi_0{}' \right] + \int_1 (\mathrm{B}_1 - \mathrm{B}_1') + \mathcal{O}(\alpha_{\mathrm{s}}^2)
\end{aligned}
$$

NLO matching summary

NLO matching can be obtained by showering the seed cross section

$$\overline{\mathrm{B}}'_n = \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_{\mathrm{rad}} \left(\mathsf{B}'_{n+1} - \mathrm{D}_{n+1}\right)\right] \mathcal{O}_{n+0} + \int d\Phi_{\mathrm{rad}} \left(\mathrm{B}_{n+1} - \mathsf{B}'_{n+1}\right) \mathcal{O}_{n+1}$$

NLO matching methods differ in the choice of B'_{n+1}:

POWHEG uses $\mathsf{B}'_{n+1} = \mathsf{B}_{n+1}$ or $\mathsf{B}'_{n+1} = \mathsf{B}_{n+1}F(\Phi)$

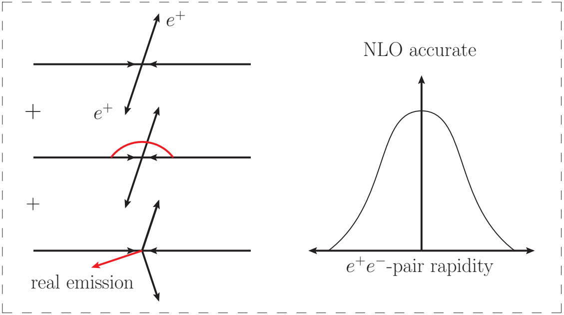
MC@NLO uses $\mathsf{B}'_{n+1} = \mathrm{B}_n \otimes P(z)\Theta(\mu_Q - \rho)$

Parton showers supply the all-order Sudakov factors.
...this calculation can only describe a very limited set of observables.
...all other observables are produced by parton showering.

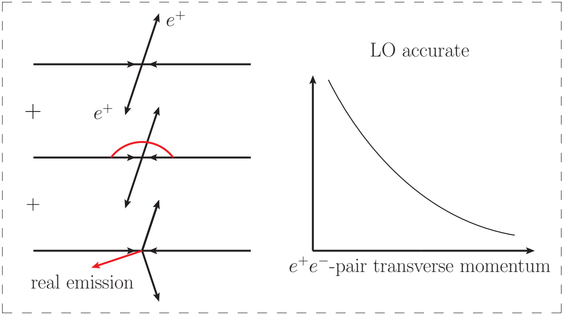
Problem: Getting many distributions "right" – what is "right"?

What is NLO accuracy?

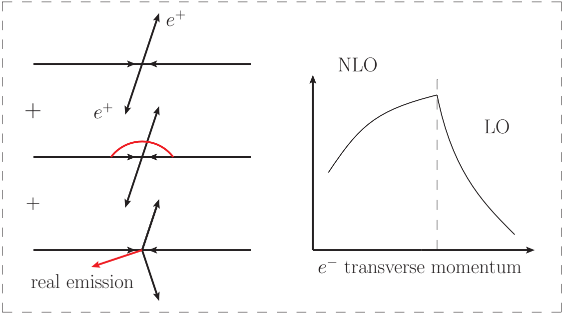
NLO accuracy is achieved when we calculate *corrections* to an observable that was already defined at a lower order.



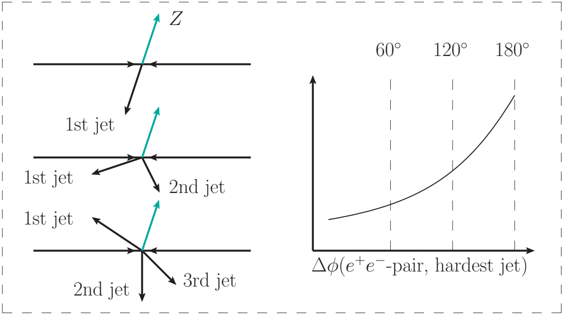
What is NLO accuracy?
...not all outcomes of an NLO calculation are "NLO accurate"



What is NLO accuracy?
NLO up to 45 GeV, LO beyond!

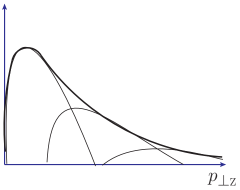


What is NLO accuracy?
How many "next-to's" do you need to describe this at least to lowesr order everywhere?



Problem: Getting many distributions "right"

We could equally well have started from B_1, and extended this calculation to describe the $p_\perp$ spectrum of the e^+e^-*parton*-system.

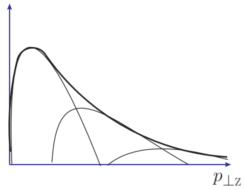


$$\mathrm{B}_1\mathcal{O}_1 \;\longrightarrow\; \mathrm{B}_1\mathcal{O}_1 \;-\; \int_1 \mathrm{B}_2\mathcal{O}_1\Pi_1 \;+\; \int_1 \mathrm{B}_2\mathcal{O}_2\Pi_1$$

Again, B_2 diverges, and Sudakov factors are needed.

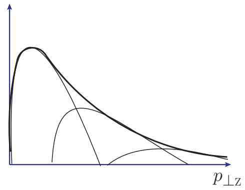
But we can't add this to B_0, since this again gives a divergent e^+e^- pair rapidity.

Do the same as before: Multiply Sudakov.



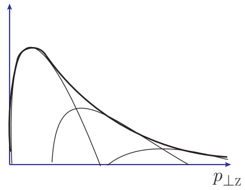
$$\mathrm{B}_1\mathcal{O}_1 \;\longrightarrow\; \Pi_0\mathrm{B}_1\mathcal{O}_1 \;-\; \int_1 \mathrm{B}_2\mathcal{O}_1\Pi_0\Pi_1 \;+\; \int_1 \mathrm{B}_2\mathcal{O}_2\Pi_0\Pi_1$$

Now we can add everything and get



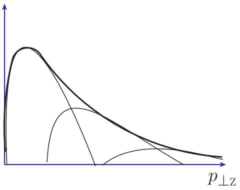
$$
\begin{aligned}
& \mathrm{B}_0\mathcal{O}_0 \\
\to\ & \mathrm{B}_0\mathcal{O}_0 \ - \int\limits_1 \mathrm{B}_1\mathcal{O}_0\Pi_0 \ + \int\limits_1 \mathrm{B}_1\mathcal{O}_1\Pi_0 \\
& \qquad\quad - \int\limits_2 \mathrm{B}_2\mathcal{O}_1\Pi_0\Pi_1 \ + \int\limits_2 \mathrm{B}_2\mathcal{O}_2\Pi_0\Pi_1
\end{aligned}
$$

...and we can keep merging more calculations



$$
\begin{aligned}
& \mathrm{B}_0\mathcal{O}_0 \\
\rightarrow \quad & \mathrm{B}_0\mathcal{O}_0 \;-\; \int_1 \mathrm{B}_1\mathcal{O}_0\Pi_0 \;+\; \int_1 \mathrm{B}_1\mathcal{O}_1\Pi_0 \\
& \qquad -\; \int_2 \mathrm{B}_2\mathcal{O}_1\Pi_0\Pi_1 \;+\; \int_2 \mathrm{B}_2\mathcal{O}_2\Pi_0\Pi_1 \\
& \qquad -\; \int_3 \mathrm{B}_3\mathcal{O}_2\Pi_0\Pi_1\Pi_2 \;+\; \int_3 \mathrm{B}_3\mathcal{O}_3\Pi_0\Pi_1\Pi_2
\end{aligned}
$$

There are again many choices that can be made …CKKW approximates this as



$$
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\rightarrow\ & \mathrm{B}_0\mathcal{O}_0\Pi_0 && + \int_1 \mathrm{B}_1\mathcal{O}_1\Pi_0\Pi_1 \\
& && + \int_2 \mathrm{B}_2\mathcal{O}_2\Pi_0\Pi_1\Pi_2 \\
& && \quad + \int_3 \mathrm{B}_3\mathcal{O}_3\Pi_0\Pi_1\Pi_2
\end{aligned}
$$

CKKW(-L)

- Only as good as Sudakov factors. If Sudakov factors do not contain (i.e. can damp) all divergences of the ME, then there are left-over divergences that need to be removed
- Solutions:
  - make PS better
  - keep add-subtract scheme
  - remove divergent phase space regions by cut-off.
- For the latter, we have "holes" in some observables.
  $\Longrightarrow$ Cut-off (merging scale) dependence.
- Fill up the "holes" left by cut-off with parton showers.

……back to the slides!

Matching vs. Merging

Matrix element matching:

- +Next-to-leading order accurate.
- +Improved description of "first" Sudakov.
- −Only possible one process at a time.
- −Multiple jets always given by PS.

Matrix element merging:

- +Process independent method.
- +Valid for any number of extra partons.
- −Only a leading-order method.

However, for data description, we need more. For example, H_T, n_{jets} are common, but "tricky" jet observables.

⇒ To describe these with small (scale/PDF...) uncertainties, combine NLO calculations!
⇒ **NLO merging**

Any leading-order method **X** contains approximate virtual corrections.

We want to use the full NLO multijet results whenever possible, e.g. have

- NLO accuracy for inclusive $W + 0$ jet observables
- NLO accuracy for inclusive $W + 1$ jet observables
- NLO accuracy for inclusive $W + 2$ jet observables

...all at the same time. And the method should be process-independent.

Any leading-order method **X** contains approximate virtual corrections.

We want to use the full NLO multijet results whenever possible, e.g. have

NLO accuracy for inclusive W + 0 jet observables
NLO accuracy for inclusive W + 1 jet observables
NLO accuracy for inclusive W + 2 jet observables

...all at the same time. And the method should be process-independent.

To do NLO multi-jet merging for your preferred LO scheme **X**, do:

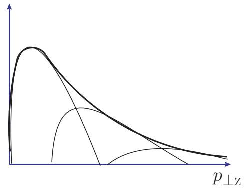
- Subtract approximate **X** $\mathcal{O}(\alpha_s)$-terms, add full NLO calculations.
- Make sure fixed-order calculations do not overlap by cutting, vetoing events, and/or vetoing emissions.
- Adjust higher orders to suit other needs.

$\Rightarrow$ **X**@NLO

Problem: Getting many distributions "right" – at NLO

...back to the blackboard (if we have time)

Let's start from LO merging again.

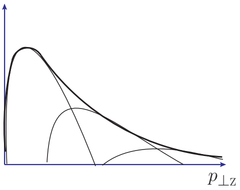


$$\mathrm{B}_0\mathcal{O}_0$$

$$\to \mathrm{B}_0\mathcal{O}_0 \; - \; \int_1 \mathrm{B}_1\mathcal{O}_0 \; \Pi_0 \qquad\qquad + \; \int_1 \mathrm{B}_1\mathcal{O}_1 \; \Pi_0$$

$$- \; \int_2 \mathrm{B}_2\mathcal{O}_1\Pi_0\Pi_1 \; + \; \int_2 \mathrm{B}_2\mathcal{O}_2\Pi_0\Pi_1$$

The lowest order does not (really) require subtractions.

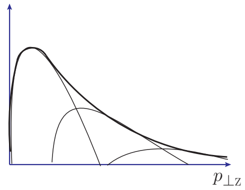


$$\mathrm{B}_0\mathcal{O}_0$$

$$\to\overline{\mathrm{B}}_0\mathcal{O}_0 \; - \int_1 \mathrm{B}_1\mathcal{O}_0 \; \Pi_0 \qquad + \int_1 \mathrm{B}_1\mathcal{O}_1 \; \Pi_0$$

$$- \int_2 \mathrm{B}_2\mathcal{O}_1\Pi_0\Pi_1 \; + \int_2 \mathrm{B}_2\mathcal{O}_2\Pi_0\Pi_1$$

One-jet states need approximate virtuals removed.

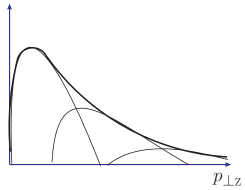


$$\mathrm{B}_0\mathcal{O}_0$$

$$\to\overline{\mathrm{B}}_0\mathcal{O}_0 - \int_1 \mathrm{B}_1\mathcal{O}_0\ \Pi_0 \qquad + \int_1 \mathrm{B}_1\mathcal{O}_1\left[\Pi_0 - \Pi_0^{(0)} - \Pi_0^{(1)}\right]$$

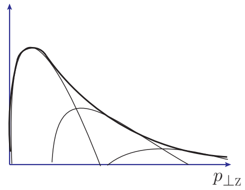
$$- \int_2 \mathrm{B}_2\mathcal{O}_1\Pi_0\Pi_1 + \int_2 \mathrm{B}_2\mathcal{O}_2\Pi_0\Pi_1$$

One-jet states need approximate virtuals removed
...but don't forget to subtract what you add!



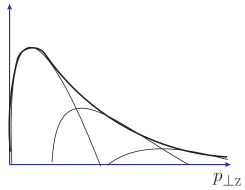
$$
\begin{aligned}
&\mathrm{B}_0\mathcal{O}_0 \\
&\to \overline{\mathrm{B}}_0\mathcal{O}_0 \;-\; \int_1 \mathrm{B}_1\mathcal{O}_0\left[\Pi_0 - \Pi_0^{(0)} - \Pi_0^{(1)}\right] \;+\; \int_1 \mathrm{B}_1\mathcal{O}_1\left[\Pi_0 - \Pi_0^{(0)} - \Pi_0^{(1)}\right] \\
&\qquad - \int_2 \mathrm{B}_2\mathcal{O}_1\Pi_0\Pi_1 \;+\; \int_2 \mathrm{B}_2\mathcal{O}_2\Pi_0\Pi_1
\end{aligned}
$$

Now we can add the full one-jet NLO calculation.



$$
\begin{aligned}
&\mathrm{B}_0\mathcal{O}_0 \\
&\rightarrow \overline{\mathrm{B}}_0\mathcal{O}_0 - \int_1 \mathrm{B}_1\mathcal{O}_0\left[\Pi_0 - \Pi_0^{(0)} - \Pi_0^{(1)}\right] + \int_1 \mathrm{B}_1\mathcal{O}_1\left[\Pi_0 - \Pi_0^{(0)} - \Pi_0^{(1)}\right] \\
&\qquad + \int_1 \overline{\mathrm{B}}_1\mathcal{O}_1 \\
&\qquad - \int_2 \mathrm{B}_2\mathcal{O}_1\Pi_0\Pi_1 + \int_2 \mathrm{B}_2\mathcal{O}_2\Pi_0\Pi_1
\end{aligned}
$$

Now we can add the full one-jet NLO calculation.
...but don't forget to subtract what you add!



$$
\begin{aligned}
&\mathrm{B}_0\mathcal{O}_0 \\
&\to \overline{\mathrm{B}}_0\mathcal{O}_0 - \int_1 \mathrm{B}_1\mathcal{O}_0\left[\Pi_0 - \Pi_0^{(0)} - \Pi_0^{(1)}\right] + \int_1 \mathrm{B}_1\mathcal{O}_1\left[\Pi_0 - \Pi_0^{(0)} - \Pi_0^{(1)}\right] \\
&\qquad - \int_1 \overline{\mathrm{B}}_1\mathcal{O}_0 + \int_1 \overline{\mathrm{B}}_1\mathcal{O}_1 \\
&\qquad - \int_2 \mathrm{B}_2\mathcal{O}_1\Pi_0\Pi_1 + \int_2 \mathrm{B}_2\mathcal{O}_2\Pi_0\Pi_1
\end{aligned}
$$

……back to the slides!

NLO merging schemes

FxFx[1]: Combine MC@NLO's by MLM jet matching@NLO
Pro: Probably fewest counter events.
Con: Restricted merging scale range. Accuracy unclear.

MEPS@NLO[2]: Combine MC@NLO's by METS@NLO
Pro: Improved Sudakovs.
Con: Restricted merging scale range.

UNLOPS[3]: Combine MC@NLO's or POWHEG's by UMEPS @NLO
Pro: Unitarity by approximate NNLO terms.
Con: Naively, many counter events.

MiNLO[4]: Get zero-jet NLO by reweighted one-jet POWHEG after integration
Pro: Improved resummation, unitary.
Con: Process-dependent, only two NLO's can be combined.

[1] Frixione, Frederix [2] Höche, Krauss, Schönherr, Siegert [3] Lönnblad, SP, Plätzer, [4] Hamilton, Nason, Oleari, Zanderighi

Summary

- Parton showers are derived from collinear factorisation and resum (leading) logarithms.
- This necessitates improvements to describe LHC data. The two main avenues for PS improvements are NLO matching, and multijet merging.
- Matrix element matching: "PS" used in conjuction with NLO calculations.
  Two schools: MC@NLO and POWHEG. Differences in exponentiation and in treatment of real corrections.
- Matrix element merging: Emphasis on combining many multijet ME's. Make fixed-order calculations additive by making them exclusive through no-emission probabilities. Then minimise the impact of arbitrary slicing parameters.
  Three schools: MLM, CKKW(-L) and UMEPS. Differences in generation (approximation of) no-emission probabilities, and in the treatment of non-showerlike configurations.
- NLO merging combines both NLO matching and multijet merging.

What I did not tell you about

...in matching

- What gets exponentiated, what is kept as "power correction"...
- Combination of matched result with full shower, PS starting conditions...

...in merging

- Handling of merging scale dependence (in some cases)...
- Cancellations between weights (in others)...

...more generally

- How is the reweighting with Sudakovs actually done?
- Treatment of resonances + resummation...
- Treatment of unordered emissions...
- New channels (incomplete histories)...
- Assigning uncertainties...
- Correlation with soft physics (MPI)...
- Electroweak logarithms...

These subtleties are important, and are the reason why competing approaches exist.

Tools / prescriptions shopping list

LO merging

- MLM available with Alpgen + (Herwig6, Pythia6/8), Madgraph + (Herwig++, Pythia6/8), Whizard + Pythia6
- CKKW no longer available (?) in Sherpa, Herwig++
- CKKW-L / METS available in Sherpa, (Alpgen, Madgraph,...) + Pythia8
- UMEPS available in (Alpgen, Madgraph,...) + (Herwig++, Pythia8)

NLO matching

NLO merging

NNLO matching

Other improvements

Tools / prescriptions shopping list

LO merging

NLO matching

- POWHEG available in Sherpa, Herwig++, POWHEG-BOX + (Herwig6/++, Pythia6/8)
- MC@NLO available in Sherpa, Herwig++, aMC@NLO + (Herwig6/++, Pythia6/8)

NLO merging

NNLO matching

Other improvements

Tools / prescriptions shopping list

LO merging

NLO matching

NLO merging

- MEPS@NLO available in Sherpa
- UNLOPS available in Herwig++, (POWHEG-BOX, aMC@NLO) + Pythia8
- FXFX available in aMC@NLO + (Herwig++, Pythia8)

NNLO matching

Other improvements

Tools / prescriptions shopping list

LO merging

NLO matching

NLO merging

NNLO matching

- UN^{2}LOPS available as plugin to Sherpa
- MiNLO-NNLOPS available through POWHEG-BOX

Other improvements

Tools / prescriptions shopping list

LO merging

NLO matching

NLO merging

NNLO matching

Other improvements

- MiNLO available through POWHEG-BOX
- Iterated ME corrections available through VINCIA
- ME reweighting available in HEJ
- KRKC proposed new NLO matching
- GENEVA proposed higher-logs + fixed-order (NLO, NNLO) + showers

References

Matrix element corrections
Pythia (PLB 185 (1987) 435, NPB 289 (1987) 810, PLB 449 (1999) 313, NPB 603 (2001) 297)
Herwig (CPC 90 (1995) 95)
Vincia (Phys.Rev. D78 (2008) 014026, Phys.Rev. D84 (2011) 054003, Phys.Rev. D85 (2012) 014013, Phys.Lett. B718 (2013) 1345-1350, Phys.Rev. D87 (2013) 5, 054033, JHEP 1310 (2013) 127)

POWHEG
JHEP 0411 (2004) 040 JHEP 0711 (2007) 070 POWHEG-BOX (JHEP 1006 (2010) 043)

MC@NLO
Original (JHEP 0206 (2002) 029) Herwig++ (Eur.Phys.J. C72 (2012) 2187)
Sherpa (JHEP 1209 (2012) 049) aMC@NLO (arXiv:1405.0301)

NLO matching results and comparisons
Plots taken from Ann.Rev.Nucl.Part.Sci. 62 (2012) 187 Plots taken from JHEP 0904 (2009) 002

Tree-level merging MLM (Mangano, http://www-cpd.fnal.gov/personal/mrenna/tuning/nov2002/mlm.pdf. Talk presented at the Fermilab ME/MC Tuning Workshop, Oct 4, 2002, Mangano et al. JHEP 0701 (2007) 013)
Pseudoshower (JHEP 0405 (2004) 040)
CKKW (JHEP 0111 (2001) 063, JHEP 0208 (2002) 015)
CKKW-L (JHEP 0205 (2002) 046, JHEP 0507 (2005) 054, JHEP 1203 (2012) 019)
METS (JHEP 0911 (2009) 038, JHEP 0905 (2009) 053)

Parton shower histories
Andre, Sjöstrand (PRD 57 (1998) 5767)

Unitarised merging
Pythia (JHEP 1302 (2013) 094) Herwig (JHEP 1308 (2013) 114) Sherpa (arXiv:1405.3607)

Intermediate step: MENLOPS
POWHEG (JHEP 1006 (2010) 039) Sherpa (JHEP 1108 (2011) 123)

FxFx: Jet matching @ NLO: JHEP 1212 (2012) 061

Merging MC@NLO calculations with MEPS@NLO
JHEP 1304 (2013) 027 JHEP 1301 (2013) 144 Plots taken from arXiv:1401.7971

UNLOPS = UMEPS@NLO: JHEP 1303 (2013) 166 arXiv:1405.1067

MiNLO: Original (JHEP 1210 (2012) 155) Improved (JHEP 1305 (2013) 082)

MiNLO-NNLOPS: JHEP 1310 (2013) 222 arXiv:1407.2940

UN^{2}LOPS: arXiv:1405.3607 arXiv:1407.3773