NLO and higher order calculations

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Part I

– *Basics of QCD: quantization and renormalization*

– QCD at work: infrared safety, factorization and evolution

– Higgs boson production

Quantum Chromodynamics

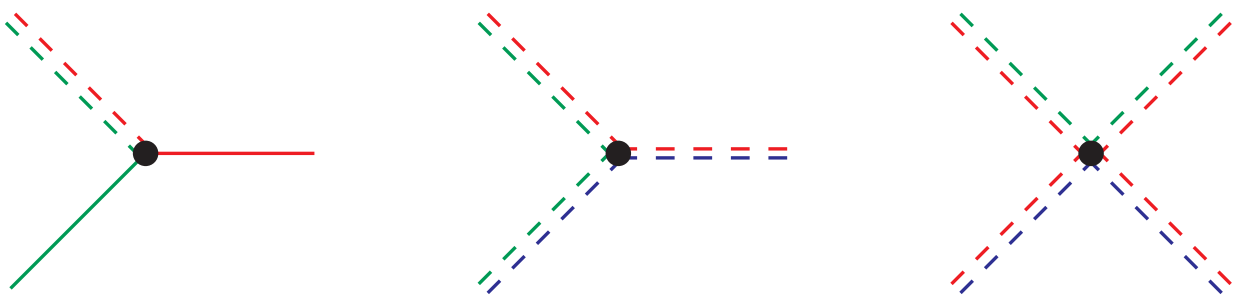
- Fundamental forces: quantum fields with gauge symmetries
- Strong interaction: color-$SU(3)$
  - quarks (antiquarks): 6 spin-1/2 flavors in 3 colors

$$\begin{bmatrix} u_{\mathrm{B}} \\ u_{\mathrm{G}} \\ u_{\mathrm{R}} \end{bmatrix} \begin{bmatrix} d_{\mathrm{B}} \\ d_{\mathrm{G}} \\ d_{\mathrm{R}} \end{bmatrix}, \quad \begin{bmatrix} s_{\mathrm{B}} \\ s_{\mathrm{G}} \\ s_{\mathrm{R}} \end{bmatrix} \begin{bmatrix} c_{\mathrm{B}} \\ c_{\mathrm{G}} \\ c_{\mathrm{R}} \end{bmatrix}, \quad \begin{bmatrix} b_{\mathrm{B}} \\ b_{\mathrm{G}} \\ b_{\mathrm{R}} \end{bmatrix} \begin{bmatrix} t_{\mathrm{B}} \\ t_{\mathrm{G}} \\ t_{\mathrm{R}} \end{bmatrix}$$

  - gluons: 8 spin-1 color-anticolor-combinations

$$g_{\mathrm{B\bar{G}}}\,,\quad g_{\mathrm{R\bar{B}}}\,,\quad g_{\mathrm{G\bar{R}}}\,,\quad \cdots\,,\quad g_{\mathrm{B\bar{B}-G\bar{G}}}\,,\quad g_{\mathrm{B\bar{B}+G\bar{G}-2\,R\bar{R}}}$$

- Interactions: Feynman diagrams



QCD Lagrangian

- Classical part of QCD Lagrangian

$$\mathcal{L}_{\rm cl} \quad = \quad -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \sum_{\rm flavors} \bar{\psi}_i \, ({\rm i}\not{D} - m_q)_{ij} \, \psi_j$$

- Matter fields $\psi_i, \bar{\psi}_j$ with $i, j = 1, \dots, 3$ (fundamental rep.)
  - covariant derivative $D_{\mu,ij} = \partial_\mu \delta_{ij} + {\rm i} g_s \, (t_a)_{ij} \, A^a_\mu$
- Field strength tensor $F^a_{\mu\nu}$ with $a = 1, \dots, 8$ (adjoint rep.)
  - covariant derivative $D_{\mu,ab} = \partial_\mu \delta_{ab} - g_s f_{abc} A^c_\mu$
  - $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f_{abc} A^b_\mu A^c_\nu$
- Formal parameters of the theory (no observables)
  - strong coupling $\alpha_s = g_s^2/(4\pi)$
  - quark masses m_q

Quantization

- Gauge fixing (Feynman gauge $\lambda = 1$) $\mathcal{L}_{\rm gauge-fix} \;=\; -\frac{1}{2\lambda} \, \left(\partial^\mu A^a_\mu\right)^2$
- Ghosts (Grassmann fields η) $\mathcal{L}_{\rm ghost} \;=\; \partial_\mu \eta^{a\,\dagger} \left(D^\mu_{ab} \eta^b\right)$
  (removal of unphysical degrees of freedom for gauge fields) Fadeev, Popov

From Lagrangian to Feynman rules

- Consider action S

$$S = \mathrm{i} \int d^4x \left(\mathcal{L}_{\mathrm{cl}} + \mathcal{L}_{\mathrm{gauge-fix}} + \mathcal{L}_{\mathrm{ghost}}\right) = S_{\mathrm{free}} + S_{\mathrm{int}}$$

- Decompose action into free S_{free} and interacting part S_{int}
  - S_{free} contains bi-linear terms in fields
  - S_{int} contains interactions
- Derivation of Feynman rules
  - inverse propagators from S_{free}
  - interacting parts from S_{int} (in perturbative expansion)

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Examples (I)

- Fermion propagator in QCD from $\bar{\psi}_i \delta_{ij} \, (\mathrm{i}\not{\partial} - m_q) \, \psi_j$
  - substitution $\partial_\mu = -\mathrm{i} p_\mu$ (Fourier transformation)
- Inverse propagator (momentum space) $\Gamma_{ij}^{\bar{\psi}\psi}(p) = -\mathrm{i} \, \delta_{ij} \, (\not{p} - m_q)$
- Check: quark propagator $\Delta_{ij}(p) = +\mathrm{i} \, \delta_{ij} \, \dfrac{1}{\not{p} - m_q + \mathrm{i}0}$
  - causality in Minkowski space: prescription $+\mathrm{i}0$

Examples (II)

- Gluon propagator in QCD from bi-linear terms in $F^a_{\mu\nu} F_a^{\mu\nu}$ and $\mathcal{L}_{\rm gauge-fix}$
  - recall $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f_{abc} A^b_\mu A^c_\nu$
  - recall $\mathcal{L}_{\rm gauge-fix} \;=\; -\dfrac{1}{2\lambda}\,\left(\partial^\mu A^a_\mu\right)^2$
- Inverse propagator (momentum space)

$$\Gamma^{AA}_{ab;\mu\nu}(p) \;=\; +{\rm i}\,\delta_{ab}\,\left[p^2 g_{\mu\nu} - \left(1-\frac{1}{\lambda}\right) p_\mu p_\nu\right]$$

- Gluon propagator $\Delta^{ab;\mu\nu}(p) \;=\; +{\rm i}\,\delta_{ab}\,\left[\dfrac{-g_{\mu\nu}}{p^2} + (1-\lambda)\dfrac{p_\mu p_\nu}{p^2}\right]$
  - Check: $\Gamma^{AA}_{ac;\mu\rho}(p)\,\Delta^{cb;\rho\nu}(p) \;=\; \delta_a^{\;b}\, g_\mu^{\;\nu}$

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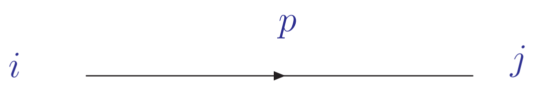
Examples (III)

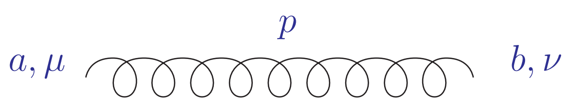
- Interactions derived from S_{int}
  - fermion-gluon interaction from $\bar\psi_i \mathrm{i} \not{A}_{ij} \psi_j \longrightarrow -\mathrm{i} t^a_{ij} \gamma_\mu$
- General rule
  - replacement of all ∂_μ by momenta p_μ
    (tedious for 3- and 4-gluon interactions)

Feynman rules (I)

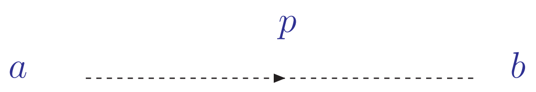
- Propagators
  - fermions, gluons, ghosts
  - covariant gauge



$$\delta^{ij}\,\frac{\mathrm{i}}{\not{p} - m}$$



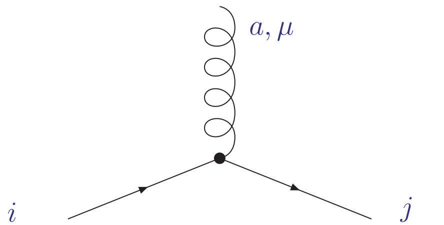
$$\delta^{ab}\,\mathrm{i}\left(\frac{-g^{\mu\nu}}{p^2} + (1-\lambda)\frac{p^\mu p^\nu}{(p^2)^2}\right)$$



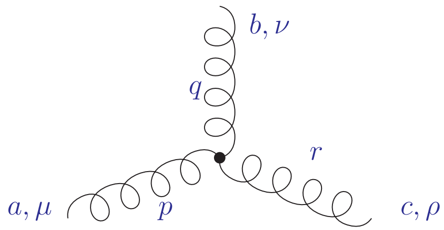
$$\delta^{ab}\,\frac{\mathrm{i}}{p^2}$$

Feynman rules (II)

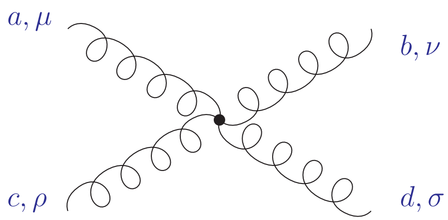
- Vertices



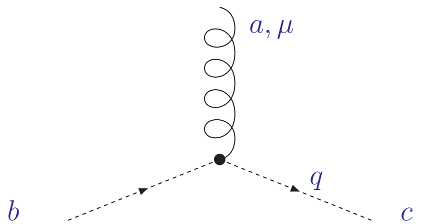
$$-\mathrm{i}\, g\ (t^a)_{ji}\ \gamma^\mu$$



$$-g\, f^{abc}\ \left((p-q)^\rho g^{\mu\nu} + (q-r)^\mu g^{\nu\rho} + (r-p)^\nu g^{\mu\rho}\right)$$



$$-\mathrm{i}\, g^2\, f^{xac} f^{xbd}\ \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}\right)$$
$$-\mathrm{i}\, g^2\, f^{xad} f^{xbc}\ \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}\right)$$
$$-\mathrm{i}\, g^2\, f^{xab} f^{xcd}\ \left(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}\right)$$



$$g\, f^{abc}\, q^\mu$$

Renormalization

Physics picture

- Parameters of Lagrangian in quantum field theory have no unique physical interpretation
- Generic quantity R depends on
  - hard scale Q, mass m_q
  - in perturbative study on coupling constant α_s
- Radiative corrections
  - resolve quantum fluctuations at given resolution length $a \sim 1/\mu$
  - induce dependence of R on scale μ

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  - resolve quantum fluctuations at given resolution length $a \sim 1/\mu$
  - induce dependence of R on scale μ
- Renormalization "group" governed by QCD describes changes R with respect to μ (differential equation of first order)

$$\left\{ \mu^2 \frac{\partial}{\partial \mu^2} + \beta\left(\alpha_s\right) \frac{\partial}{\partial \alpha_s} - \gamma_m\left(\alpha_s\right) m_q \frac{\partial}{\partial m_q} \right\} R\left(\frac{Q^2}{\mu^2}, \alpha_s, \frac{m_q^2}{Q^2} \right) = 0$$

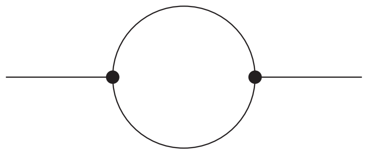
  - partial derivatives $\beta\left(\alpha_s\right) = \frac{\partial}{\partial \mu^2} \alpha_s$ and $\gamma_m\left(\alpha_s\right) m_q = \frac{\partial}{\partial \mu^2} m_q$
  - solution of differential equation requires initial conditions $\longrightarrow$ definition of renormalization scheme

Renormalization

Technicalities

- Radiative corrections require integration over loop momenta
  - loop integrals can diverge in ultraviolet $l \to \infty$
  - power counting reveals divergence in ultraviolet
- Example: self-energy in scalar field theory (off-shell momentum $q^2 \neq 0$)

$$\int d^4l \, \frac{1}{l^2(l-q)^2}$$



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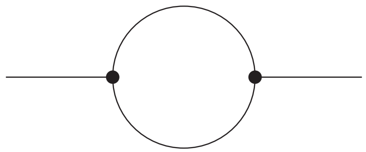
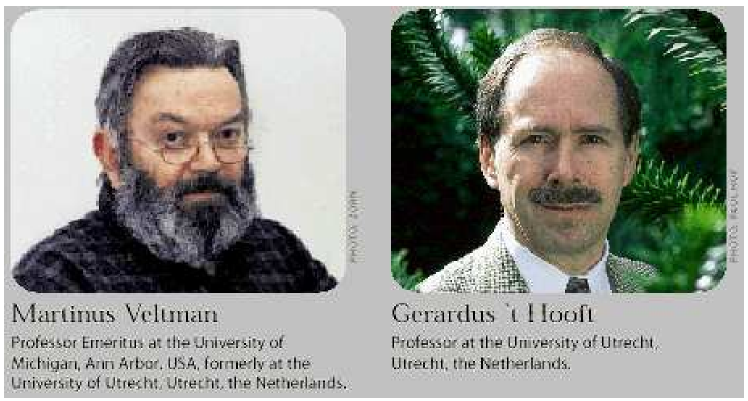
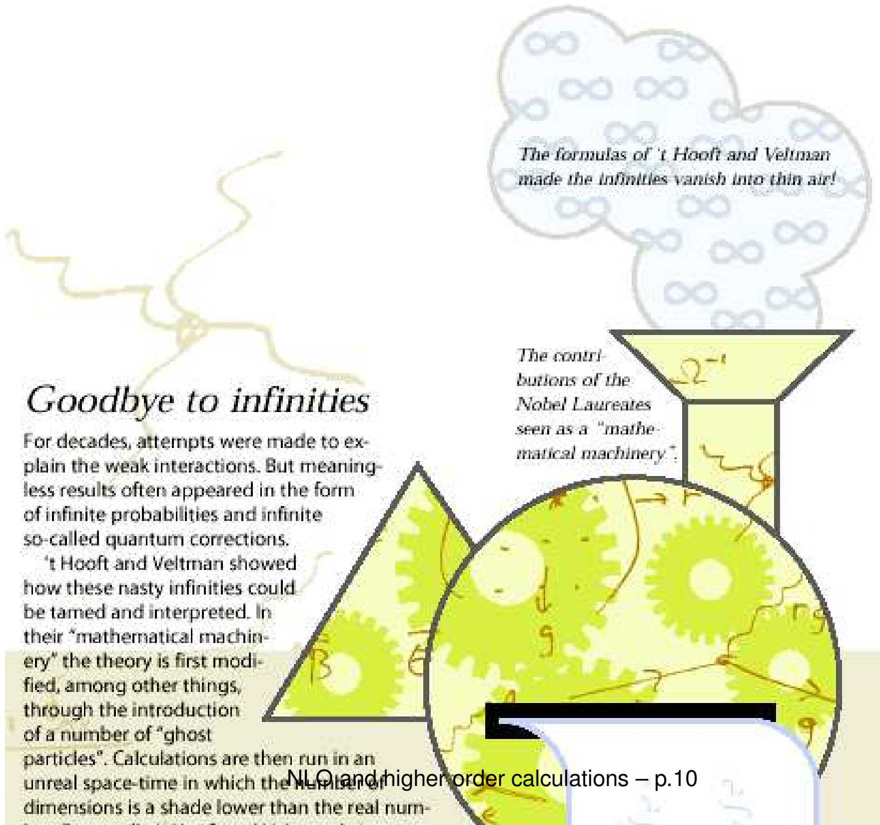


Nobel prize 1999

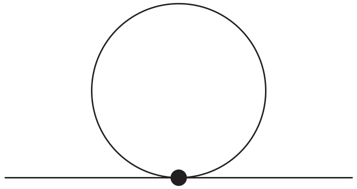




Regularization

- Example for UV divergent loop integral (Euclidean region):

$$\int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 + m_q^2}$$



- Various regularization methods
  - cut-off $l \leq \Lambda_{\mathrm{cut-off}}$; Pauli-Villars $\frac{1}{l^2} \rightarrow \frac{1}{l^2 - M^2}$; lattice; ...

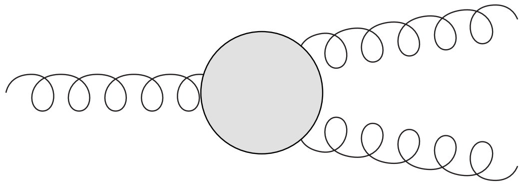
Dimensional regularization

- Lorentz invariance and $SU(N)$ gauge invariance manifest
- Anlytical continuation in space-time dimension $D = 4 - 2\epsilon$
  - loop integral $\int \frac{d^4 l}{(2\pi)^4} \rightarrow \int \frac{d^D l}{(2\pi)^D}$
  - Lorentz index $\mu \in \{0, 1, 2, 3\} \rightarrow \{0, 1, \ldots, D\}$
  - Lorentz vector $p^\mu \in \left(p^0, p^1, p^2, p^3\right) \rightarrow \left(p^0, p^1, \ldots, p^{D-1}\right)$
  - metric $g^{\mu\nu} g_{\mu\nu} = g^\mu_\mu = D$
  - Dirac algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ and $\gamma^\mu \gamma^\nu \gamma_\mu = (2 - D)\gamma^\nu$

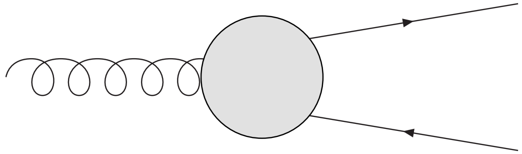
Renormalization

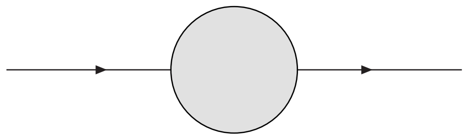
In a nut-shell

- compute vertex corrections
  (one-particle irreducible diagrams)
  - three-gluon vertex
    $V_{A_\mu A_\nu A_\rho} = g_s A_\mu A_\nu A_\rho$

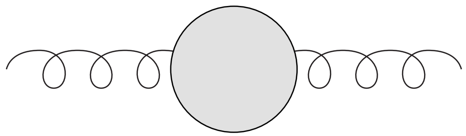


  - gluon-quark vertex
    $V_{\psi\bar{\psi}A_\mu} = g_s A_\mu \bar{\psi}\psi$





- compute self-energy corrections



- Redefinition of fields, vertices and parameters in Lagrangian
  - vertices $V^{\mathrm{b}}_{A_\mu A_\nu A_\rho} = Z_1 V^{\mathrm{r}}_{A_\mu A_\nu A_\rho}$ and $V^{\mathrm{b}}_{\psi\bar{\psi}A_\mu} = Z_{1F} V^{\mathrm{r}}_{\psi\bar{\psi}A_\mu}$
  - fields $\psi^{\mathrm{b}} = (Z_2)^{1/2}\, \psi^{\mathrm{r}}$ and $A^{\mathrm{b}}_\mu = (Z_3)^{1/2}\, A^{\mathrm{r}}_\mu$
  - parameters coupling constant $g_s^{\mathrm{b}} = Z_g g_s^{\mathrm{r}}$ and mass $m^{\mathrm{b}} = Z_m m^{\mathrm{r}}$

Renormalization

Gauge invariance

- Renormalization of vertex corrections imply

$$Z_1 \;=\; Z_g \left(Z_3\right)^{3/2} \,, \qquad\qquad Z_{1F} \;=\; Z_g \left(Z_3\right)^{1/2} Z_2$$

- Combinations of Z-factors fixed by $SU(N)$ gauge invariance of QCD
  - Slavnov-Taylor (or Ward) identites

$$Z_g \left(Z_3\right)^{1/2} \;=\; \frac{Z_1}{Z_3} \;=\; \frac{Z_{1F}}{Z_2}$$

Renormalized Lagrangian

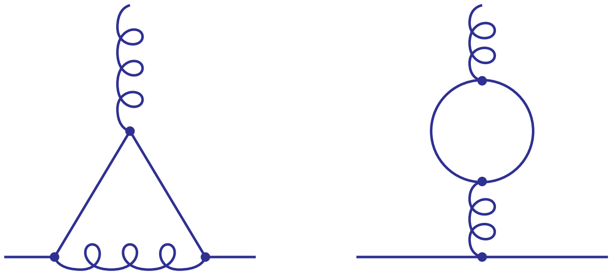
- Construction of renormalized QCD Lagrangian $\mathcal{L}^{\rm ren}$ with rescaled fields, parameters. etc.
- $\mathcal{L}^{\rm bare}$ decomposed into $\mathcal{L}^{\rm ren}$ and counter term $\mathcal{L}^{\rm ct}$

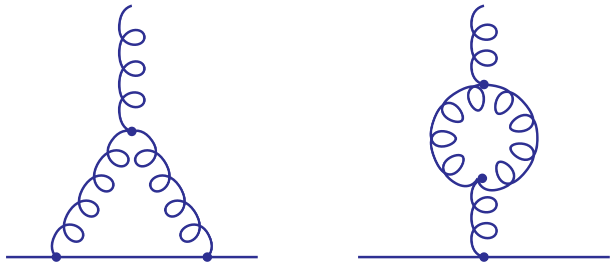
$$\mathcal{L}^{\rm bare}\left(\psi^{\rm b}, \bar{\psi}^{\rm b}, A_\mu^{\rm b}\right) \;=\; \mathcal{L}^{\rm ren}\left(\psi^{\rm r}, \bar{\psi}^{\rm r}, A_\mu^{\rm r}\right) + \mathcal{L}^{\rm ct}$$

  - $\mathcal{L}^{\rm ct}$ contains all parameters with factors $(Z_i - 1)$
  - ultraviolet divergences absorbed by $\mathcal{L}^{\rm ct}$

Running coupling

- Effective coupling constant α_s depends on resolution
- QCD distinguished by self-interaction of gluons; e.g. vertex corrections





– screening (like in QED)

– anti-screening (color charge of g)

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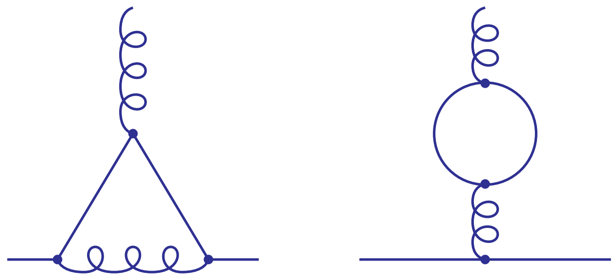

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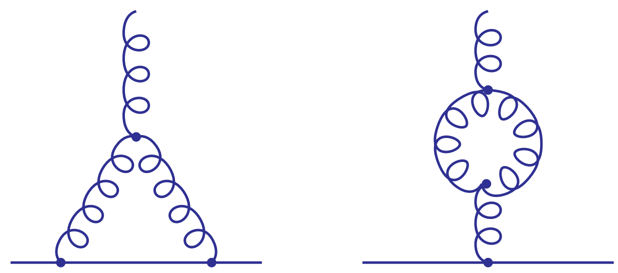


– anti-screening (color charge of g)

- Scale dependence governed by β-function of QCD

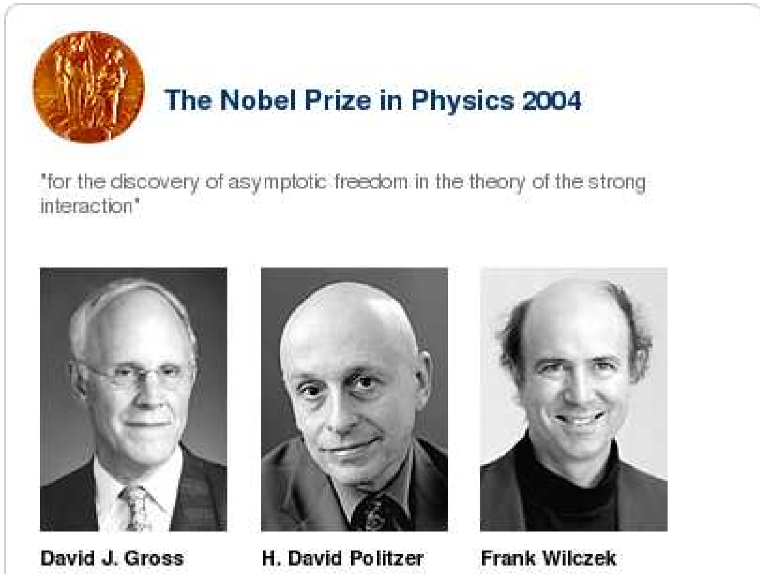
$$\mu^2 \frac{d}{d\mu^2} \alpha_s(\mu) = \beta(\alpha_s) = -\beta_0 \alpha_s^2 - \beta_1 \alpha_s^3 - \beta_2 \alpha_s^4 - \beta_3 \alpha_s^5 - \ldots$$

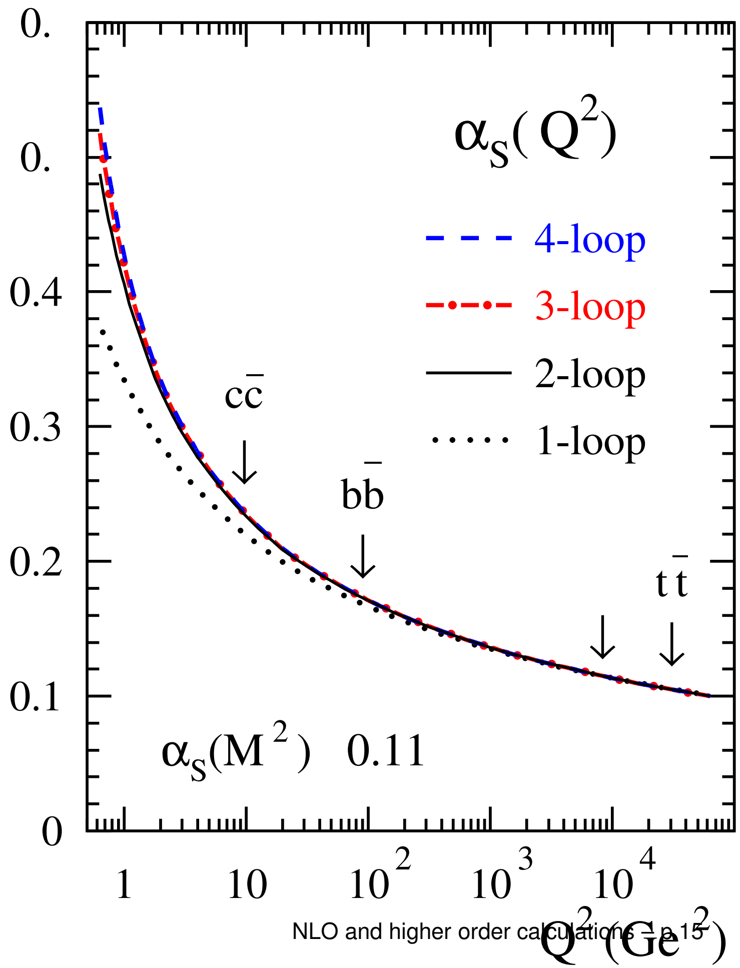
  - QCD β-function has negative sign
  - perturbative expansion with coefficients $\beta_0, \beta_1, \beta_2, \ldots$

$$\beta_0 = \frac{1}{4\pi} \left(\frac{11}{3} C_A - \frac{2}{3} n_f \right) = \frac{1}{4\pi} \Big(7 \Big) \qquad \text{(for } n_f = 6)$$

Asymptotic freedom

- Solution of QCD β-function
  - perturbative expansion to four loops van Ritbergen, Vermaseren, Larin '97
  - very good convergence of perturbative series even at low scales (but $\alpha_s \gg \alpha_{\text{QED}}$)

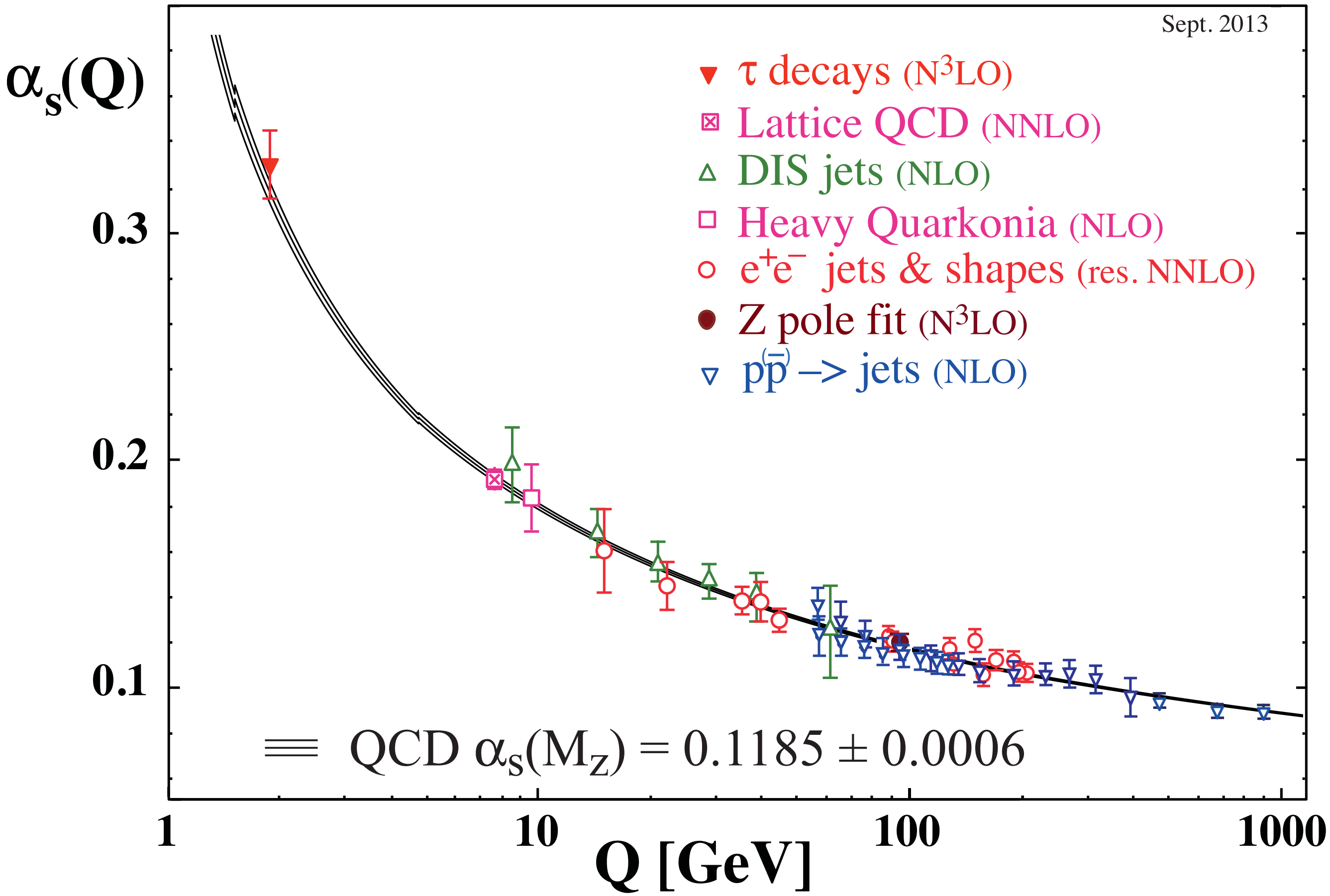




Summary of α_s measurements 2014

Bethke for PDG 2014

- Summary of α_s-determinations at various scales



Treatment of heavy quarks

- Light quarks: $m_u, m_d \ll \Lambda_{\rm QCD}, \qquad m_s < \Lambda_{\rm QCD}$
  - neglect "light quark" masses in hard scattering process
- Heavy quarks: $m_c, m_b, m_t \gg \Lambda_{\rm QCD}$
  - mass effects important

Example

- Different kinematical regions for m_c
  - $Q \not\gg m_c$: partons u, d, s, g with $n_f = 3$

    massive charm quark and terms $m_c/Q \neq 0$ are sizable
  - $Q \gg m_c$: partons u, d, s, c, g with $n_f = 4$

    massless charm quark and terms $m_c/Q \to 0$ are neglected

Decoupling

In a nut-shell

- QCD with different number of quarks can be related $\longrightarrow$ matching of two distinct theories
- Heavy quarks can be decoupled in limit $m_q \to \infty$ Appelquist, Carrazone '74
- Consider QCD parameters in both theories and match at scale μ
  - n_l light flavors + n_h heavy quarks of masses m_q at low scales
  - $n_l + n_h$ light flavors at high scales
- Example: running coupling constant

$$\alpha_s^{n_l} \longrightarrow \alpha_s^{(n_l+n_h)}$$

$\overline{\mathrm{MS}}$ *scheme*

- Decoupling theorem in the $\overline{\mathrm{MS}}$-scheme not true in naive sense
  - mass effects not $1/m_q$ suppressed in theory with n_l light and n_h heavy flavors
  - anomalous dimensions exhibit discontinuities at flavor thresholds
- Decoupling constants in the $\overline{\mathrm{MS}}$ scheme
  Larin, van Ritbergen, Vermaseren '94; Chetyrkin, Kniehl, Steinhauser '97

α_s *with flavor thresholds*

- Solution of QCD β-function
  - discontinuities for $n_f = 3 \to n_f = 4 \to n_f = 5$
- Big picture

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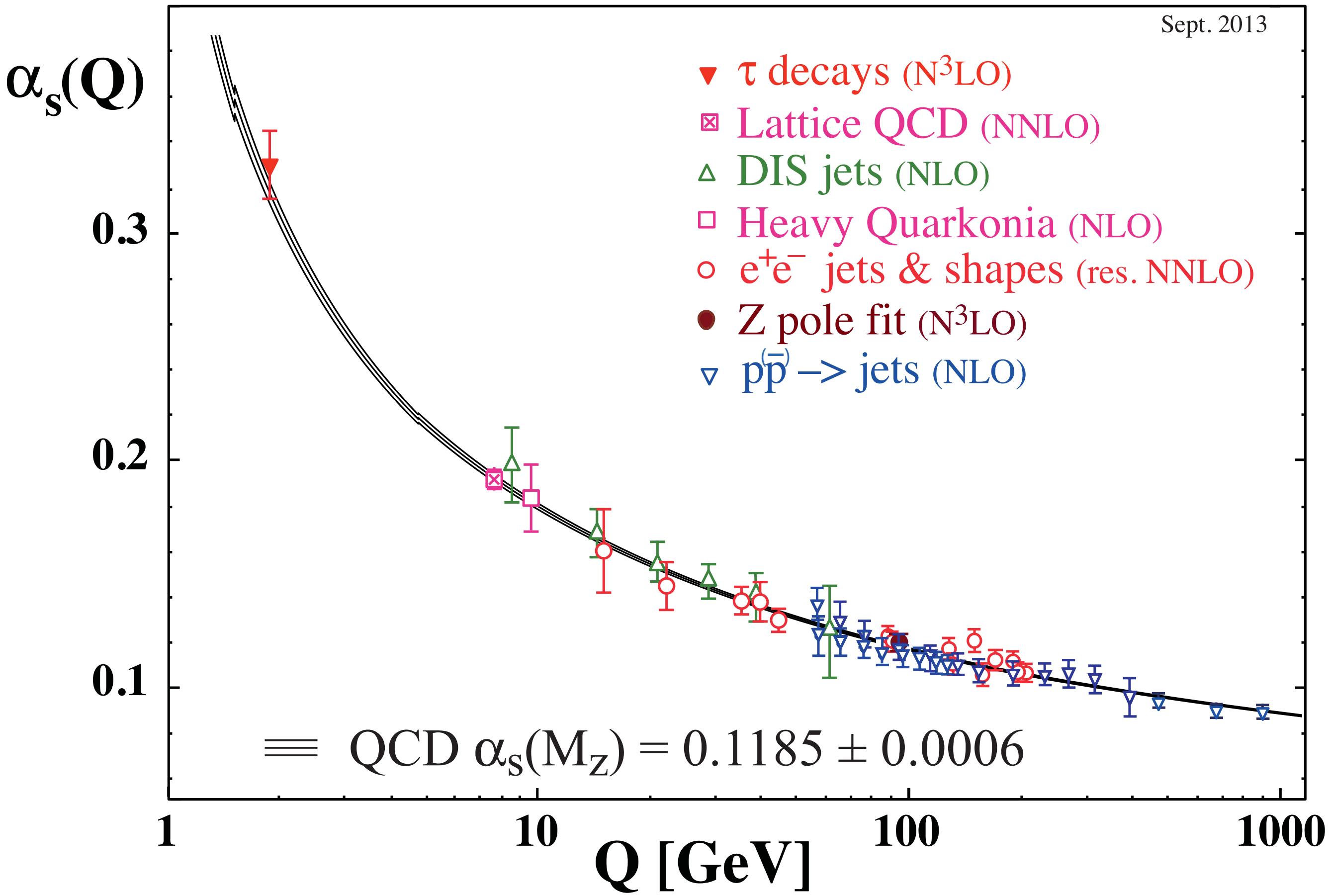
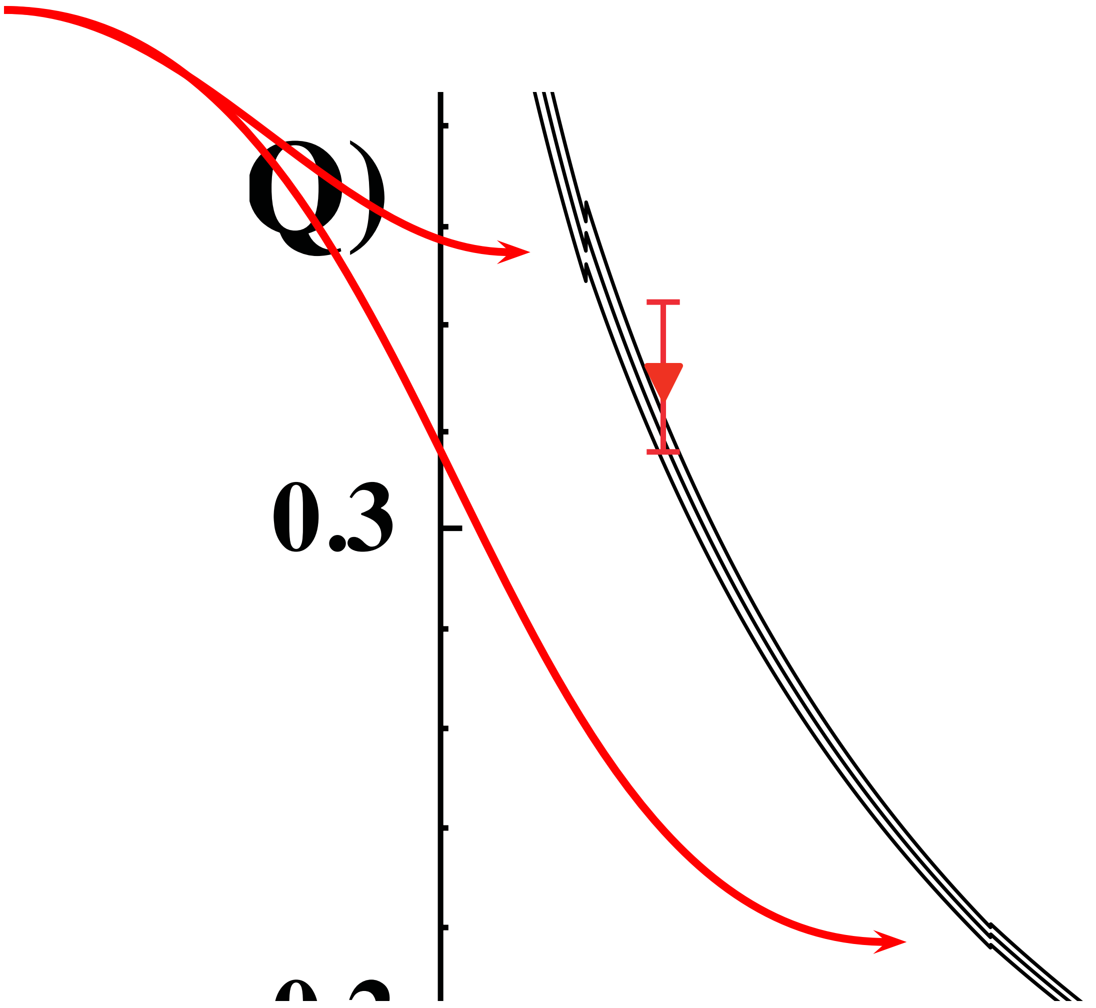


α_s *with flavor thresholds*

- Solution of QCD β-function
  - discontinuities for $n_f = 3 \to n_f = 4 \to n_f = 5$
- Zoom



Summary (part I)

QCD: the gauge theory of the strong interaction

- Quarks and gluons as classical degrees of freedom
- Quantum corrections determine dynamical properties
  - scale dependence of observables
  - running coupling constant and asymptotic freedom
- Renormalization required by quantum corrections
  - subtraction of ultraviolet singularities
  - definition of renormalization scheme
  - parameters of Lagrangian are not observables $\alpha_s, m_q, \ldots$

Part II

- Basics of QCD: quantization and renormalization
- *QCD at work: infrared safety, factorization and evolution*
- Higgs boson production

Perturbative QCD at Work

- QCD – the gauge theory of the strong interactions
- QCD covers dynamics in a large range of scales
  - asymptotically free theory of quarks and gluons at short distances
  - confining theory of hadrons at long distances
- Essential and established part of toolkit for discovering new physics
  - Tevatron and LHC
  - we no longer "test" QCD

Basic concepts of perturbative QCD

- Theoretical framework for QCD predictions at high energies relies on few basic concepts
  - infrared safety
  - factorization
  - evolution

Infrared safety

- Small class of cross sections at high energies and decay rates directly calculable in perturbation theory
- Infrared safe quantities
  - free of long range dependencies at leading power in large momentum scale Q Kinoshita '62; Lee, Nauenberg '64
- General structure of cross section
  - large momentum scale Q, renormalization scale μ

$$Q^2\,\hat{\sigma}\left(Q^2,\mu^2,\alpha_s(\mu^2)\right) = \sum_n \alpha_s^n\, c^{(n)}(Q^2/\mu^2)$$

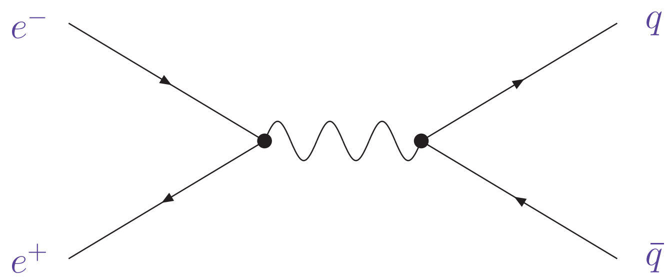
- Examples
  - total cross section in $e^+\,e^-$-annihilation

$$R^{\rm had}(s) = \frac{\sigma(e^+\,e^- \to \text{hadrons})}{\sigma(e^+\,e^- \to \mu^+\,\mu^-)}$$

  - jet cross sections in $e^+\,e^-$-annihilation
  - total width of Z-boson

Soft and collinear singularities

- e^+e^--annihilation (massless quarks)
  - Born cross section $\sigma^{(0)} = \dfrac{4\pi\alpha^2}{3s}$



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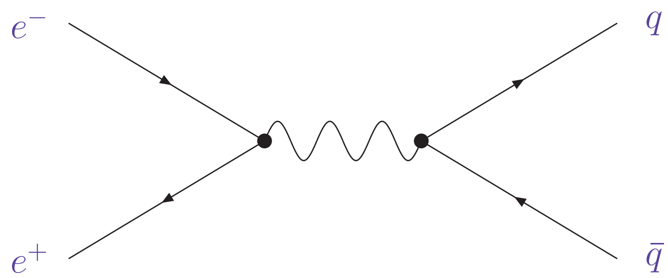
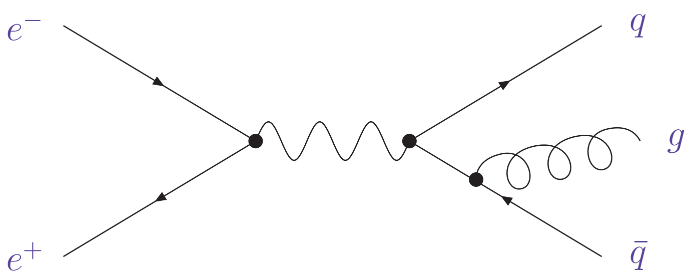
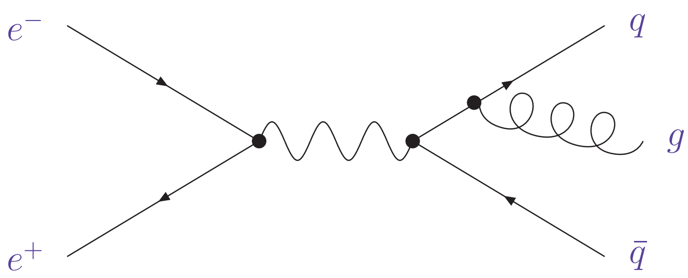


- Study QCD corrections (real emissions)



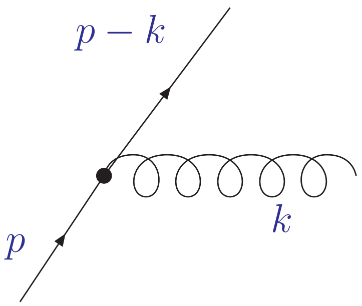


- Cross section
  - dimensional regularization $D = 4 - 2\epsilon$ (with $f(\epsilon) = 1 + \mathcal{O}(\epsilon^2)$)

$$\sigma^{q\bar{q}g} = \sigma^{(0)}\, 3 \sum_q e_q^2\, f(\epsilon)\, C_F\, \frac{\alpha_s}{2\pi} \int dx_1 dx_2\, \frac{x_1^2 + x_2^2 - \epsilon(2 - x_1 - x_2)}{(1-x_1)^{1+\epsilon}\,(1-x_2)^{1+\epsilon}}$$

- scaled energies $x_1 = 2\dfrac{E_q}{\sqrt{s}}$ and $x_2 = 2\dfrac{E_{\bar{q}}}{\sqrt{s}}$

- Soft and collinear divergencies ($0 \leq x_1, x_2 \leq 1$ and $x_1 + x_2 \geq 1$)



$$1 - x_1 = x_2 \frac{E_g}{\sqrt{s}} (1 - \cos\theta_{2g})$$ and

$$1 - x_2 = x_1 \frac{E_g}{\sqrt{s}} (1 - \cos\theta_{1g})$$

- Integrate over phase space for real emission contributions

$$\sigma^{q\bar{q}g} = \sigma^{(0)} \, 3 \sum_q e_q^2 \, f(\epsilon) \, C_F \, \frac{\alpha_s}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} + \mathcal{O}(\epsilon) \right)$$

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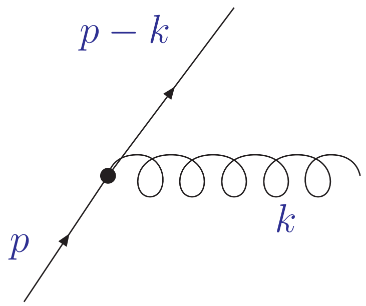

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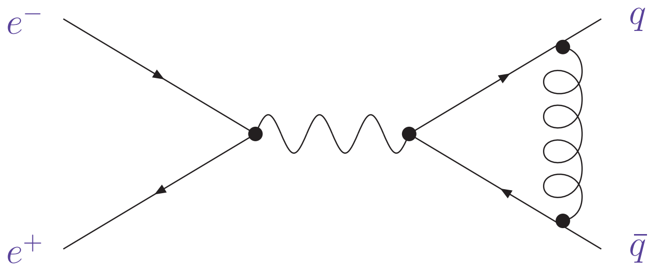
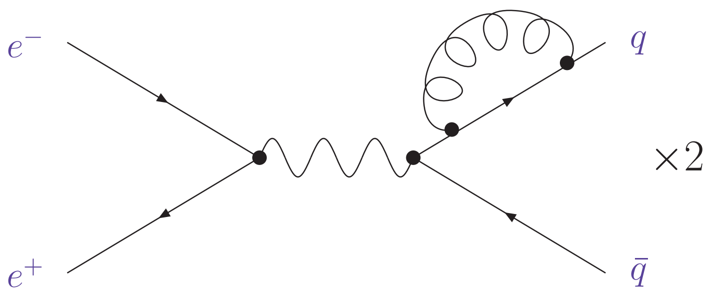
- Divergencies cancel against virtual contributions





$$\sigma^{q\bar{q}(g)} \quad = \quad \sigma^{(0)}\, 3 \sum_q e_q^2\, f(\epsilon)\, C_F\, \frac{\alpha_s}{2\pi} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \mathcal{O}(\epsilon) \right)$$

Infrared safety

Infrared safety

- Total cross section ($R(s)$) is directly calculable in perturbation theory (finite)

$$R(s) \quad = \quad 3 \sum_q e_q^2 \left\{ 1 + \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right\}$$

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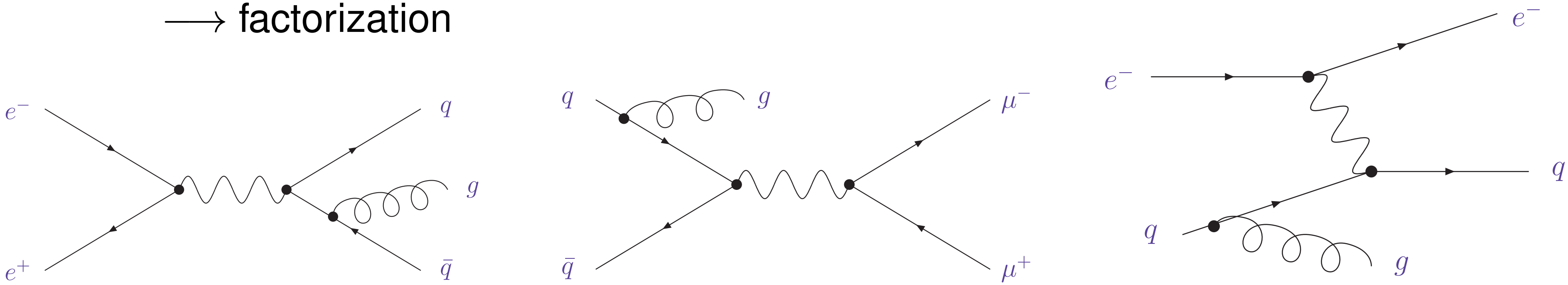
Collinear singularities

- Collinear divergencies remain for hadronic observables $\longrightarrow$ factorization



- Left: single-hadron inclusive e^+e^--annihilation (time-like kinematics)
- Center: Drell-Yan process in pp-scattering (space-like kinematics)
- Right: Deep-inelastic e^-p-scattering (space-like kinematics)

Factorization

- Large class of hard-scattering reactions with initial state hadrons
  - cross section not infrared safe
  - dependent on quark and gluon degrees of freedom in hadron
  - sensitive to nonperturbative processes at long distances
- Factorization of cross section
  - infrared safe hard part $\hat{\sigma}_{\rm pt}$ calculable in perturbative QCD
  - nonperturbative function f determined from data
  - f parametrizes hadron structure
- General structure of cross section
  - large momentum scale Q, factorization scale μ

$$Q^2 \, \sigma_{\rm phys}\,(Q) = \hat{\sigma}_{\rm pt}\,(Q/\mu, \alpha_s(\mu)) \, \otimes \, f\,(\mu)$$

  - convolution $\otimes$ in suitable kinematical variables
- Factorization
  - generalization of operator product expansion

Classic example

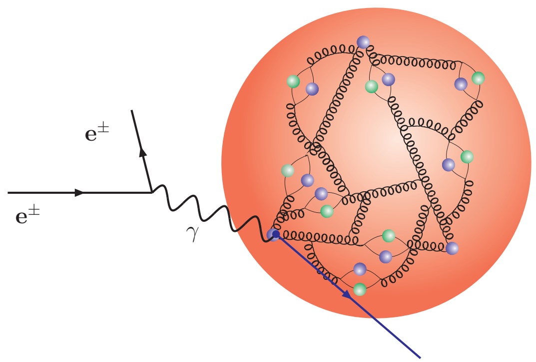
- Deep-inelastic scattering
  - test parton dynamics at factorization scale μ

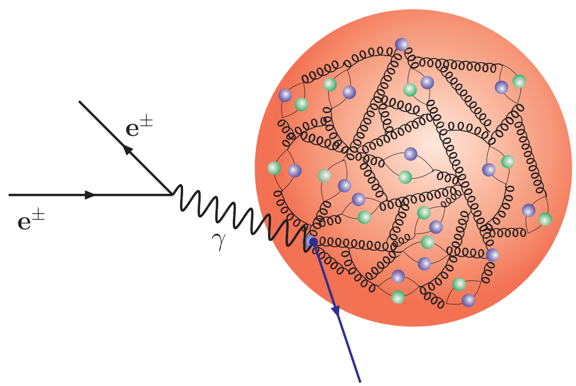
$$\sigma_{\gamma p \to X} = \sum_i f_i(\mu^2) \otimes \hat{\sigma}_{\gamma i \to X}\left(\alpha_s(\mu^2), Q^2, \mu^2\right)$$

Physics picture

- QCD factorization
  - constituent partons from proton interact at short distance
  - photon momentum $Q^2 = -q^2$, Bjorken's $x = Q^2/(2p \cdot q)$
  - low resolution

high resolution

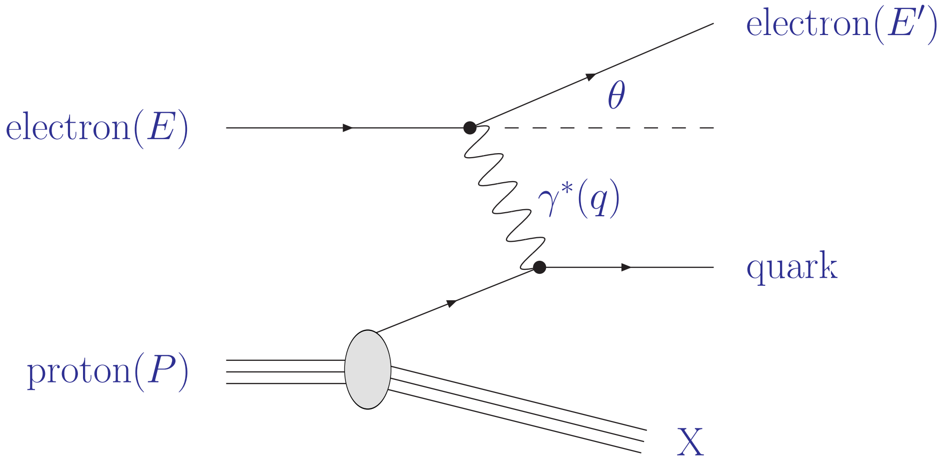




Once upon a time . . .

- HERA: deep structure of proton at highest Q^2 and smallest x

Inelastic electron-proton scattering



- Virtuality of photon: resolution

$$Q^2 \equiv -q^2 = 4EE' \sin^2(\theta/2)$$

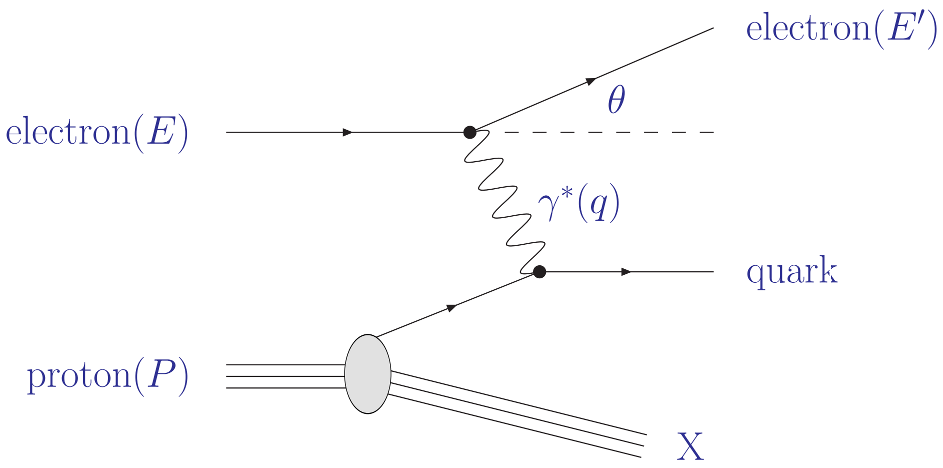
- Bjorken variable: inelasticity

$$x = \frac{Q^2}{2P \cdot q} < 1$$

- Cross section (X inclusive): proton structure function F_i^p

$$(E - E') \frac{d\sigma}{d\Omega\, dE'} \stackrel{\text{lab}}{=} \underbrace{\frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}}}_{\text{Mott-scattering (point-like)}} \left\{ F_2^p(x, Q^2) + \tan^2 \frac{\theta}{2}\, F_1^p(x, Q^2) \right\}$$

Inelastic electron-proton scattering



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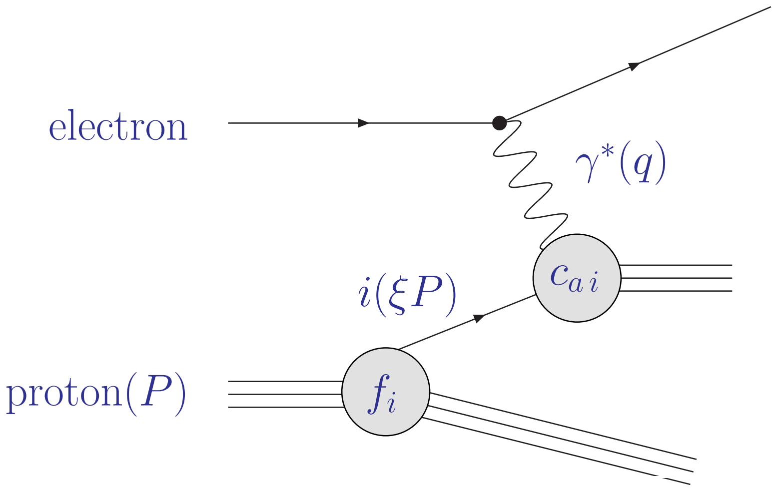
- Deep-inelastic scattering (Bjorken limit: $Q^2 \to \infty$ and x fixed)
  Parton modell (quasi-free point-like constituents, incoherence)

$$F_2(x, Q^2) \simeq F_2(x) = \sum_i e_i^2\, x f_i(x)$$

  - $x f_i(x)$ distribution for momentum fraction x of parton i

QCD corrections in deep-inelastic scattering



- Structure function F_2 (up to terms $\mathcal{O}(1/Q^2)$)
  - Renormalization/factorization scale $\mu = \mathcal{O}(Q)$

$$x^{-1} F_2^p(x, Q^2) = \sum_i \int_x^1 \frac{d\xi}{\xi}\, c_{2,i}\left(\frac{x}{\xi}, \alpha_s(\mu^2), \frac{\mu^2}{Q^2}\right) f_i^p(\xi, \mu^2)$$

- Coefficient functions c_a

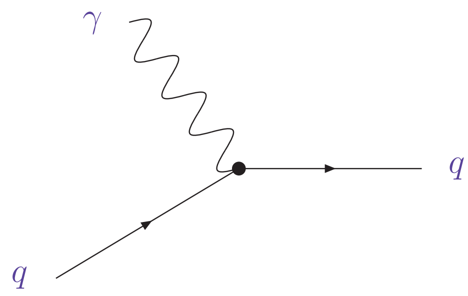
$$c_a = \underbrace{\alpha_s^{n_a}\left[c_a^{(0)} + \alpha_s\, c_a^{(1)}\right.}_{} + \left.\alpha_s^2\, c_a^{(2)} + \ldots \right]$$

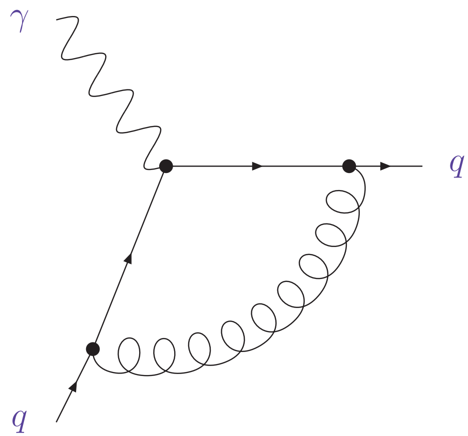
NLO: standard approximation (large uncertainties)

Radiative corrections in a nutshell

- Leading order
  - partonic structure function

$$\hat{F}^{(0)}_{2,q} = e_q^2 \delta(1-x)$$

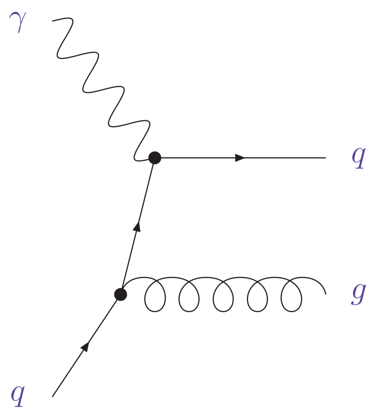


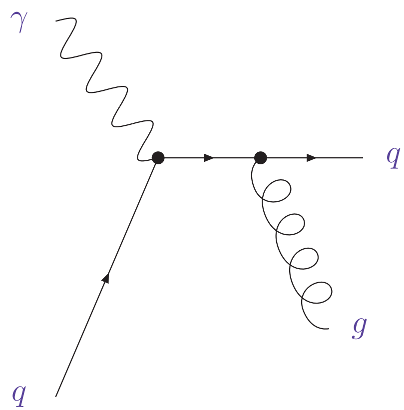


- Next-to-leading order
  - virtual correction
    (infrared divergent; proportional to Born)
  - dimensional regularization $D = 4 - 2\epsilon$

$$\hat{F}^{(1),v}_{2,q} = e_q^2 \, C_F \, \frac{\alpha_s}{4\pi} \, \delta(1-x) \left(\frac{\mu^2}{Q^2}\right)^\epsilon \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \zeta_2 + \mathcal{O}(\epsilon)\right)$$

- Next-to-leading order





  - add real and virtual corrections $\hat{F}_{2,q}^{(1)} = \hat{F}_{2,q}^{(1),r} + \hat{F}_{2,q}^{(1),v}$
  - collinear divergence remains splitting functions $P_{qq}^{(0)}$

$$
\begin{aligned}
\hat{F}_{2,q}^{(1)} \quad = \quad & e_q^2 \, C_F \, \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \Bigg\{ \frac{1}{\epsilon} \left(\frac{4}{1-x} - 2 - 2x + 3\delta(1-x) \right) \\
& +4 \frac{\ln(1-x)}{1-x} - 3 \frac{1}{1-x} - (9 + 4\zeta_2)\delta(1-x) \\
& \quad -2(1+x)(\ln(1-x) - \ln(x)) - 4 \frac{1}{1-x} \ln(x) + 6 + 4x \\
& +\mathcal{O}(\epsilon) \Bigg\}
\end{aligned}
$$

- Structure of NLO correction
  - absorb collinear divergence $P_{qq}^{(0)}$ in renormalized parton distributions

$$\hat{F}_{2,q}^{(1),\text{bare}} = e_q^2 \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \left\{ \frac{1}{\epsilon} P_{qq}^{(0)}(x) + c_{2,q}^{(1)}(x) + \mathcal{O}(\epsilon) \right\}$$

$$q^{\text{ren}}(\mu_F^2) = q^{\text{bare}} - \frac{\alpha_s}{4\pi} \frac{1}{\epsilon} P_{qq}^{(0)}(x) \left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}$$

  - partonic (physical) structure function at factorization scale μ_F

$$\hat{F}_{2,q} = e_q^2 \left(\delta(1-x) + \frac{\alpha_s}{4\pi} \left\{ c_{2,q}^{(1)}(x) - \ln\left(\frac{Q^2}{\mu_F^2}\right) P_{qq}^{(0)}(x) \right\} \right)$$

QCD evolution

- Evolution formulates dependence of cross sections for observable on momentum transfer
- Classic example: scaling violations of structure functions
  Gross, Wilczek '73; Politzer '73
- Physical cross section in factorization ansatz cannot depend on μ
  $Q^2\,\sigma_{\rm phys}\,(Q) = \hat{\sigma}_{\rm pt}\,(Q/\mu, \alpha_s(\mu))\, \otimes\, f\,(\mu)$
  - factorization scale μ arbitrary $\mu \frac{d\,\sigma_{\rm phys}}{d\mu} = 0$
- Immediate consequence **DGLAP**: Altarelli, Parisi '77

$$\mu \frac{d\,f(\mu)}{d\mu} = P\,(\alpha_s(\mu))\, \otimes\, f(\mu) + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

$$\mu \frac{d\,\hat{\sigma}_{\rm pt}\,(Q/\mu, \alpha_s(\mu))}{d\mu} = -P\,(\alpha_s(\mu))\, \otimes\, \hat{\sigma}_{\rm pt}\,(Q/\mu, \alpha_s(\mu)) + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

- PDF evolution from renormalization group equation

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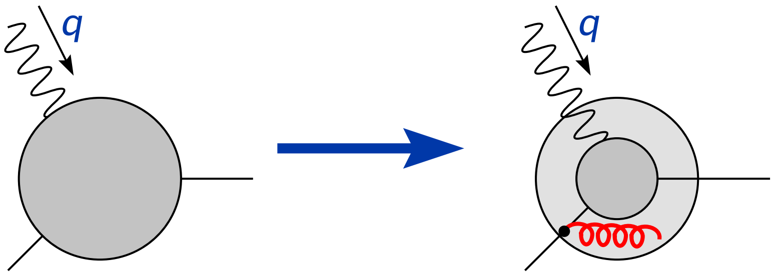
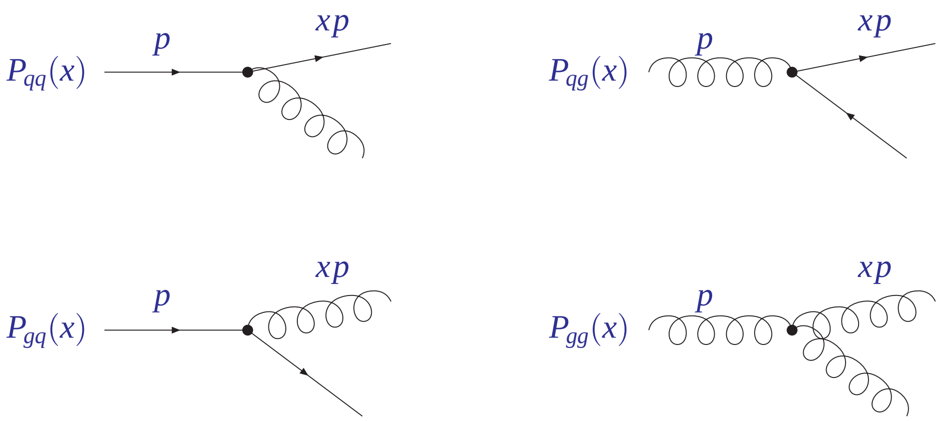
- PDF evolution from renormalization group equation
  - splitting functions calculable in QCD

Parton evolution



- Proton in resolution $1/Q \longrightarrow$ sensitive to lower momentum partons

- Feynman diagrams in leading order



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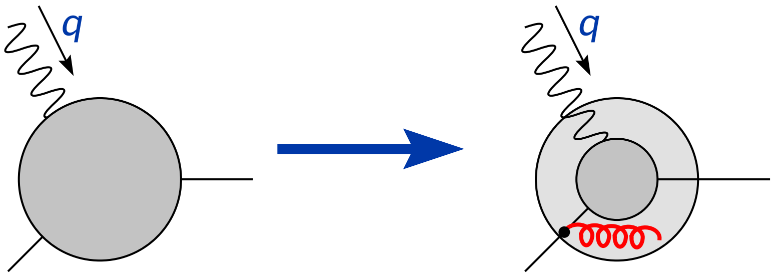

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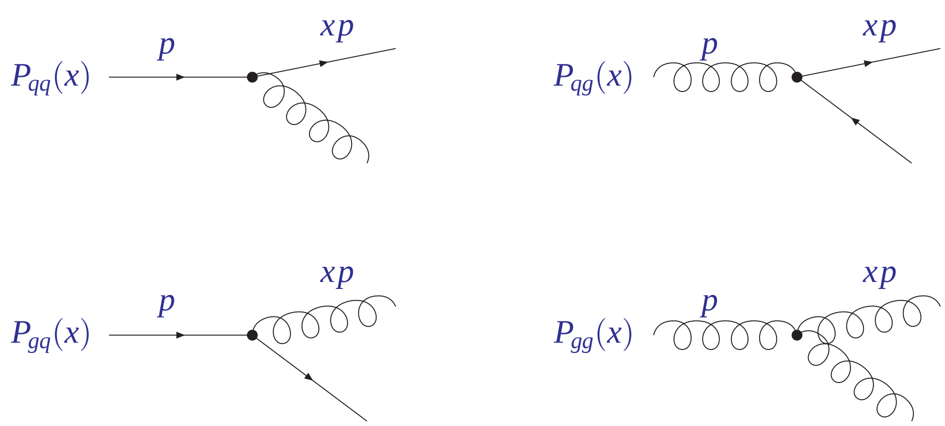


- Proton in resolution $1/Q \longrightarrow$ sensitive to lower momentum partons

- Evolution equations for parton distributions f_i
  - predictions from fits to reference processes (universality)

$$\frac{d}{d\ln\mu^2} f_i(x,\mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2)\right](x)$$

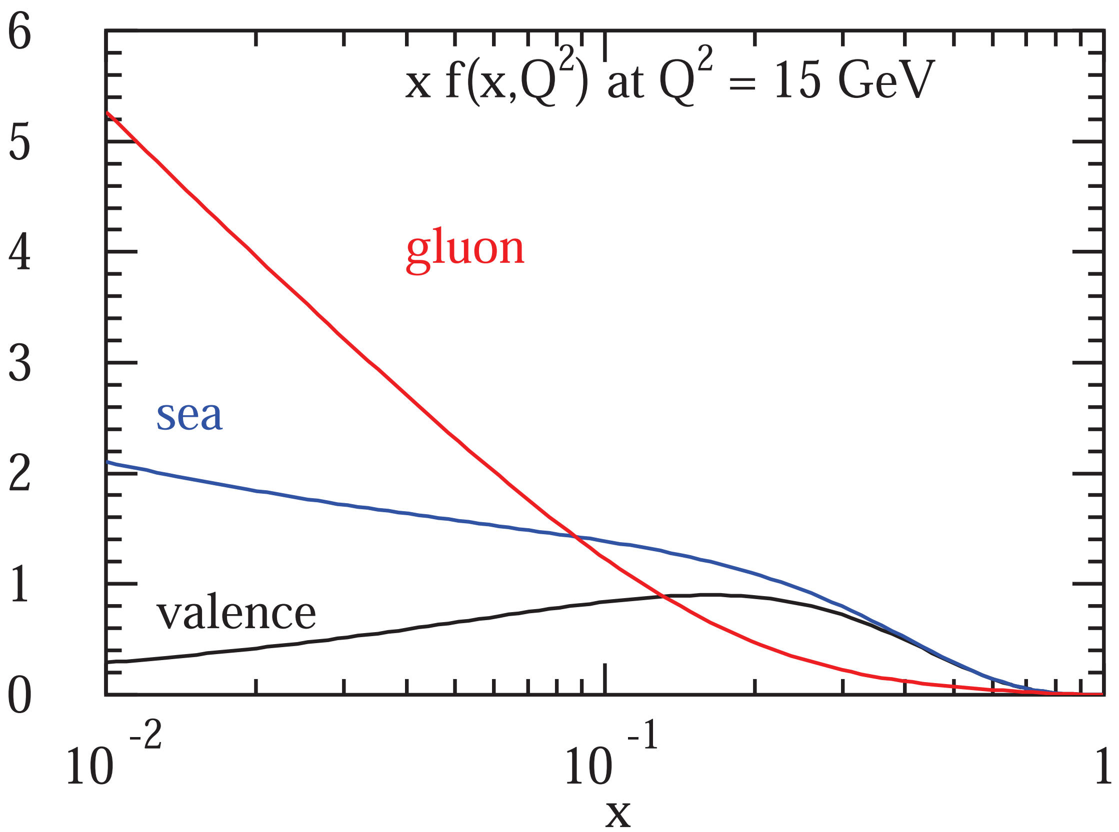
- Splitting functions P

$$P = \underbrace{\alpha_s\, P^{(0)} + \alpha_s^2\, P^{(1)}}_{} + \alpha_s^3\, P^{(2)} + \dots$$

NLO: standard approximation (large uncertainties)

Parton distributions in proton

- Valence $q - \bar{q}$ (additive quantum numbers) sea (part with $q + \bar{q}$)



- Parameterization (bulk of data from deep-inelastic scattering)
  - structure function $F_2 \longrightarrow$ quark distribution
  - scale evolution (perturbative QCD) $\longrightarrow$ gluon distribution

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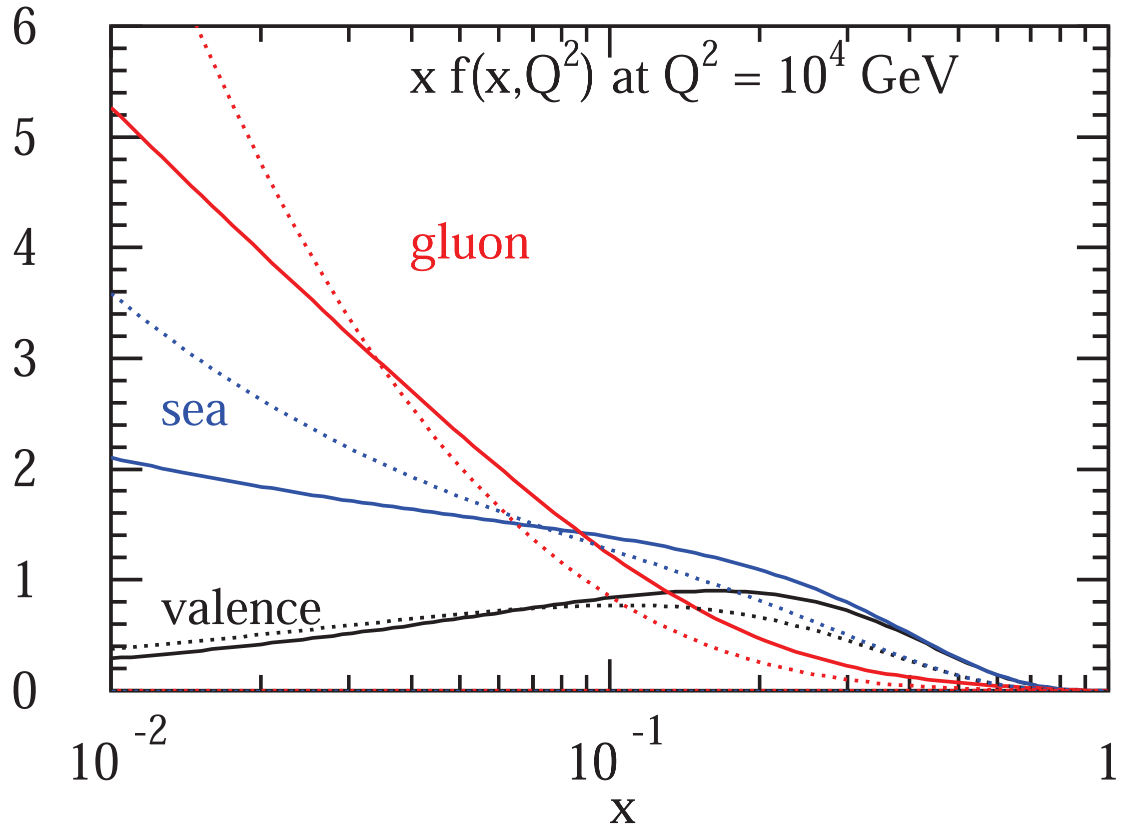

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Parton distribution fits

Global PDF fits

- PDF sets currently available
  - ABM12 Alekhin, Blümlein, S.M. '13
  - CT10 Gao et al. '13
  - HERAPDF (v1.5) H1 & ZEUS Coll. '11
  - JR09 Jimenez-Delgado, Reya '09
  - MSTW Martin, Stirling, Thorne, Watt '09
  - NNPDF (NN23) Ball et al. '12

Iterative cycle of PDF fits

i) check of compatibility of new data set with available world data

ii) study of potential constraints due to addition of new data set to fit

iii) perform high precision measurement of the non-perturbative parameters
  - parton distributions
  - strong coupling $\alpha_s(M_Z)$
  - heavy quark masses

LHC measurements

General remarks

- QCD corrections important
  - require theory predictions to NNLO accuracy
- PDF fits with 3-flavors for DIS, 5-flavors for LHC data (matching from 3 to 5-flavors)
  - QCD evolution over large range

Benchmark processes

- Complete NNLO QCD corrections available for
  - $W^{\pm}$- and Z-boson production
    Hamberg, van Neerven, Matsuura '91; Harlander, Kilgore '02
  - top-quark hadro-production Czakon, Fiedler, Mitov '13
- Jet data from Tevatron and LHC
  - QCD corrections only NLO known
  - possible impact of jet definition and algorithm
  - ongoing effort towards NNLO (corrections expected to be as big as $\mathcal{O}(15-20\%)$) Gehrmann-De Ridder, Gehrmann, Glover, Pires '13

Example: ABM PDFs

Data considered in the fit

- Analysis of world data for deep-inelastic scattering and fixed-target data for Drell-Yan process
  - inclusive DIS data HERA, BCDMS, NMC, SLAC
  - semi-inclusive DIS charm production data (HERA)
  - Drell-Yan data (fixed target) E-605, E-866
  - neutrino-nucleon DIS data (di-muon production) CCFR/NuTeV
  - LHC data for $W^{\pm}$- and Z-boson production

Theory considerations

- Consistent theory description for consistent data sets
  - low scale DIS data with account of higher twist
- Determination of PDFs and strong coupling constant α_s to NNLO QCD
- Consistent scheme for treatment of heavy quarks
  - fixed-flavor number scheme for $n_f = 3, 4, 5$
  - $\overline{\mathrm{MS}}$-scheme for quark masses and α_s
- Full account of error correlations

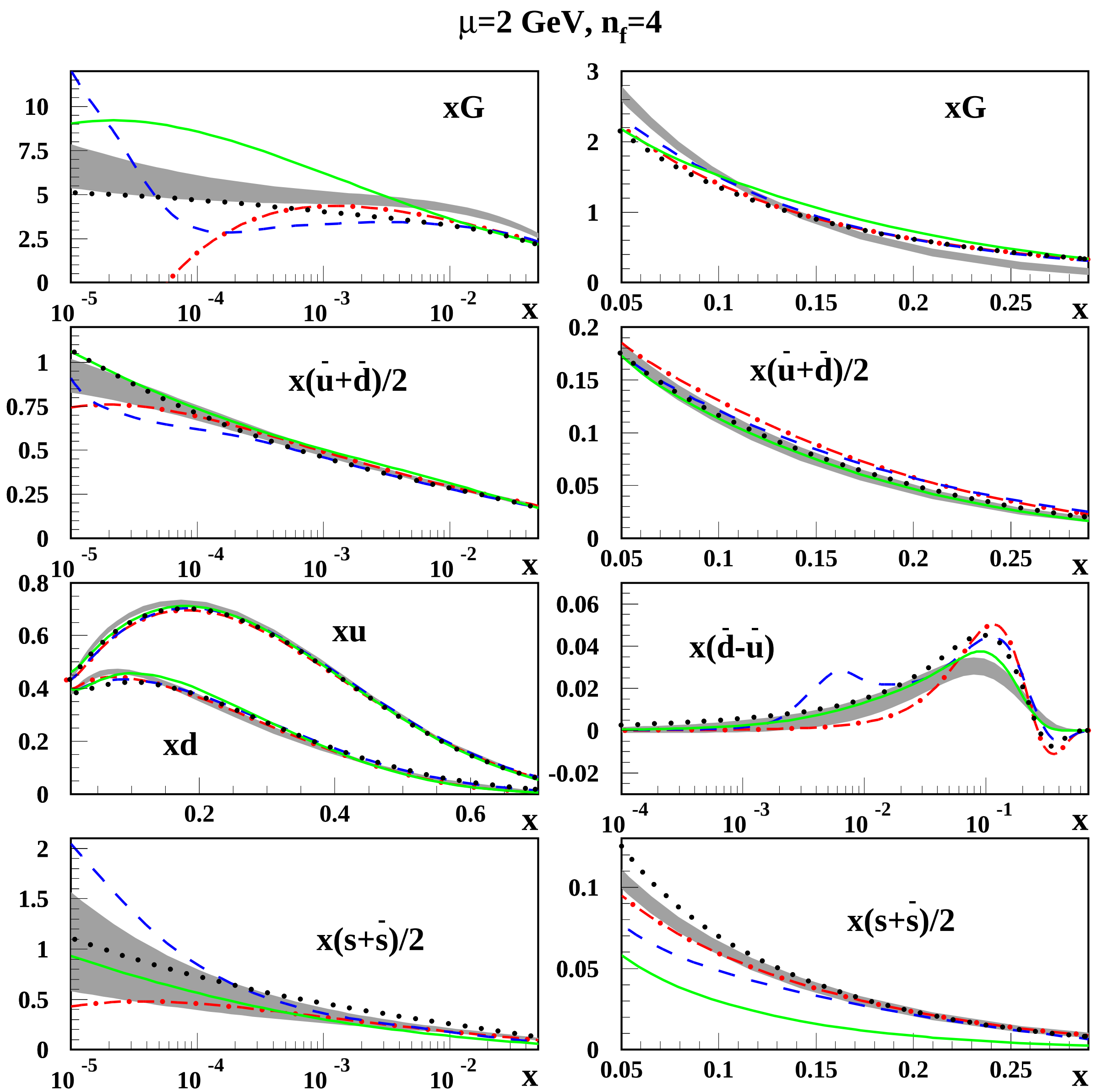
ABM PDF ansatz

- PDFs parameterization at scale $Q_0 = 3\mathrm{GeV}$ in scheme with $n_f = 3$
  Alekhin, Blümlein, S.M. '12
  - ansatz for valence-/sea-quarks, gluon with polynomial $P(x)$
  - strange quark is taken in charge-symmetric form
  - 24 parameters in polynomials $P(x)$
  - 4 additional fit parameters: $\alpha_s^{(n_f=3)}(\mu = 3$ GeV), m_c, m_b and deuteron correction
  - simultaneous fit of higher twist parameters (twist-4)

$$
\begin{aligned}
xq_v(x,Q_0^2) &= \frac{2\delta_{qu}+\delta_{qd}}{N_q^v} x^{a_q}(1-x)^{b_q} x^{P_{qv}(x)} \\
xu_s(x,Q_0^2) = x\bar{u}_s(x,Q_0^2) &= A_{us} x^{a_{us}}(1-x)^{b_{us}} x^{a_{us}\,P_{us}(x)} \\
x\Delta(x,Q_0^2) = xd_s(x,Q_0^2) - xu_s(x,Q_0^2) &= A_\Delta x^{a_\Delta}(1-x)^{b_\Delta} x^{P_\Delta(x)} \\
xs(x,Q_0^2) = x\bar{s}(x,Q_0^2) &= A_s x^{a_s}(1-x)^{b_s}\,, \\
xg(x,Q_0^2) &= A_g x^{a_g}(1-x)^{b_g} x^{a_g\,P_g(x)}
\end{aligned}
$$

- Ansatz provides sufficient flexibility; no additional terms required to improve the quality of fit

Parton distributions tuned to LHC data



- 1σ band for ABM12 PDFs (NNLO, 4-flavors) at $\mu = 2$ GeV
  Alekhin, Blümlein, S.M.'13
- comparison with:
  JR09 (solid lines),
  MSTW (dashed dots),
  NN23 (dashes) and
  CT10 (dots)
- Some interesting observations to be made . . .

Summary (part II)

Perturbative QCD at work

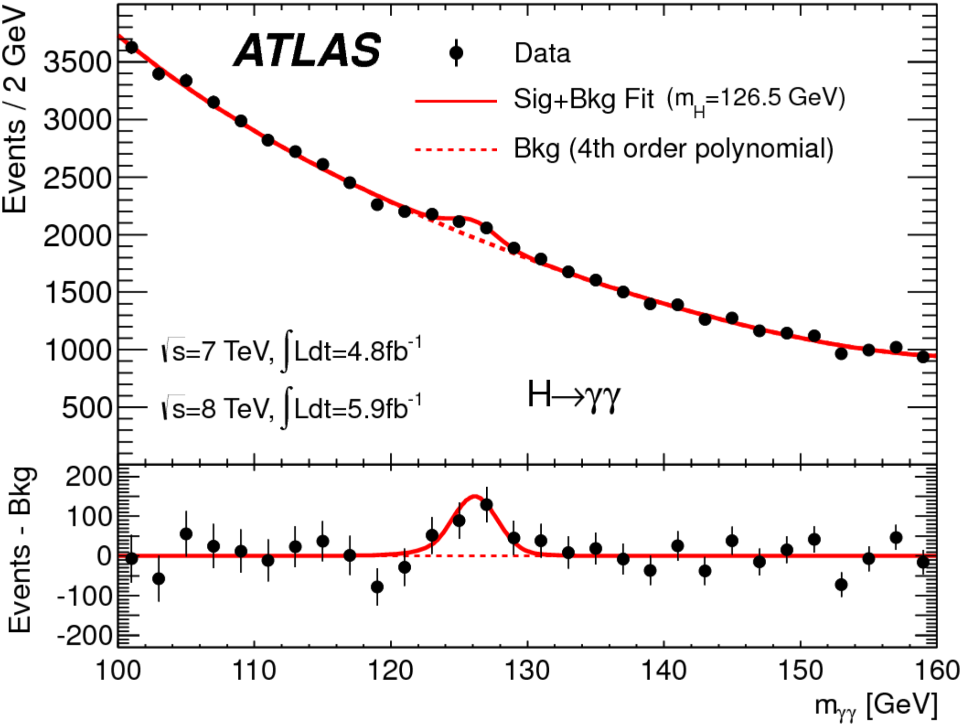
- Basics concepts of QCD
- Infrared safety
  - cancellation of soft and collinear singularities in inclusive observables
  - example $e^+e^- \to$ hadrons at NLO
- Factorization
  - scattering with initial state hadrons requires collinear factorization
  - separation of long and short distance physics
  - parton distribution function
  - example $ep \to X$ in DIS at NLO
- Evolution
  - factorization induces evolution equations via renormalization group

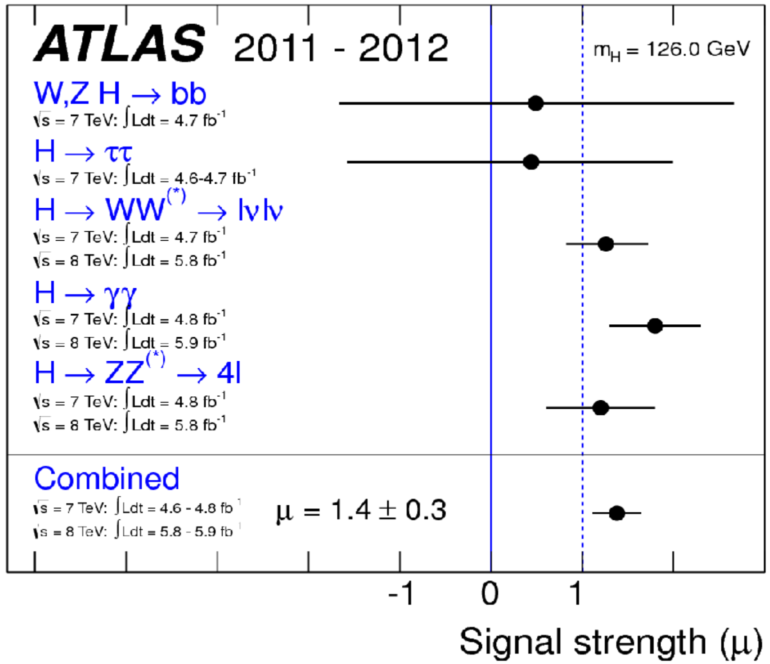
Part III

- Basics of QCD: quantization and renormalization
- QCD at work: infrared safety, factorization and evolution
- *Higgs boson production*

LHC measurements

Atlas coll. July 2012





- Measured $H \to \gamma\gamma$ decay mode (left)
- Signal strength of all analyzed decay modes normalized to SM expectation (right)
- Agreement with SM for $H \to ZZ$; excess of $H \to \gamma\gamma$ (new physics ?)

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Nobel Prizes and Laureates

Physics Prizes ‹ 2013 ›

▾ About the Nobel Prize in Physics 2013

- Summary
- Prize Announcement
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- Advanced Information
- Popular Information

▸ François Englert

▸ Peter W. Higgs

All Nobel Prizes in Physics

All Nobel Prizes in 2013

The Nobel Prize in Physics 2013

François Englert, Peter W. Higgs

The Nobel Prize in Physics 2013

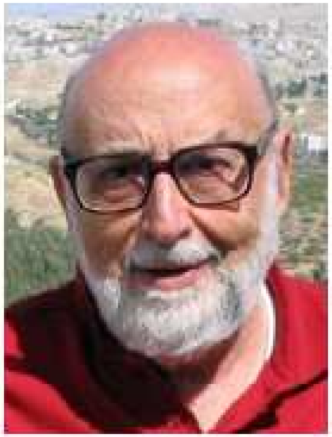


Photo: Pnicolet via Wikimedia Commons

François Englert

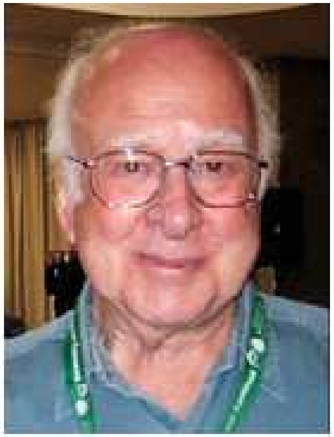


Photo: G-M Greuel via Wikimedia Commons

Peter W. Higgs

The Nobel Prize in Physics 2013 was awarded jointly to François Englert and Peter W. Higgs *"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"*

Highest energies at colliders until 202x

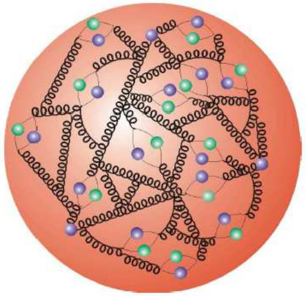
Energy frontier

- Search for Higgs boson, new massive particles at highest energies

$$E = m\,c^2$$

Hadron colliders

- Proton–proton collisions reach TeV-scale
  - LHC run I $\sqrt{S} = 7$ and 8TeV, run II $\sqrt{S} = 13$TeV



- Proton: composite multi-particle bound state
  - collider: "wide-band beams" of quarks and gluons
  - understand SM background (LHC is a QCD machine)
- Theory has to match or exceed accuracy of LHC data
  - perturbative QCD is essential and established part of toolkit
  - electroweak corrections important for precision predictions

Challenges

- Solve master equation

$$\text{new physics} = \text{data} - \text{Standard Model}$$

- LHC explores the energy frontier
  - searches require understanding of SM background
  - theory has to match or exceed accuracy of LHC data

Challenges

- Solve master equation

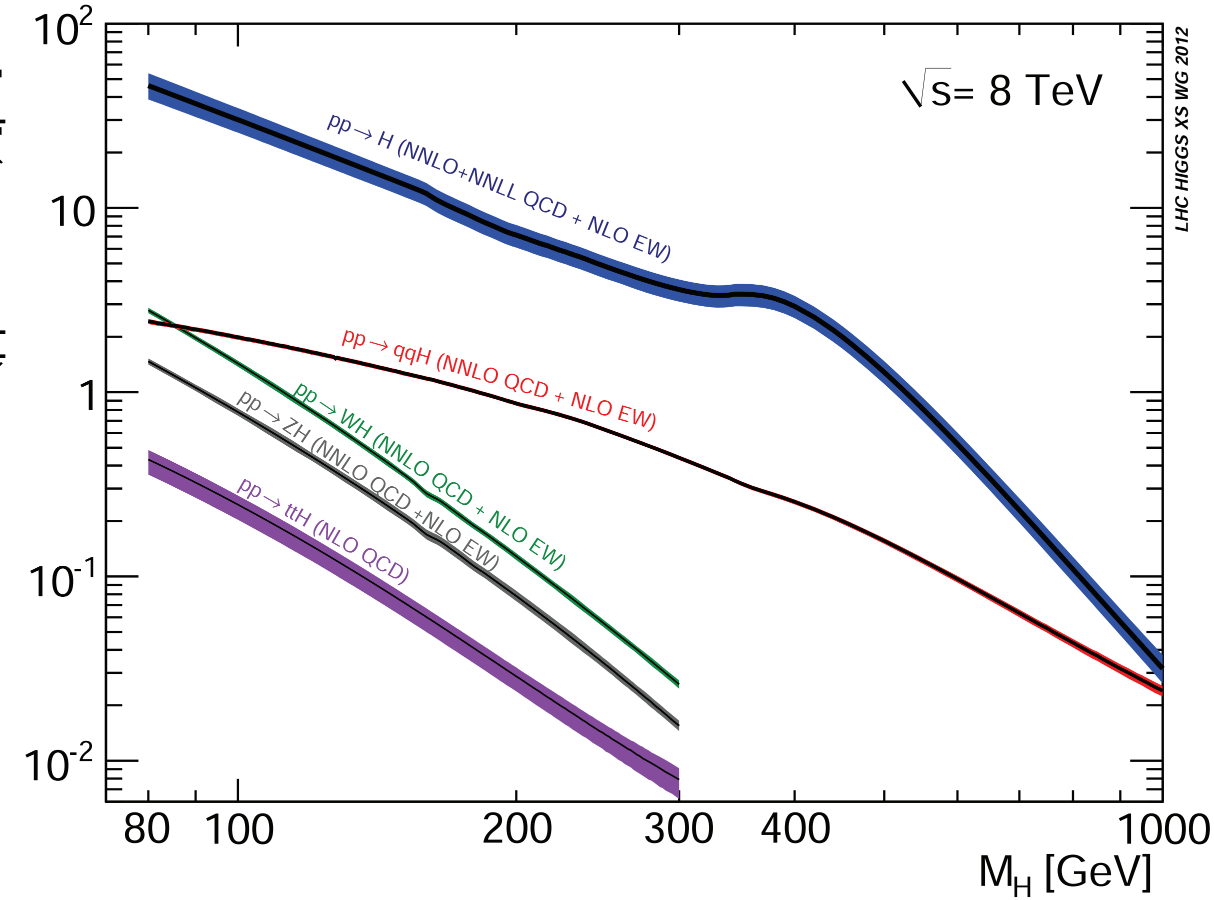
$$\text{new physics} = \text{data} - \text{Standard Model}$$

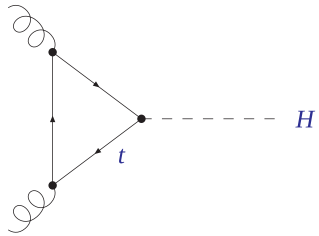
- LHC explores the energy frontier
  - searches require understanding of SM background
  - theory has to match or exceed accuracy of LHC data

Higgs cross section

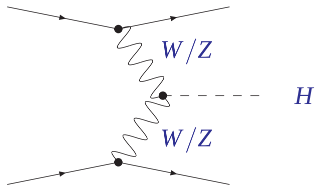
Cross section for Higgs production at the LHC

- Dominant channels for Higgs boson production LHC Higgs XS WG '12

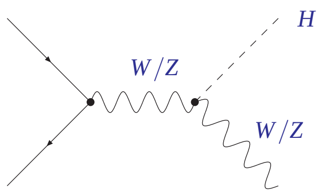




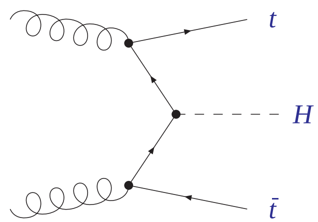
gg-fusion



weak boson fusion



Higgs strahlung

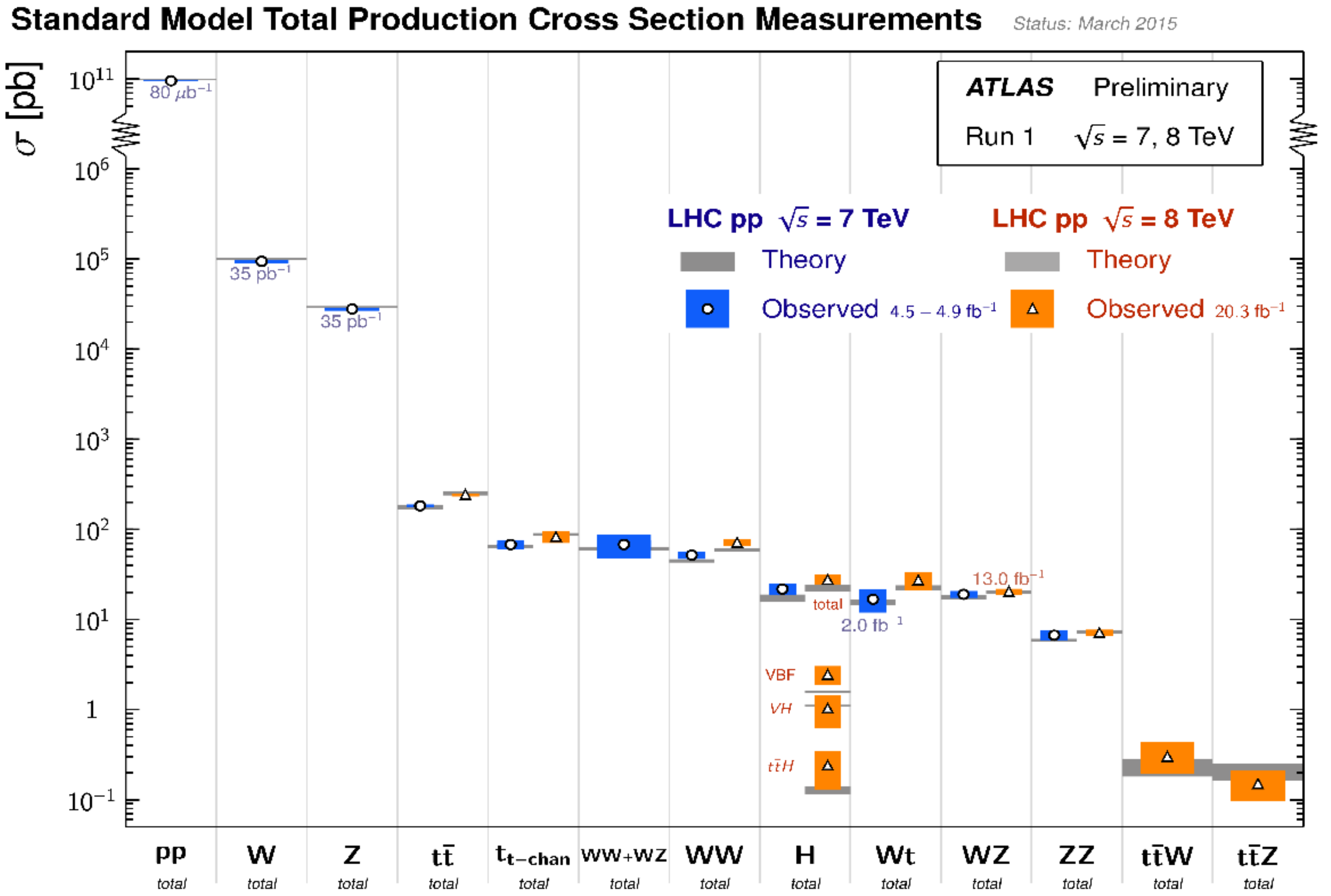


$t\bar{t}H$-channel

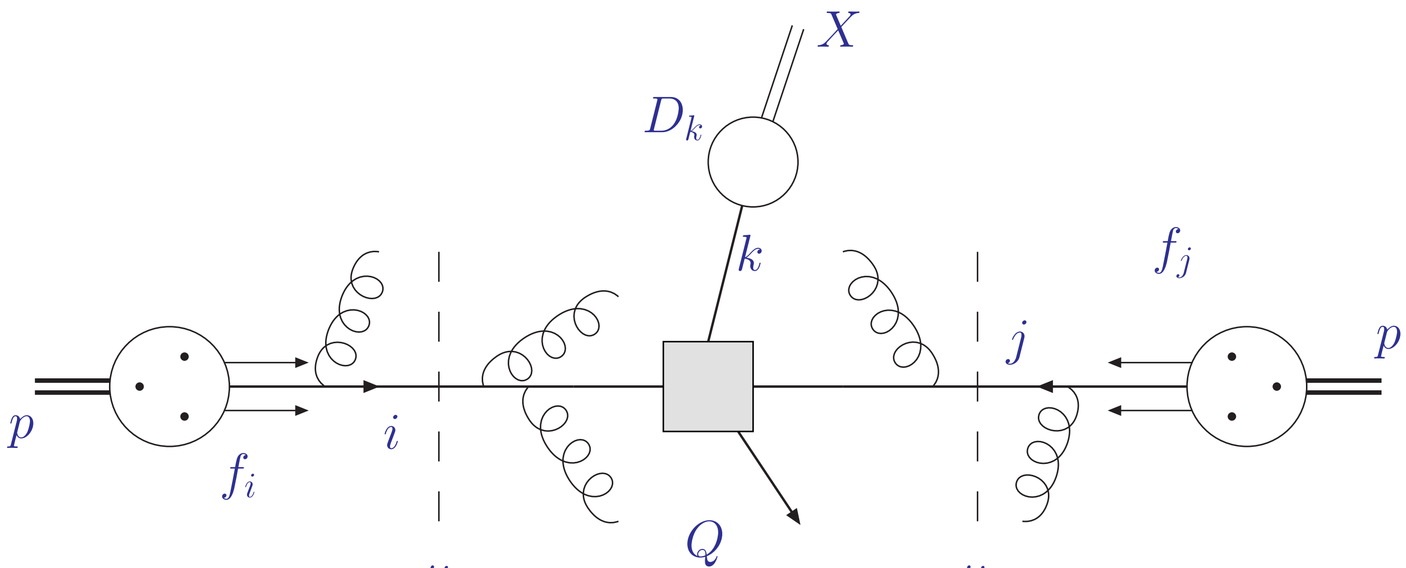
Standard Model cross sections

Cross sections for Standard Model processes at the LHC

- All dominant channels for Higgs boson production observed Atlas coll. '15



QCD factorization

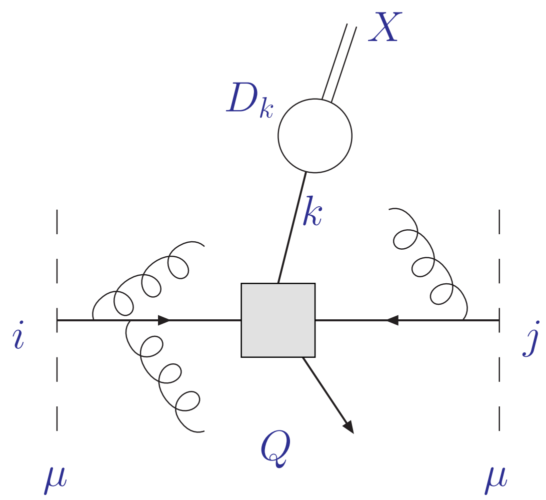


$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Factorization at scale μ
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - cross section $\hat{\sigma}_{ij\to k}$ for parton types i, j and hadronic final state X
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, . . .

Hard scattering cross section

- Parton cross section $\hat{\sigma}_{ij\to k}$ calculable pertubatively in powers of α_s
  - known to NLO, NNLO, . . . ($\mathcal{O}(\text{few}\%)$ theory uncertainty)



- Accuracy of perturbative predictions
  - LO (leading order) ($\mathcal{O}(50 - 100\%)$ unc.)
  - NLO (next-to-leading order) ($\mathcal{O}(10 - 30\%)$ unc.)
  - NNLO (next-to-next-to-leading order) ($\lesssim$ $\mathcal{O}(10\%)$ unc.)
  - N^3LO (next-to-next-to-next-to-leading order)
  - . . .

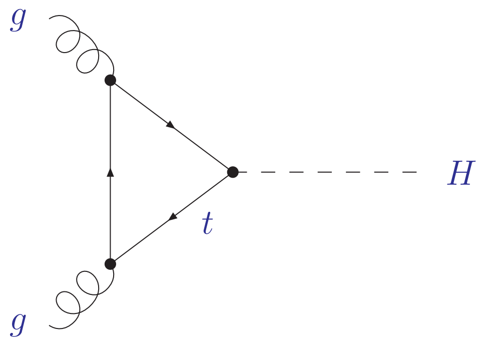
Perturbation theory at work

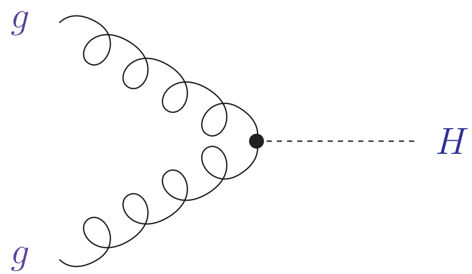
Particle physics

- Perturbative approach straightforward in principle
  - draw all Feynman diagrams and evaluate them,
  - use standard reduction techniques for tree/loop amplitudes
- (Extremely) hard in practice
  - intermediate expressions more complicated than final results
- Known bottlenecks
  - **many diagrams** — many diagrams are related by gauge invariance
  - **many terms in each diagram** — nonabelian gauge boson self-interactions are complicated
  - **many kinematic variables** — allowing the construction of very complicated expressions
- Computer algebra programs are a standard tool

Higgs production in gg-fusion

Effective theory





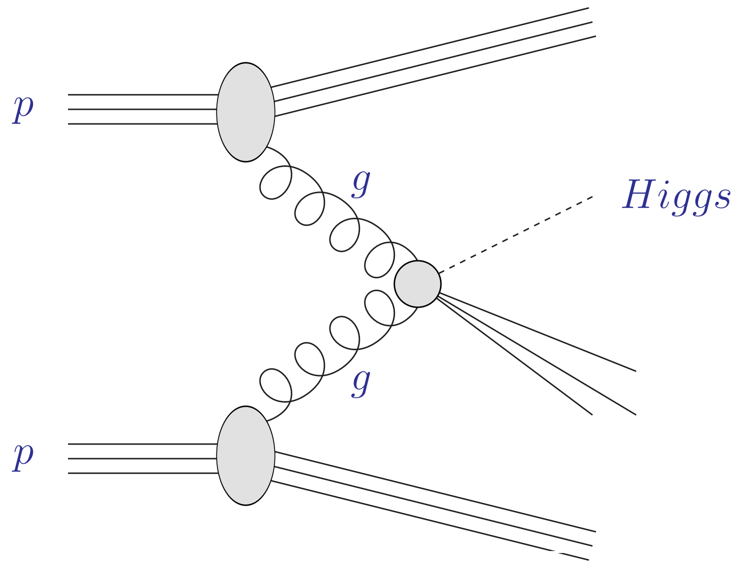
- Integration of top-quark loop (finite result)
  - decay width $H \to gg$ ($m_q = 0$ for light quarks, m_t heavy)

$$\Gamma_{H \to gg} = \frac{G_\mu \, m_H^3}{64 \sqrt{2} \, \pi^3} \, \alpha_s^2 \, f\left(\frac{m_H^2}{4m_t^2}\right)$$

- Effective theory in limit $m_t \to \infty$; Lagrangian $\mathcal{L} = -\frac{1}{4} \, \frac{H}{v} \, C_H \, G^{\mu\nu\, a} \, G^a_{\mu\nu}$
  - operator $HG^{\mu\nu\, a} \, G^a_{\mu\nu}$ relates to stress-energy tensor
  - additional renormalization proportional to QCD β-function required

    Kluberg-Stern, Zuber '75; Collins, Duncan, Joglekar '77

QCD corrections to ggF



- Hadronic cross section $\sigma_{pp\to H}$ with $\tau = m_H^2/S$
  - renormalization/factorization (hard) scale $\mu = \mathcal{O}(m_H)$

$$\sigma_{pp\to H} = \sum_{ij} \int_\tau^1 \frac{dx_1}{x_1} \int_{x_1}^1 \frac{dx_2}{x_2} f_i\left(\frac{x_1}{x_2}, \mu^2\right) f_j\left(x_2, \mu^2\right) \hat{\sigma}_{ij\to H}\left(\frac{\tau}{x_1}, \frac{\mu^2}{m_H^2}, \alpha_s(\mu^2)\right)$$

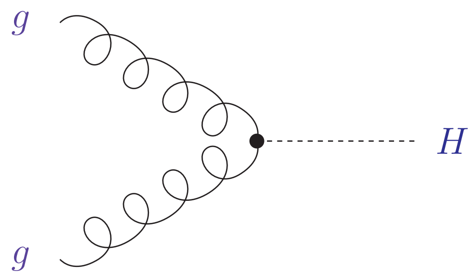
- Partonic cross section $\hat{\sigma}_{ij\to H}$

$$\hat{\sigma}_{ij\to H} = \underbrace{\alpha_s^2 \Big[\hat{\sigma}^{(0)}_{ij\to H} + \alpha_s\, \hat{\sigma}^{(1)}_{ij\to H}}_{} + \alpha_s^2\, \hat{\sigma}^{(2)}_{ij\to H} + \dots \Big]$$

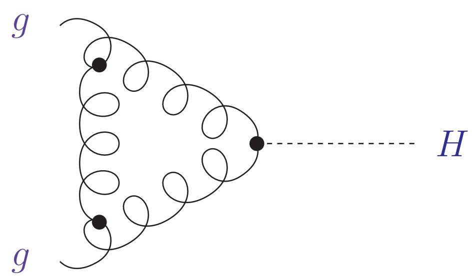
NLO: standard approximation (large uncertainties)

Radiative corrections in a nutshell

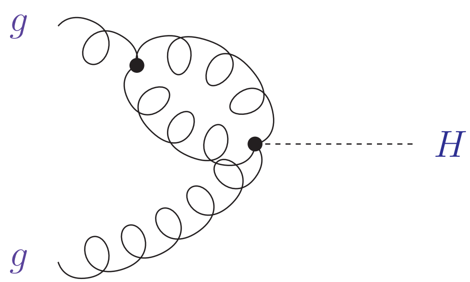
- Leading order
  - partonic cross section $x = \tau/x_1 = M_H^2/\hat{s}$
    $\hat{\sigma}^{(0)}_{gg\to H} = \delta(1-x)$



- Next-to-leading order (virtual corrections)
  kinematics proportional to Born
  - vertex correction

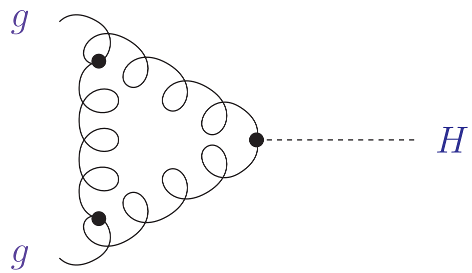


  - self-energy correction
    (massless tadpole)



NLO virtual corrections

- Contribution of vertex correction to ggF cross section
  - kinematics proportional to Born
  - gluon form factor in time-like kinematics
  - infrared divergent; $D = 4 - 2\epsilon$

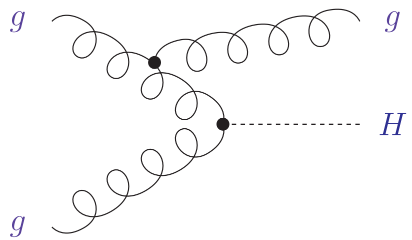


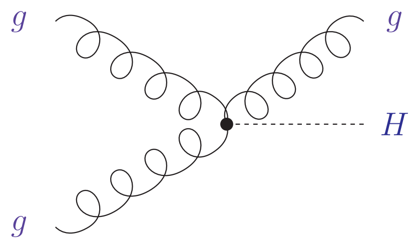
$$\hat{\sigma}^{(1),v}_{gg\to H} \quad = \quad C_A \frac{\alpha_s}{4\pi}\,\delta(1-x)\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon}\left(-\frac{2}{\epsilon^2}+\frac{7}{6}\,\pi^2+\mathcal{O}(\epsilon)\right)$$

- Effective theory requires UV renormalization
  - additional contribution from renormalization of effective operator

$$\alpha_s^{\rm bare} \quad = \quad \alpha_s^{\rm ren}\left\{1-\frac{\beta_0}{\epsilon}\frac{\alpha_s^{\rm ren}}{4\pi}+\mathcal{O}(\alpha_s^2)\right\}$$

- Next-to-leading order (real emission)



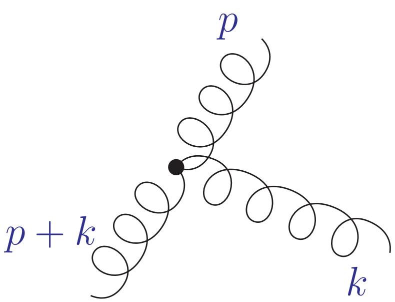


  - real corrections $\hat{\sigma}^{(1),r}_{gg\to H}$ require integration over phase space

$$\hat{\sigma}^{(1),r}_{gg\to H} \quad = \quad \int dLIPS^{(m)}\, |\mathcal{M}_{gg\to Hg}|^2$$

Soft and collinear singularities

- Soft/collinear regions of phase space
  - massless partons



$$\frac{1}{(p+k)^2} \quad = \quad \frac{1}{2p\cdot k} = \frac{1}{2E_{g_i}E_{g_o}(1-\cos\theta_{gg})}$$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \quad \longrightarrow \quad \alpha_s \int dE_{g_o}\, d\sin\theta_{gg} \frac{1}{2E_{g_i}E_{g_o}(1-\cos\theta_{gg})}$$

$$\longrightarrow \quad \alpha_s \frac{1}{\epsilon^2} \times (\ldots)$$ in dim. reg. $D = 4 - 2\epsilon$

- Complete NLO corrections
  - add real and virtual corrections $\hat{\sigma}^{(1)}_{gg\to H} = \hat{\sigma}^{(1),r}_{gg\to H} + \hat{\sigma}^{(1),v}_{gg\to H}$
  - collinear divergence remains: splitting functions $P^{(0)}_{gg}$

$$\hat{\sigma}^{(1)}_{gg\to H} = \frac{\alpha_s}{4\pi}\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon}\Bigg\{$$

$$\frac{1}{\epsilon}\,C_A\left(\frac{8}{1-x}+\frac{8}{x}-8(2-x+x^2)+\frac{22}{3}\delta(1-x)\right)-\frac{1}{\epsilon}\,n_f\,\frac{4}{3}\delta(1-x)$$

$$+\mathcal{O}(1)\Bigg\}$$

- Complete NLO corrections
  - add real and virtual corrections $\hat{\sigma}^{(1)}_{gg\to H} = \hat{\sigma}^{(1),r}_{gg\to H} + \hat{\sigma}^{(1),v}_{gg\to H}$
  - large threshold logarithms for $x = M_H^2/s \simeq 1$: resummation

$$\hat{\sigma}^{(1)}_{gg\to H} = \frac{\alpha_s}{4\pi}\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon}\Bigg\{$$

$$\frac{1}{\epsilon}C_A\left(\frac{8}{1-x}+\frac{8}{x}-8(2-x+x^2)+\frac{22}{3}\delta(1-x)\right)-\frac{1}{\epsilon}n_f\,\frac{4}{3}\delta(1-x)$$

$$+C_A\left(16\frac{\ln(1-x)}{1-x}+\left(\frac{22}{3}+\frac{4}{3}\pi^2\right)\delta(1-x)-16x(2-x+x^2)\ln(1-x)\right.$$

$$\left.-8\frac{(1-x+x^2)^2}{1-x}\ln(x)-\frac{22}{3}(1-x)^3\right)+\mathcal{O}(\epsilon)\Bigg\}$$

- Structure of NLO correction
  - absorb collinear divergence $P_{gg}^{(0)}$ in renormalized parton distributions

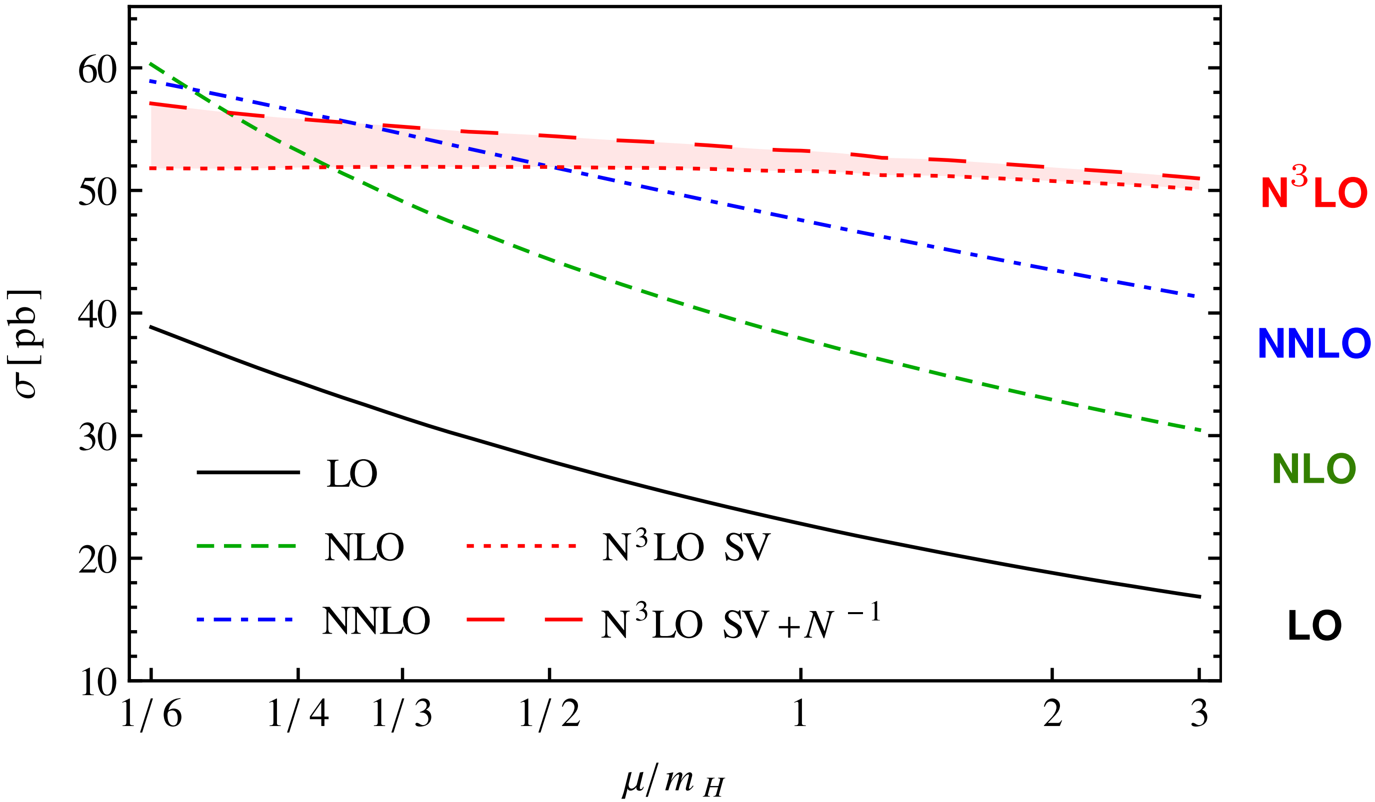
$$\hat{\sigma}_{gg\to H}^{(1),\mathrm{bare}} = \frac{\alpha_s}{4\pi}\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon}\left\{\frac{1}{\epsilon}\,2\,P_{gg}^{(0)}(x) + \hat{\sigma}_{gg\to H}^{(1)}(x) + \mathcal{O}(\epsilon)\right\}$$

$$g^{\mathrm{ren}}(\mu_F^2) = g^{\mathrm{bare}} - \frac{\alpha_s}{4\pi}\,\frac{1}{\epsilon}\,P_{gg}^{(0)}(x)\left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}$$

  - partonic (physical) structure function at factorization scale μ_F

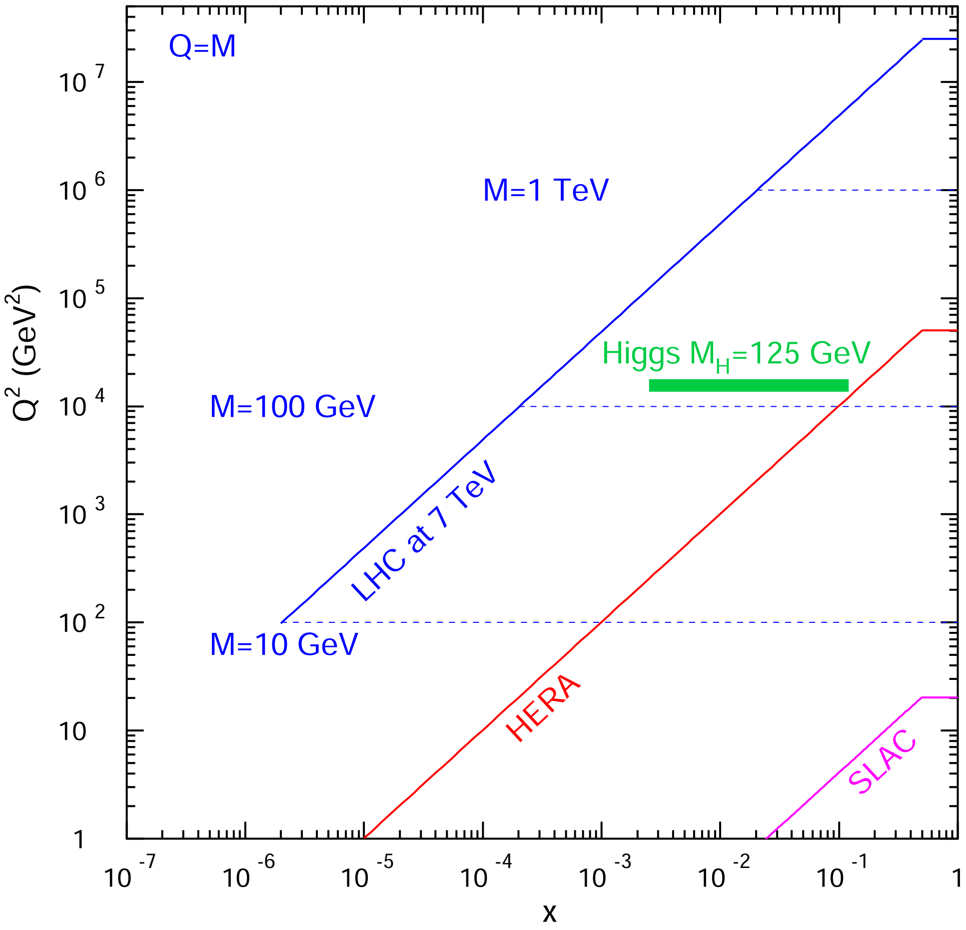
$$\hat{\sigma}_{gg\to H} = \delta(1-x) + \frac{\alpha_s}{4\pi}\left\{\hat{\sigma}_{gg\to H}^{(1)}(x) - \ln\left(\frac{m_H^2}{\mu_F^2}\right)\,2\,P_{gg}^{(0)}(x)\right\}$$

QCD precision predictions



- Simultaneous variation of scales $\mu_R = \mu_F$ around m_H at $\sqrt{s} = 14$ TeV
  - perturbative stability under renormalizaion scale variation
- Apparent convergence of perturbative expansion
  - NNLO corrections still large
    Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03
  - improvement through approximate N^3LO corrections
    S.M., Vogt '05; de Florian, Mazitelli, S.M., Vogt '14

Parton luminosity at LHC



- LHC run at $\sqrt{s}$ = 7/8 TeV
  - parton kinematics well covered by HERA and fixed target experiments
- Parton kinematics at effective $\langle x \rangle = M/\sqrt{S}$
  - 100 GeV physics: small-x, sea partons
  - TeV scales: large-x

Dependence on parton distributions

- Cross section $\sigma(H)$ at NNLO with uncertainties: $\sigma(H) + \Delta\sigma(\mathrm{PDF} + \alpha_{\mathrm{s}})$

ABM11	ABM12	CT10	MSTW	NN23
39.58 ± 0.77	39.70 ± 0.84	$41.84\ ^{+1.30}_{-1.69}$	$42.12\ ^{+0.44}_{-0.63}$	43.75 ± 0.41

- Comparison for PDF sets at NNLO
  - ABM11, ABM12 Alekhin, Blümlein, S.M. '13, CT10 Gao et al. '13, MSTW Martin, Stirling, Thorne, Watt '09, NNPDF (NN23) Ball et al. '12
- Large spread for predictions from different PDFs $\sigma(H) = 39.6 \ldots 43.8$
- PDF and α_s differences between sets amount to up to 10%
  - significantly larger than residual theory uncertainty due to incomplete N^3LO QCD corrections
- Observed spread due to differences in theory considerations and analysis procedures $\longrightarrow$ correlations between α_s, $g(x)$ and m_q
  - target mass corrections and higher twist in DIS
  - treatment of heavy quarks
  - error correlations among data sets
  - fits to compatible data sets

Summary (part III)

Standard Model

- Successful experimental program at LHC relies crucially on detailed understanding of Standard Model processes
- QCD at work
  - illustration of factorization, infrared safety and evolution for $gg \to H$
  - resummation of large logarithms near threshold

Higgs measurements

- Precision predictions for Higgs production at LHC available
  - radiative corrections (higher orders) important
  - essential to control theory uncertainties