

## Exercise 3 - Introduction

- Schedule:
  - Tuesday - Exercise 1:
    - Random numbers
    - MC method
    - MC integration
  - Wednesday - Exercise 2:
    - Sudakov form factor
    - MC solution of evolution equation
  - Thursday - Exercise 3
    - Calculation & simulation of Higgs production
    - Using MC solution of evolution equation → calculation of pt spectrum of Higgs at LHC

# Original References...

PHYSICS REPORTS (Review Section of Physics Letters) 58, No. 5 (1980) 269–395. North-Holland Publishing Company

## HARD PROCESSES IN QUANTUM CHROMODYNAMICS

Yu.L. DOKSHITZER, D.I. DYAKONOV and S.I. TROYAN

*Leningrad Nuclear Physics Institute, Gatchina,  
Leningrad 188350, U.S.S.R.*

Received 28 May 1979

Nuclear Physics B154 (1979) 427–440  
© North-Holland Publishing Company

## SMALL TRANSVERSE MOMENTUM DISTRIBUTIONS IN HARD PROCESSES

G. PARISI

*INFN, Laboratori Nazionali di Frascati, Italy*

R. PETRONZIO\*

*CERN, Geneva, Switzerland*

Received 8 February 1979

# All order resummation - kinematics

Ellis, Fleishon, Stirling, PRD 24, 1386 (1981)

- impose kinematic constraints ... delta function for  $k_t$

$$\frac{1}{\sigma} \frac{d\sigma^{(N)}}{dp_t^2} \sim \prod_{i=1}^N \left[ \int d^2 k_{ti} dx_i M^{(N)} \right] \delta \left( \sum \vec{k}_{ti} + \vec{p}_t \right)$$

- with for soft gluons  $M^{(N)} \sim \prod \frac{\alpha_s}{k_{ti}^2}$

→ in limit of strong ordering:

$$\frac{1}{\sigma} \frac{d\sigma^{(N)}}{dp_t^2} \sim \prod_{i=1}^N \left[ \int \frac{d^2 k_{ti}}{k_{ti}^2} \log \frac{s}{k_{ti}^2} \right] \delta \left( \sum \vec{k}_{ti} + \vec{p}_t \right)$$

$$\frac{1}{\sigma} \frac{d\sigma^{(N)}}{dp_t^2} \sim p_t^2 \log \frac{s}{p_t^2} \int \frac{d^2 k_{t,N-1}}{k_{t,N-1}^2} \log \frac{s}{k_{t,N-1}^2} \int \frac{d^2 k_{t,N-2}}{k_{t,N-2}^2} \log \frac{s}{k_{t,N-2}^2} \dots$$

# All order resummation

- Iterative procedure:

$$\frac{d\sigma^{(1)}}{dp_t^2} \simeq \frac{1}{p_t^2} \log \frac{s}{p_t^2} = A$$

$$\frac{d\sigma^{(2)}}{dp_t^2} \simeq A \int \frac{d^2 k_t}{k_t^2} \log \frac{s}{k_t^2} = A \frac{1}{2} \log^2 \frac{s}{k_t^2}$$

$$\frac{d\sigma^{(3)}}{dp_t^2} \simeq \int \frac{d^2 k_t}{k_t^2} \log \frac{s}{k_t^2} \frac{d\sigma^{(2)}}{dp_t^2} = A \frac{1}{2} \int \frac{d^2 k_t}{k_t^2} \log^3 \frac{s}{k_t^2} = A \frac{1}{2} \left( \frac{1}{2} \log^2 \frac{s}{k_t^2} \right)^2$$

$$\frac{d\sigma^{(4)}}{dp_t^2} \simeq \int \frac{d^2 k_t}{k_t^2} \log \frac{s}{k_t^2} \frac{d\sigma^{(3)}}{dp_t^2} = A \frac{1}{2} \frac{1}{4} \int \frac{d^2 k_t}{k_t^2} \log^5 \frac{s}{k_t^2} = A \frac{1}{2} \frac{1}{3} \left( \frac{1}{2} \log^2 \frac{s}{k_t^2} \right)^3$$

$$\frac{d\sigma^{(N)}}{dp_t^2} \simeq A \frac{1}{(N-1)!} \left( \frac{1}{2} \log^2 \frac{s}{k_t^2} \right)^{N-1}$$

- $\chi$ -section for up to  $N$  gluons:

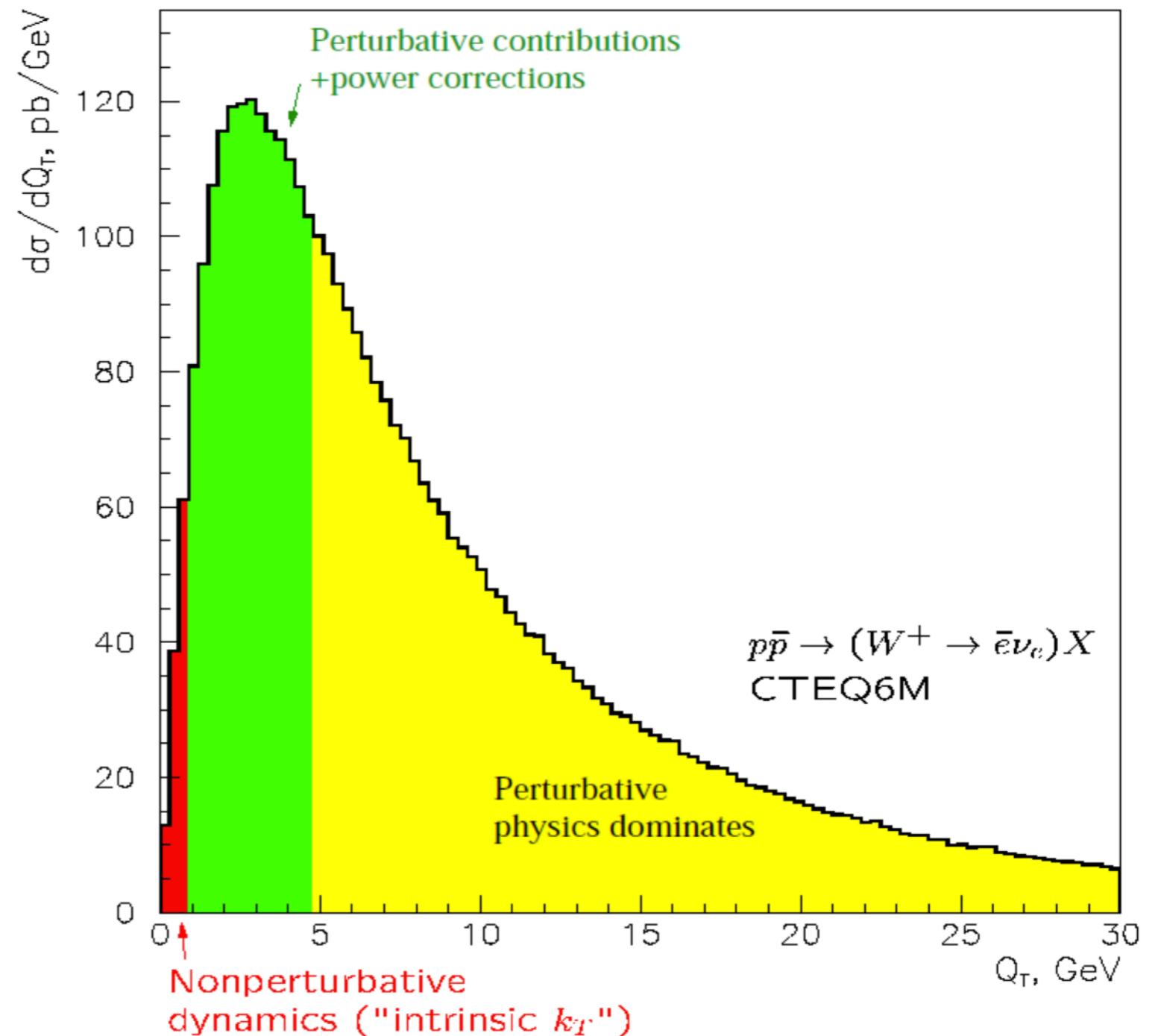
$$\frac{d\sigma}{dp_t^2} = \sum_i \frac{d\sigma^{(i)}}{dp_t^2} \simeq A \sum_i \frac{1}{(i-1)!} \left( \frac{1}{2} \log^2 \frac{s}{p_t^2} \right)^{i-1}$$

$$\simeq A \exp \left[ \frac{1}{2} \log^2 \frac{s}{p_t^2} \right]$$

Fred Olness, CTEQ summerschool  
2003

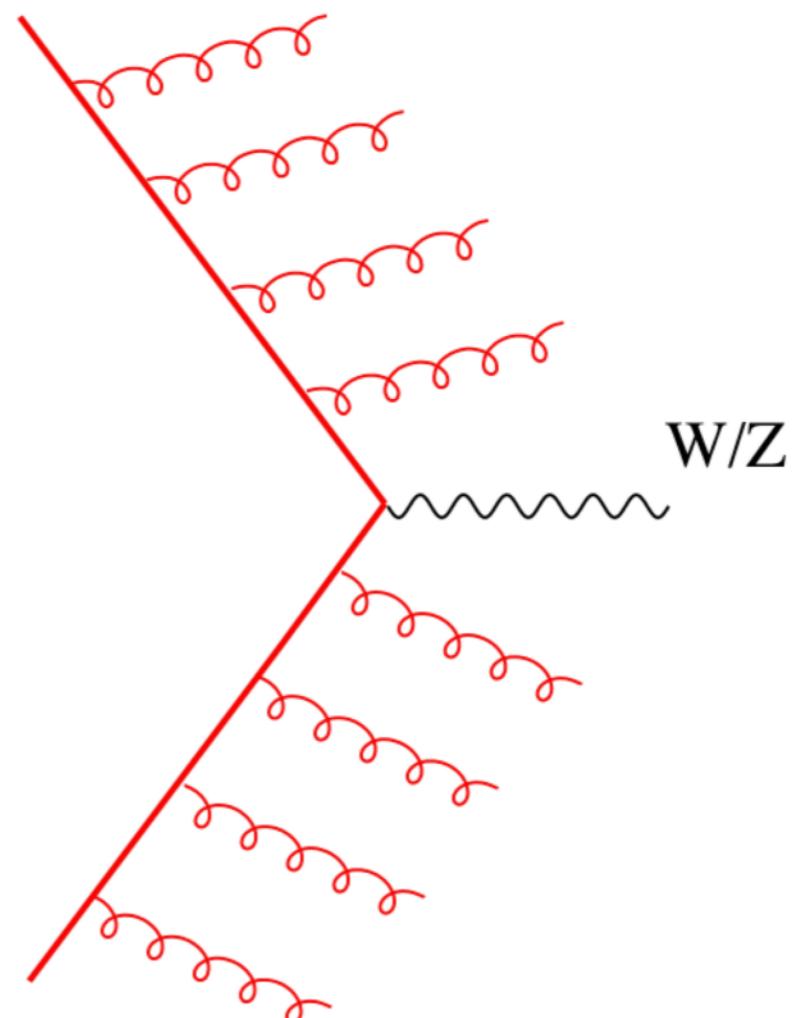
## The complete $P_T$ spectrum for the W boson

The full  $P_T$  spectrum  
for the W-boson  
showing the different  
theoretical regions



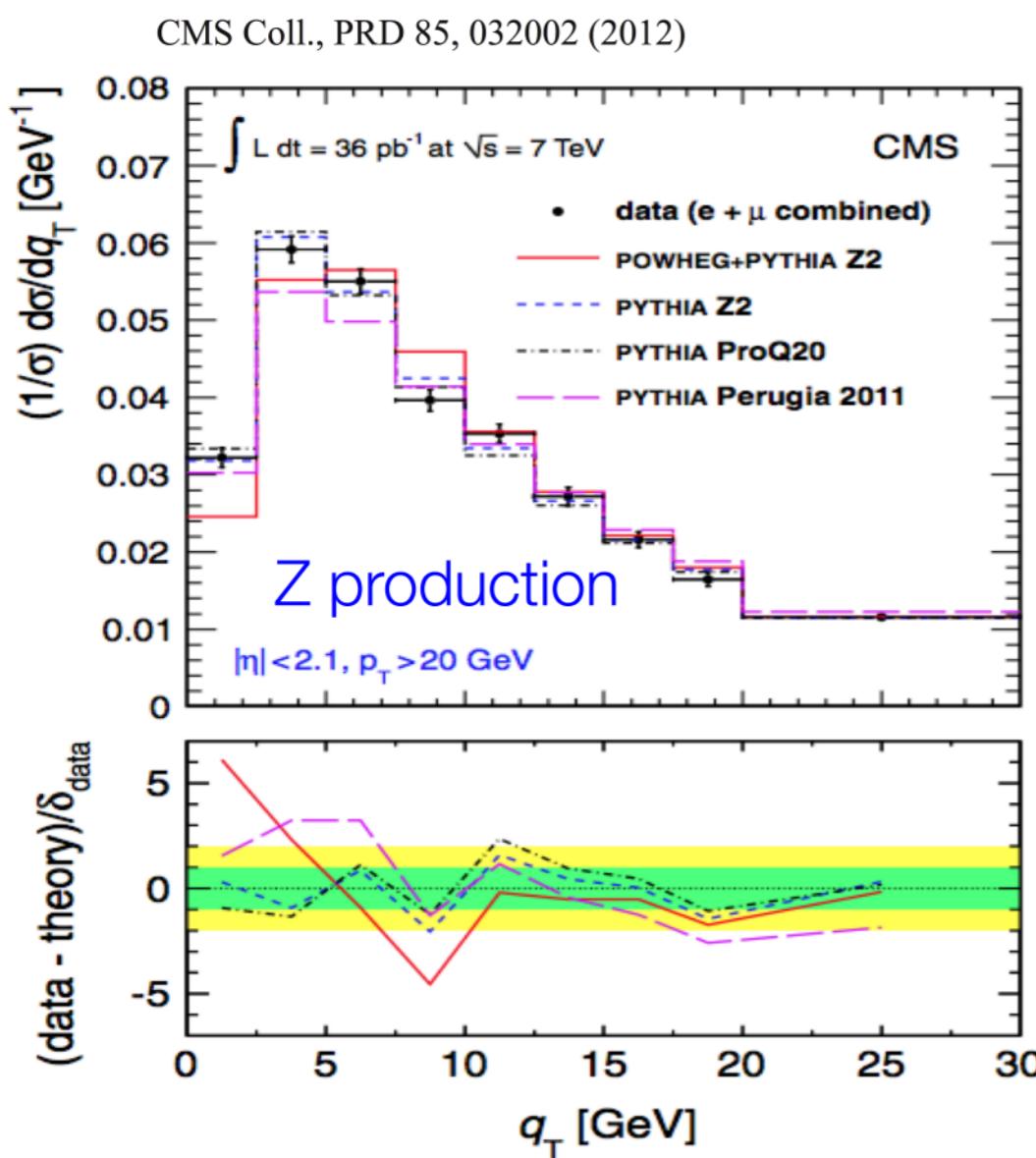
# Monte Carlo approach

- simulate explicitly parton radiation with evolution of parton densities
- advantage to include properly energy momentum conservation in each step
- perform resummation numerically
- will be done in exercise !!!

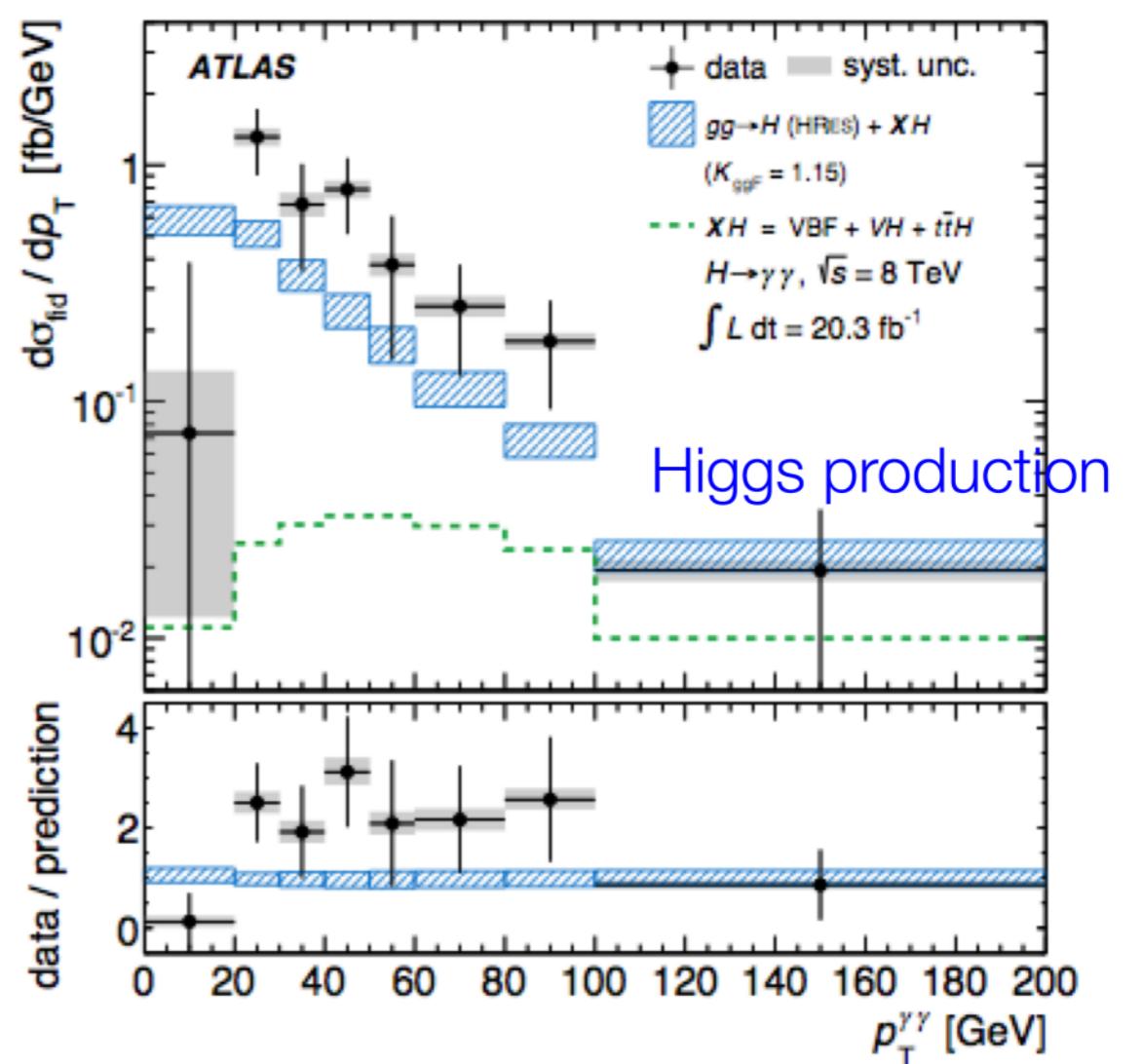


# Transverse Momentum of Z and Higgs

CMS Coll. Measurement of the Rapidity and Transverse Momentum Distributions of Z Bosons in pp Collisions at  $\sqrt{s} = 7$  TeV.  
 Phys.Rev., D85:032002, 2012.



Differential cross sections of the higgs boson measured in the diphoton decay channel using 8 TeV pp collisions. ATLAS arXiv 1407.4222



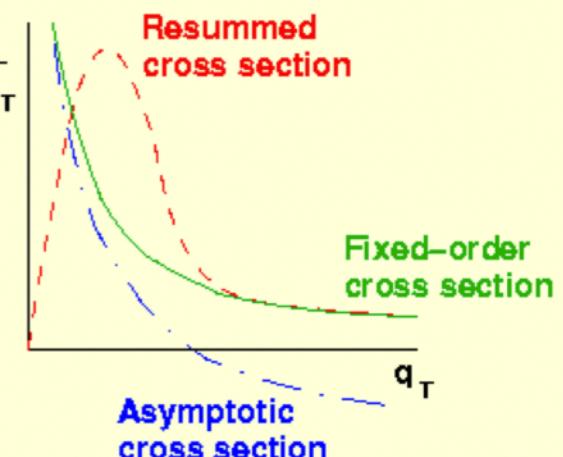
# $Q_t$ - Resummation



On this website, you can plot transverse momentum distributions for cross sections of several particle reactions. Currently, the following processes are implemented ( $p$  corresponds both to protons and antiprotons):

- Massive vector boson production:  $pp \rightarrow W^\pm X$ ,  $pp \rightarrow Z^0 X$
- Photon pair production:  $pp \rightarrow \gamma\gamma X$
- $Z$ -boson pair production:  $pp \rightarrow Z^0 Z^0 X$
- SM Higgs boson production  $pp \rightarrow H^0 X$

The output figure shows distributions  $d\sigma/dQ^2 dy dq_T$  for the production of *on-shell* particles (or pairs of *on-shell* particles in the case of the  $\gamma\gamma$  and  $ZZ$  production) with specified invariant mass  $Q$ , rapidity  $y$  and transverse momentum  $q_T$  in the lab frame (the center-of-mass frame of the hadron beams). You can plot resummed, fixed-order and asymptotic cross sections. For a short explanation of these quantities, visit [this page](#) (for a detailed explanation see, for instance, a paper by J.C. Collins, D.E. Soper and G. Sterman in *Nucl. Phys. B250, 199 (1985)*).

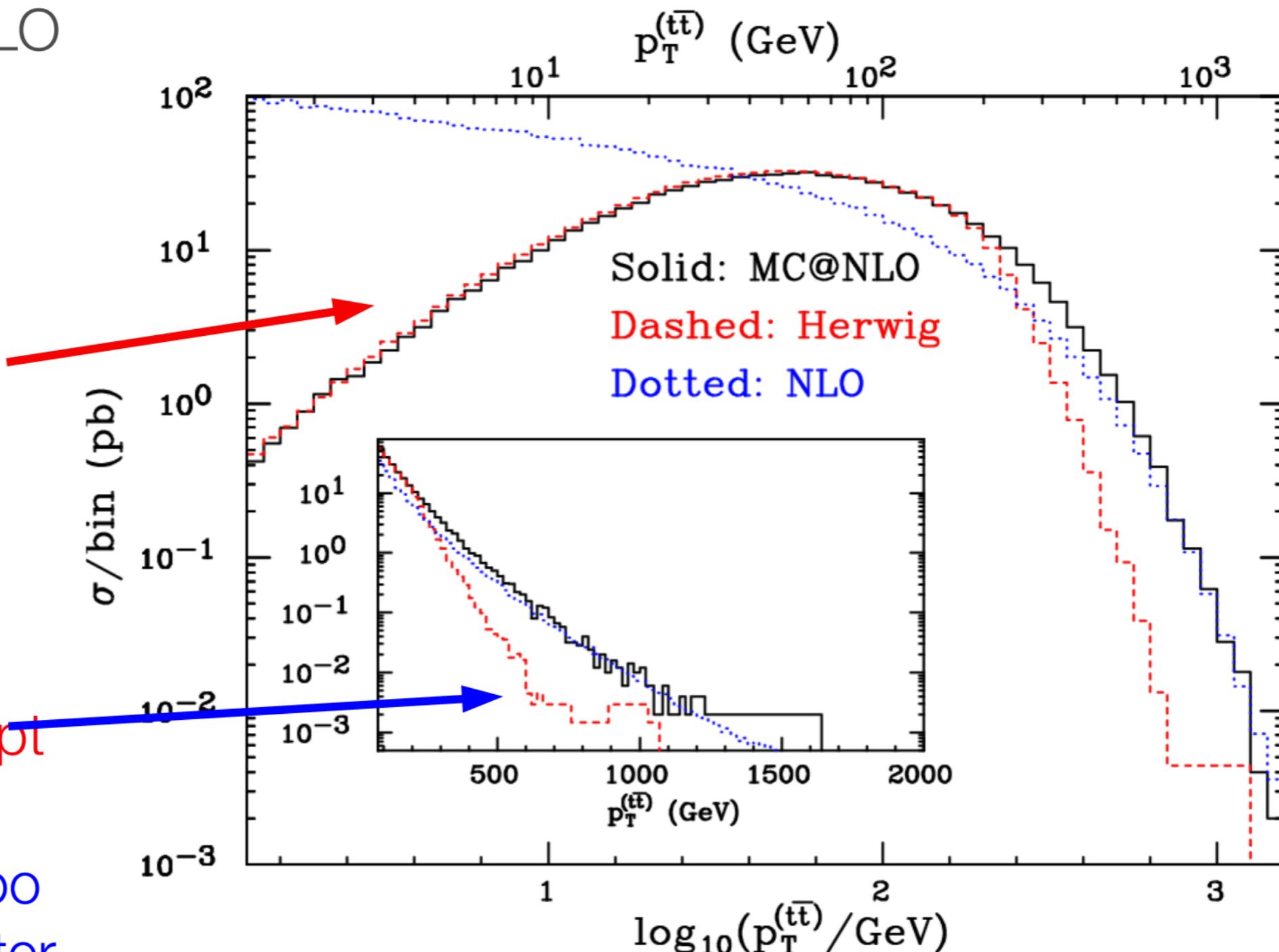


<http://hep.pa.msu.edu/wwwlegacy/>

# Small pt in heavy quark prod.

Frixione et al, hep-ph/035252

- Compare fixed NLO calculation of top production with resummed calculation from Monte Carlo
- Similar effects at small pt are observed:  
Suppression of xsection at small pt
- At large pt, resummation is too small, NLO is better



# Exercise 3

## Exercises 3 (16. April 2015)

8. Calculate  $\sigma(p + p \rightarrow h)$  (Higgs production via gluon fusion) in lowest order. Take  $\sqrt{s} = 7000$  GeV. Calculate the total cross section, and plot  $x_1$ ,  $x_2$  and  $y_h$ . Require  $120 < m_h < 130$  GeV. Plot the transverse momenta of the incoming partons. Use for simplicity parton density of the form  $xg(x) = 3(1 - x)^5$ .  
 The Higgs cross section is:

$$\sigma(g + g \rightarrow h) = \alpha_s^2 \frac{\sqrt{2}}{\pi} \frac{G_F}{576}$$

with  $G_F = 1.166 \cdot 10^{-5}$  GeV $^{-1}$  and  $\alpha_s = 0.1$ .

Use a Breit-Wigner form for the Higgs:

$$P(m) = \frac{1}{2\pi} \frac{\Gamma_h}{(m - m_h)^2 + \Gamma_h^2/4}$$

with  $m_h = 125$  GeV and  $\Gamma_h = 0.4$  GeV. Calculate the cross section.

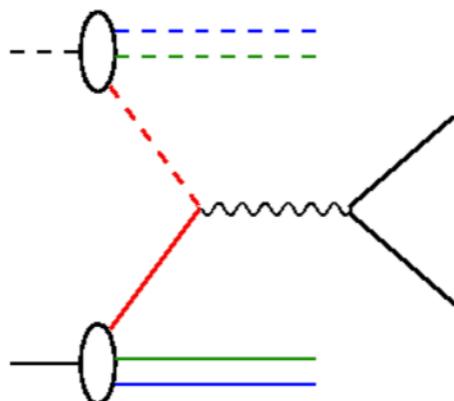
Include in the calculation a small intrinsic transverse momentum from both of the incoming partons. Assume  $h(k_t) = \exp(-bk_t^2)$ . Using  $b = 1$  corresponds to a gauss distribution with  $\mu = 0$  and  $\sigma \sim 0.7$ . Plot the transverse momentum  $k_t$  and the transverse momentum squared  $k_t^2$  of both incoming partons and the resulting  $h$ . Write the code in a modular way, such that it can be used for the last exercise.

9. Use the evolved pdf (from previous exercise) to calculate higgs production from above. Set the scale  $t = 10000$  GeV $^2$ . Use for simplicity the a gluon density  $xg(x) = 3(1 - x)^5$  as a starting distribution and use  $P_{gg}$ . Calculate the transverse momentum of the incoming partons and calculate the transverse momentum of the Higgs. Plot the  $x$ -values of the incoming partons and the transverse momenta.

# Rapidities and all that ...

- define rapidity:

$$y = \frac{1}{2} \log \frac{E + p_z}{E - p_z}$$



$$y(l^+ l^-) \text{ pair} = y(q\bar{q}) \text{ pair}$$

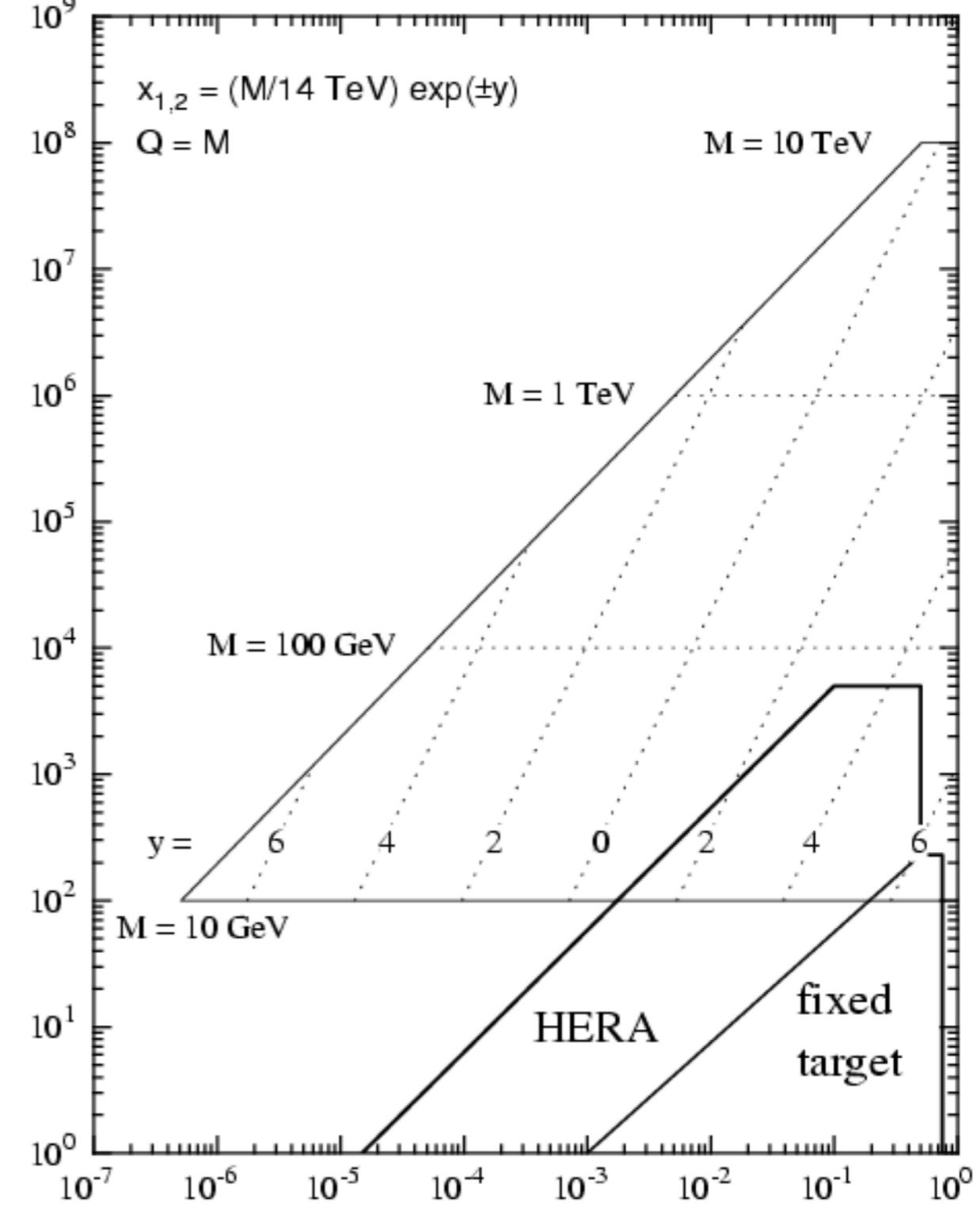
- gives:  $y = \frac{1}{2} \log \frac{x_1}{x_2}$

$$\bullet \text{ using: } \tau = \frac{M^2}{s} = x_1 x_2$$

- gives:

$$x_{1,2} = \sqrt{\tau} \exp(\pm y) = \frac{M}{\sqrt{s}} \exp(\pm y)$$

Hard interactions of quarks and gluons: a primer for LHC physics J M Campbell et al 2007 Rep. Prog. Phys. 70 89-193



# How to get started

- utilities:

`courselib.h`: include headers

`ranlxd.h, ranlxd.cc`: random number generator `ranlux`

- initialize ROOT (needed for plotting)

`module avail`

`module load root/5.34`

- copy all the templates (be careful, do not to overwrite ... )

`cp -rp /afs/desy.de/user/s/school30/public/Exercises .`

- compiling and running:

`cd exercise-1`

`make -f makefile-example-1`

`./example-1`

- templates are provided which include the general structure – you only have to fill the interesting – important parts ... good luck

## Computing setup

- Connect either to eduroam or to the school network:  
Name: terascale  
WPA/WPA2-PSK: XxPWjNH7
- All will get school accounts for naf:
  - for example: ssh -X school30@naf-school01.desy.de
  - create folder:  
`cd public`
  - copy all templates:  
`cp -rp /afs/desy.de/user/s/school30/public/Exercises .`
- Writeup, Exercise sheets, templates and solutions at:  
<http://www.desy.de/~jung/mcschool2015/>