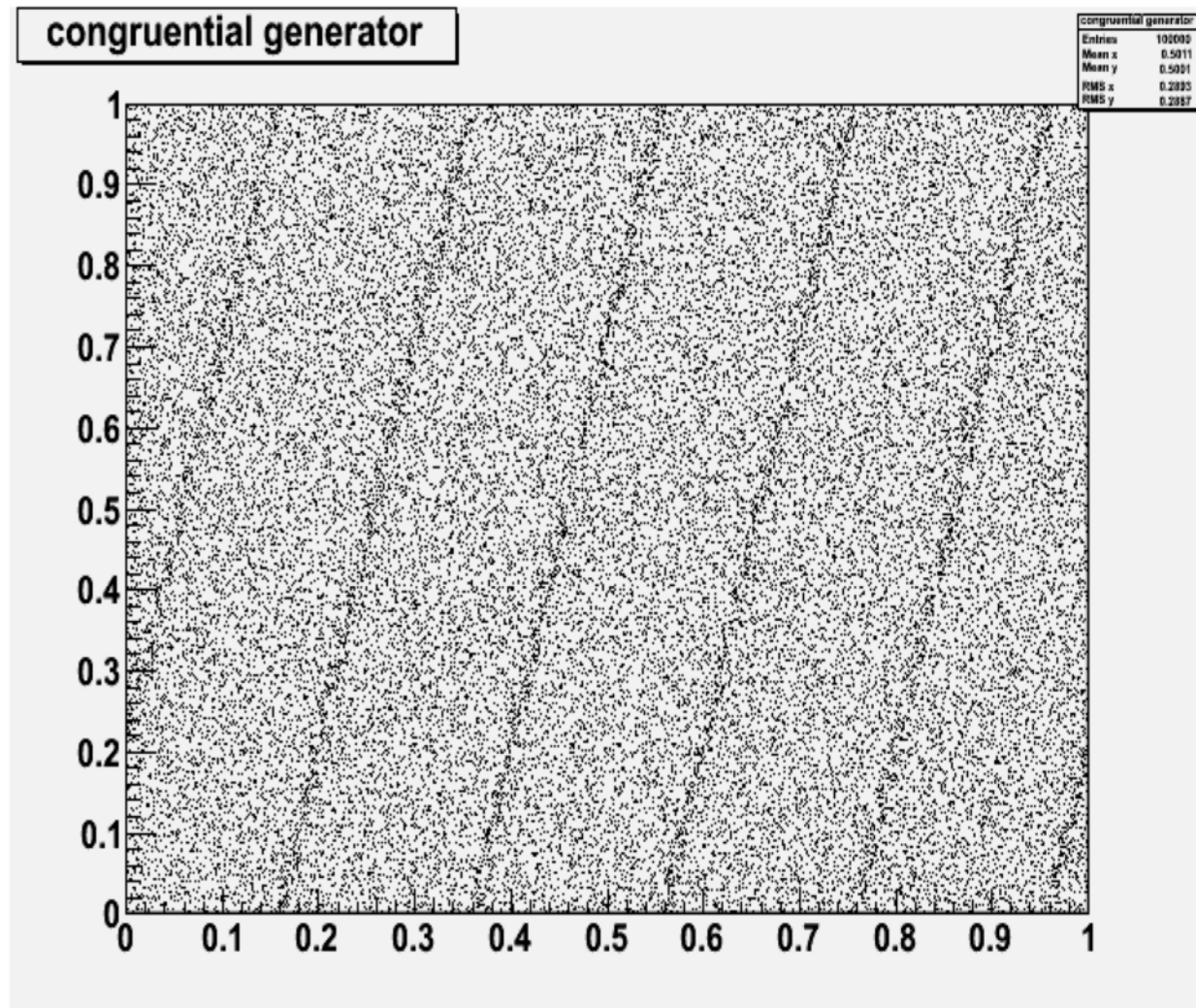


Wrapping-up exercises

- Schedule:
 - Tuesday - Exercise 1:
 - Random numbers
 - MC method
 - MC integration
 - Wednesday - Exercise 2:
 - Sudakov form factor
 - MC solution of evolution equation
 - Thursday - Exercise 3
 - Calculation & simulation of Higgs production
 - Using MC solution of evolution equation → calculation of p_t spectrum of Higgs at LHC

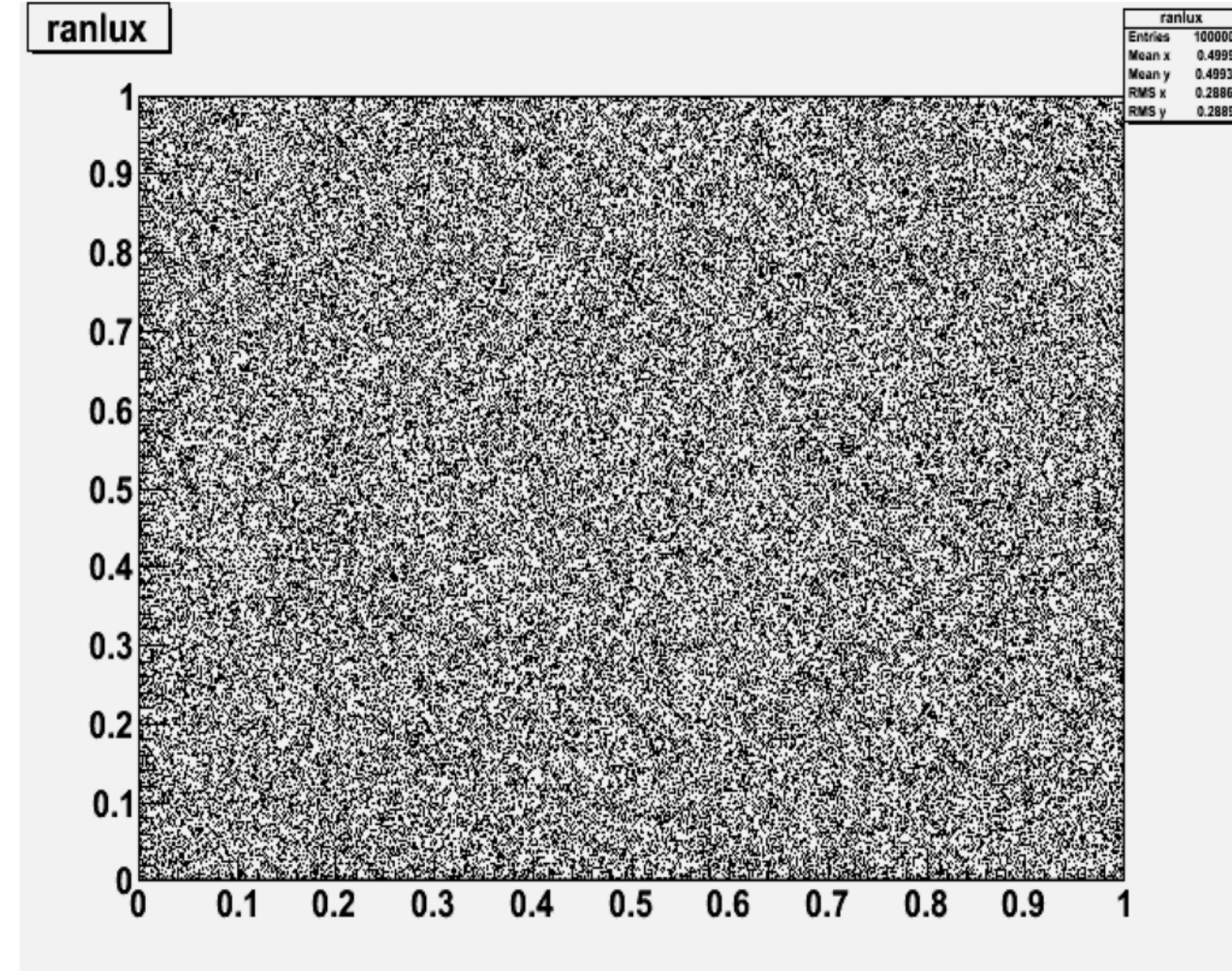
Randomness tests

- Congruential generator



- RANLUX

M. Lüscher, A portable high-quality random number generator for lattice field theory simulations, Computer Physics Communications 79 (1994) 100
<http://luscher.web.cern.ch/luscher/ranlux/index.html>

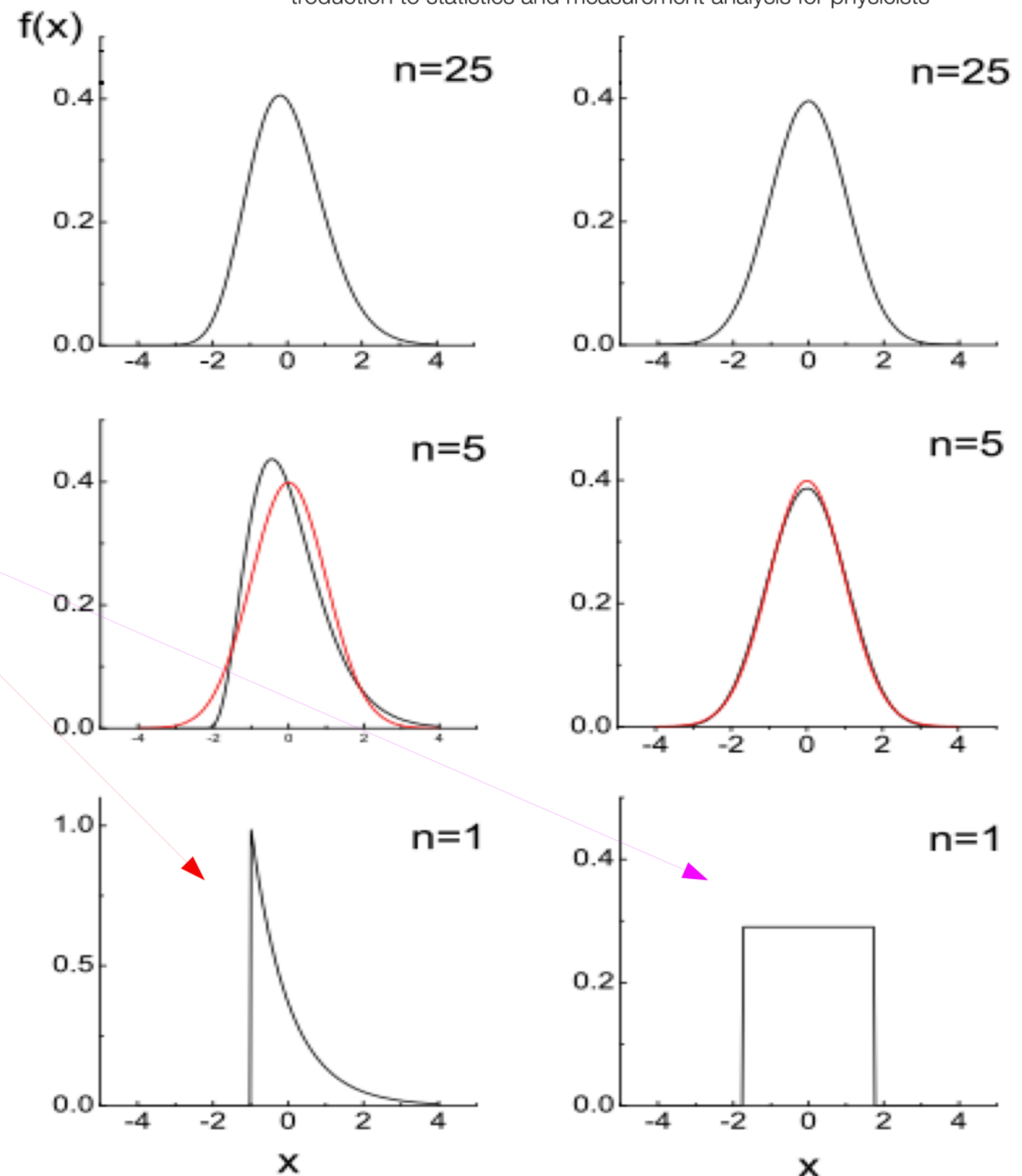


➔ RANLUX much more sophisticated. Developed and used for QCD lattice calcs

Central Limit Theorem

G. Böhm, G. Zechin
Introduction to statistics and measurement analysis for physicists

- **Central Limit Theorem**
for large N the sum of independent random variables is **always** normally (**Gaussian**) distributed
 - for any starting distribution
 - for uniform distribution
 - for exponential distribution



Central Limit Theorem

http://onlinestatbook.com/stat_sim/sampling_dist/index.html

[RVLS home](#) > [Simulations and Demonstrations](#) > **Sampling Distributions**

Thanks to Juan,
 who found this
 page

Sampling Distribution

[Begin](#)

[Instructions](#)

[Exercises](#)

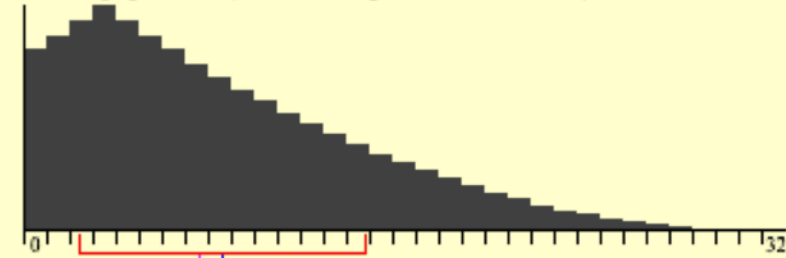
This is a new version written in Javascript to avoid the security problems with Java. There are still a few bugs to work out. For example, kurtosis does not appear to be calculated correctly.

[Original Java version](#)

mean= 8.08
 median= 7.00
 sd= 6.22
 skew= 0.83
 kurtosis= 0.06

Reps= 10000
 mean= 8.06
 median= 8.00
 sd= 2.76
 skew= 0.38
 kurtosis= 0.02

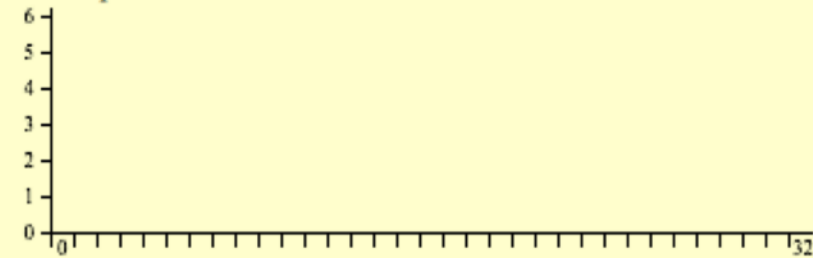
Parent population (can be changed with the mouse)



[Clear lower 3](#)

Skewed [↕](#)

Sample Data



Sample:

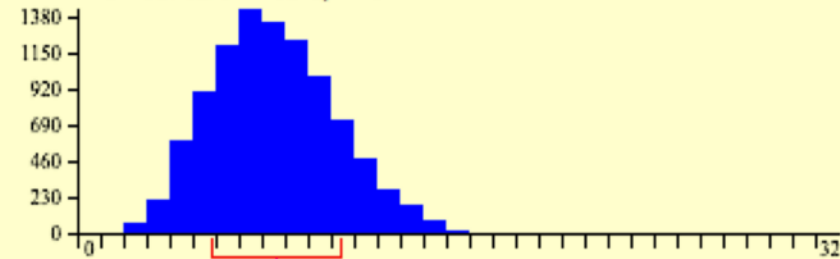
[Animated](#)

[5](#)

[10,000](#)

[100,000](#)

Distribution of Means, N=5



Mean [↕](#)

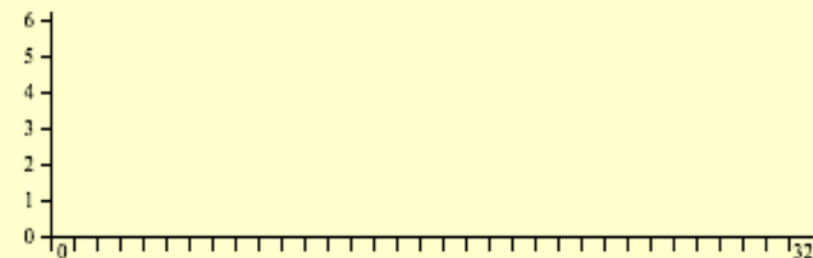
N=5 [↕](#)

☐ Fit normal

None [↕](#)

N=5 [↕](#)

☐ Fit normal



Importance Sampling

- MC calculations most efficient for small weight fluctuations:

$$f(x)dx \rightarrow f(x) dG(x)/g(x)$$

- chose point according to $g(x)$ instead of uniformly
- f is divided by $g(x) = dG(x)/dx$
- **generate x according to:**

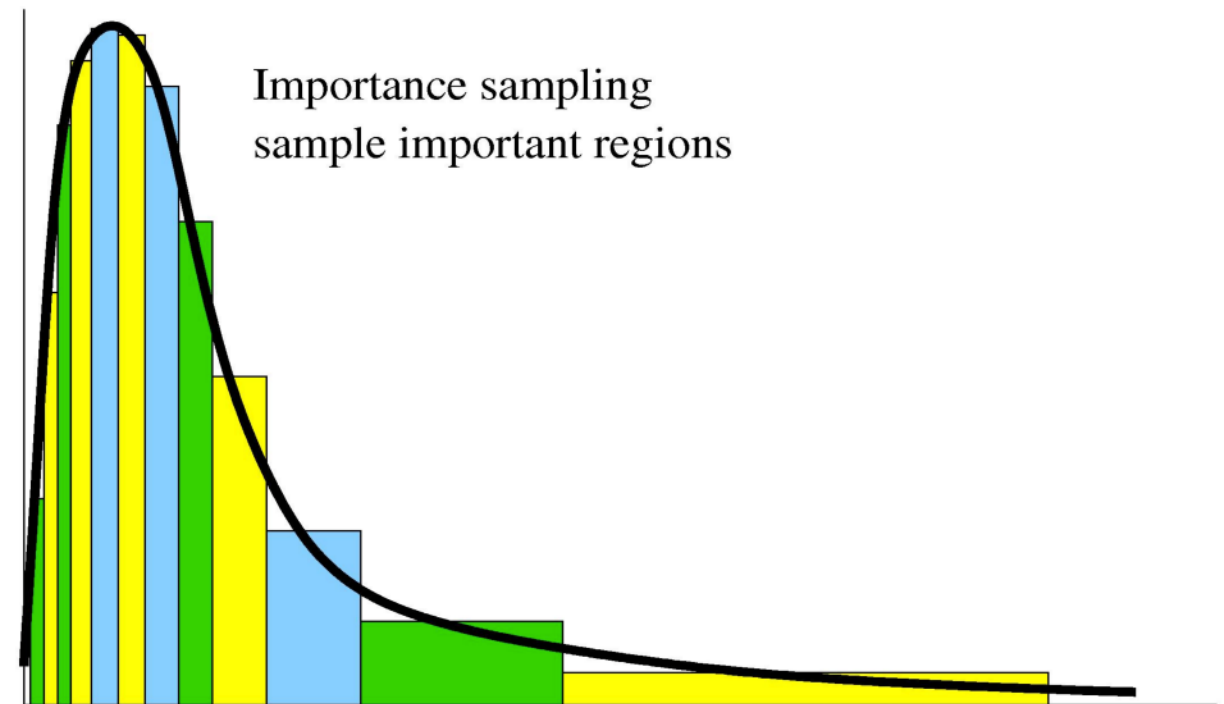
$$R \int_a^b g(x') dx' = \int_a^x g(x') dx'$$

- **relevant variance is now $V(f/g)$:**

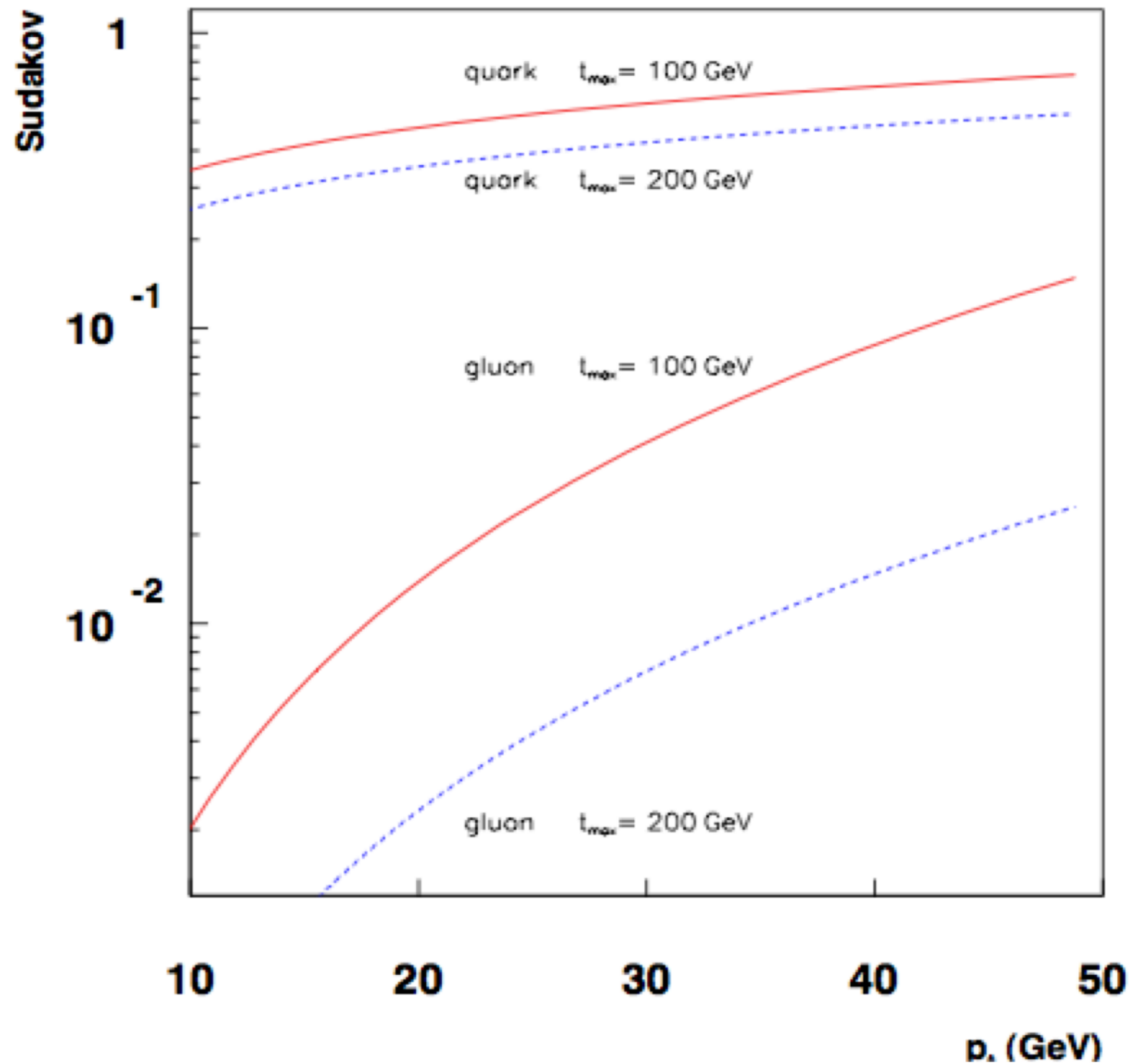
small if $g(x) \sim f(x)$

- **how-to get $g(x)$**

- (1) $g(x)$ is probability: $g(x) > 0$ and $\int dG(x) = 1$
- (2) integral $\int dG(x)$ is known analytically
- (3) $G(x)$ can be inverted (solved for x)
- (4) $f(x)/g(x)$ is nearly constant, so that $V(f/g)$ is small compared to $V(f)$



Sudakov form factors



Evolution equation and MC

$$f(x, t) = f(x, t_0) \Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via iteration:

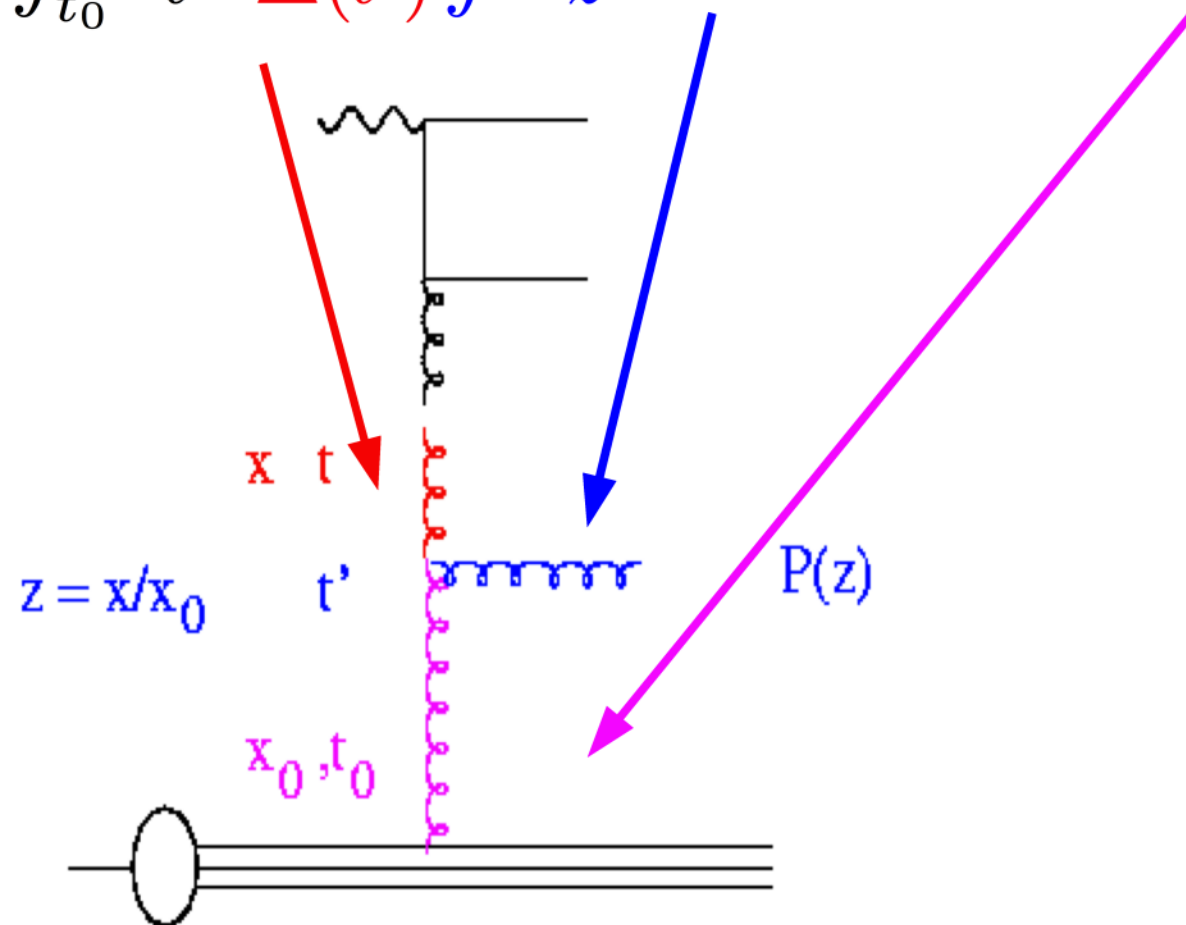
$$f_0(x, t) = f(x, t_0) \Delta(t)$$

from t' to t
w/o branching

branching at t'

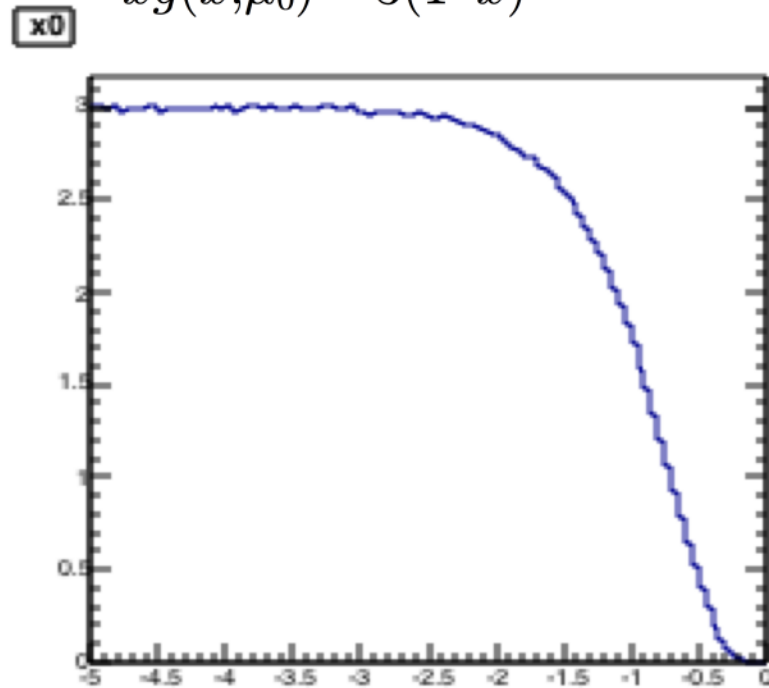
from t_0 to t'
w/o branching

$$f_1(x, t) = f(x, t_0) \Delta(t) + \int_{t_0}^t \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z, t_0) \Delta(t')$$

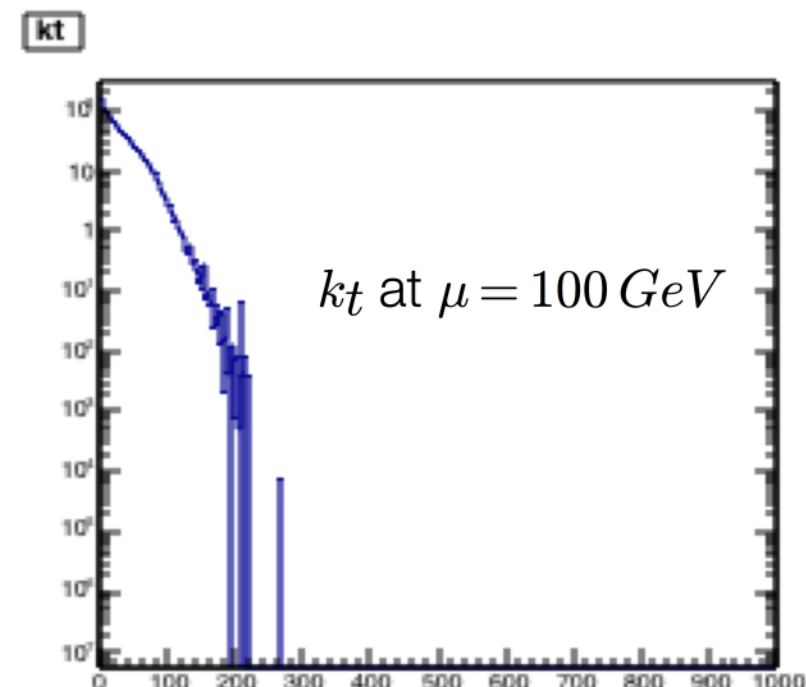
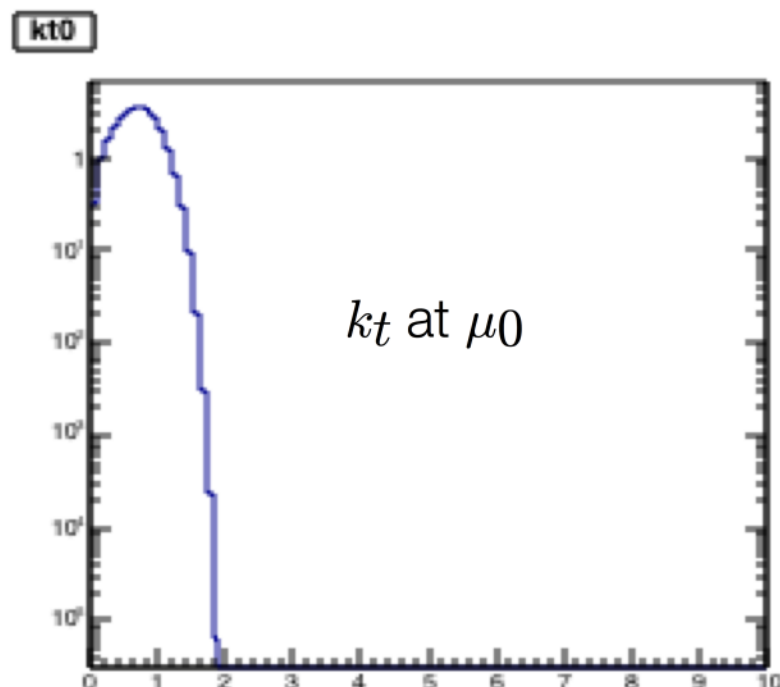
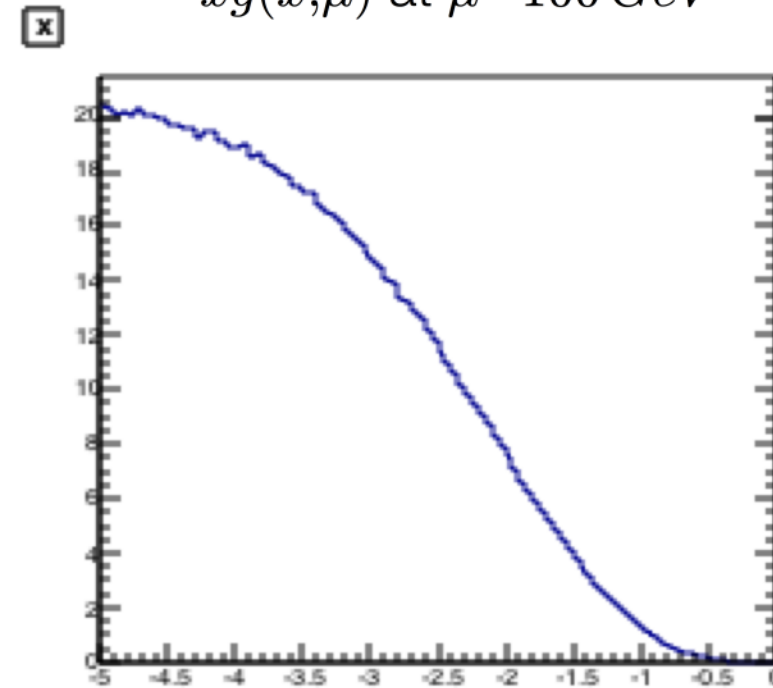


Evolution of gluon density

Starting distribution:
 $xg(x, \mu_0) = 3(1-x)^5$

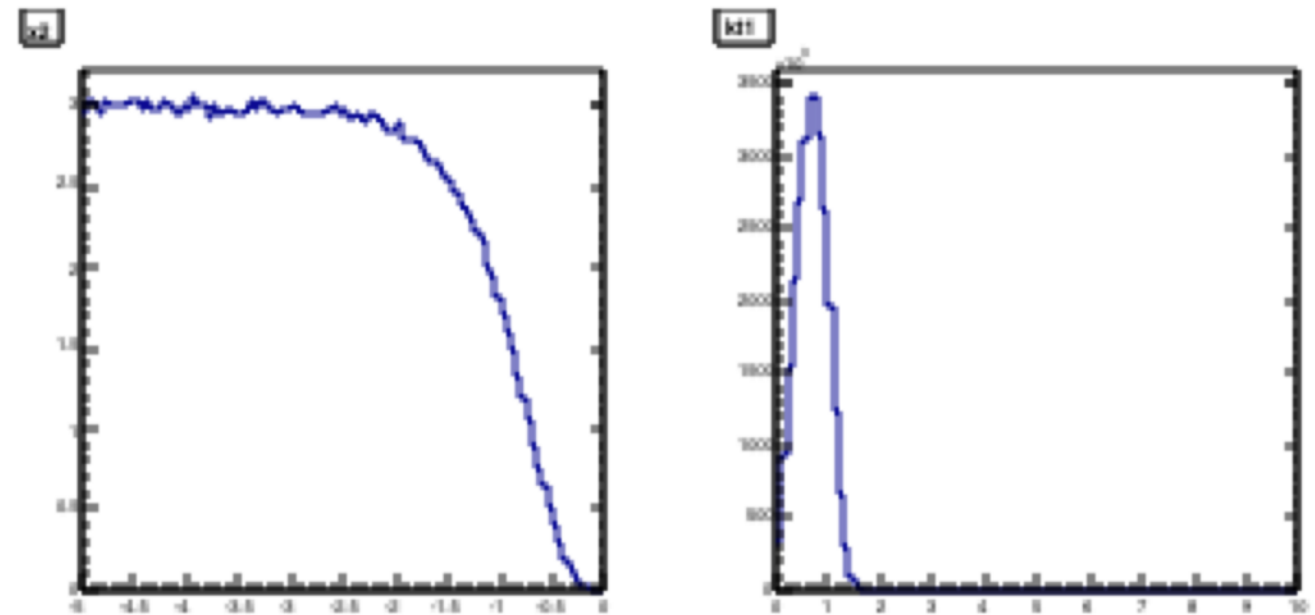


Evolved distribution:
 $xg(x, \mu)$ at $\mu = 100 \text{ GeV}$

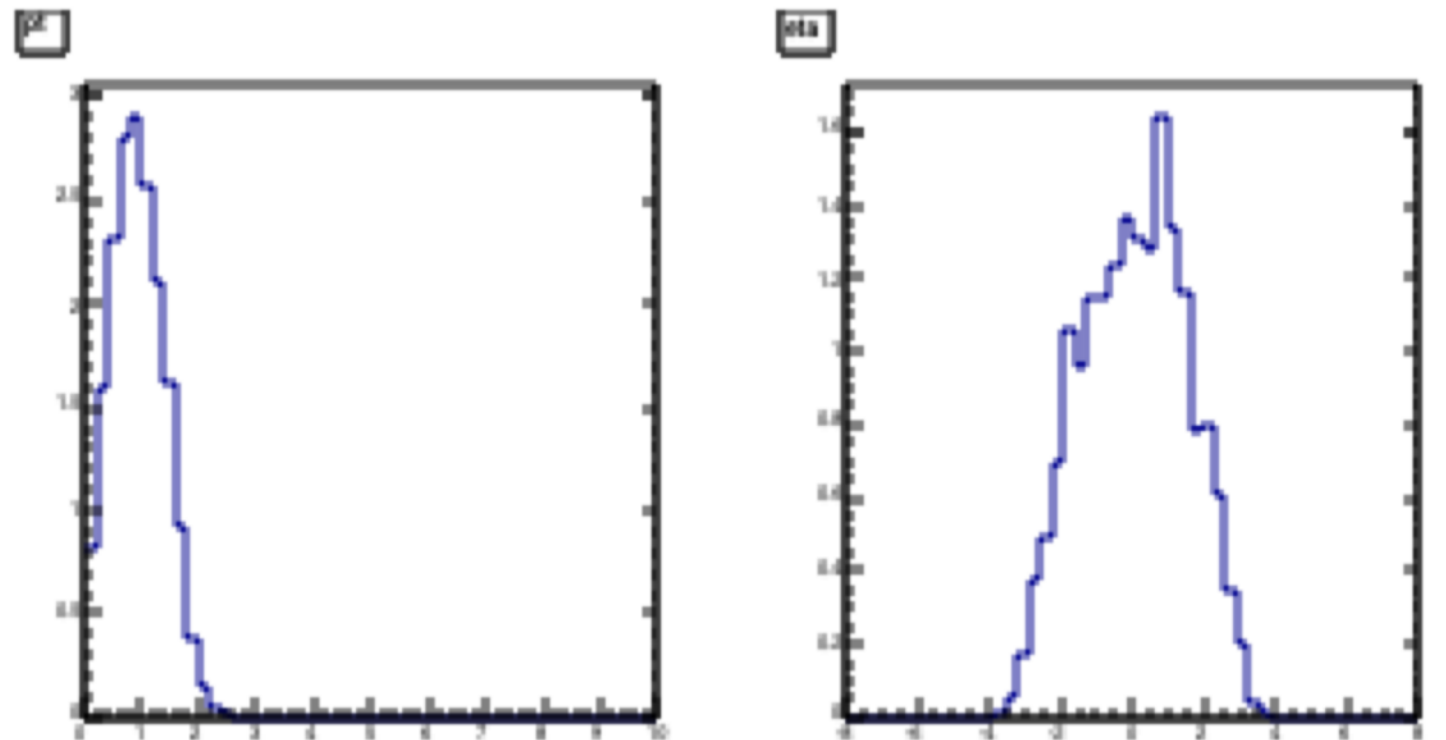


Higgs p_t at LO w/o evolution

Gluon distribution

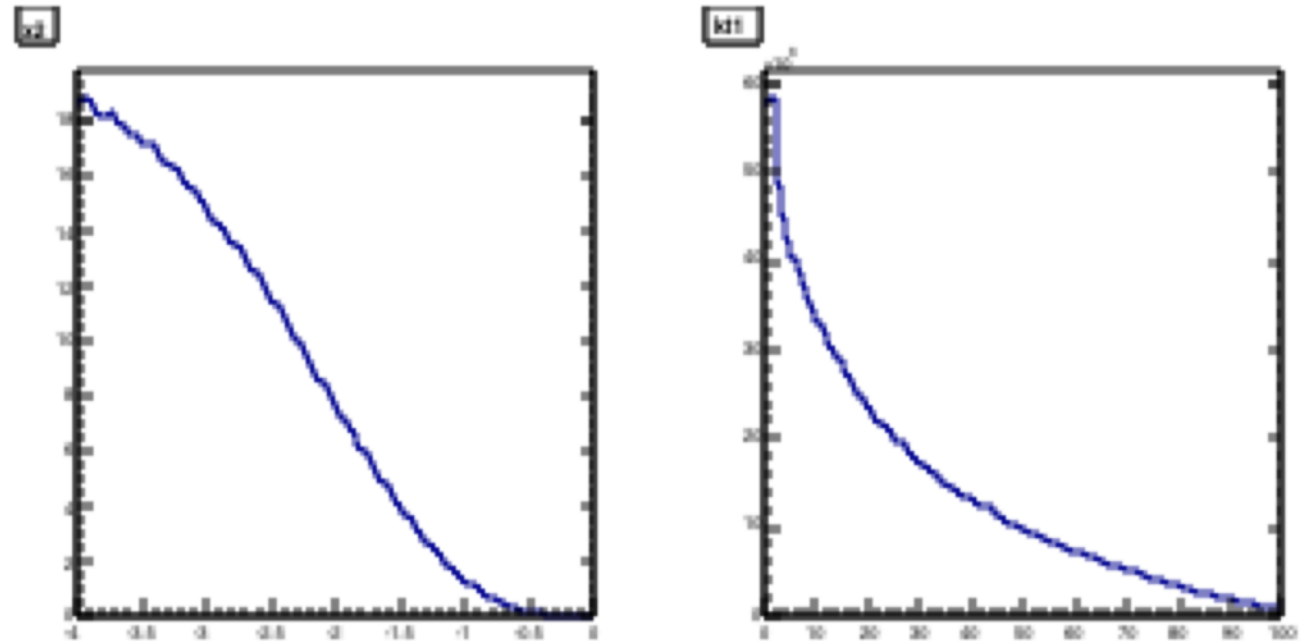


Higgs p_t and y



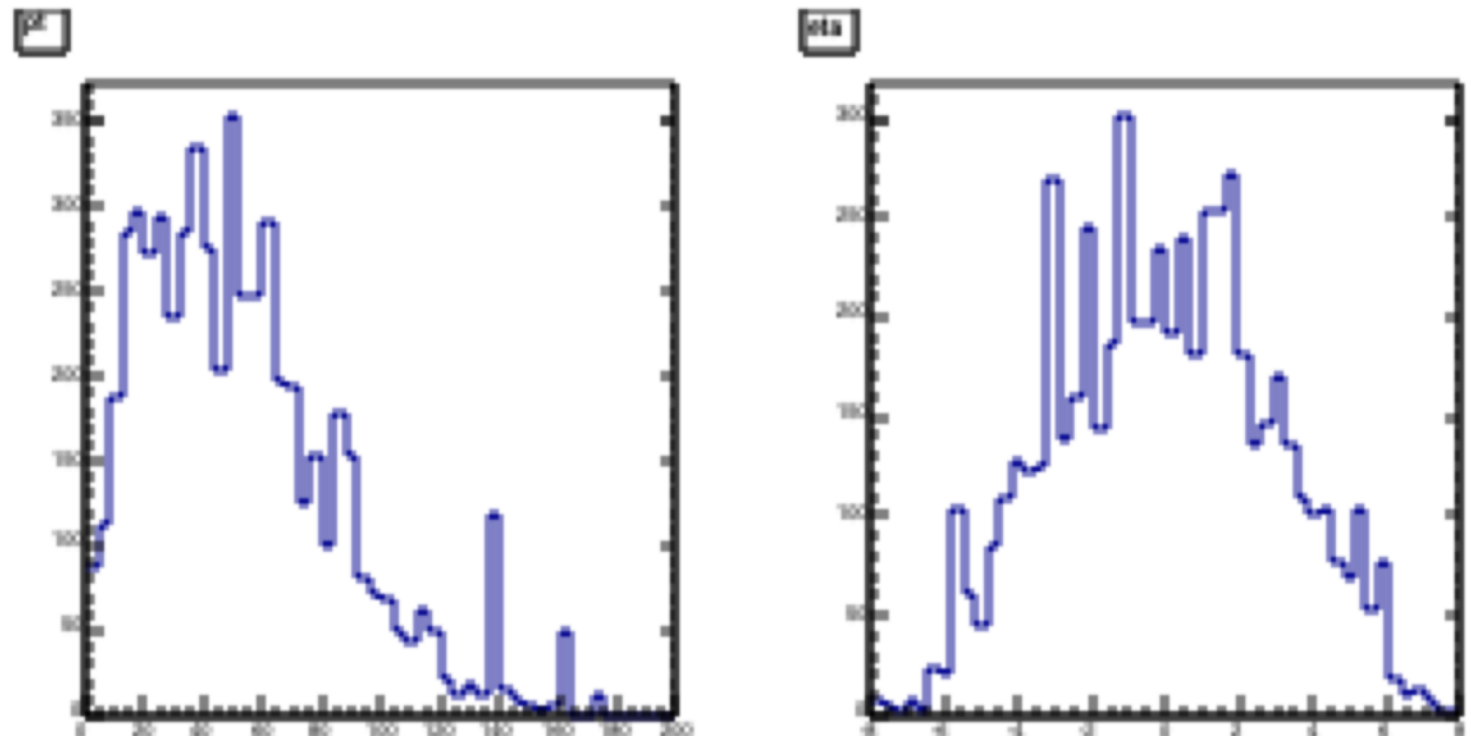
Higgs p_t at LO with evolution

Gluon distribution



Higgs p_t and y

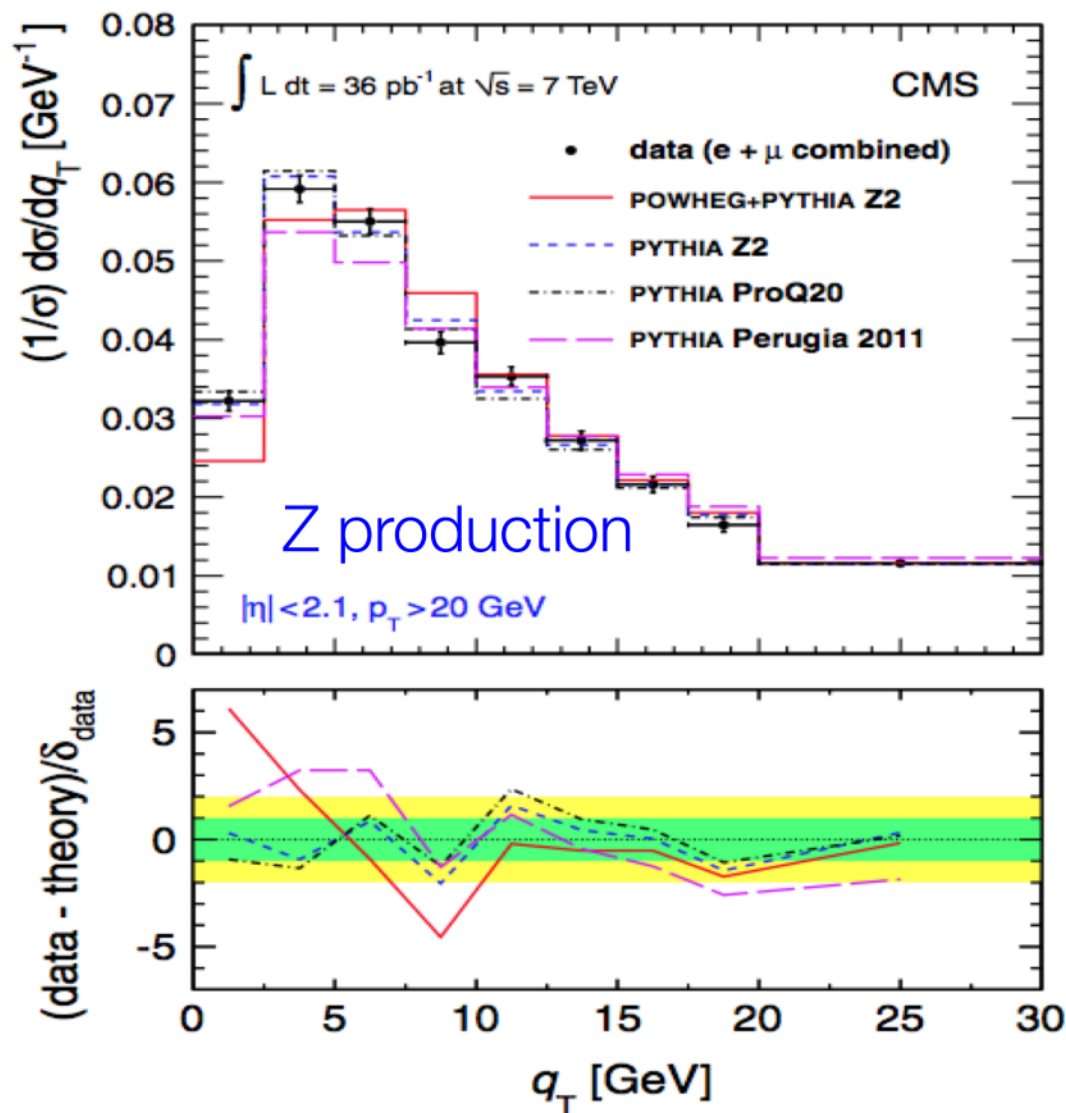
- peak at $p_t \sim 20 \text{ GeV}$
- tail to large p_t



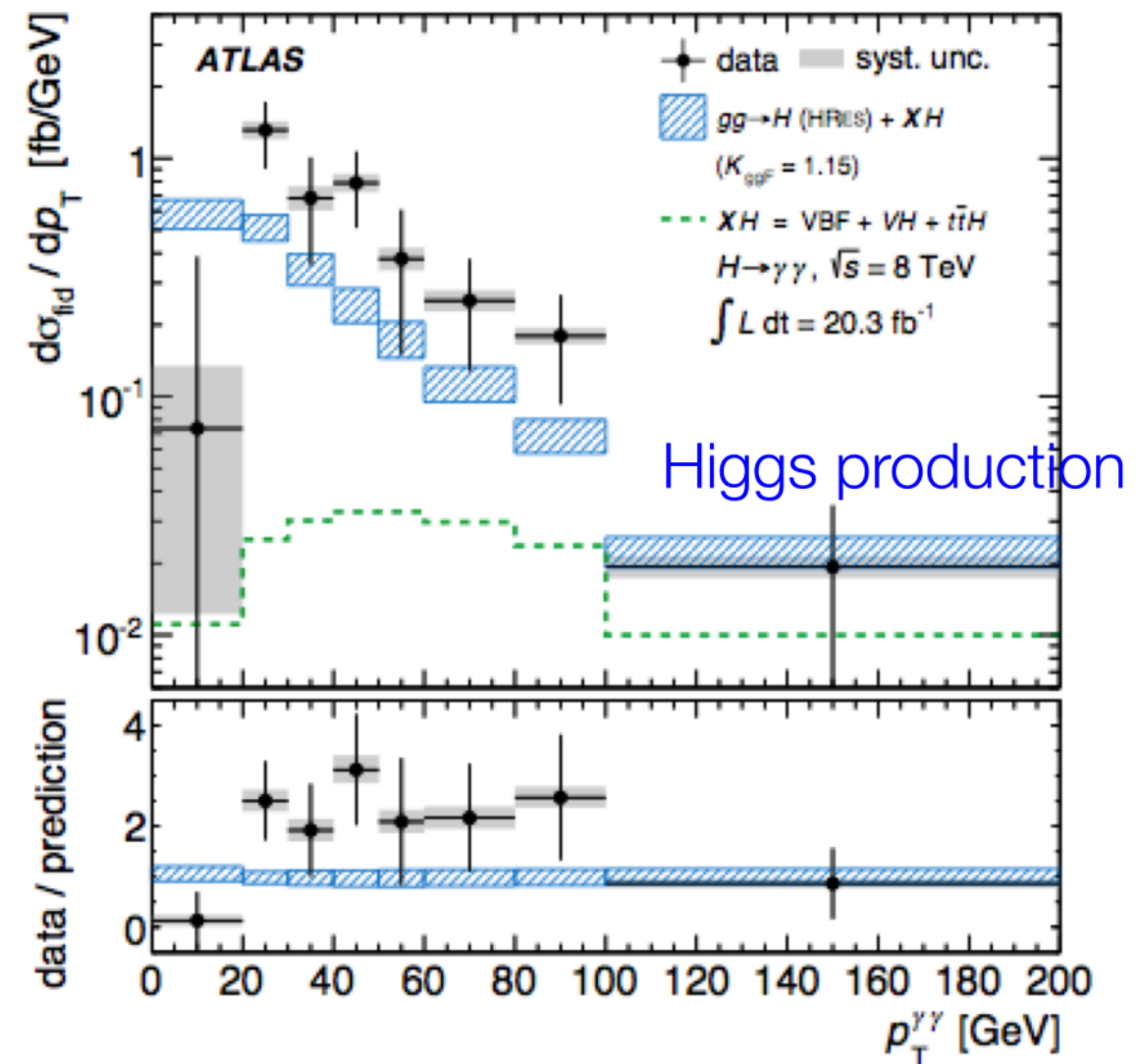
Transverse Momentum of Z and Higgs

CMS Coll. Measurement of the Rapidity and Transverse Momentum Distributions of Z Bosons in pp Collisions at $\sqrt{s} = 7$ TeV. Phys.Rev., D85:032002, 2012.

CMS Coll., PRD 85, 032002 (2012)



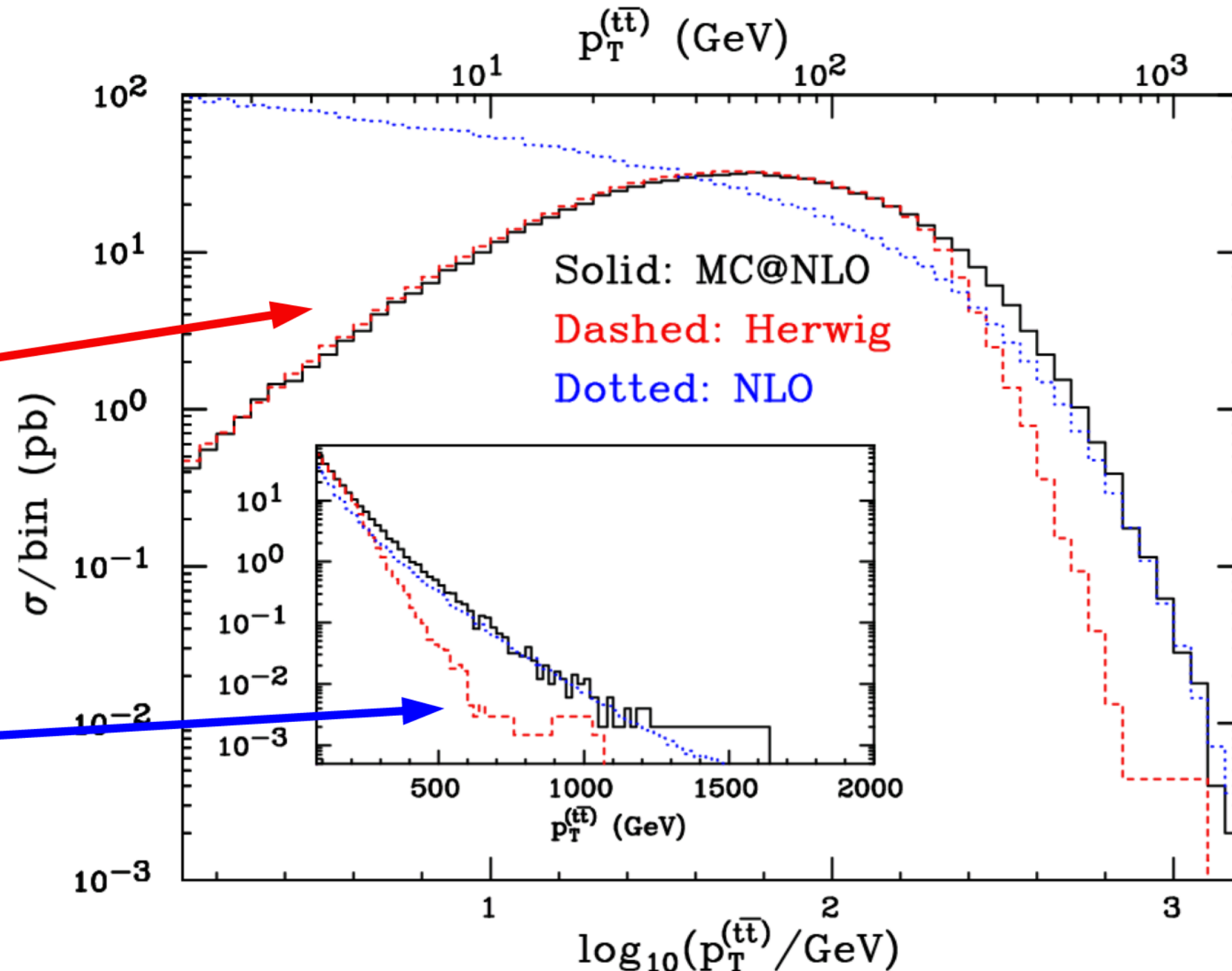
Differential cross sections of the higgs boson measured in the diphoton decay channel using 8 TeV pp collisions. ATLAS arXiv 1407.4222



Small pt in heavy quark prod.

Frixione et al, hep-ph/035252

- Compare fixed NLO calculation of top production with resummed calculation from Monte Carlo
- Similar effects at small pt are observed:
Suppression of xsection at small pt
- At large pt, resummation is too small, NLO is better



- Exercises & solutions are on indico page [link](#)
- If there are questions (even later ...)
 - please contact us: hannes.jung@desy.de
- Comments ?
- Questions ?