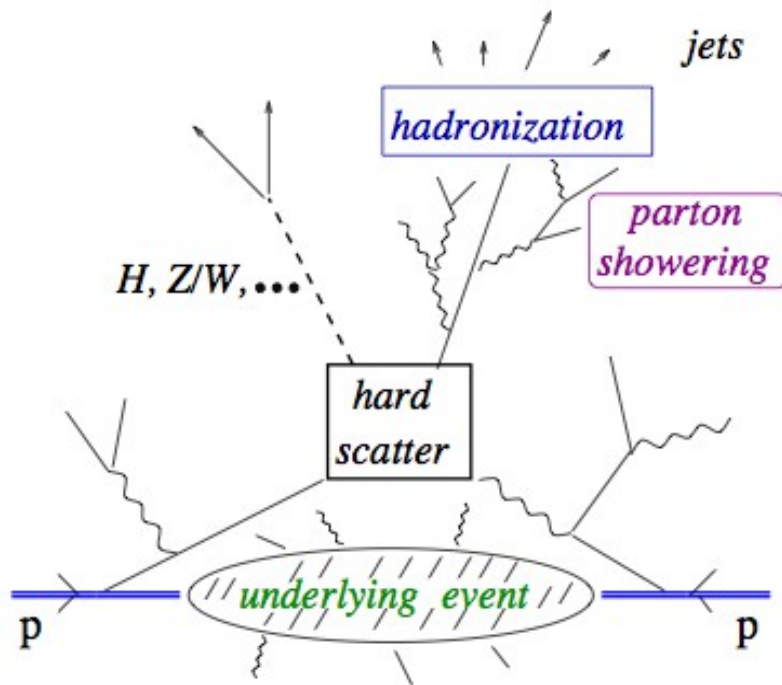


Lecture 1

MOTIVATION

BASIC QCD FACTORIZATION RESULTS



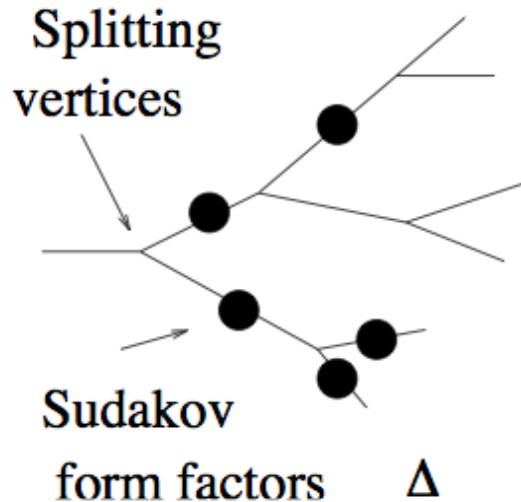
Lecture 2

PARTON SHOWERING

EXAMPLES

Lecture 2

PARTON SHOWERING INCLUDING TMDs



Based on reviews in
H, Acta Phys. Pol. B40 (2009) 2139;
ibid. B44 (2013) 761

COLLINEAR RADIATION:

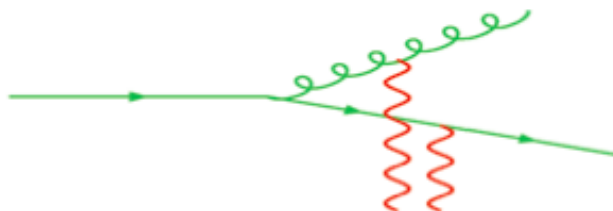
$$d\mathcal{P} = \int \frac{dq^2}{q^2} \int dz \alpha_S P(z) \Delta(q^2, q_0^2)$$

- based on dominance of collinear evolution of jets developing (both “forwards” and “backwards”) from hard event
- Factorization of QCD cross sections in collinear limit
→ probabilistic picture
- summation of logarithmically enhanced radiative contributions
 $(\alpha_S \ln p_T/\Lambda)^n$

NEXT: INCLUDE SOFT RADIATION

A) Soft gluon radiation by coherent branching

▷ soft gluons radiated over long times \longrightarrow quantum interferences



✓ factorization in soft limit

$$|M_{n+1}^{a_1 \dots a_n a}(p_1, p_n, q)\rangle = \mathbf{J}^a |M_n^{a_1 \dots a_n}(p_1, p_n)\rangle, \quad \mathbf{J}^{a\mu} = \sum_i \mathbf{Q}_i^a \frac{p_i^\mu}{p_i \cdot q}, \quad \mathbf{Q} = \text{color charge}$$

interference terms \downarrow

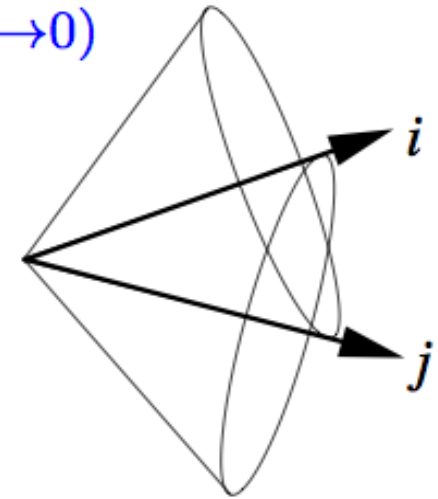
$$d\sigma_{n+1} = d\sigma_n \frac{d^3 q}{(q^0)^3} \sum_{i,j} \mathbf{Q}_i \cdot \mathbf{Q}_j w_{ij}, \quad w_{ij} = \frac{(q^0)^2 p_i \cdot p_j}{(p_i \cdot q)(p_j \cdot q)}$$

\rightarrow spoils probabilistic picture? **NO**, owing to soft-gluon coherence \hookrightarrow

♣ single-emission: separate singularities along emitters' directions

$$\frac{(q^0)^2 p_i \cdot p_j}{(p_i \cdot q)(p_j \cdot q)} \equiv \frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}}$$
$$= \frac{1}{2} \left(\frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}} - \frac{1}{\zeta_{jq}} + \frac{1}{\zeta_{iq}} \right) + \frac{1}{2} \left(\frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}} - \frac{1}{\zeta_{iq}} + \frac{1}{\zeta_{jq}} \right)$$

where $\zeta_{nk} \equiv \frac{p_n \cdot p_k}{p_n^0 p_k^0} \simeq 1 - \cos \theta_{nk} \quad (m \rightarrow 0)$



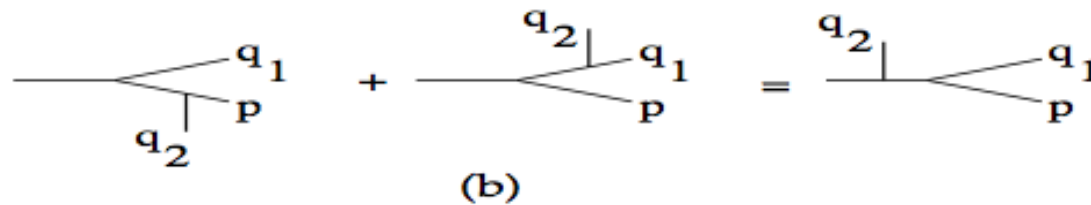
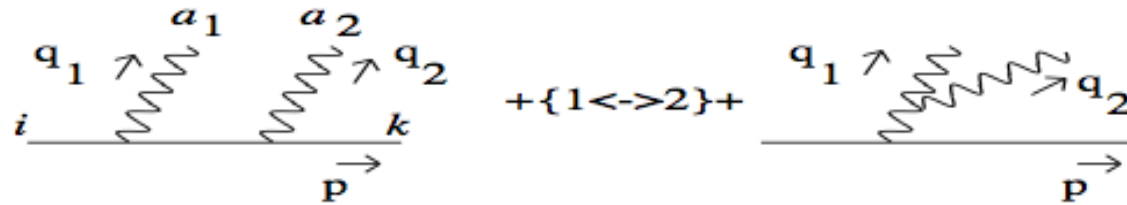
→ by azimuthal average

$$\left\langle \frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}} \right\rangle = \frac{1}{\zeta_{iq}} \Theta(\zeta_{ij} - \zeta_{iq}) + \frac{1}{\zeta_{jq}} \Theta(\zeta_{ij} - \zeta_{jq})$$

- large-angle emissions of soft gluons sum coherently outside angular-ordered cones

♣ multiple emission: (q_1, q_2 with $q_2^0 \ll q_1^0$)

$$\mathbf{J}_1^{\mu a_1} = Q_p^{a_1} \frac{p^\mu}{p \cdot q_1}, \quad \mathbf{J}_2^{\mu a_2} = Q_p^{a_2} \frac{p^\mu}{p \cdot q_2} + Q_{q_1}^{a_2} \frac{q_1^\mu}{q_1 \cdot q_2}$$



$$\begin{aligned} \mathcal{M}_{ki}^{a_1 a_2} &= g_s^2 \langle a_1 k | \mathbf{J}_2 \cdot \varepsilon_2 | a' i' \rangle \langle i' | \mathbf{J}_1 \cdot \varepsilon_1 | i \rangle \\ &= g_s^2 \frac{p \cdot \varepsilon_1}{p \cdot q_1} \left(\frac{p \cdot \varepsilon_2}{p \cdot q_2} t^{a_2} t^{a_1} + \frac{q_1 \cdot \varepsilon_2}{q_1 \cdot q_2} [t^{a_1}, t^{a_2}] \right)_{ki} \end{aligned}$$

- small angle: bremsstrahlung cones
- large angle ($\theta_{pq_2} \gg \theta_{pq_1}$): sees total charge $Q_p + Q_{q_1}$

B) Coherence in the high energy limit

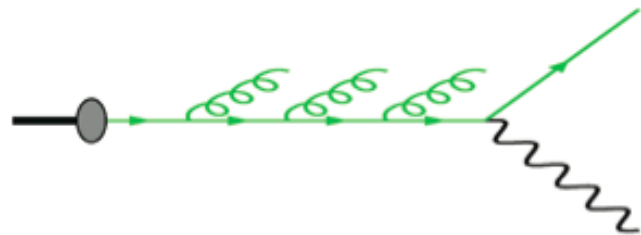
Arguments on soft vector emission current from **external** legs \rightarrow

- leading IR singularities

[J.C. Taylor, 1980]

- fully appropriate in single-scale hard processes

Dokshitzer, Khoze, Mueller and Troian, RMP (1988); Webber, A. Rev. Nucl. Part. (1986)



multi-scale: $s = q_1^2 \gg \dots \gg q_n^2 \gg \Lambda^2$
[e.g.: LHC final states with multi-jets]



▷ **internal** emissions non-negligible

▷ current also factorizable at high-energy: *[Ciafaloni 1998; 1988]*

$$\begin{aligned} |M^{(n+1)}(k, p)|^2 &= \{ [M^{(n)}(k+q, p)]^\dagger [\mathbf{J}^{(R)}]^2 M^{(n)}(k+q, p) \\ &\quad - [M^{(n)}(k, p)]^\dagger [\mathbf{J}^{(V)}]^2 M^{(n)}(k, p) \} \quad . \quad \text{BUT...} \triangleright \end{aligned}$$

- ▷ ...
 - \mathbf{J} depends on total transverse momentum transmitted
 \Rightarrow matrix elements and pdf at fixed k_\perp (“TMDs”)
 - virtual corrections not fully represented by Δ form factor
 \Rightarrow modified branching probability $P(z, k_\perp)$ as well

▷ enhanced terms $\mathcal{O}(\alpha_s^k \ln^m s / p_T^2)$

◇ Note: superleading logs $m > k$ cancel in fully inclusive quantities

e.g: high-energy corrections to anomalous dimensions γ^{ij}
 at most single-logarithmic

$$\gamma^{ij}(\alpha_s, \omega) = \frac{\alpha_s}{\omega^p} c_0^{ij} \left[1 + \sum_{n=1}^{\infty} c_n^{ij} \left(\frac{\alpha_s}{\omega} \right)^n + \mathcal{O} \left(\alpha_s \left(\frac{\alpha_s}{\omega} \right)^{n-1} \right) \right]$$

ω - moment conjugate to $\ln s$

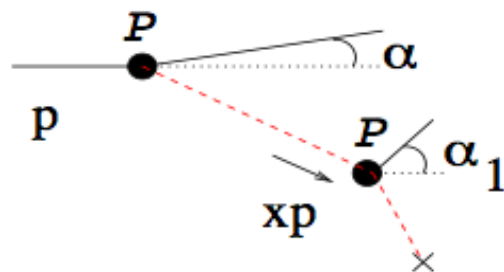
BFKL; Jaroszewicz; Catani et al.

◇ but cancellations do not apply in exclusive final-state correlations

K_{\perp} -DEPENDENT PARTON BRANCHING

- MC for (almost-)NLO QCD evolution at unintegrated level
proposed in [Jadach & Skrzypek, arXiv:0905.1399 \[hep-ph\]](#)
- $\{x \rightarrow 0\} \oplus \{x \rightarrow 1\}$ gluon branching eq. (leading-logarithms, all orders in α_s)
CCFM evolution equation [Marchesini et al., 1990's]

$$\mathcal{G}(x, k_T, \mu) = \mathcal{G}_0(x, k_T, \mu) + \int \frac{dz}{z} \int \frac{dq^2}{q^2} \Theta(\mu - zq) \\ \times \underbrace{\Delta(\mu, zq)}_{\text{Sudakov}} \underbrace{\mathcal{P}(z, q, k_T)}_{\text{unintegr. splitting}} \mathcal{G}\left(\frac{x}{z}, k_T + (1-z)q, q\right)$$



$$, \quad \text{---} \bullet \text{---} = \text{---} \text{---} + \text{---} \text{---} + \dots$$

▷ Monte Carlo implementations CASCADE, LDCMC, ...

- unintegrated quark with k_T -dependent branching
↪ ongoing work

CCFM exclusive evolution

→ Catani-Ciafaloni-Fiorani-Marchesini (1990's)

$$x\mathcal{A}(x, k_t, q) = x\mathcal{A}(x, k_t, q_0)\Delta_s(q) + \int dz \int \frac{dq'}{q'} \cdot \frac{\Delta_s(q)}{\Delta_s(q')} \tilde{P}(z, k_t, q') \frac{x}{z} \mathcal{A}\left(\frac{x}{z}, q'\right)$$

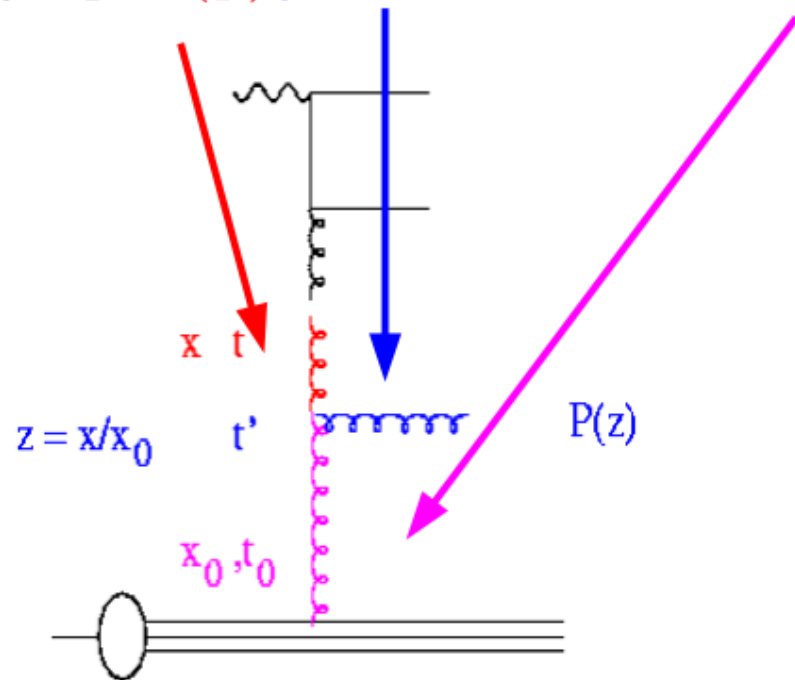
- solve integral equation via iteration:

$$x\mathcal{A}_0(x, k_t, q) = x\mathcal{A}(x, k_t, q_0)\Delta(q)$$

from q' to q
w/o branching
branching at q'
from q_0 to q'
w/o branching

$$x\mathcal{A}_1(x, k_t, q) = x\mathcal{A}(x, k_t, q_0)\Delta(q) + \int \frac{dq'}{q'} \frac{\Delta(q)}{\Delta(q')} \int dz \tilde{P}(z) \frac{x}{z} \mathcal{A}(x/z, k'_t, q_0)\Delta(q')$$

- Note: evolution equation formulated with Sudakov form factor is equivalent to “plus” prescription, **but** better suited for numerical solution for **treatment of kinematics**

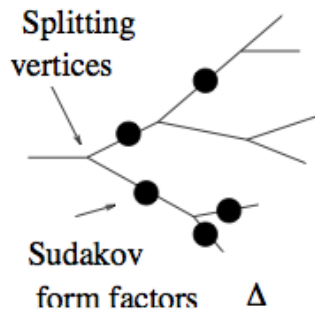


- evolution code **uPDFevolv**
[Jung, Taheri Monfared & H,
arXiv:1407.5935]

FROM QCD TO MONTE CARLO EVENT GENERATORS

- Factorizability of QCD x-sections \longrightarrow probabilistic branching picture

◇ QCD evolution by “parton showering” methods:

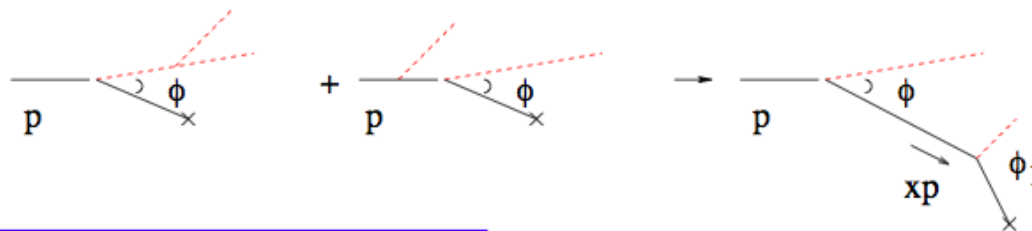


$$d\mathcal{P} = \int \frac{dq^2}{q^2} \int dz \alpha_S(q^2) P(z) \Delta(q^2, q_0^2)$$

\hookrightarrow collinear, incoherent emission

◇ Soft emission \longrightarrow interferences \longrightarrow ordering in decay angles:

\hookrightarrow gluon coherence for $x \sim 1$



◇ Gluon coherence for $x \ll 1 \Rightarrow$ corrections to angular ordering:

\hookrightarrow k_{\perp} -dependent parton showers

CCFM equation is TMD branching equation which contains both Sudakov physics and BFKL physics

EXAMPLES:

**TMD kinematic effects
in parton shower evolution**

Kinematic effects in parton shower evolution

S. Dooling, talk at DIS 2013, Marseille

Longitudinal Momentum Shift

In SMC:

hard subprocess is generated with full 4-momentum for the external lines

Momentum of the partons initiating the hard scatter:

$$k_j^{(0)} = x_j p_j$$

on-shell and fully collinear with the incoming momenta

Applying
shower algorithm

Complete final states:

$$k_j \neq x_j p_j$$

no longer collinear

Energy momentum conservation \triangleright Reshuffling in x_j (long. mom fraction)

Collinear approximation \otimes energy momentum conservation



=

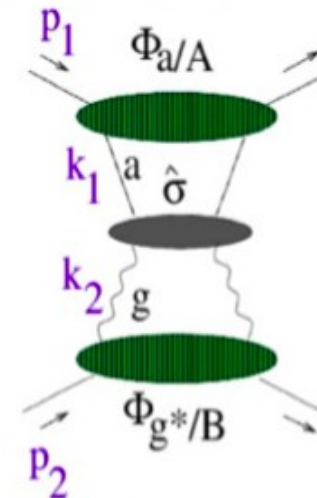


kinematic shift in longitudinal momentum distribution due to showering

Dooling et al.
arXiv:1212.6264

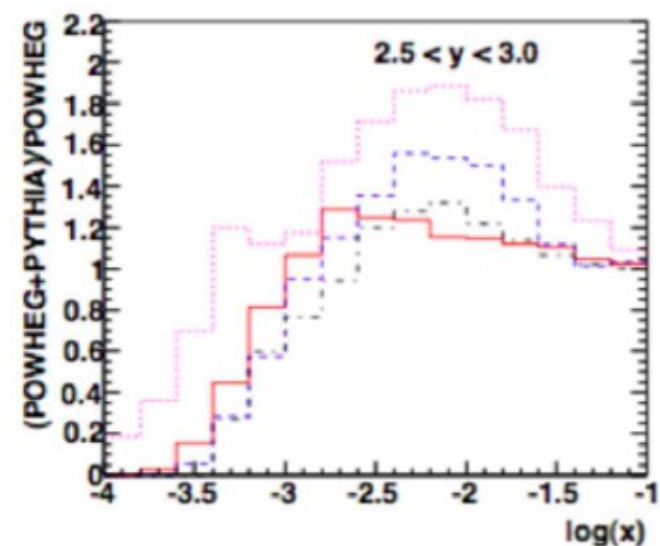
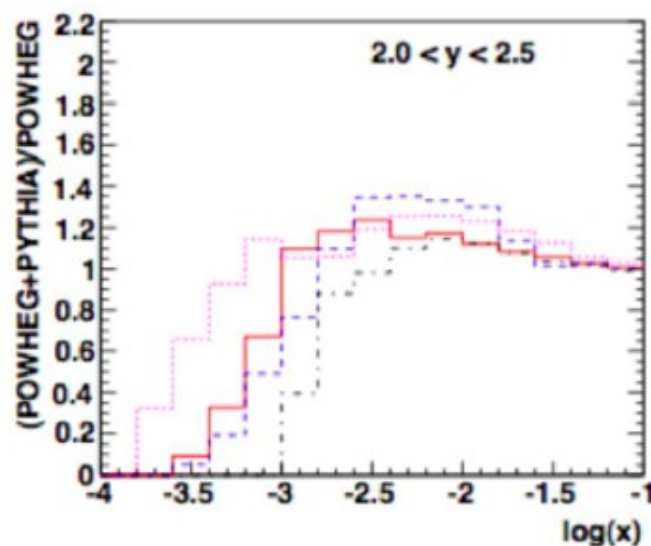
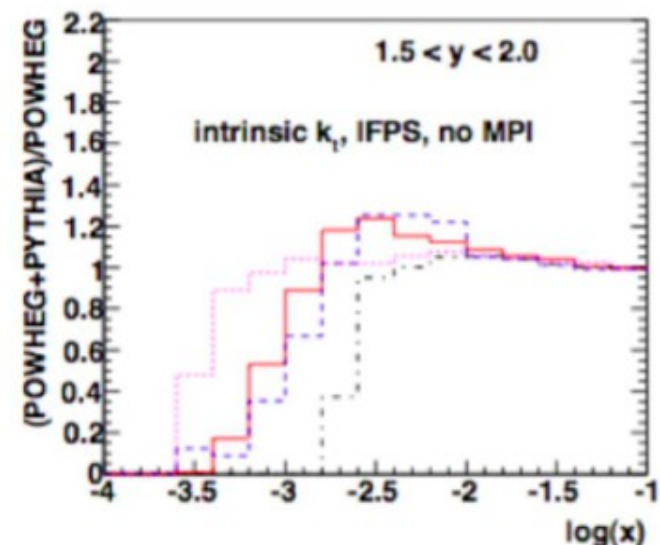
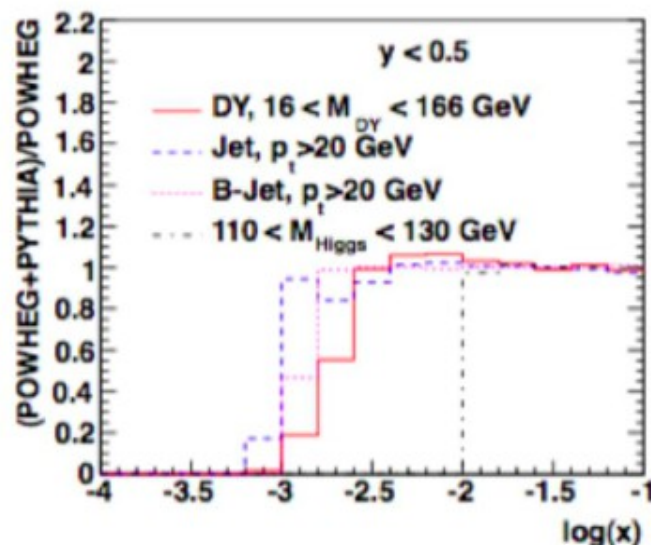


Factorized jet cross section
at high rapidity



TMD effects in pp collisions

- Transverse momentum dependent (TMD) effects are relevant for many processes at the LHC
- parton shower matched with NLO generates additional k_t , leading to energy-momentum mismatch
- avoided by using formulation with TMD distributions from the outset



NONPERTURBATIVE (NP) AND SHOWERING (PS) CORRECTIONS

- Estimates using leading order (LO-MC):

$$K_0^{NP} = N_{LO-MC}^{(ps+mpi+had)} / N_{LO-MC}^{(ps)}$$

[CMS, PRL 107 (2011) 132001; ATLAS, PRD86 (2012) 014022]

— natural definition with LO-MC

— but affected by potential inconsistency if combined with NLO parton-level results

- Alternatively, assign NP correction factors by using NLO-MC:

[Dooling, Gunnellini, Jung & H, arXiv:1212.6164 [hep-ph]]

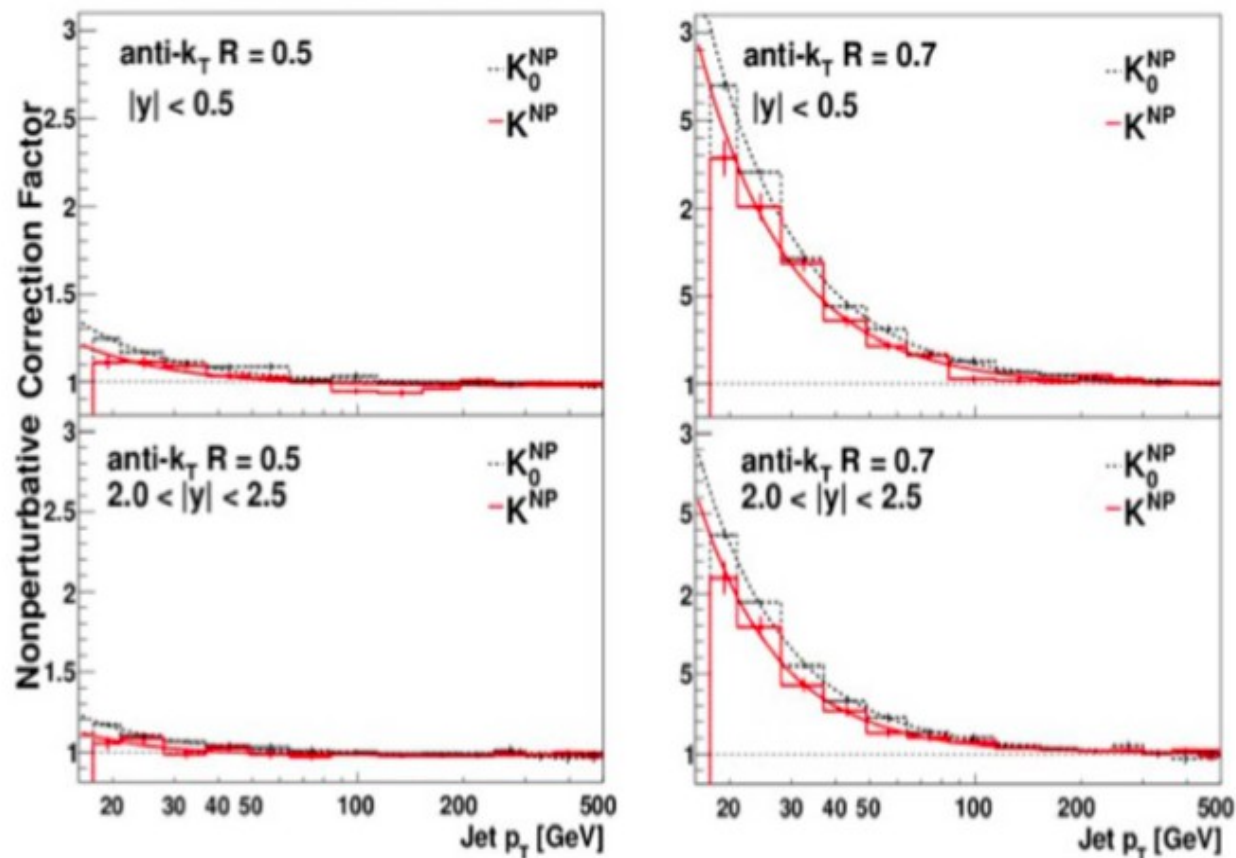
$$K^{NP} = N_{NLO-MC}^{(ps+mpi+had)} / N_{NLO-MC}^{(ps)}$$

$$K^{PS} = N_{NLO-MC}^{(ps)} / N_{NLO-MC}^{(0)}$$

♣ K^{NP} differs from K_0^{NP}

♣ K^{PS} is new

Nonperturbative Correction



Non-negligible effect from nonperturbative effects at small p_T

Difference between LO and NLO correction

► Matching of MPI to the NLO calculation because the MPI p_T scale is different in LO and NLO

$$K_0^{NP} = K_{LO-MC}^{(ps+mpi+had)} / K_{LO-MC}^{(ps)}$$

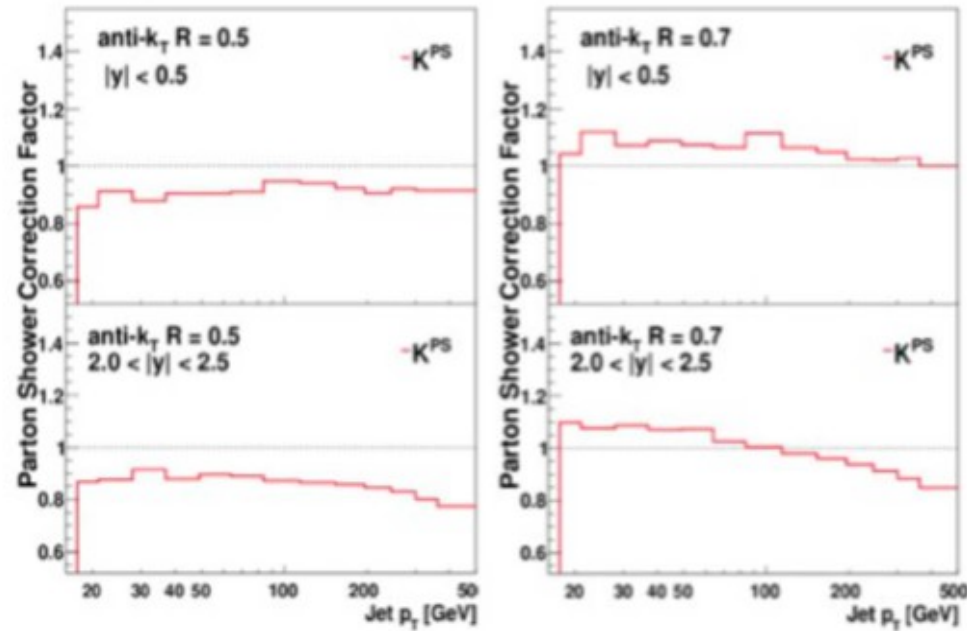
$$K^{NP} = K_{NLO-MC}^{(ps+mpi+had)} / K_{NLO-MC}^{(ps)}$$

Dooling et al.
arXiv:1212.6264



S. Dooling, talk at DIS 2013, Marseille

Parton Shower Correction



$$K^{PS} = K_{NLO-MC}^{(ps)} / K_{NLO-MC}^{(0)}$$

- Depends on rapidity and p_T especially in the forward region

- Finite effect also at large p_T

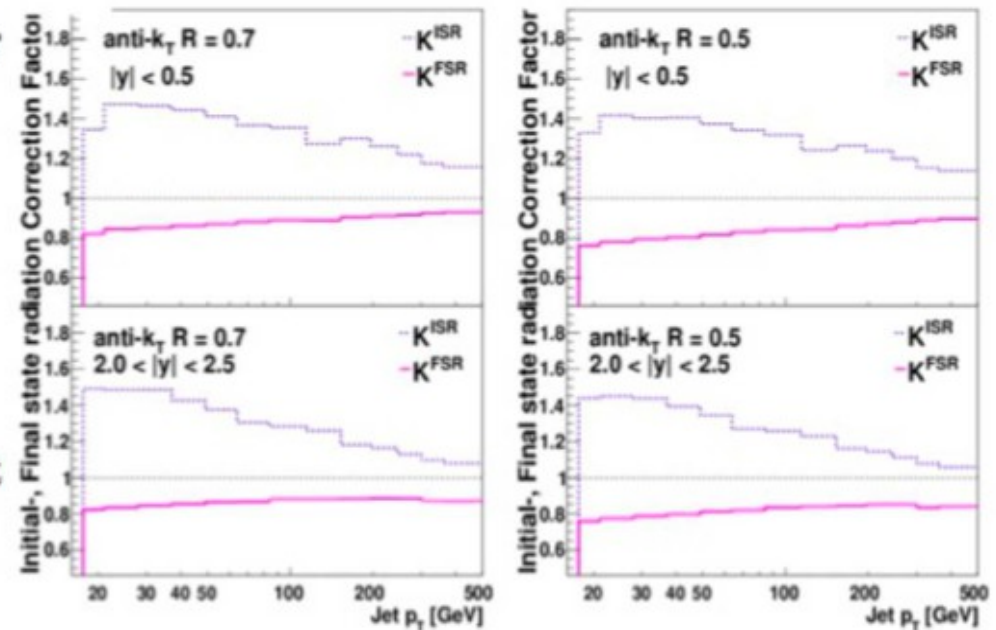
- Initial and Final State Parton Shower considered independently

- But they are interconnected:

The combined effects cannot be obtained by adding the individual contributions

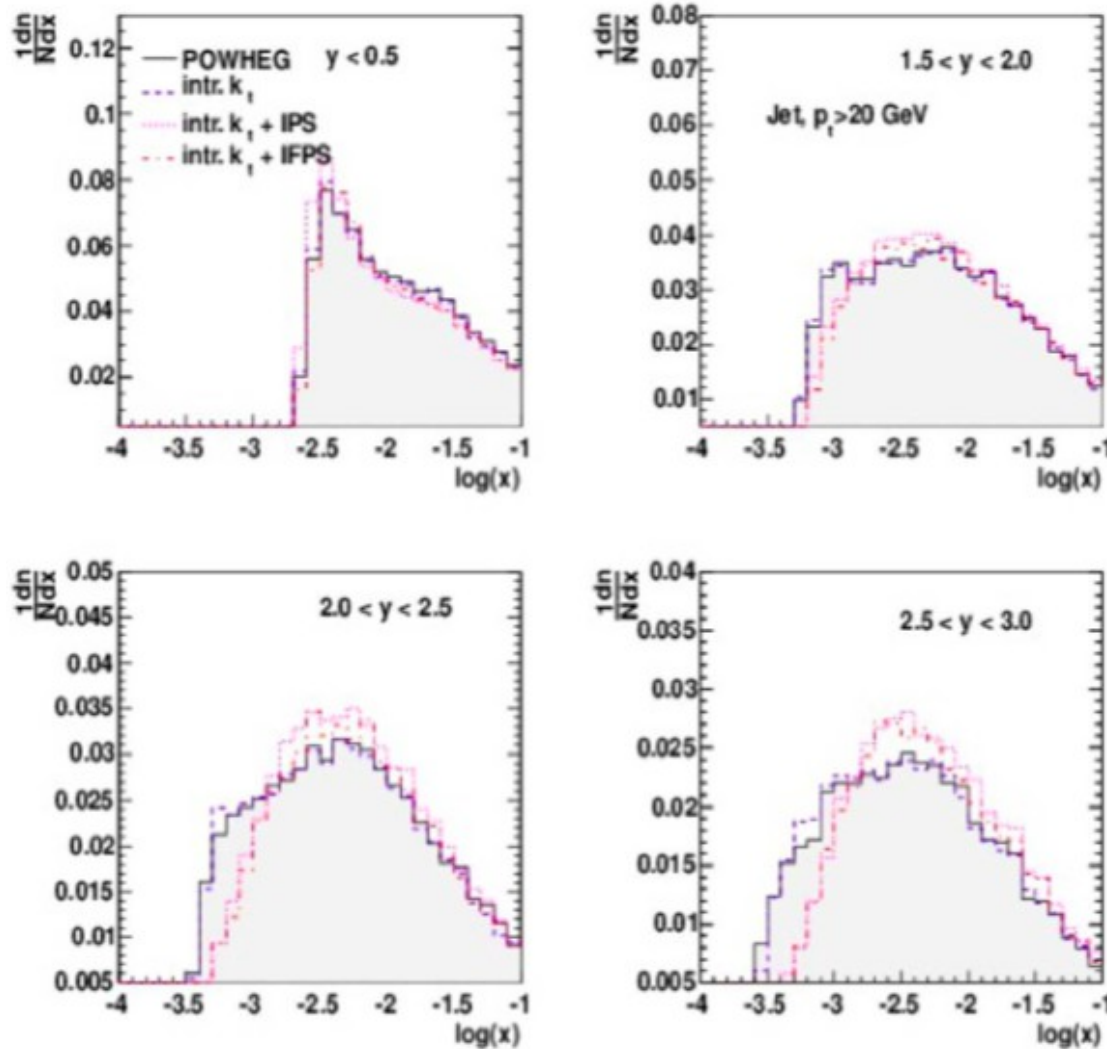
- ISR largest at low p_T , FSR significant for all p_T

[S. Dooling, talk at DIS 2013]



Longitudinal Momentum Shift – Inclusive Jets

Jet measurement in the rapidity range $y < 2.5$



Compute x_j from POWHEG before parton showering and after parton showering (using PYTHIA6)

Kinematic reshuffling in x is negligible for central rapidities but becomes significant for $y > 1.5$

► Kinematic shift can affect predictions through the PDFs

Dooling et al.
arXiv:1212.6264



[S. Dooling, talk at DIS 2013]

EXAMPLES:
VECTOR BOSON + JETS
FINAL STATES

Application to vector bosons + jets

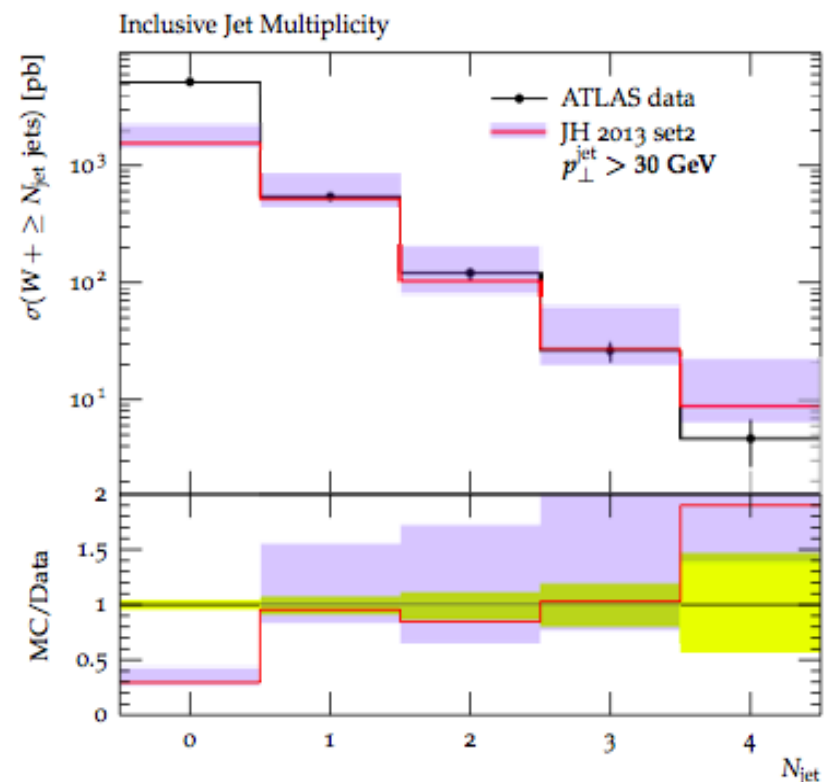
- Motivation: effects of not only collinear-ordered emissions but also non-ordered region which opens up at high s / pt^2 (and large pt).
- Finite angle multi-gluon radiation.
- Push limits of high-energy expansion beyond small- x region.

- Jet multiplicities associated with W boson production

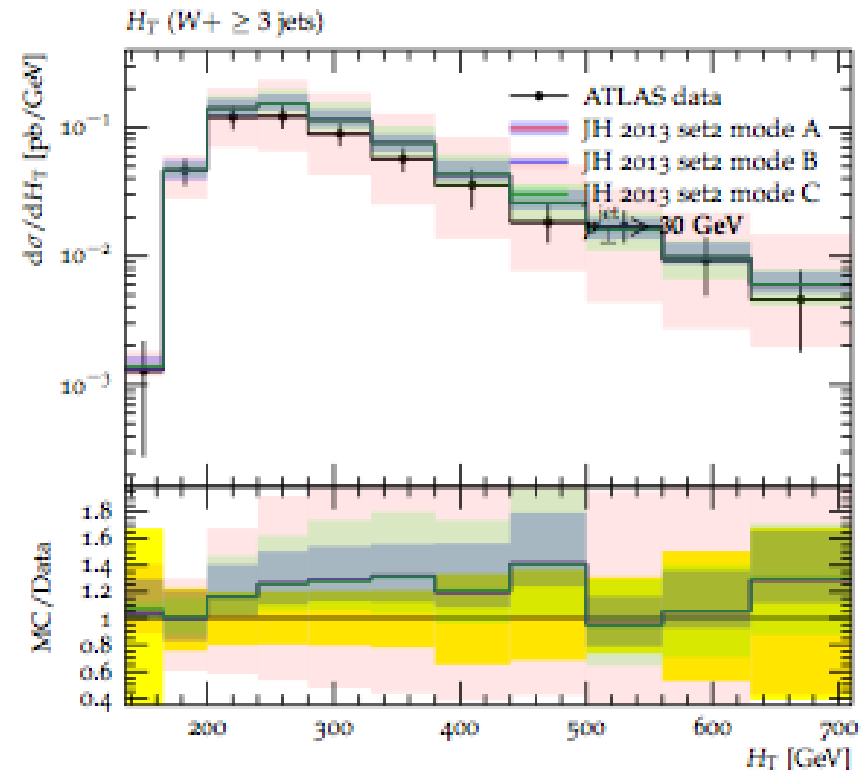
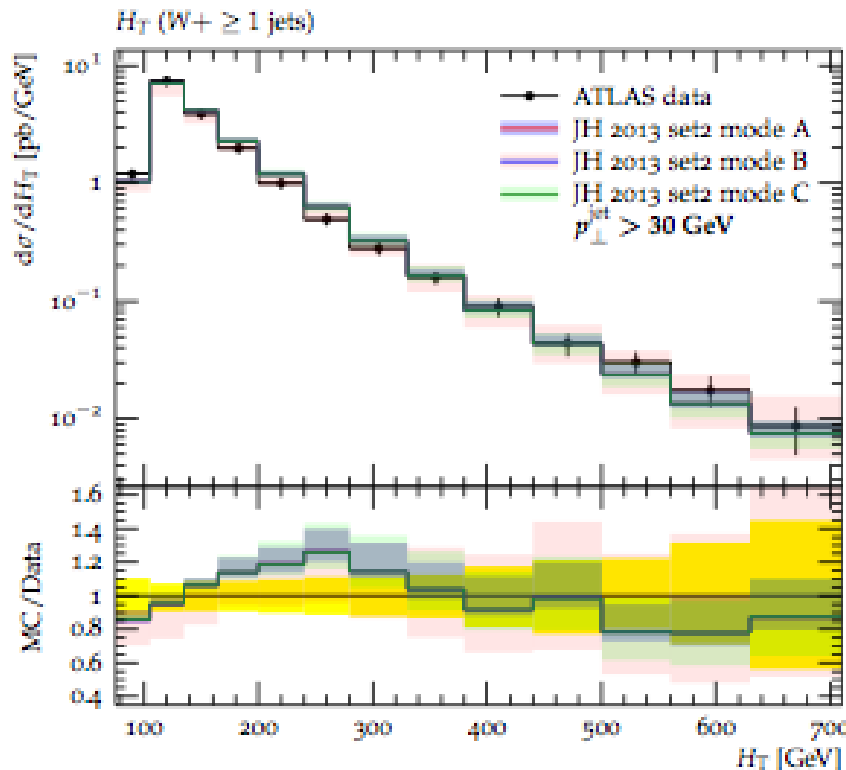
Atlas data PRD85 (2012) 092002:
 $jet |y| < 4.4$

Note: pt -ordered shower (eg, Pythia) cannot predict higher jet multiplicities

- Role of transverse-momentum kinematics on jets produced at moderately non-central rapidities



Theoretical uncertainties larger for larger H_T (increasing x) and,
at fixed H_T , for higher jet multiplicities

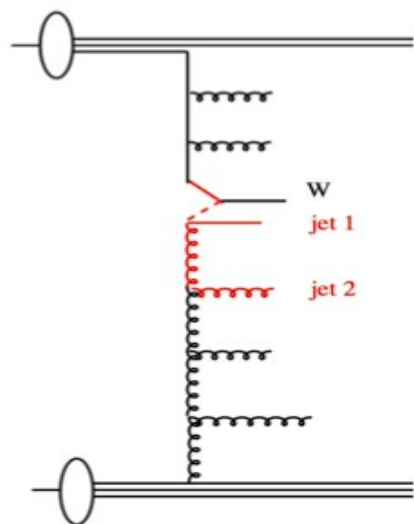


$$\mu^2 = m^2 + qT^2$$

Dooling, Jung & H, Phys. Lett. B736 (2014) 293

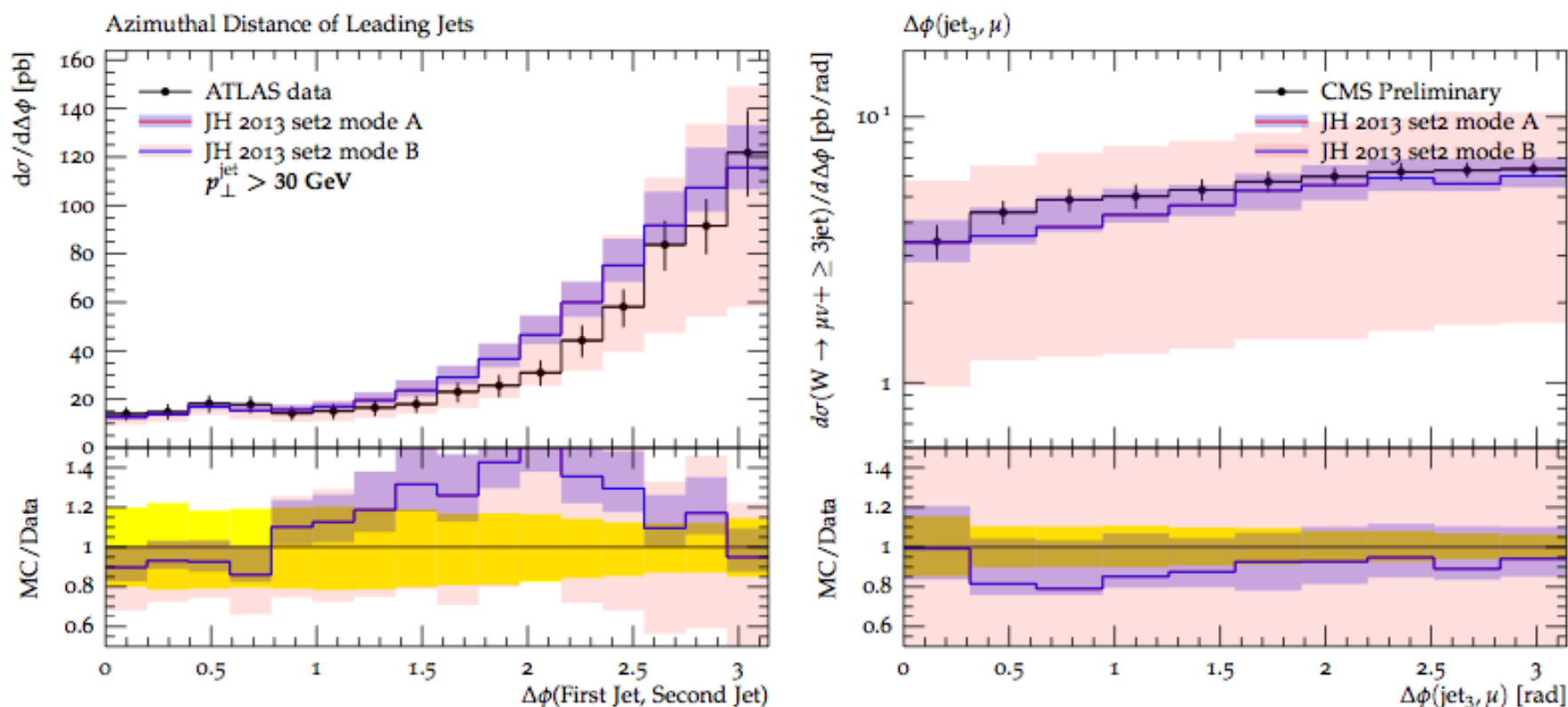
Mode C: vary transverse part of μ^2 by factor 2 above and below central value
(more closely related to standard collinear calculations)

Mode B: include variation of longitudinal component
(more conservative estimate – unlike standard collinear approximations)



Angular correlations in $W + n$ jets final states

Dooling, Jung & H, Phys. Lett. B 736 (2014) 293



(left) Delta-phi between two hardest jets; (right) vector boson - third jet correlation

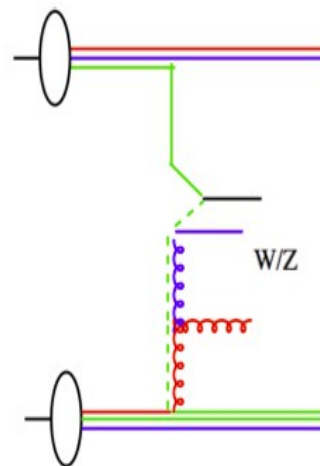
What do we gain?

- Uses of TMD pdfs + kt-dependent shower:
 - matching with $2 \rightarrow n$ off-shell parton calculations
(automated method, see van Hameren, Kotko & Kutak JHEP 1301 (2013) 078)
- Opens possibility for full LHC phenomenology of QCD, EWK and BSM processes

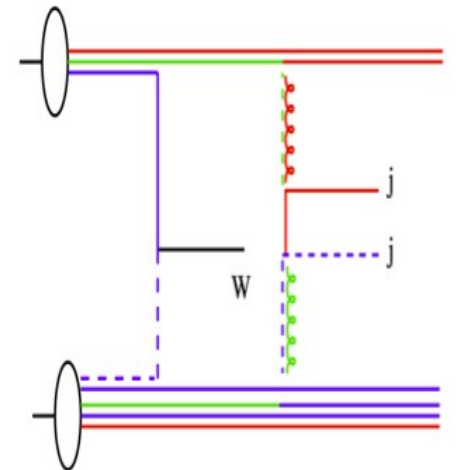
$W + 2$ jets as signal
of double parton interactions

- Influence of TMD corrections to shower evolution on analysis of DPI?

- Formalism interpolates from low p_T to high p_T
- Incorporates experimental information from high-precision DIS measurements
- Takes into account transverse momentum kinematics without approximations in the branching

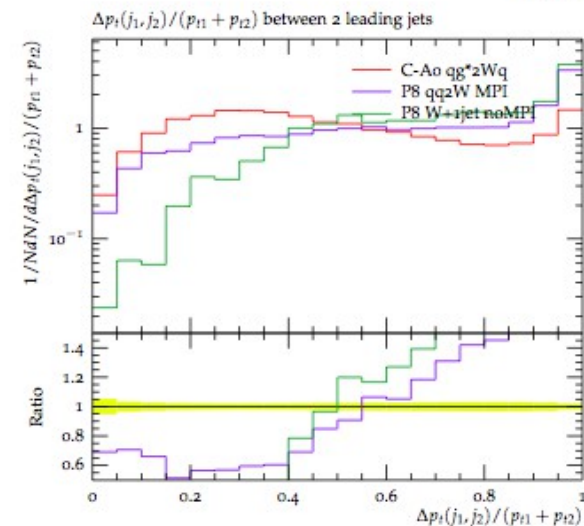
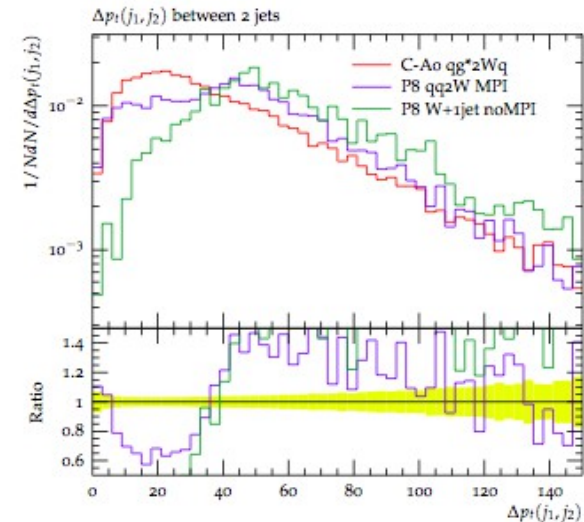
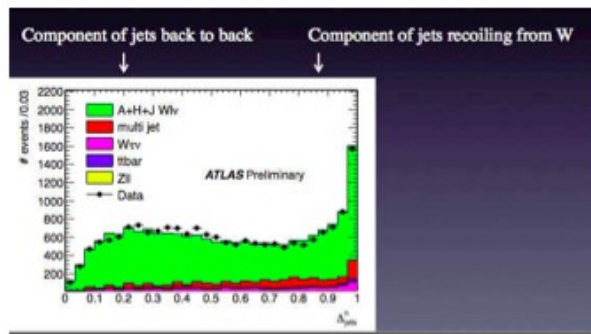
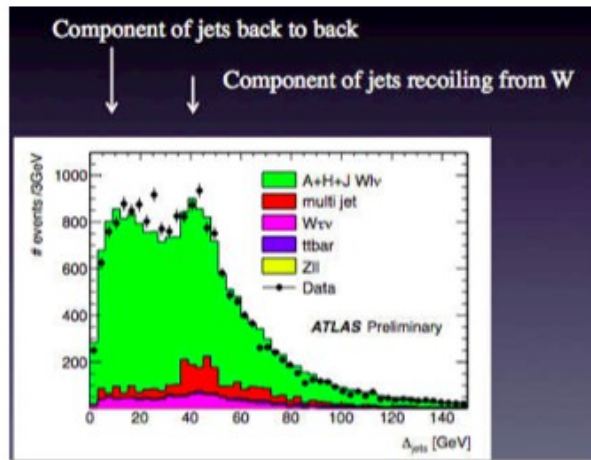


Single chain



Double chain

W + 2 jets: signal for double parton interactions?



[E. Dobson, talk at MPI-TAU Workshop, October 2012]

ATLAS, New J Phys 15 (2013) 033038

For jet $p_T = O(20 \text{ GeV})$ effects from higher orders in kt-shower significant

Conclusion

- TMD parton distributions and showers relevant to both large p_T and small p_T processes, high x and low x :
TMDlib platform <http://tmdlib.hepforge.org/>
- First determination of TMD gluon from combined high-precision DIS data, including uncertainties [\rightarrow herafitter]
- The approach has far reaching implications for LHC physics:
treatment of kinematic corrections to parton showers;
studies of theor uncertainties in multi-particle final states;
ex.: $W + \text{jets}$ p_T spectra and angular correlations