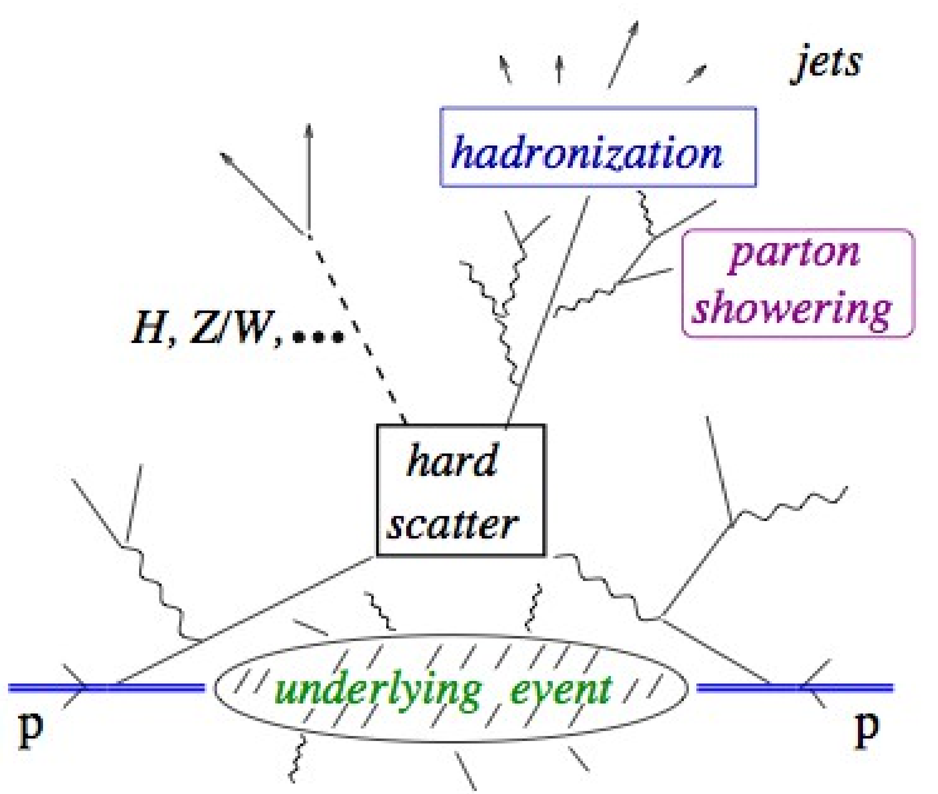
F Hautmann

# TWO LECTURES ON QCD PARTON SHOWERS INCLUDING TRANSVERSE MOMENTUM DEPENDENT PARTON DISTRIBUTIONS



# Lecture 1

MOTIVATION

BASIC QCD FACTORIZATION RESULTS

# Lecture 2

PARTON SHOWERING

EXAMPLES

# Lecture 1

# MOTIVATION

## Electroweak gauge boson hadroproduction spectra:

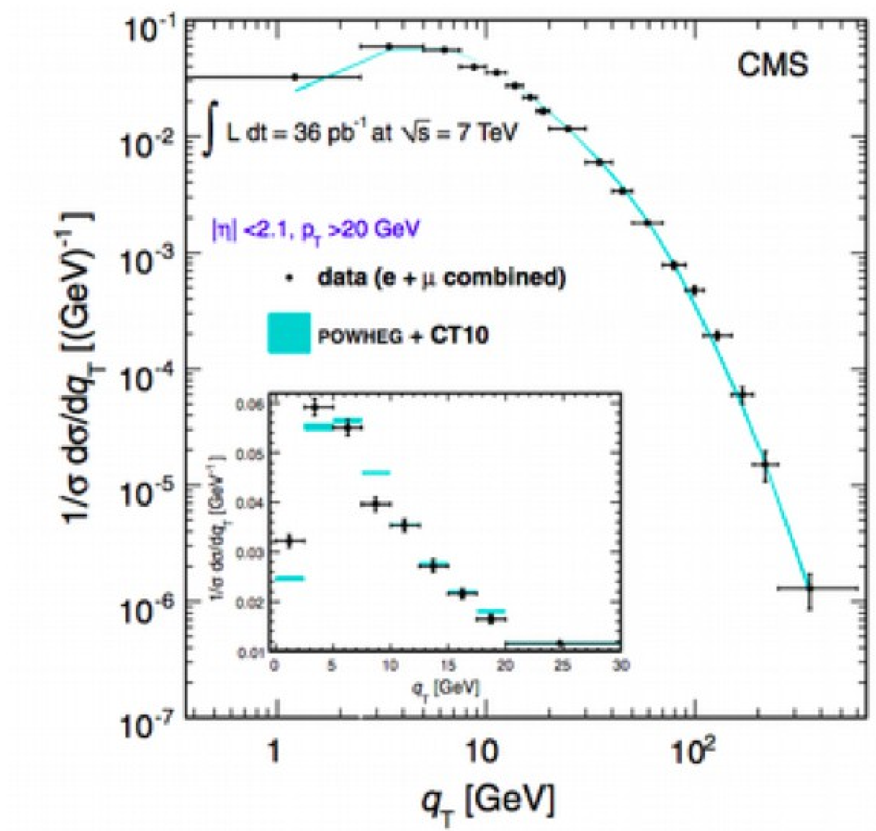
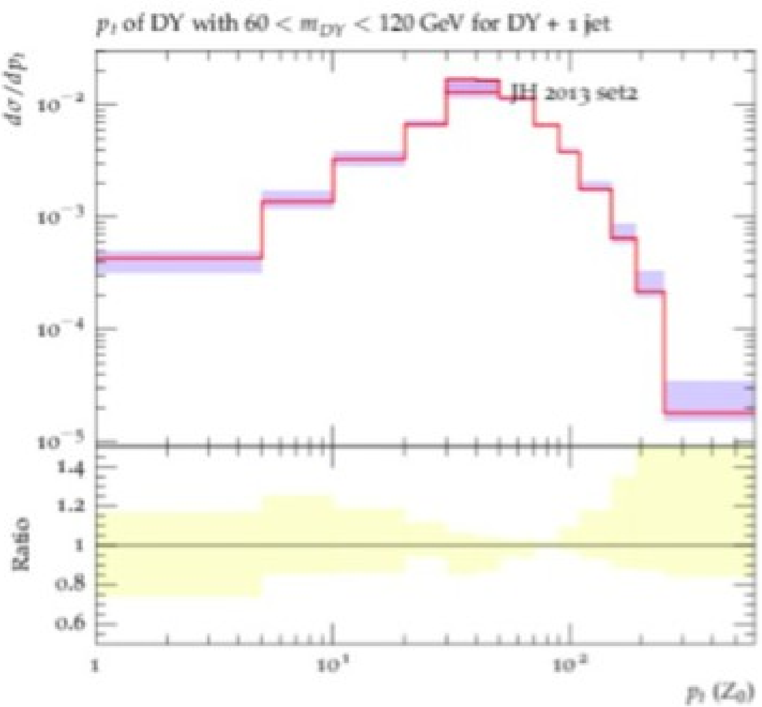



*Fig 2: Differential cross section as a function of the transverse momentum for the production of a Z boson in pp collisions* [6].

# MOTIVATION

## Gauge boson + jet hadroproduction:



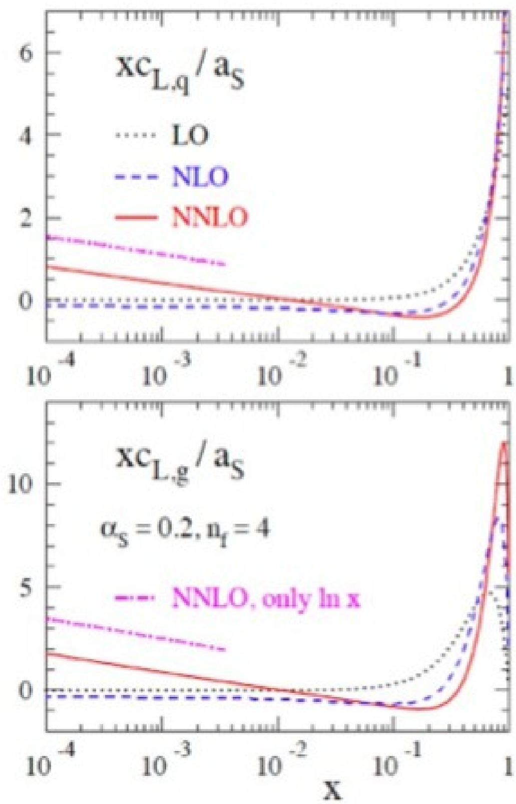

- DY + jet, $p_t{}^{jet} > 30$ GeV
  - peak shifted to larger $p_t$
  - "mini-jet" resummation
  - need TMDs in truly perturbative region

# MOTIVATION

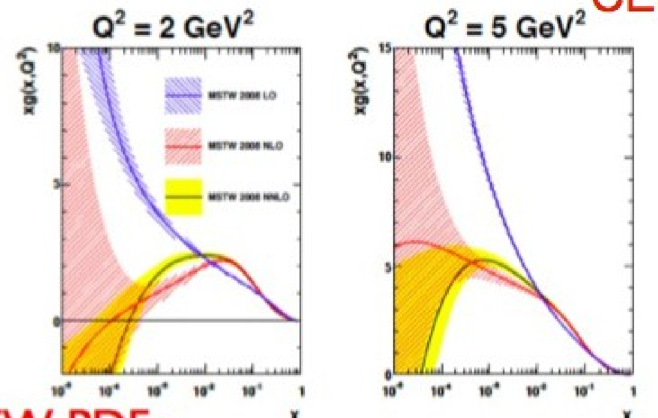
## Proton's structure functions at high energies:

A Stasto, talk at VHECR2014, CERN, Aug 2014

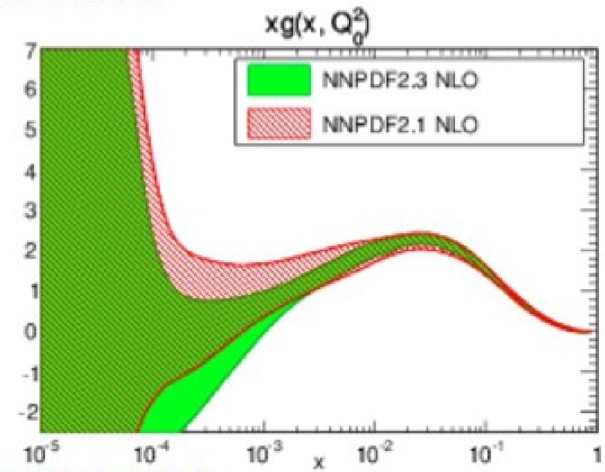
$$x^{-1}F_L = C_{L,\mathrm{ns}} \otimes q_{\mathrm{ns}} + \langle e^2 \rangle \left( C_{L,\mathrm{q}} \otimes q_{\mathrm{s}} + C_{L,\mathrm{g}} \otimes g \right)$$




Moch, Vermaseren, Vogt



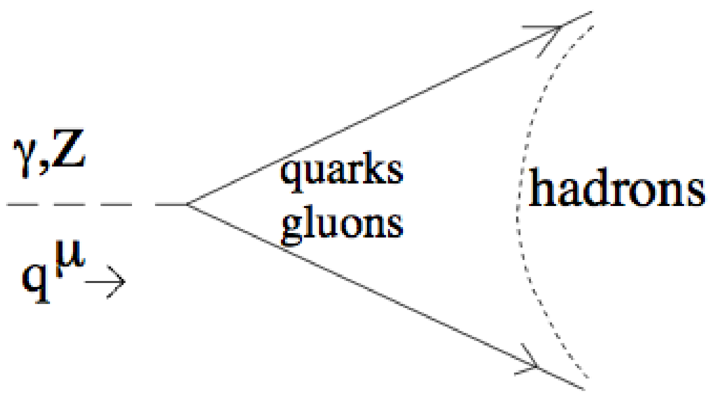

MSTW PDFs




NNPDF2.3 PDFs

# PRINCIPLES OF FACTORIZATION

## A) $\underline{V \to \text{hadrons}}$




$\sqrt{Q^2} = \sqrt{q^\mu q_\mu} \gg$ "hadronic scale" $\approx \Lambda_{\rm QCD} \approx 1$ fermi $^{-1}$

- Separate short-time and long-time dynamics: $(\Delta t)_{partonic} \approx Q^{-1} \ll (\Delta t)_{hadroniz.} \approx \Lambda_{\rm QCD}^{-1}$

$$\Rightarrow \quad P(e^+e^- \to h) = P(e^+e^- \to q\bar{q}) P(q\bar{q} \to h)$$

- Next use completeness : $\sum_h P(i \to h) = 1 \;\Rightarrow$

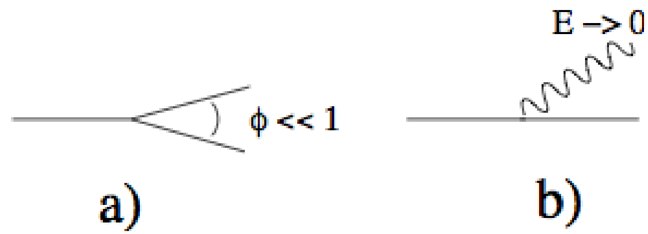
$$\Rightarrow \quad \sigma_{\rm tot}(e^+e^- \to h) \;\equiv\; \sum_h P(e^+e^- \to h)$$

$$= \; P(e^+e^- \to q\bar{q}) \sum_h P(q\bar{q} \to h) = P(e^+e^- \to q\bar{q})$$

▷ almost right — but not quite: rhs is IR-divergent in PT... $\hookrightarrow$

$\hookrightarrow$ particle number nonconservation $\Rightarrow$ add in multi-particle states ($q\bar{q}g$ to 1st order)

$\Rightarrow \sigma(e^+e^- \to q\bar{q}) + \sigma(e^+e^- \to q\bar{q}g)$ insensitive to long-time interactions:




i.e., insensitive to collinear and soft parton emission

• perturbative calculability ( = "IR-safety" )

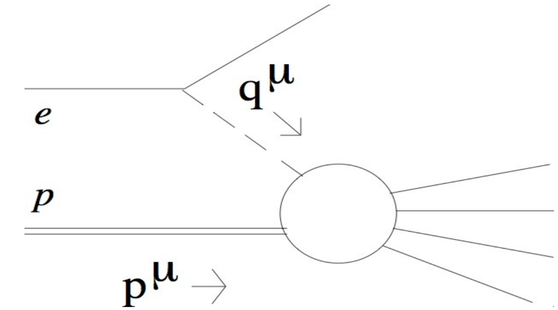
♠ valid to *any* order in $\alpha_s$:

$$\sigma_{\rm tot}(e^+e^- \to h) = \sigma_0 \left[ 1 + \frac{\alpha_s}{\pi} + 1.4092 \left(\frac{\alpha_s}{\pi}\right)^2 - 12.805 \left(\frac{\alpha_s}{\pi}\right)^3 + \dots \right]$$

♠ valid for large classes of *infrared-safe* observables:

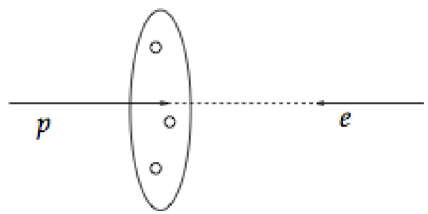
▷ accurate determinations of QCD running coupling $\alpha_s(Q^2)$ from LEP $e^+e^-$ experiments on jet physics (1990's)

## $B$) Hadron scattering. E.g., DIS




- necessarily sensitive to long timescales, BUT

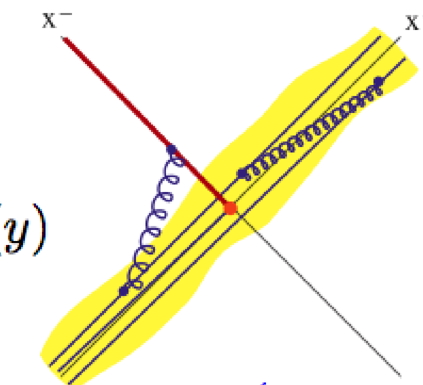
$$\sigma(Q, m) = C(Q, \text{parton momenta } > \mu) \otimes f(\text{parton momenta } < \mu, m)$$




$\delta t_{\text{scatter}} \ll \tau_{\text{parton}}$ in "infinite-momentum" frame

The collinear parton density functions:

$$\text{Pdf's}: \quad f(x, \mu) = \int \frac{dy^-}{2\pi} \, e^{-ixp^+y^-} \, \widetilde{f}(y)$$

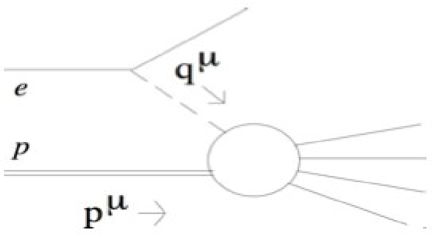



$$\widetilde{f}(y) = \langle \, P \mid \overline{\psi}(y) \, V_y^\dagger(n) \, \gamma^+ \, V_0(n) \, \psi(0) \mid P \, \rangle \quad , \qquad y = (0, y^-, 0)$$

$$V_y(n) = \mathcal{P} \exp\left( i g_s \int_0^\infty d\tau \; n \cdot A(y + \tau \, n) \right)$$

↖ correlation of parton fields at lightcone distances

# Hadron scattering. E.g.: DIS




◇ Renormalization group invariance ⇒

$$\frac{d}{d\ln\mu}\,\sigma = 0 \quad \Rightarrow \quad \frac{d}{d\ln\mu}\,\ln f = \gamma = -\frac{d}{d\ln\mu}\,\ln C$$

↪ DGLAP evolution equations [Altarelli-Parisi
Dokshitzer
Gribov-Lipatov]

$$f = f_0 \times \exp \int \frac{d\mu}{\mu}\,\gamma(\alpha_s(\mu))$$

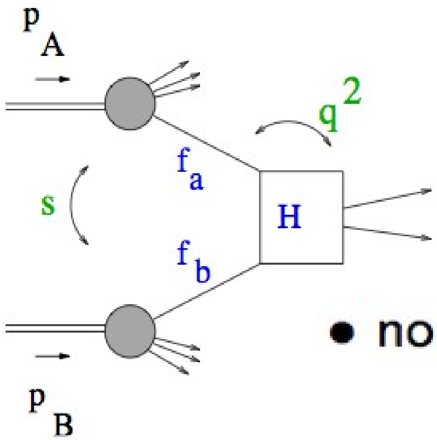
↗ resummation of $(\alpha_s \ln Q/\Lambda_{\mathrm{QCD}})^n$ to all orders in PT

Note: expansions $\gamma \simeq \gamma^{(LO)}\,(1 + b_1\alpha_s + b_2\alpha_s^2 + \ldots)$

$$C \simeq C^{(LO)}\left(1 + c_1\alpha_s + c_2\alpha_s^2 + \ldots\right)$$

give LO, NLO, NNLO, ... logarithmic corrections

# C) Multi-scale hard scattering at LHC energies




$$s \gg q_1^2 \gg \cdots q_n^2 \gg \Lambda_{\mathrm{QCD}}^2$$

- nonperturbative components probed near kinematic boundaries

$$(x \to 0,\ 1 - x \to 0)$$

- more complex, potentially large corrections to all orders in $\alpha_s$, $\sim \ln^k(q_i^2/q_j^2)$

$$e.g.\ \ C \simeq C^{(LO)}\,(1 + c_1\alpha_s + \ldots + c_{n+m}\alpha_s^m(\alpha_s\ L)^n + \ldots)\ ,\ L = \text{"large log"}$$

$\hookrightarrow$ yet summable by QCD techniques that

▷ generalize RG factorization

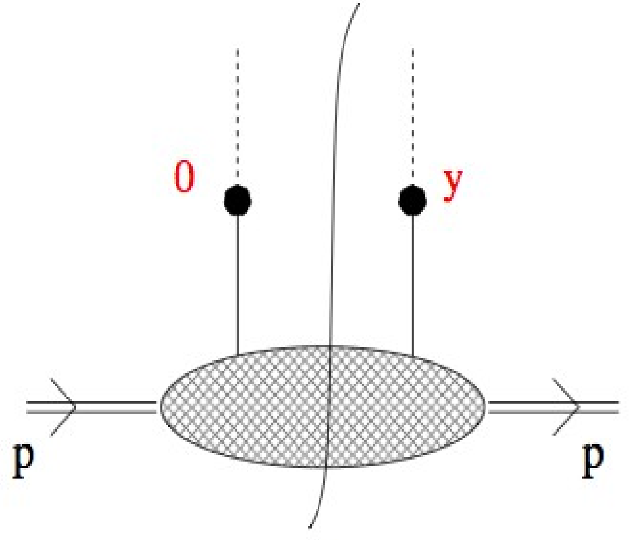
▷ extend parton correlation functions off the lightcone

$\Rightarrow$ unintegrated (or TMD) pdf's

♠ new nonperturbative information; generalized evolution equations

# Transverse momentum dependent (TMD) parton density functions

Generalize matrix element to non-lightlike distances:




$$p = (p^+, m^2/2p^+, 0_\perp)$$

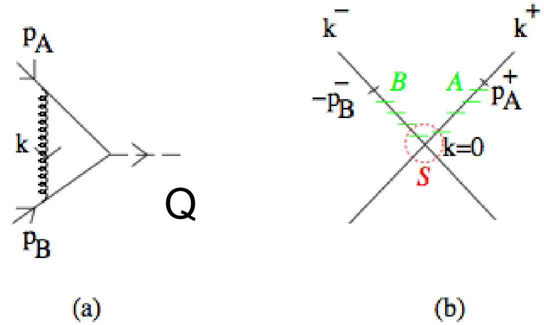
$$\tilde{f}(y) = \langle P \mid \overline{\psi}(y)\, V_y^\dagger(n)\, \gamma^+\, V_0(n)\, \psi(0) \mid P \rangle \quad , \qquad y = (0, y^-, y_\perp)$$

TMD pdf's:

$$f(x, k_\perp) = \int \frac{dy^-}{2\pi} \frac{d^{d-2} y_\perp}{(2\pi)^{d-2}} e^{-ixp^+ y^- + ik_\perp \cdot y_\perp} \tilde{f}(y)$$

# Examples: generalized evolution equations

- Sudakov form factor $S$:

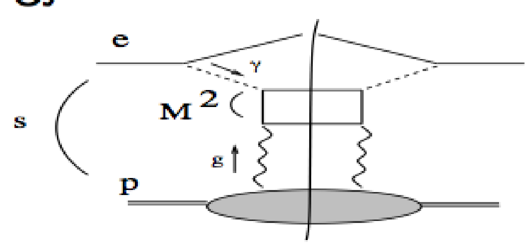



▷ entering Drell-Yan production, W/Z boson $q_\perp$ distribution, . . .

$\Rightarrow \partial S/\partial\eta = K \otimes S$ **CSS evolution equations** [Collins-Soper-Sterman]

↖ resums $\alpha_s^n \ln^m Q/q_T$

- High-energy resummation: $s \gg M^2 \gg \Lambda^2_{\mathrm{QCD}}$




◇ energy evolution: **BFKL equation** [Balitsky-Fadin-Kuraev-Lipatov]

↖ resums $(\alpha_s \ln \sqrt{s}/M)^n$

↪ corrections down by $1/\ln s$ rather than $1/M$

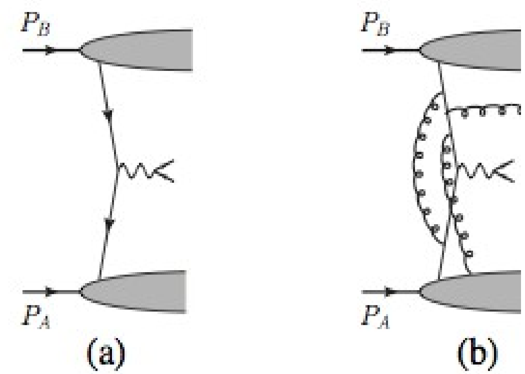
# Example I:
# Drell Yan production at low pT

- CSS formalism

$$\frac{d\sigma}{d^4q} = \sum_{ij} H_{ij}(Q^2/\mu^2, \alpha_s(\mu)) \int d^2b_\perp \, e^{iq_\perp \cdot b_\perp} \; f_i(x_1, b_\perp; \zeta_1, \mu) \; f_j(x_2, b_\perp; \zeta_2, \mu)$$

$$+ \, Y\text{−term} \, + \, \mathcal{O}\left(\Lambda^2_{\rm QCD}/Q^2\right)$$

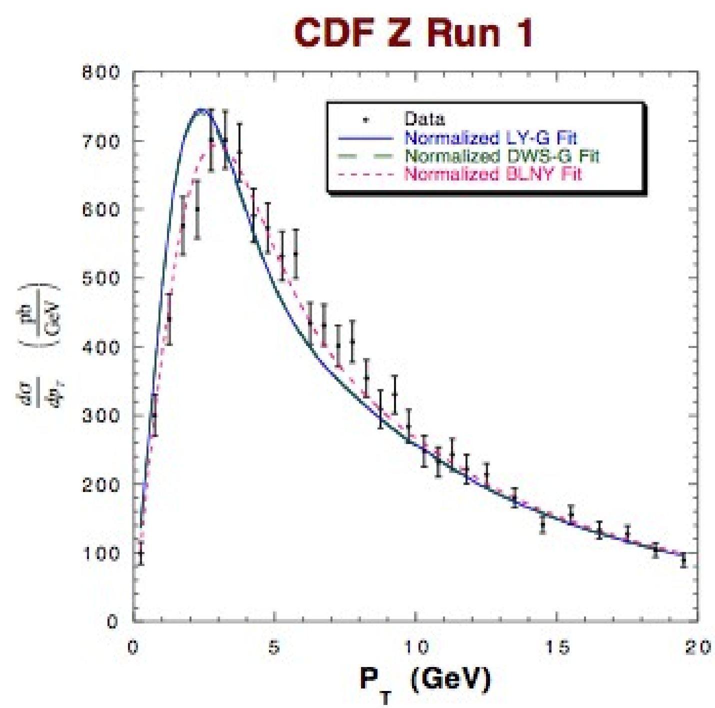
TMD factorization




where $\dfrac{\partial \ln f}{\partial \ln \sqrt{\zeta}} = K(b_\perp, \mu)$ and $\dfrac{d \ln f}{d \ln \mu} = \gamma_f(\alpha_s(\mu), \zeta/\mu^2)$

$$\frac{dK}{d\ln\mu} = -\gamma_K(\alpha_s(\mu))$$

cusp anomalous dimension



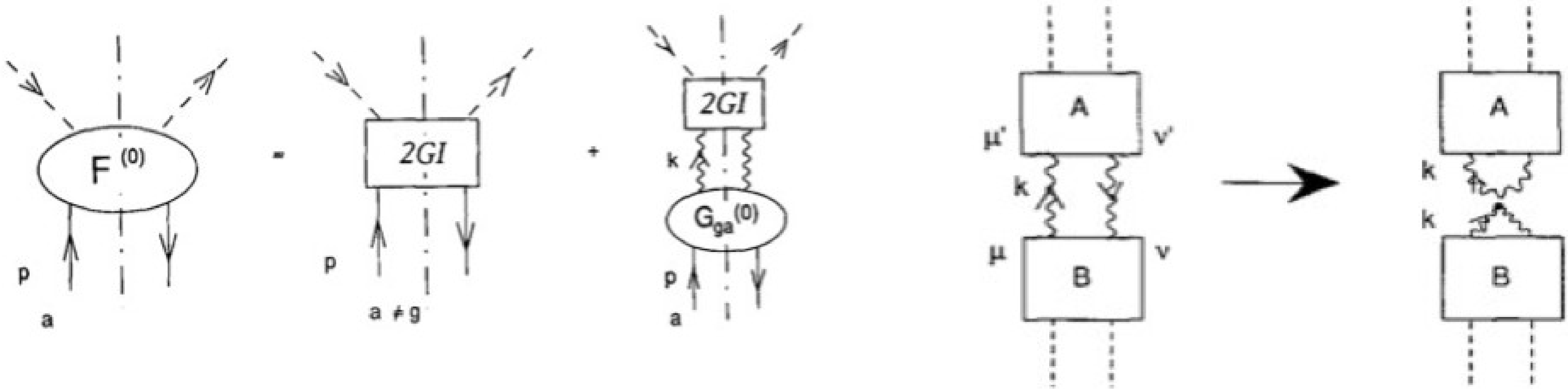

Landry et al.

- Soft Collinear Effective Theory (SCET) provides alternative approach to comparable results

*[Echevarria, Idilbi, Scimemi 2012; Chiu, Jain, Neill, Rothstein 2012; Becher, Neubert 2011; Mantry, Petriello 2011]*

# Example II:

# DIS at high energies




transverse momentum dependent
high-energy factorization :

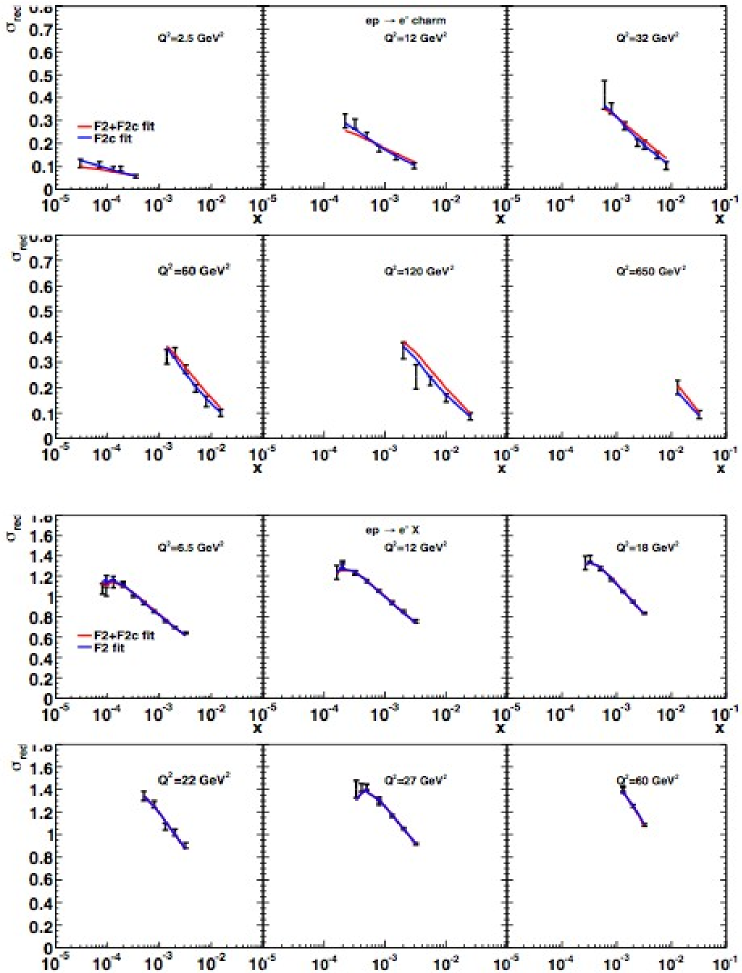
$$F_j(x,Q^2) = \int_x^1 \frac{dz}{z} \int d^{2+2\varepsilon}\mathbf{k} \; \underbrace{\hat{\sigma}_j\left(x/z, \mathbf{k}/Q, \alpha_s(Q/\mu)^\varepsilon, \varepsilon\right)}_{2GI\ kernel} \; \mathcal{A}(z,\mathbf{k},\mu,\varepsilon) \qquad j = 2, L$$

where $\mathcal{A}(z,\mathbf{k},\mu,\varepsilon) = \int \frac{dk^2}{2(2\pi)^{4+2\varepsilon}} P_{\mu\nu}^{(H)} \; G^{\mu\nu}(k,p)$

↖ *unintegrated (TMD) gluon density*

↖ high-energy projector (spin and momentum)

# kT-dependent gluon density from precision DIS data




[Jung & H, Nucl. Phys. B 883 (2014) 1]

- Good description of inclusive DIS data with TMD gluon
- Sea quark yet to be included at TMD level
- Fit performed with herafitter package https://www.herafitter.org/ arXiv:1410.4412 [hep-ph]

| | $\chi^2/ndf\,(F_2^{(\mathrm{charm})})$ | $\chi^2/ndf\,(F_2)$ | $\chi^2/ndf\left(F_2 \text{ and } F_2^{(\mathrm{charm})}\right)$ |
|---|---|---|---|
| 3-parameter | 0.63 | 1.18 | 1.43 |
| 5-parameter | 0.65 | 1.16 | 1.41 |

# "The TMDlib project" http://tmdlib.hepforge.org/

- a platform for theory and phenomenology of TMD pdfs
- library of fits and parameterizations LHApdf style

DESY 14-059
NIKHEF 2014-024
RAL-P-2014-009
YITP-SB-14-24
Dec 2014



**TMDlib and TMDplotter:**
**library and plotting tools for**
**transverse-momentum-dependent parton distributions**

F. Hautmann[1,2], H. Jung[3,4], M. Krämer[3],
P. J. Mulders[5,6], E. R. Nocera[7], T. C. Rogers[8,9], A. Signori[5,6]

[1] Rutherford Appleton Laboratory, UK
[2] Dept. of Theoretical Physics, University of Oxford, UK
[3] DESY, Hamburg, FRG
[4] University of Antwerp, Belgium
[5] Department of Physics and Astronomy, VU University Amsterdam, the Netherlands
[6] Nikhef, the Netherlands
[7] Università degli Studi di Genova and INFN Genova, Italy
[8] C.N. Yang Institute for Theoretical Physics, Stony Brook University, USA
[9] Department of Physics, Southern Methodist University, Dallas, Texas 75275, USA

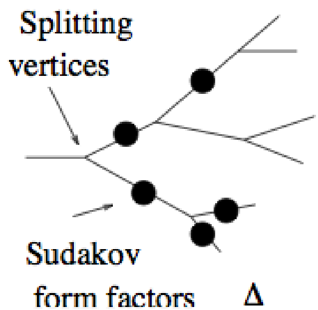
**Abstract**

Transverse-momentum-dependent distributions (TMDs) are extensions of collinear parton distributions and are important in high-energy physics from both theoretical and phenomenological points of view. In this manual we introduce the library TMDlib, a tool to collect transverse-momentum-dependent parton distribution functions (TMD PDFs) and fragmentation functions (TMD FFs) together with an online plotting tool, TMDplotter. We provide a description of the program components and of the different physical frameworks the user can access via the available parameterisations.

# Next lecture's preview:
# FROM QCD TO MONTE CARLO EVENT GENERATORS

- Factorizability of QCD x-sections $\longrightarrow$ probabilistic branching picture

  ◇ QCD evolution by "parton showering" methods:

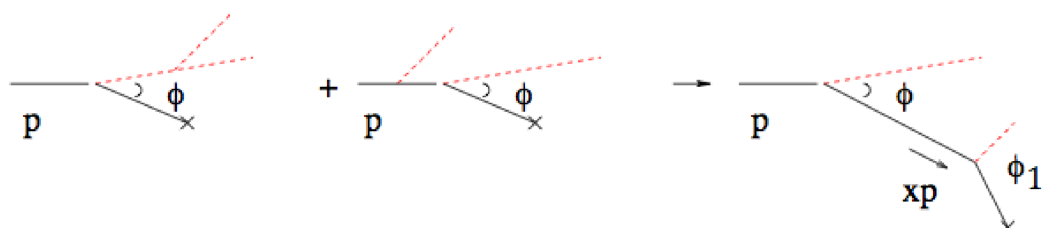



$$d\mathcal{P} = \int \frac{dq^2}{q^2} \int dz \; \alpha_S(q^2) \; P(z) \; \Delta(q^2, q_0^2)$$

$\hookrightarrow$ collinear, incoherent emission

◇ Soft emission $\longrightarrow$ interferences $\longrightarrow$ ordering in decay angles:

$\hookrightarrow$ gluon coherence for $x \sim 1$




◇ Gluon coherence for $x \ll 1$ $\Rightarrow$ corrections to angular ordering:

$\hookrightarrow$ $k_\perp$-dependent parton showers

CCFM equation is TMD branching equation which contains both Sudakov physics and BFKL physics