Monte Carlo Techniques and Event Generation
Lecture 3: Hadronization & Underlying Event

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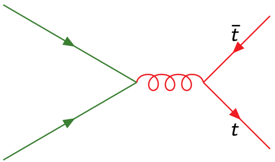
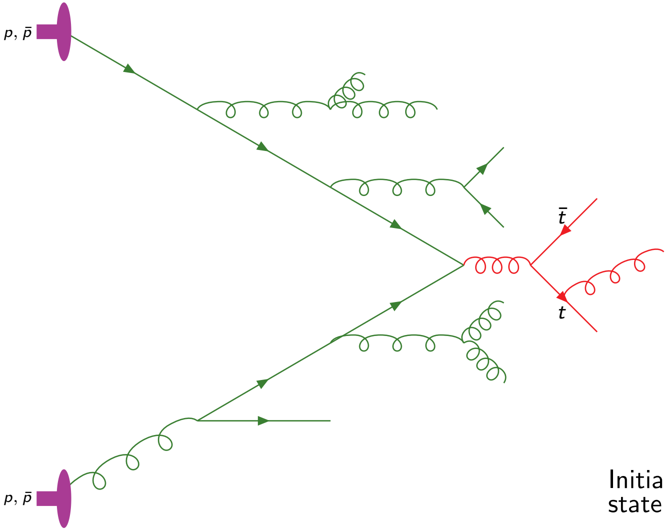
A Monte Carlo Event



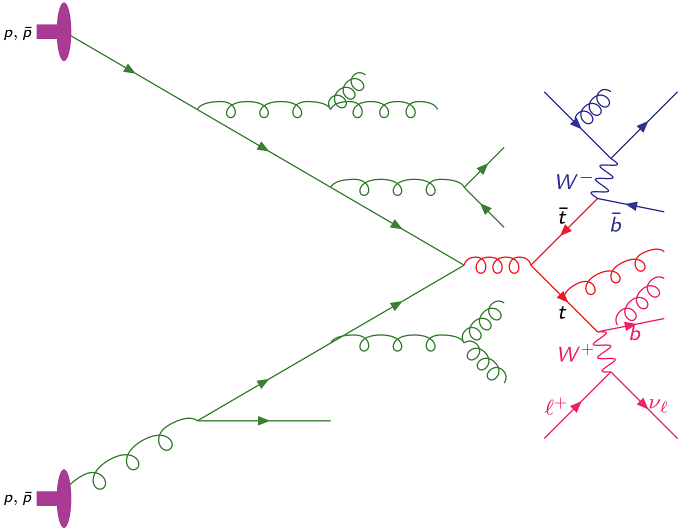
Hard Process, usually calculated at leading order

A Monte Carlo Event



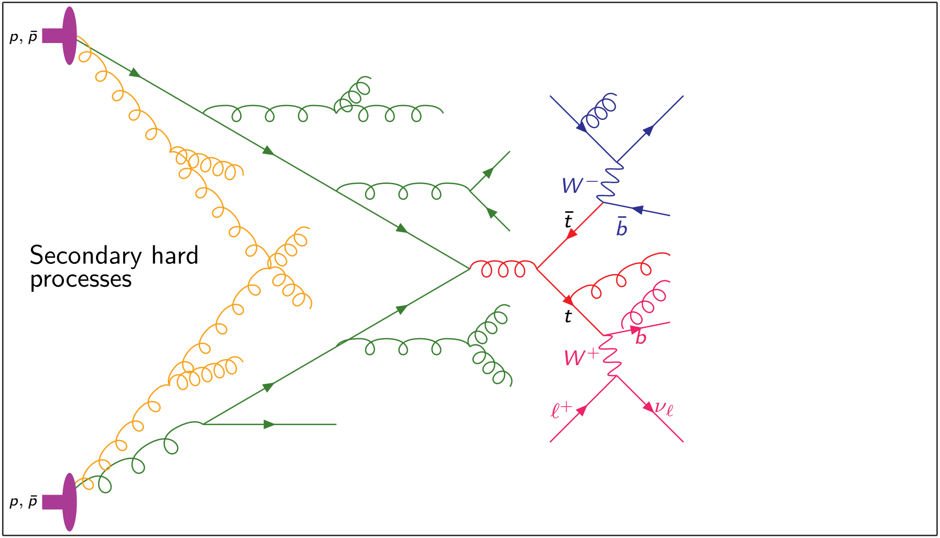
Initial- and final-state parton shower

A Monte Carlo Event

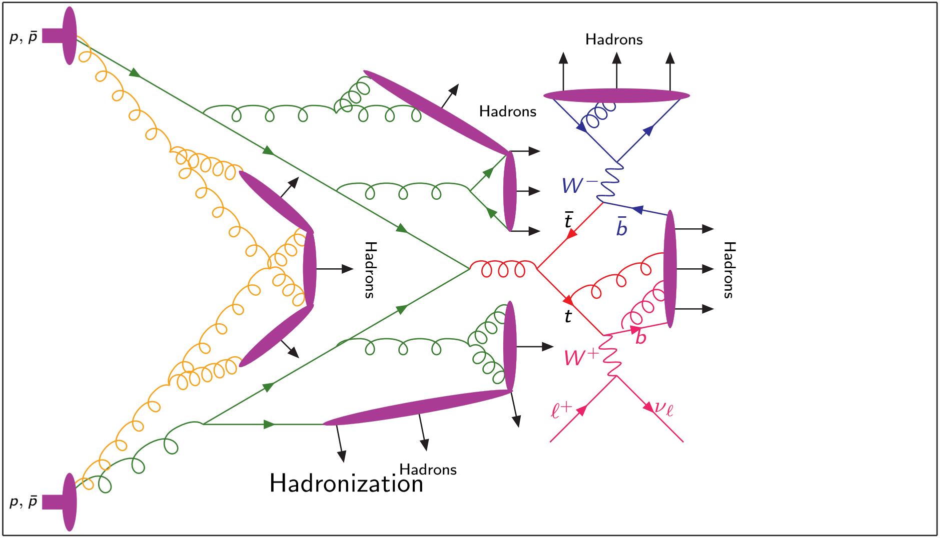


Perturbative decays of heavy particles

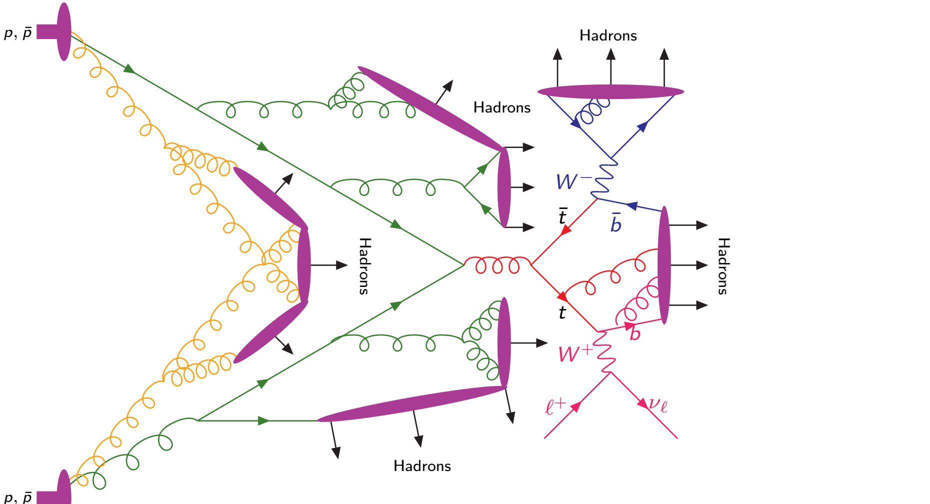
A Monte Carlo Event



A Monte Carlo Event



A Monte Carlo Event



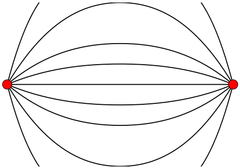
Hadron Decays

Hadronization

- Partons aren't physical particles: they can't propagate freely.
- We therefore need to describe the transition of the quarks and gluons in our perturbative calculations into the hadrons which can propagate freely.
- We need a phenomenological model of this process.
- There are two models which are commonly used:
  - Lund String Model;
  - Cluster Model.

Confinement

- We know that at small distances we have asymptotic freedom and the force between a quark-antiquark pair is like that between an e^+e^- pair.



- But at long distances the self interactions of the gluons make the field lines attract each other.

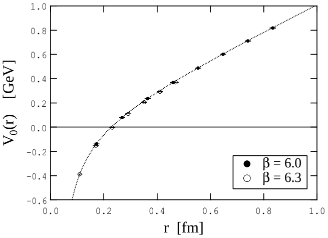
Confinement

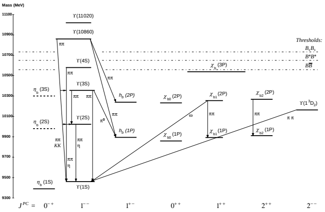
- Gives $1/r$ potential at short distances
- Linear potential at long distances and confinement.

$$V(r) \sim -\frac{4}{3}\frac{\alpha_S}{r} + \kappa r \sim -\frac{0.13}{r} + r$$

for $\alpha_S \sim 0.5$, r in fm and V in GeV.

- Either phenomenogically from quarkonium or lattice QCD.





Lund String Model

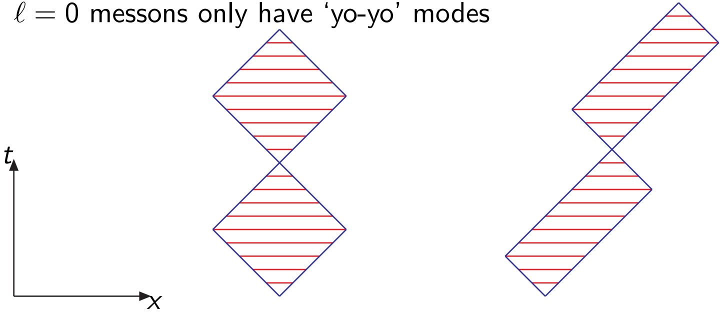
- Assume $\frac{1}{r}$ important for hadron structure but not production.
- In QCD the field lines seem to be compressed into a tube-like region, looks like a string.
- So we have linear confinement with a string tension,

$$F(r) \approx \text{const} = \kappa \approx 1\,\text{Gev/fm}.$$

- Separate the transverse and longitudinal degrees of freedom gives a simple description as a 1+1 dimensional object, the string, with a Lorentz invariant formalism.

Mesons

- In the string model mesons are light $q\bar{q}$ pairs connected by a string.
- $\ell = 0$ messons only have 'yo-yo' modes



- Area law $m^2 = 2\kappa^2$.

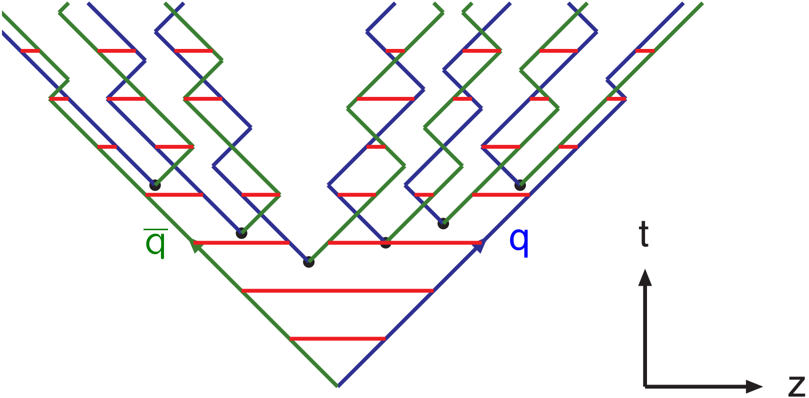
Lund String Model

- Start by considering a $q\bar{q}$ pair produced in e^+e^- annihilation.
- Ignore gluon radiation for the time being.
- q and $\bar{q}$ joined by a string.
- $q\bar{q}$ pairs are created by tunnelling in the intense chromomagnetic field of the string.

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}x\mathrm{d}t} \propto \exp\left(-\pi\frac{m_q^2}{\kappa}\right)$$

- The string breaks into mesons long before the yo-yo point.
- Gives a simple but powerful picture of hadron production.

Lund String Model



Lund Fragmentation Function

- Fermi motion is a gaussian transverse momentum distribution
- The tunnelling probability becomes

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}x\mathrm{d}t} \propto \exp\left[-b\left(m_q^2 + p_\perp^2\right)\right]$$

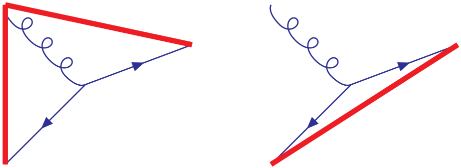
- The string picture constrains the fragmentation function
  - Lorentz invariance
  - Acausality
  - Left–right symmetry
- The function has the form

$$f(z) \propto z^{a_\alpha - a_\beta - 1}(1-z)^{a_\beta}$$

  where $a_{\alpha,\beta}$ are adjustable parameters for quarks α and β.
- a, b and m_q are the main tuneable parameters of the model.

Three-jet Events

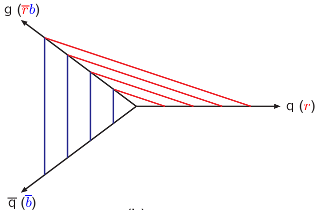
- So far we have only considered the hadronization of $q\bar{q}$ pairs, what about gluons?
- The gluon gives a kink on the string.

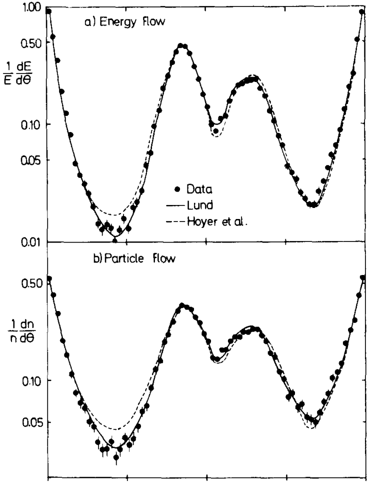


- the string effect
- The string model has an infrared safe matching with the parton shower.
- Gluons with $k_\perp < \frac{1}{\text{string width}}$ irrelevant.

String Effect

- Less radiation between the quark and antiquark.
- Either non-perturbatively via the string model.
- Can get the same result perturbatively via colour coherence.



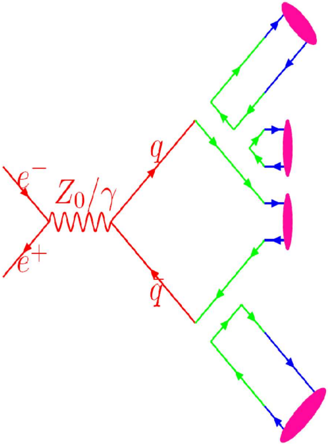


Summary of the String Model

- String model strongly physically motivated.
- Very successful fit to data.
- Universal: fitted to e^+e^- data little freedom elsewhere.
- How does motivation translate to prediction?
- $\sim$one free parameter per hadron/effect!
- Blankets too much perturbative information?
- Can we get by with a simpler model?

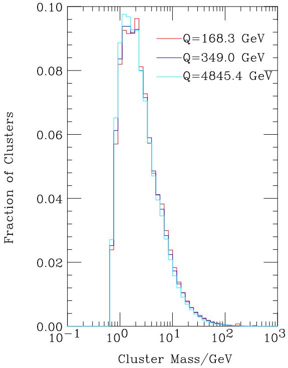
Preconfinement

- In the planar approximation, large number of colours limit: Gluon = colour-anticolour pair
- We can follow the colour structure of the parton shower.
- At the end colour-singlet pairs end up close in phase space.
- Non-perturbatively split the gluons into quark-antiquark pairs.



Preconfinement

- The mass spectrum of colour-singlet pairs is asymptotically independent of energy and the production mechanism.
- It peaks at low mass, of order the cut-off Q_0.
- Decreases rapidity for large cluster masses.



Cluster Model

- Project the colour-singlet clusters onto the continuum of high-mass mesonic resonances (=clusters).
- Decay to lighter well-known resonances and stable hadrons using a pure 2-body phase-space decay and phase space weight.

$$W \propto (2s_1+1)(2s_2+1)\frac{2p^*}{m}$$

- The hadron-level properties are fully determined by the cluster mass spectrum, i.e. by the properties of the parton shower.
- Heavier hadrons, including baryons and strange hadrons supressed.
- The cut-off Q_0 is the crucial parameter of the model.

Cluster Model: Problems

1. Tail of high-mass clusters for which cluster decay is not a good approximation.
   - Split heavy clusters into two lighter clusters along "string" direction.
   - $\sim 15\%$ of clusters in e^+e^- collisions at m_Z but gives$\sim \frac{1}{2}$ of the hadrons.
2. Sensitivity to particle content.
   - only include complete multiplets.
   - change model so adding new heavy particles doesn't effect decay of light clusters.
3. Leading hadrons are too soft
   - Perturbative quarks remember their direction

$$P(\theta^2) \sim \exp\left(-\frac{\theta^2}{2\theta_0^2}\right)$$

   - String like and extra parameter.
4. Problems with particle correlations.

The "Beliefs"

- There are two main schools of thought in the event generator community.

PYTHIA

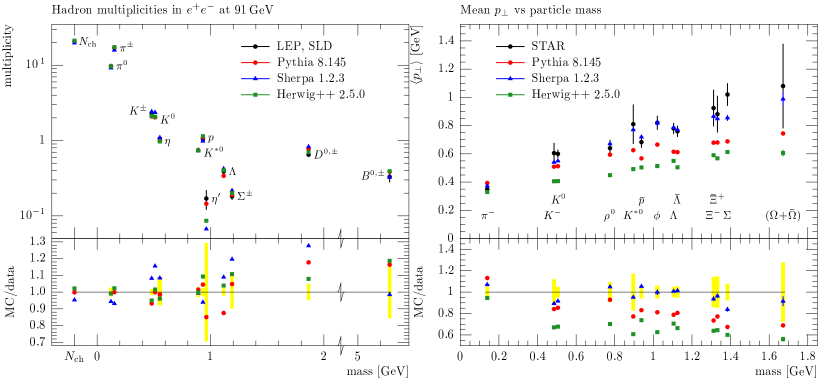
- Hadrons are produced by hadronization. You must get the nonperturbative dynamics right.
- Better data has required improvements to the perturbative simulation.

HERWIG

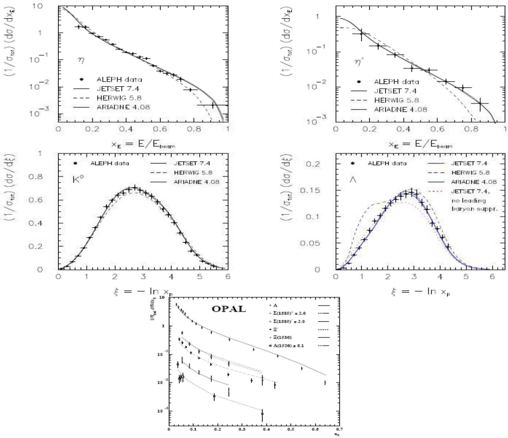
- Get the perturbative physics right and any hadronization model will be good enough
- Better data has required changes to the cluster model to make it more string-like.

- There ain't no such thing as a good parameter-free description.

Hadrochemistry



Identified Particle Spectra

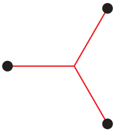


The facts?

- All the generators give good agreement for event shapes.
- HERWIG has less parameters to tune the flavour composition and tends to be worse for identified particle spectra.
- Baryon production is often a problem.

Baryon Production

- All the models have some problems with baryon production.
- In the Lund model baryons are picture as quark quarks attached to a common centre, a colour source/sink



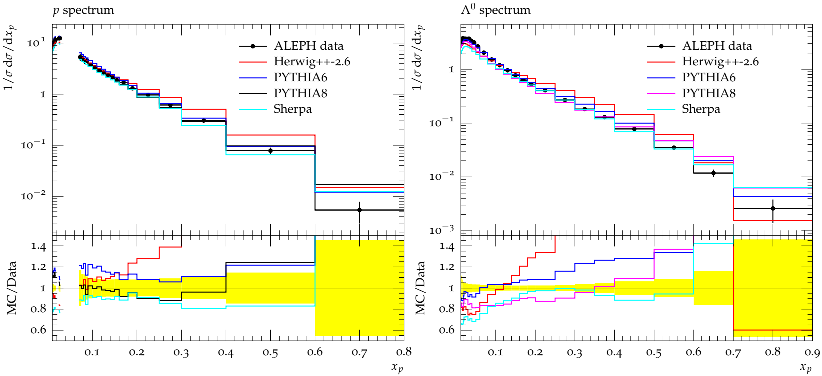
- At large separation two of the quarks are tightly bound, a diquark.

- The diquark is treated as a colour antitriplet ($3 \otimes 3 = \bar{3} \oplus 6$)

Baryon Production

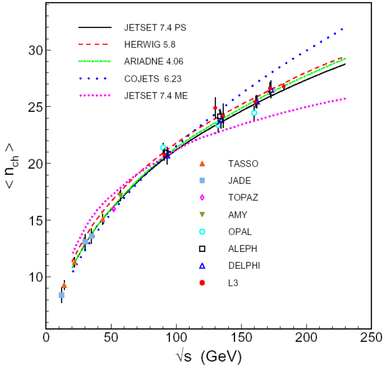
- Two quarks can tunnel nearby in phase space: baryon–antibaryon pair
- In the string model either use diquarks, with an extra parameter for each diquark.
- or the pop-corn model.
- In the cluster model allow diquarks to be produced in cluster decay (always) or non-pertrubative gluon splitting (allowed in some variants).

Baryon Production



Universality

- Evolution to a universal, low hadronization scale ensures the hadronization parameters are universal.
- Don't need to retune at each energy.
- Only have to tune the new hadron specific parameters in hadronic collisions.



Hadron Properties

- Hadronization produces hadrons so we need both the hadron properties: quark content; spin; mass; width; etc..
- and to decide which hadrons to produce.
- Many of the hadrons produced during hadronization (primary hadrons) are unstable so we also need to know how they decay to secondary hadrons.
- Not just a matrer of typing in the PDG:
  - not all resonances in a given multiplet have been measured;
  - measured branching fractions rarely add up to exactly 100%;
  - measured branching fractions rarely exactly respect isospin;
- Also need to make a lot of choices for the matrix elements to describe the various decay modes.

Hadron Properties

- Often not even numerical values for partial widths.
- Particles decaying into final-states which are'nt allowed for on-shell masses, *e.g.* $h_1' \to K\bar{K}^*$.
- In some cases the choice of decay modelling effects the decay tables, *e.g.* $a_1 \to \rho\pi$ vs. $a_1 \to \pi\pi\pi$.

$h_1(1170)$ $I^G(J^{PC}) = 0^-(1^{+-})$

$h_1(1170)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\rho\pi$	seen

$h_1(1380)$ $I^G(J^{PC}) = ?^-(1^{+-})$

OMITTED FROM SUMMARY TABLE
Seen in partial-wave analysis of the $K\overline{K}\pi$ system. Needs confirmation.

$h_1(1380)$ DECAY MODES

	Mode
Γ_1	$K\overline{K}^*(892)+$ c.c.

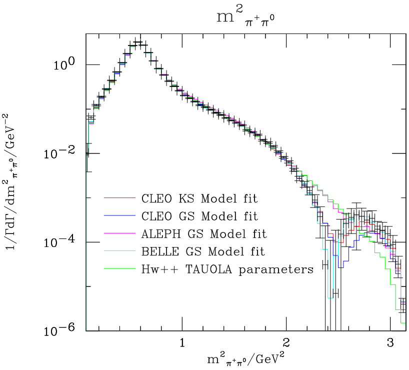
$a_1(1260)$ [k] $I^G(J^{PC}) = 1^-(1^{++})$

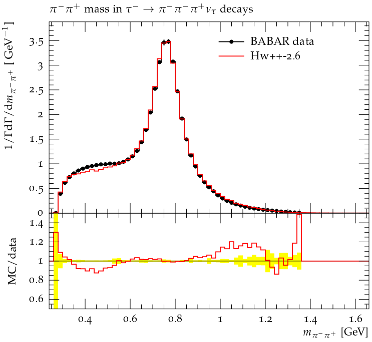
$a_1(1260)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$(\rho\pi)_{S-\text{wave}}$	seen	353
$(\rho\pi)_{D-\text{wave}}$	seen	353
$(\rho(1450)\pi)_{S-\text{wave}}$	seen	†
$(\rho(1450)\pi)_{D-\text{wave}}$	seen	†
$\sigma\pi$	seen	–
$f_0(980)\pi$	not seen	179
$f_0(1370)\pi$	seen	†
$f_2(1270)\pi$	seen	†
$K\overline{K}^*(892)+$ c.c.	seen	†
$\pi\gamma$	seen	608

Hadron Decays

- FORTRAN event generators typically used external packages:
  - TAUOLA τ lepton decays;
  - PHOTOS QED radiation in decays;
  - EVTGEN hadron, especially B meson decays.
- Originally expected more of this in the new generation of programs.
- But better modelling requires passing more information between the different stages of event generation.
- Also many problems with interfaces.
- Net result: better simulation of hadron and τ lepton decays in all the new event generators and less use of external packages.

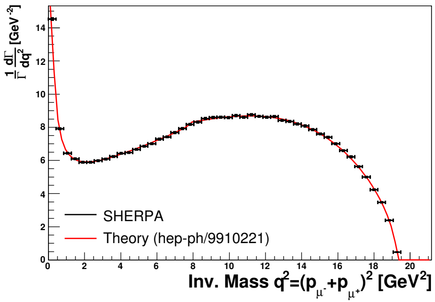
$\tau \to \rho(a_1)\nu_\tau \to \pi\pi(\pi)\nu_\tau$



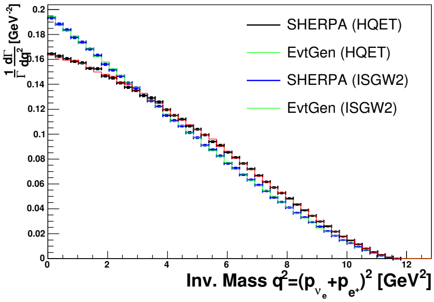


B decays

$B^+ \to K^{*+}\mu^+\mu^-$



$B \to \bar{D}\ell\nu_\ell$

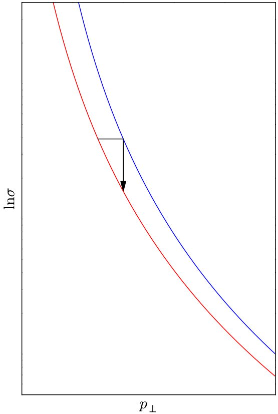


Summary

- Hadronization is described by non-perturbative models.
- Modern hadronization models give a good description of a wide range of processes.
- The parameters are universial allowing predictions once they are tuned to data.
- Don't forget about the hadron properties and decays.

Introduction

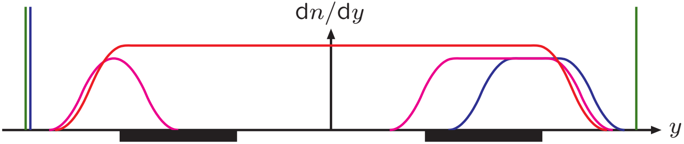
- As well as the hard perturbative scattering there is additional hadronic activity.
- This must be modelled as it is both observable and can have a large effect on jet energies.
- Before we can discuss the models we will first need to understand the definitions of the various types of event.
- We will then discuss the various different models.



Hadronic Cross Sections

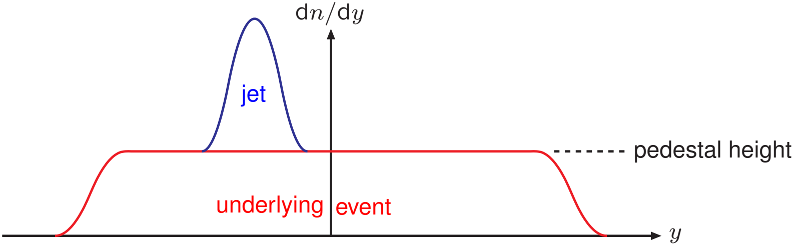
- The total hadronic cross section consists of various components

$$\sigma_{\text{total}} = \sigma_{\text{elastic}} + \sigma_{\text{single-diffractive}} + \sigma_{\text{double-diffractive}} + \cdots + \sigma_{\text{non-diffractive}}$$



- Experimentally minimum bias $\approx$ all events with no bias from trigger conditions
- Theoretically $\sigma_{\text{min-bias}} \approx \sigma_{\textit{double-diffractive}} + \sigma_{\textit{non-diffractive}}$

Hadronic Cross Sections



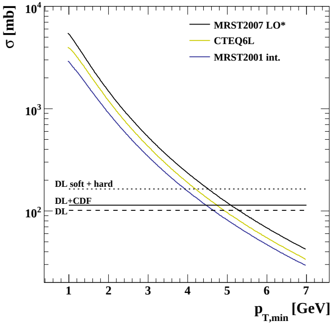
- The underlying is the additional activity from soft interactions in additional to the primary hard partonic process.
- This is a theoretical definition and such a separation is model dependent.
- However we except the description to be similar to the one we need for the bulk of non-diffractive events.

Multiparton Interaction Models

- The cross-section for $2 \to 2$ scattering is dominated by t-channel channel gluon exchange.
- It diverges like

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2} = \frac{1}{p_\perp^4} \quad \text{for} \quad p_\perp \to 0$$

- This must be regulated used a cut $p_\perp > p_\perp^{\min}$.



- For small values of $p_\perp^{\min}$ this is larger than the total hadron–hadron cross section.
- More than one parton-parton scattering per hadron collision.

Matter Distribution

- Hadrons are extended objects so we also need the matter distribution.
- Assume the dependence in x ($\parallel$ to the beam) and b ($\perp$ to the beam) factorizes

$$G_i(x, \vec{b}; \mu^2) = f_i(x; \mu^2) S(\vec{b}).$$

and the n-parton distributions are "independent"

$$G(x_i, x_j, \vec{b}_i, \vec{b}_j, \mu^2) = G_i(x_i, \vec{b}_i; \mu^2) G_j(x_j, \vec{b}_j; \mu^2)$$

Matter Distribution

- The inclusive cross section for $pp \to \mathrm{jets}$ is

$$\sigma_{\mathrm{inc}} = \int_{p_\perp^{\mathrm{min}}}^{\frac{E_{\mathrm{CMF}}}{2}} \int \mathrm{d}x_1 \int \mathrm{d}x_2 \sum_{ij} f_i(x_1, p_\perp^2) f_j(x_2, p_\perp^2) \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_\perp}$$

- The b dependence from

$$A(b) = \int \mathrm{d}^2 b_1 S(b_1) \int \mathrm{d}^2 b_2 S(b_2) \delta(b - b_1 + b_2)$$

is normalised such that $\int \mathrm{d}b^2 A(b) = 1$.
- If we assume the separate scatters are uncorrelated, *i.e.* they obey Poissonian statistics.
- The average number of scatters per event is

$$\langle n \rangle = \frac{\sigma_{\mathrm{inc}}}{\sigma_{\mathrm{nd}}}.$$

Matter Distribution

- Alternatively the probability of m scatters is

$$P_m = \frac{[A(b)\sigma_{\rm inc}]^m}{m!} \exp\left(-A(b)\sigma_{\rm inc}\right).$$

- The total cross (non-diffractive) cross section is

$$\sigma_{\rm nd} = \int {\rm d}b^2 \sum_{m=1}^{\infty} P_m = \int {\rm d}b^2 \left[1 - \exp\left(-A(b)\sigma_{\rm inc}\right)\right]$$

- Therefore

$$\langle n \rangle = \frac{\int {\rm d}b^2 \sum_{m=1}^{\infty} m P_m}{\int {\rm d}b^2 \sum_{m=1}^{\infty} P_m} = \frac{\int {\rm d}b^2 \langle n(b) \rangle}{\int {\rm d}b^2 \left[1 - \exp\left(-\langle n(b) \rangle\right)\right]} = \frac{\sigma_{\rm inc}}{\sigma_{\rm nd}}$$

Matter Distribution

- Use either the electromagnetic form factor

$$S_P(\vec{b}) = \int \frac{\mathrm{d}^2\vec{k}}{2\pi} \frac{e^{i\vec{k}\cdot\vec{b}}}{1+|\vec{k}|}$$

giving

$$A(b) = \frac{\mu^2}{96\pi}(\mu b)^2 K_3(\mu b).$$

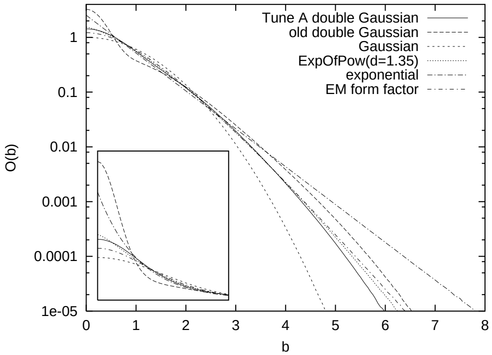
- or an empirical double Gaussian double Gaussian

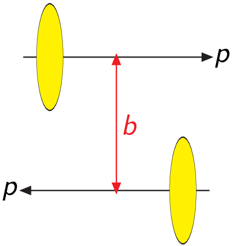
$$\rho_{\mathrm{matter}}(r) = N_1 \exp\left(-\frac{r^2}{r_1^2}\right) + N_2 \exp\left(-\frac{r^2}{r_2^2}\right)$$

where $r_1 \neq r_2$ gives "hot spots" and

$$A(b) = \int \mathrm{d}^3\mathrm{d}t\,\rho_{1,\mathrm{matter}}^{\mathrm{boosted}}(x,t)\rho_{2,\mathrm{matter}}^{\mathrm{boosted}}(x,t)$$

Matter Distribution





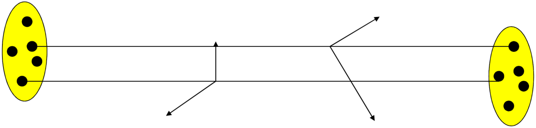
- Average activity at b proportional to $A(b)$
- Central collisions more active, broader than Poissonian
- Peripheral collisions normally give few if any collisions.

Multiparton Interaction Models

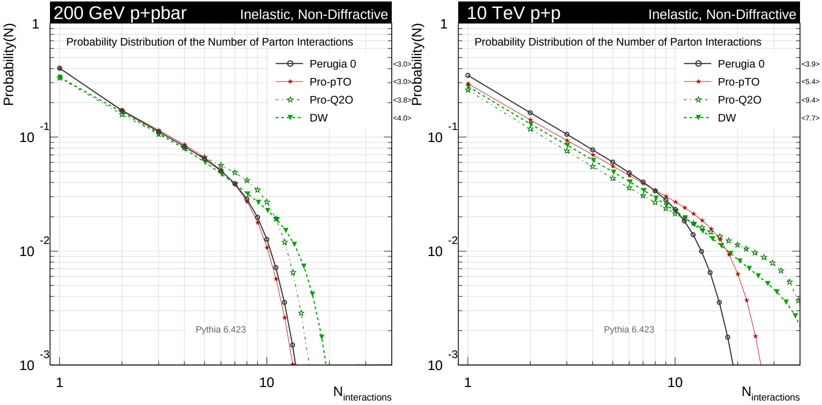
- If the interactions occur independently obeys Poissonian statistics

$$P_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

- However energy-momentum conservation tends to suppressed large numbers of parton scatterings.



Number of Interactions



PYTHIA Model

- Don't use a strict cut-off in $p_\perp$

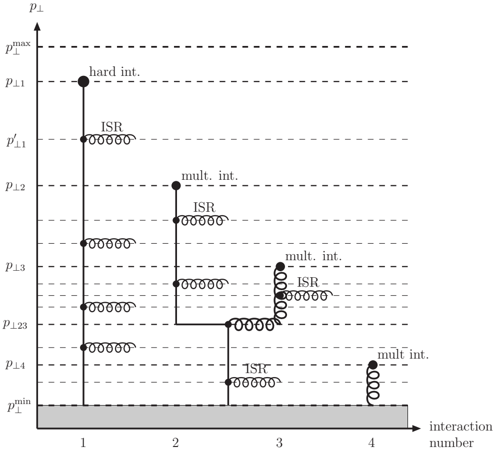
$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2} \propto \frac{\alpha_S^2(p_\perp^2)}{p_\perp^4} \to \frac{\alpha_S^2(p_\perp^2)}{(+pp_\perp^2)^2}$$

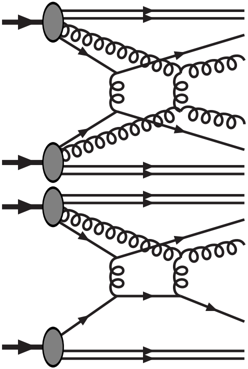
- double Gaussian matter distribution,
- PDFfs rescaled for momentum conservation
- Trace flavour content of remnant, including baryon number.
- Colour arrangement among outgoing partons
- Interactions ordered in decreasing $p_\perp$, and evolution interleaved with ISR

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}p_\perp} = \left(\frac{\mathrm{d}\mathcal{P}_{\mathrm{MPI}}}{\mathrm{d}p_\perp} + \frac{\mathrm{d}\mathcal{P}\mathrm{ISR}}{\mathrm{d}p_\perp}\right) \exp\left(-\int_{p_\perp}^{p_{\perp,i-1}} \left[\frac{\mathrm{d}\mathcal{P}_{\mathrm{MPI}}}{\mathrm{d}p_\perp} + \frac{\mathrm{d}\mathcal{P}\mathrm{ISR}}{\mathrm{d}p_\perp}\right] \mathrm{d}p'_\perp\right)$$

- Includes rescattering

PYTHIA Model





Herwig++ Model

- In terms of the eikonal function $\chi(b, s)$.

$$\sigma_{\rm tot} = 2\int_0^\infty \mathrm{d}b^2 \left[1 - e^{-\chi(b,s)}\right] \quad \sigma_{\rm ela} = \int_0^\infty \mathrm{d}b^2 \left|1 - e^{-\chi(b,s)}\right|^2$$

$$\sigma_{\rm inel} = \int_0^\infty \mathrm{d}b^2 \left[1 - e^{-2\chi(b,s)}\right]$$

- Take eikonal + partonic scattering seriously

$$\sigma_{\rm tot} = 2\int \mathrm{d}^2 b \left(1 - \exp\left[-\frac{1}{2}A(b)\sigma_{\rm inc}\right]\right)$$

- Given the form of the matter distribution predict $\sigma_{\rm inc}$

Herwig++ Model

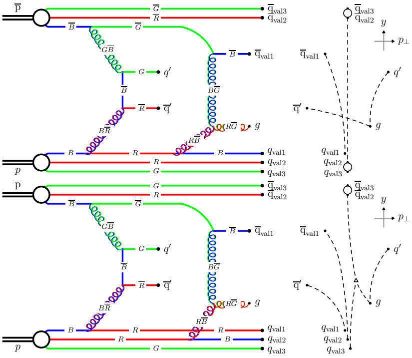
- Too restrictive

$$\sigma_{\rm tot} = 2\int \mathrm{d}^2 b \left(1 - \exp\frac{1}{2}\left[A_{\rm soft}(b)\sigma_{\rm soft,inc} + A_{\rm hard}(b)\sigma_{\rm hard,inc}\right]\right)$$

- Gives two free parameters.
- Independent perturbative scattering above $p_\perp^{\rm min}$
- Gluon scattering below $p_\perp^{\rm min}$ with $\sigma_{\rm soft,inc}$ and a Gaussian $p_\perp$ distribution.
- $\frac{\mathrm{d}\sigma}{\mathrm{d}p_\perp}$ continuous at $p_\perp^{\rm min}$.
- Includes colour reconnection of the partons in clusters produced via MPI.

Colour Correlations

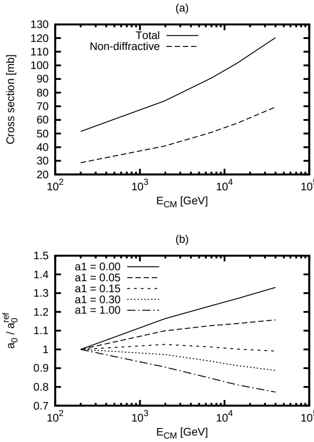
- Colour correlations can have a big influence on the final state.
- In particular $\langle p_\perp \rangle$ vs n_{ch} is very sensitive to the colour flow.
- Long string to remnants many charged particles
- Short strings less charged particles.



x-Dependent Matter Distributions

- Most models have a factorization of the x and b matter dependence.
- Corke & Sjöstrand JHEP 1105 (2011) 009 consider a Gaussian matter distribution with width

$$a(x) = a_0 \left(1 + a_1 \ln \frac{1}{x}\right)$$



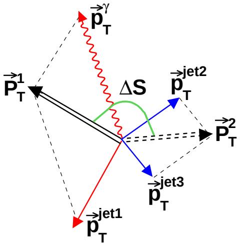
Measurements

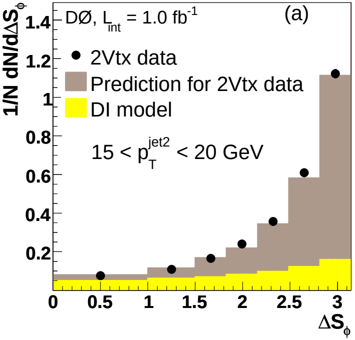
- In principle all measurements at hadron collisions can be sensitive to the underlying event.
- There are three main types of measurement which are used to study, constrain, and fit the parameters of the models.
  1. Measurements which are sensitive to a second hard scattering of a particular type.
  2. Measurements of particle numbers, $p_\perp$, etc. in phase-space regions where we don't expect perturbative radiation in hard events.
  3. Measurements of min-bias events.

Double-Parton Scattering

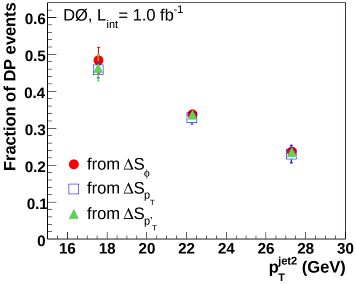
- Look at γ+jets events.
- One pure QCD scattering and one γ+jet.
- Define an effective cross section s.t.

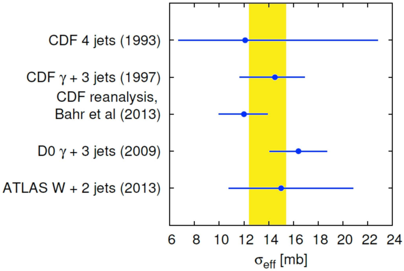
$$\sigma_{ab} = \frac{\sigma_a \sigma_b}{\sigma_{\rm eff}}$$





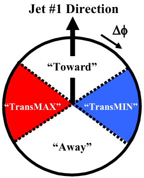
Double-Parton Scattering

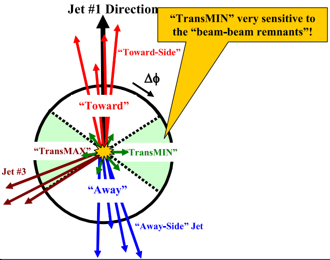




Underlying event measurements

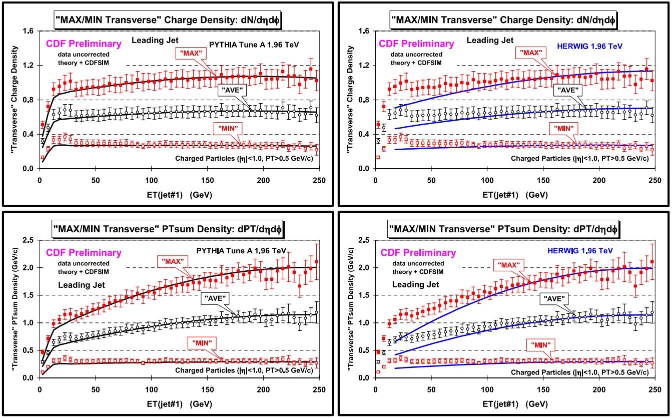
- Classic approach is to define the event using a hard jet, or other particle, *e.g.* Z^0.
- The define toward, away, transverse max and transverse min regions.
- The transverse min region is most sensitive to the underlying event, while transverse max can also be sensitive to perturbative radiation.





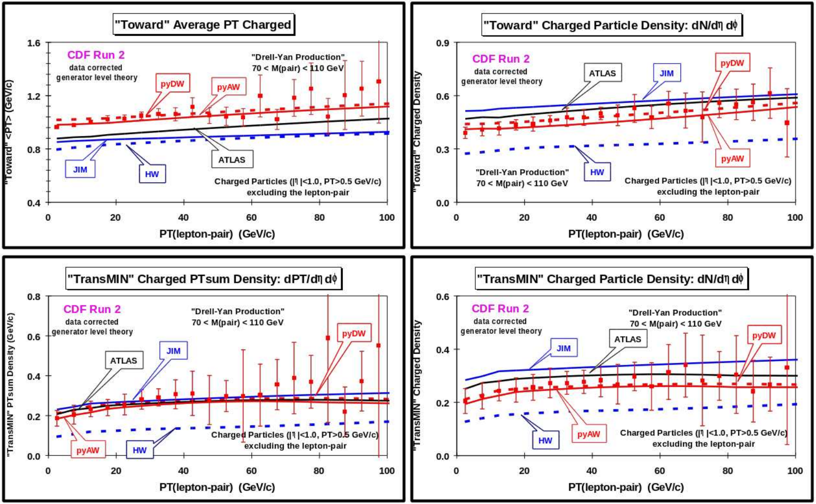
CDF Results Jets

PYTHIA Tune A   HERWIG

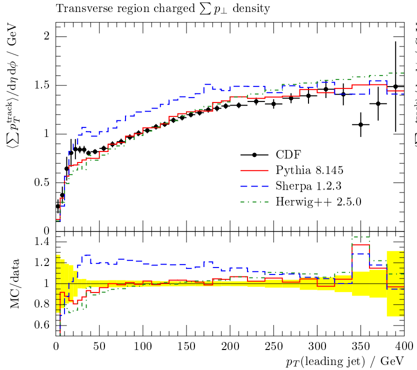


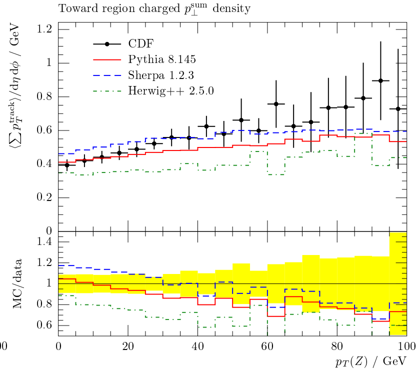
Charged particle density and PTsum density for "leading jet" events versus E_T(jet#1) for PYTHIA Tune A and HERWIG.

CDF Results Drell-Yan



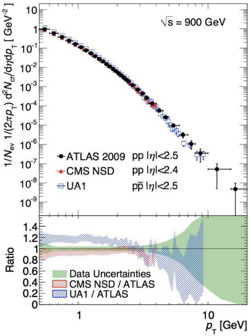
CDF Results

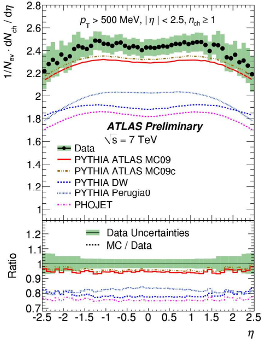

Transverse region charged $\sum p_\perp$ density
$\langle \sum p_T^{\text{track}} \rangle / \mathrm{d}\eta\,\mathrm{d}\phi$ / GeV
CDF
Pythia 8.145
Sherpa 1.2.3
Herwig++ 2.5.0
MC/data
p_T(leading jet) / GeV


Toward region charged $p_\perp^{\text{sum}}$ density
$\langle \sum p_T^{\text{track}} \rangle / \mathrm{d}\eta\,\mathrm{d}\phi$ / GeV
CDF
Pythia 8.145
Sherpa 1.2.3
Herwig++ 2.5.0
MC/data
$p_T(Z)$ / GeV

First LHC Results

Charged Particle Multiplicities at √s=0.9, 7 TeV





Monte Carlo underestimates the track multiplicity seen in ATLAS

Christophe Clement

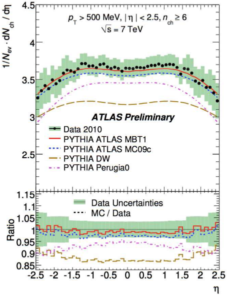
Physics at LHC, DESY, June 9th, 2010 – ATLAS First Physics Results

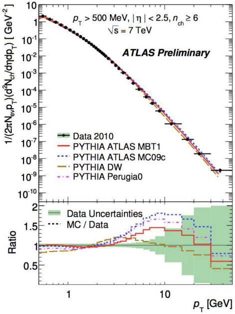
First LHC Results

Pythia Tune to ATLAS MinBias and Underlying Event

Used for the tune

ATLAS UE data at 0.9 and 7 TeV
ATLAS charged particle densities at 0.9 and 7 TeV
CDF Run I underlying event analysis (leading jet)
CDF Run I underlying event "Min-Max" analysis
D0 Run II dijet angular correlations
CDF Run II Min bias
CDF Run I Z pT





Result

This tune describes most of the MinBias and the UE data
Significant improvement compared to pre-LHC tunes
Biggest remaining deviation in → $\frac{1}{N_{\rm ev}} \cdot \frac{1}{2\pi p_{\rm T}} \cdot \frac{{\rm d}^2 N_{\rm ch}}{{\rm d}\eta {\rm d}p_{\rm T}}$
These deviations could not be removed
Needs further investigations

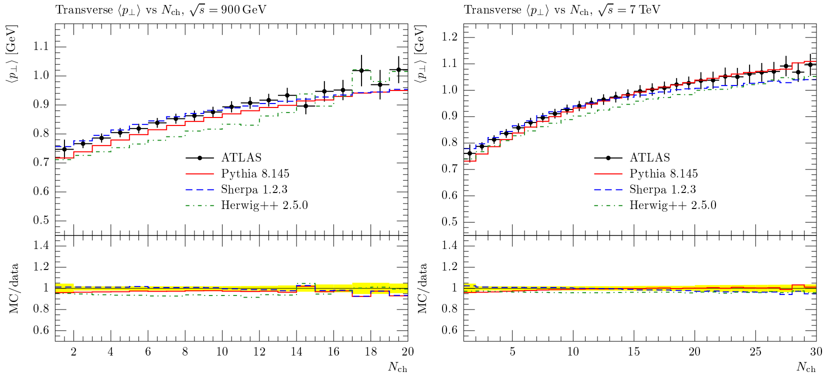
Christophe Clement

Physics at LHC, DESY, June 9th, 2010 – ATLAS First Physics Results

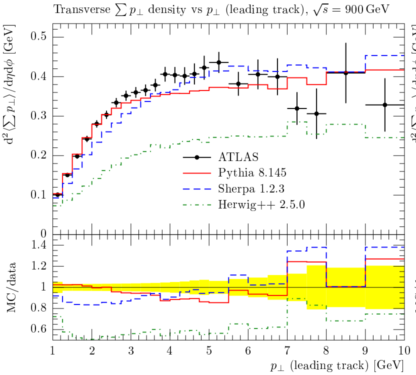
First LHC Results

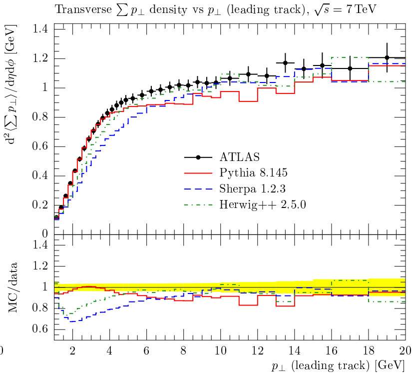
- Before the LHC start there was some worry that the models would completely fail.
- In reality in good agreement with the early data.
- Better agreement now after some tuning of the parameters.
- In both Herwig++ and PYTHIA this needs the $p_\perp^{\min}$ parameter to be energy dependent.
- Older soft models don't describe the data.

Average transverse $p_\perp$ vs $N_{\rm ch}$

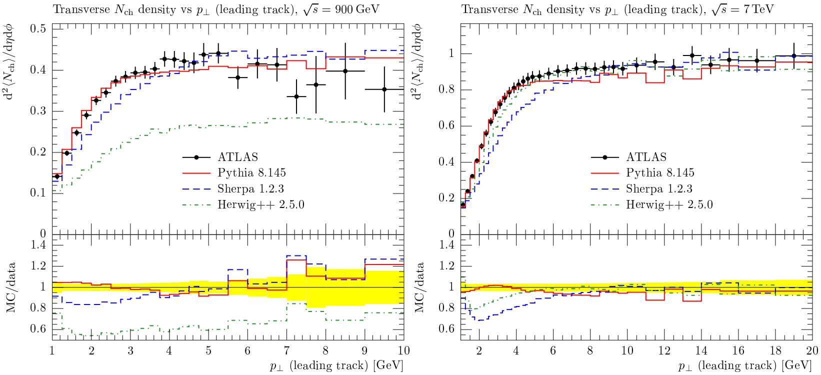


Transverse $p_\perp$ density vs $p_\perp$

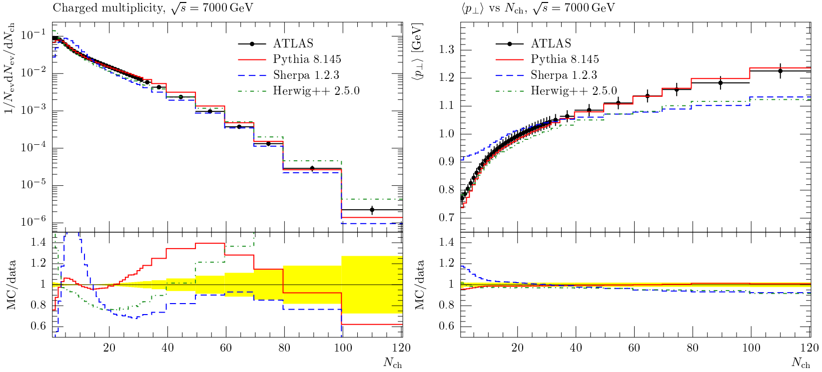




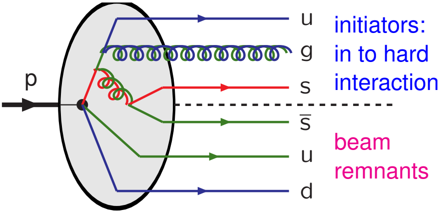
Transverse N_{ch} density vs $p_\perp$



Charged Multiplicity



Beam Remnants



Need to assign:

- correlated flavours
- correlated $x_i = p_{zi}/p_{z\text{tot}}$
- correlated primordial $k_{\perp i}$
- correlated colours
- correlated showers

- PDF after preceding MI/ISR activity:
  1. Squeeze range $0 < x < 1$ into $0 < x < 1 | \sum_i x_i$
  2. Valence quarks reduce by the number already kicked out.
  3. Introduce companion quark $q/\bar{q}$ to each kicked-out sea quark $q/\bar{q}$,with x based on assumed $g \to q\bar{q}$ splitting
  4. Gluon and sea: rescale for total momentum conservation.
- Colour flow connects hard scattering to beam remnants which can have consequences.

Summary

- Underlying event is one of the least least understood aspects of event generation.e
- Modelled and only weakly constrained by existing data.
- Models based on MPI describe the data well with a number of refinements.