Corrections to single Z boson production

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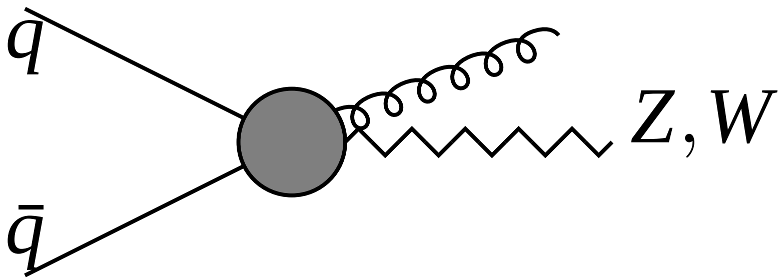
in collaboration with A. Kotikov and J. H. Kühn, Nucl. Phys. B788 (2008) 47

- introduction
- 2-loop virtual vertex correction
- calculation of 2-loop vertex functions
- construction of harmonic and nonharmonic basis
- conclusions

Drell–Yan production of Z and W bosons



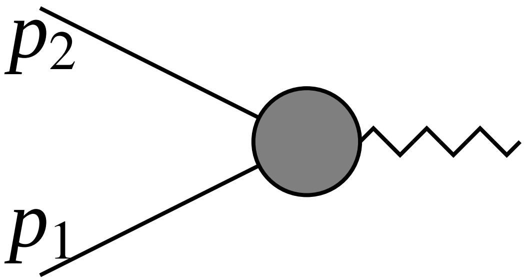
- important test of the Standard Model
- probe for PDF's over a wide range of x
- luminosity measerements
- theoretically:

  large corrections come into play

theoretical status for single vector boson production

- NLO and NNLO QCD correction
- 1-loop electroweak corrections
- mixed QCD/electroweak corrections for large p_T
- resummations of small p_T gluons
- leading (Sudakov) electroweak logarithms in 2-loop
- to complete NNLO mixed QCD/EW is is missing:
  - mixed QCD/EW virtual contributions
  - soft/collinear gluon/photon radiation at NNLO

two loop vertex with masses

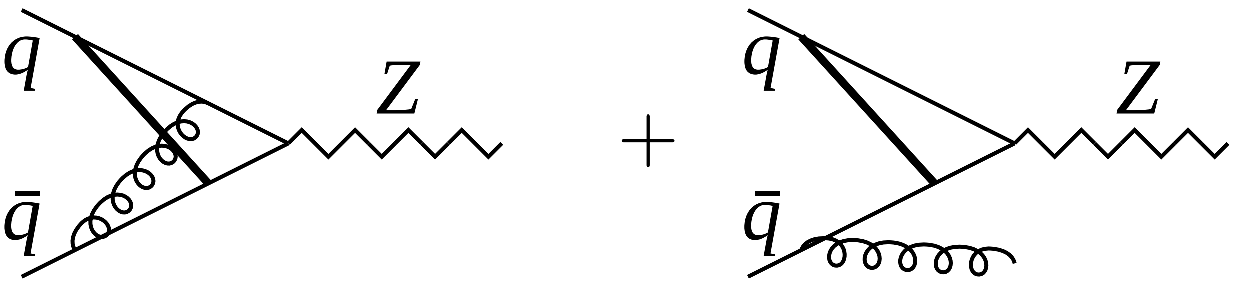


$$q = p_1 + p_2$$

$$p_1^2 = 0$$

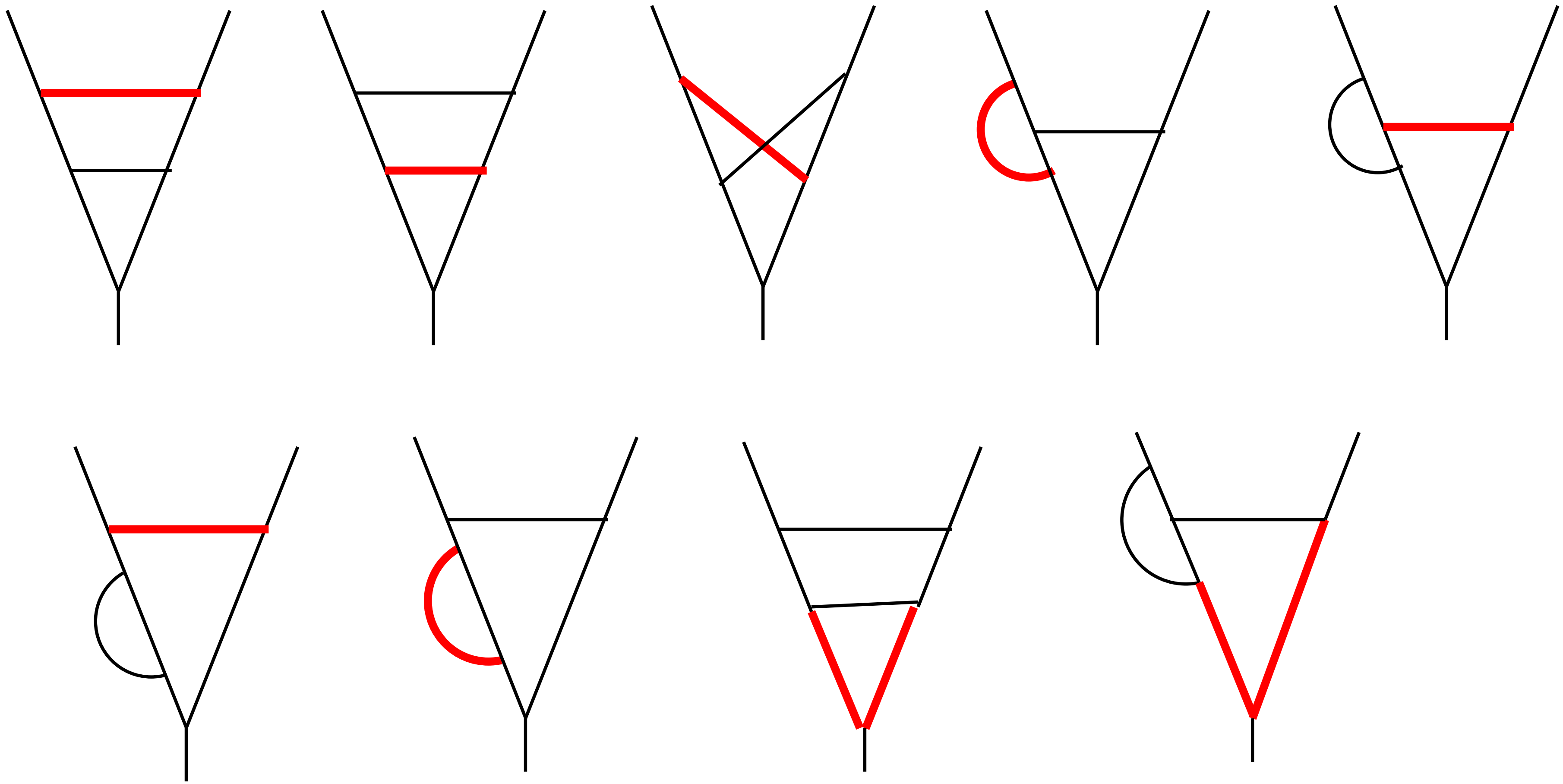
$$p_2^2 = 0$$

- $O(\alpha\alpha_s)$ corrections to Drell–Yan production of gauge bosons



- abelian formfactor at $q^2 \to \infty$
  — main component in Sudakov regime calculations

diagram prototypes



red lines — massive
black lines — massless

evaluation (I)

To evaluate: about thousand of vertex integrals
with numerators and up to 6 internal lines

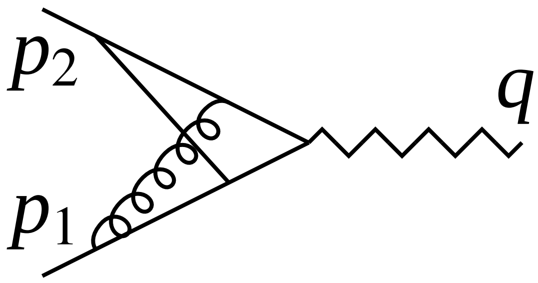
$$\int \frac{dk_1\, dk_2 \prod(k_i k_j)(k_r p_n)}{D_1 \ldots D_6}$$

- using *integration by part identities*
  perform reduction to master integrals or brute force method
  — evaluate each integral

- evaluation of masterintegrals:
  - differential equation
  - asymptotic expansion

$\Leftarrow \Leftarrow \Leftarrow$
still need to calculate
masterintegrals!!!

expansions



$$q = p_1 + p_2$$

$$\begin{cases} p_1^2 = 0 \\ p_2^2 = 0 \\ z = q^2/M^2 \end{cases}$$

<u>approximations:</u>

- large q^2 (Sudakov regime) $\longleftarrow$

  difficult to obtain even <u>first</u> term,

  requires special threatment, e.g. Mellin–Barnes technique

- small q^2 (or large mass expansion) $\longleftarrow$

  easy to get <u>tens</u> of terms

ask: **how to get *exact* result?**

evaluation (II)

1. large mass expansion (expansion parameter $z = q^2/M^2$)

$$\text{Diagram}(z) = \sum_{n=1}^{n_{\max}} c_n z^n \qquad (*)$$

2. use ansatz with *basis functions* $\phi_i(z)$ and *unknowns* x_i

$$\text{Ansatz}(z) = x_1\phi_1(z) + x_2\phi_2(z) + \ldots + x_N\phi_N(z) \qquad (**)$$

3. make them equal

$$\text{Diagram}(z) = \text{Ansatz}(z) \qquad (***)$$

4. expand eq. (***) in z and solve system for *unknowns* x_i

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ask: what are the *basis functions*?

Harmonic bases. Example: fli+.basis (up to weight 4)

fli1+	fli2+	fli3+	fli4+
$\log(1-z)$	$\log^2(1-z)$	$\log^3(1-z)$	$\log^4(1-z)$
	$\mathrm{Li}_2(z)$	$\mathrm{Li}_2(z)\log(1-z)$	$\mathrm{Li}_2(z)\log^2(1-z)$
		$\mathrm{Li}_3(z)$	$\mathrm{Li}_3(z)\log(1-z)$
		$S_{12}(z)$	$S_{12}(z)\log(1-z)$
			$\mathrm{Li}_2^2(z)$
			$\mathrm{Li}_4(z)$
			$S_{13}(z)$
			$S_{22}(z)$

Expansion in z always gives structure:

$$\sum_{n=1}^{\infty} z^n \times (\text{product of harmonic sums})$$

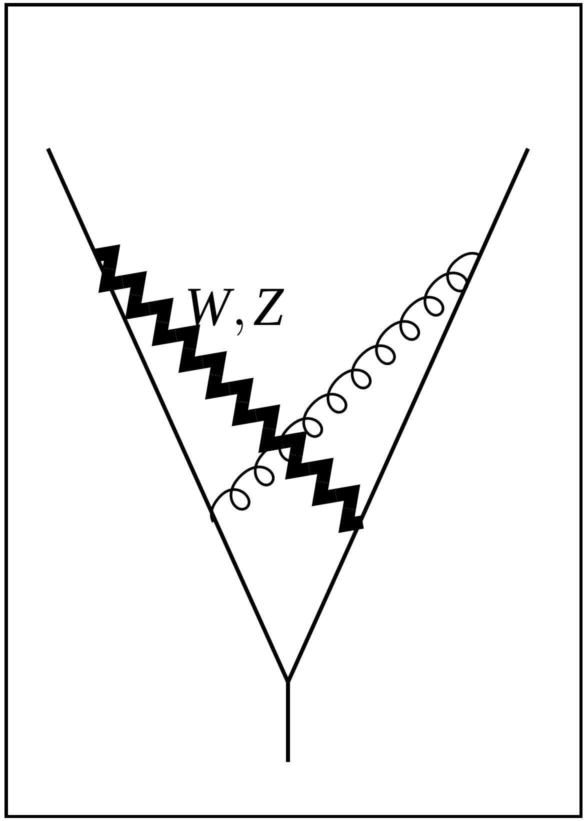
Harmonic bases (II)

Numerators in diagrams shift summation index: $\sum_{n=1}^{\infty} z^n \longrightarrow \sum_{n=1}^{\infty} z^{n-1}$
Alternating harmonic sums: fli+.basis $\longrightarrow$ fli-.basis
Additional structures in nonplanar diagram: $H_{-1,0,1}(-z)$, $H_{-1,0,1,1}(-z)$, $H_{-1,0,0,1}(-z)$, $H_{-1,-1,0,1}(-z)$, $H_{0,-1,0,1}(-z)$

- fli+.basis $\longrightarrow \sum_{n=1}^{\infty} z^n \times$ (product of harmonic sums)
- fli+p.basis $\longrightarrow \sum_{n=1}^{\infty} z^{n-1} \times$ (product of harmonic sums)
- fli-.basis $\longrightarrow \sum_{n=1}^{\infty} (-z)^n \times$ (product of harmonic sums)
- fli-p.basis $\longrightarrow \sum_{n=1}^{\infty} (-z)^{n-1} \times$ (product of harmonic sums)
- N5.basis $\longrightarrow \sum_{n=1}^{\infty} (-z)^n \times$ (product of harmonic sums)

Solution. Generated code for FORM (example).

```
id N5(k2.p2,1,1,1,1,1,0) =
+ 1 * 1/z^0 * ( 2*LOG-3*zt2+3/2-3/4*LOG^2 )
+ 1 * 1/z^0 * (
    + (2*zt2-3+1/2*LOG^2) * 1 * log(1-z)
    + (LOG-2) * 1 * Li2(z)
    + 1 * 1 * log(1-z)*Li2(z)
    + (-1) * 1 * Li3(z)
    + 2 * 1 * S12(z)
    + (-2*zt2+3-1/2*LOG^2) * 1/z * log(1-z)
    + (-LOG+zt2-1+1/4*LOG^2) * 1/z * Li2(z)
    + (-1) * 1/z * log(1-z)*Li2(z)
    + (-LOG+1) * 1/z * Li3(z)
    + (-2) * 1/z * S12(z)
    + 1/4 * 1/z * Li2(z)^2
    + 3/2 * 1/z * Li4(z)
    + (-1) * 1/z * S22(z)
        );
```



abelian vector formfactor

$$F(q,M)\gamma_\mu = \int dx e^{-ixq} \langle \psi' | J_\mu(x) | \psi \rangle$$

A.F. absorbs large Sudakov logratims $\ln \frac{q^2}{M^2}$ as $q^2 \to \infty$

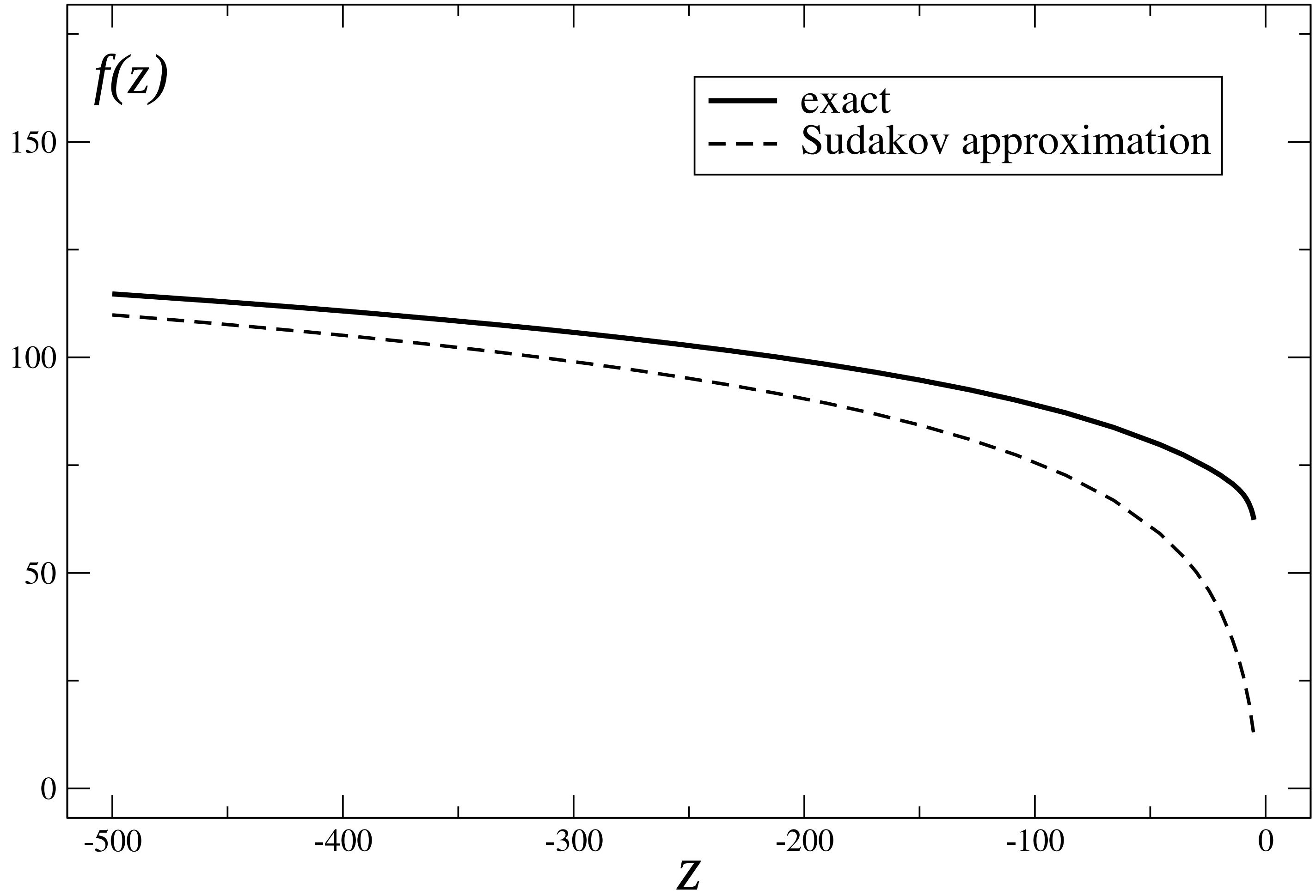
in a theory with massless (gluon!) and massive (Z, W) bosons

$$F(\alpha, \alpha', q, M) = F_{\mathrm{massless}}(\alpha', q) F_{\mathrm{massive}}(\alpha, \alpha', q, M)$$

factorization

$$F_{\mathrm{massive}}(\alpha, \alpha', q, M) = 1 + \alpha g(q^2/M^2) + \alpha\alpha' f(q^2/M^2) + \ldots$$

how good is Sudakov approximation



$(z = q^2/M_Z^2)$

nonabelian case

- problem: new structures appear — *binomial sums*

$$\sum_{n=1} \frac{1}{\binom{2n}{n}} W_k(n-1) \frac{z^n}{n^a}$$

- with new *nonharmonic* sums (W_k), e.g.

$$W_k(n) = \sum_{j=1}^{n} \binom{2j}{j} \frac{1}{j^k}$$

- new *basis functions* , e.g.

$$h_{\ldots,-r,\ldots}(z) = \int \ldots \int_0^{x_k} \frac{dx_{k-1}}{\sqrt{4(4+x_{k-1})}} \int \ldots$$

nonabelian case

- introduce

$$H_{a,b,\ldots,c}(z) = \int_0^z \frac{dx_1}{x_1 - a} \int_0^{x_1} \frac{dx_2}{x_2 - b} \cdots\cdots \int_0^{x_k} \frac{dx_2}{x_2 - c}$$

in case $a, b, \ldots, c = +1, -0, 1$ harmonic polylogarithms

- nonabelian diagrams require functions with
$a, b, \ldots, c = +1, 0, -1, +e^{i\pi/3}, -e^{i\pi/3}$

or equviv. factors $\int_0^{x_j} dx_j / \sqrt{x_j(4 + x_j)}$

(all together 4 new functions)

relations to polylogarithms

we can express these functions in terms of polylogarithms of new nonlinear argument

$$y = \frac{1 - \sqrt{z/(z-4)}}{1 + \sqrt{z/(z-4)}}$$

e.g.

$$\int_0^{-z} \frac{dx_1}{\sqrt{x_1(x_1+4)}} \int_0^{x_1} \frac{dx_2}{\sqrt{x_2(x_2+4)}} \int_0^{x_2} \frac{dx_3}{1+x_3}$$

$$= -\tfrac{1}{6}\ln^3 y + \tfrac{1}{2}\zeta(2)\ln y + \tfrac{2}{3}\zeta(3) + \mathrm{Li}_3(-y) - \tfrac{1}{9}\mathrm{Li}_3(-y^3)$$

Z-production

for a ligth quark

$$F(q^2)_\mu = \gamma_\mu \frac{1+\gamma_5}{2} F_R(q^2) + \gamma_\mu \frac{1-\gamma_5}{2} F_L(q^2)\,.$$

$$\begin{aligned}
F_R &= i\frac{e}{s_W}\left(1 + C_F \frac{\alpha_s}{4\pi} f^{(0,1)}\right)\left[g_R + \frac{\alpha}{4\pi s_W^2} g_R^3 \rho_{\mathrm{A}}(q^2/m_Z^2, m_Z^2) + C_F \frac{\alpha_s}{4\pi}\frac{\alpha}{4\pi s_W^2} g_R^3 \phi_{\mathrm{A}}(q^2/m_Z^2)\right], \\
F_L &= i\frac{e}{s_W}\left(1 + C_F \frac{\alpha_s}{4\pi} f^{(0,1)}\right)\Bigg[g_L \\
&+ \frac{\alpha}{4\pi s_W^2}\left(g_L^3 \rho_{\mathrm{A}}(q^2/m_Z^2, m_Z^2) + \frac{g_L}{2}\rho_{\mathrm{A}}(q^2/m_W^2, m_W^2) + c_W \frac{I_3}{2}\rho_{\mathrm{NA}}(q^2/m_W^2, m_W^2)\right) \\
&+ C_F \frac{\alpha_s}{4\pi}\frac{\alpha}{4\pi s_W^2}\left(g_L^3 \phi_{\mathrm{A}}(q^2/m_Z^2) + \frac{g_L}{2}\phi_{\mathrm{A}}(q^2/m_W^2) + c_W \frac{I_3}{2}\phi_{\mathrm{NA}}(q^2/m_W^2)\right)\Bigg],
\end{aligned}$$

Z-production (II)

$$\begin{aligned}
\phi_{\rm A}(1+i0) &= 14+72\zeta_2 l_2-64\zeta_2 l_2^2-\frac{16}{3}l_2^4+22\zeta_2-28\zeta_3+16\zeta_4-128\mathrm{Li}_4(\tfrac{1}{2}) \\
&\quad +i\pi(85+32l_2+24l_2^2-\frac{32}{3}l_2^3+14\zeta_2-120\zeta_3) \\
&= -2.1073-19.0331i\,, \\
\phi_{\rm NA}(1+i0) &= -16-144\zeta_2 l_2+128\zeta_2 l_2^2+\frac{32}{3}l_2^4+\frac{70}{3}\zeta_2+\frac{184}{3}\zeta_3-236\zeta_4 \\
&\quad +26\frac{\pi}{\sqrt{3}}+256\mathrm{Li}_4(\tfrac{1}{2})-84\frac{1}{\sqrt{3}}\mathrm{Ls}_2(\tfrac{\pi}{3})-\frac{16}{3}\pi\mathrm{Ls}_2(\tfrac{\pi}{3})+96\Big(\mathrm{Ls}_2(\tfrac{\pi}{3})\Big)^2 \\
&\quad +i\pi(54-64l_2-48l_2^2+\frac{64}{3}l_2^3-28\zeta_2+48\zeta_3) \\
&= -7.5880+16.7194i\,,
\end{aligned}$$

with $l_2=\log 2$. For the actual masses of the W- and Z-bosons ($z=m_Z^2/m_W^2=1.2856$):

$$\phi_{\rm A}(1.2856+i0)=-1.3598-30.4095i\,,\qquad \phi_{\rm NA}(1.2856+i0)=-10.1248+35.0336i$$

outlook

- DY single vector boson production plays an important role in at the LHC experiment and the test of the Standard Model

- we consider 2-loop on-shell vertex function at arbitrary q^2 by expansion in the large mass regime

- framework for the analytical evaluation is constructed in the case with one and two (nonabelian case) massive lines

- this covers e.g. $O(\alpha\alpha_s)$ contribution to the abelian formfactor and mixed $O(\alpha\alpha_s)$ correction to the production of Z-boson

- ingridients for the mixed complete $O(\alpha\alpha_s)$ analysis are available. combain all together ...