Three Loop Corrections to the MSSM Higgs Boson Mass

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in collaboration with
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Motivation

why consider Higgs sector of MSSM?

- quite predictive
  two new free parameters at tree level

$$M_A, \ \tan\beta$$

- important for possible discovery of MSSM
  light supersymmetric Higgs might be first trace of supersymmetry
- experimental accuracy
  $\delta M_h \approx 100 - 200$MeV at LHC, $\delta M_h \approx 50$MeV at ILC

Higgs Mass in the MSSM

Higgs Potential

$$V = m_1^2 \left|H_1\right|^2 + m_2^2 \left|H_2\right|^2 + m_{12}^2 \left(\epsilon_{ab} H_1^a H_2^b + \epsilon_{ab} H_1^{a*} H_2^{b*}\right)$$
$$+ \frac{1}{8}\left(g_1^2 + g_2^2\right)\left(\left|H_1\right|^2 - \left|H_2\right|^2\right)^2 + \frac{1}{2} g_2^2 \left|H_1^* H_2\right|^2$$

spontaneous symmetry breaking: H_1, H_2 aquire vacuum expectation values ⇒ gauge bosons and fermions aquire masses.

difference to the SM : quartic terms fixed by gauge couplings

$$\Rightarrow M_h \text{ can be predicted, tree level: } M_h \leq M_z$$

Radiative Corrections to M_h

- 1-loop corrections from top and stop loops $\propto M_t^4$

  [Ellis,Ridolfi,Zwirner 1991; Haber,Hempfling 1991; Okada,Yamaguchi,Yanagida 1991]

significant shift of up to 35GeV

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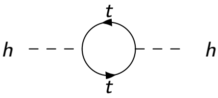
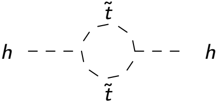
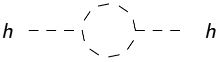


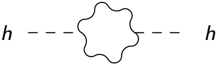


significant shift of up to 35GeV

- from other sectors of the MSSM

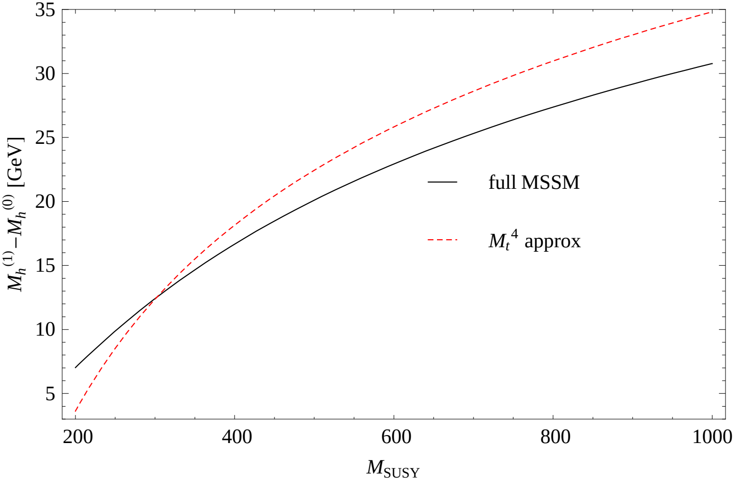
  [Chankowski,Pokorski,Rosiek 1994; Dabelstein 1995; Bagger,Matchev,Pierce,Zhang 1997]





small against the corrections from tops and stops

Relevance of M_t^4 Approximation



2loop Corrections to M_h

- corrections in effective potential approximation ($p^2 = 0$)

  [Brignole, Carena, Casas, Dedes, Degrassi, Espinosa, Haber, Hempfling, Heinemeyer, Hoang, Hollik, Martin, Quirós, Riotto, Rzehak, Slavich, Wagner, Weiglein, Zhang, Zwirner, ... '94-today]

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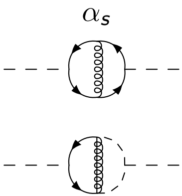
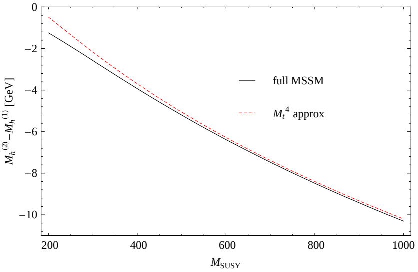
  contributions from t and $\tilde{t}$ loops becomes even more important!





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- momentum dependence $\approx$ 160MeV [Martin '05]

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- momentum dependence ≈ 160MeV [Martin '05]
- 3loop LL and NLL through Renormalisation Group [Martin '07]

remaining uncertainty: $3 - 5$GeV
against $100 - 200$MeV at LHC, 50MeV at ILC

Aim: Genuine Three-Loop Calculation

simplifying assumptions

- effective potential approximation
- restrict to t, $\tilde{t}$ sector
  t, $\tilde{t}$, g, $\tilde{g}$, q and $\tilde{q}$ as internal particles

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- effective potential approximation
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  t, $\tilde{t}$, g, $\tilde{g}$, q and $\tilde{q}$ as internal particles
- assume mass hierarchy

$$
\begin{aligned}
m_q = 0, \quad & m_t \ll m_{\tilde{t}_1} = m_{\tilde{t}_2} = m_{\tilde{g}} = m_{\tilde{q}} \\
& m_t \ll m_{\tilde{t}_1} = m_{\tilde{t}_2} = m_{\tilde{g}} \ll m_{\tilde{q}} \\
& m_t \ll m_{\tilde{t}_1} \ll m_{\tilde{t}_2} = m_{\tilde{g}} \ll m_{\tilde{q}}
\end{aligned}
$$

asymptotic expansion in small ratios of masses

Calculational Setup

- lots and lots of diagrams (30, 717) ...
- ... require automatisation: QGRAF, Q2E, EXP, MINCER, MATAD, FORM

[Nogueira; Harlander, Seidensticker; Larin, Tkachov; Steinhauser; Vermaseren]

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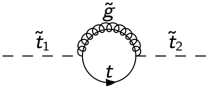
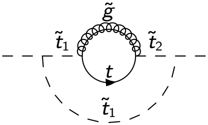
  [Nogueira; Harlander, Seidensticker; Larin, Tkachov; Steinhauser; Vermaseren]

- regularisation through dimensional reduction [Siegel '79]
  ε-scalars implemented, $M_\varepsilon = 0$ on-shell

subleties

- degenerate $\tilde{t}$ masses, no mixing between $\tilde{t}_1$ and $\tilde{t}_2$
  mixing angle receives radiative corrections





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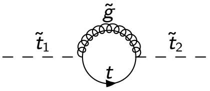
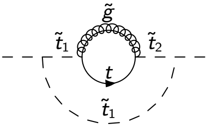
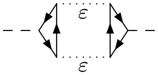




  cannot force degenerate bare $\tilde{t}$ masses
  $\Rightarrow$ expand integrals in $m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}$
- infrared problems can hide amongst the ε-scalars



Result

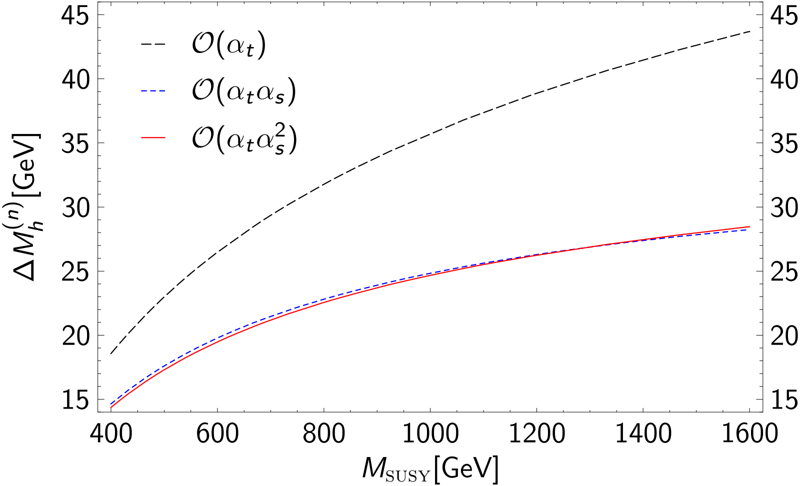
$$
\begin{aligned}
\Delta M_h \quad = \quad & -\frac{3 G_F M_t^4}{\sqrt{2}\pi^2}\Bigg\{ - L_{tS} + \frac{\alpha_s}{\pi}\left[4 L_{tS} - 2 L_{tS}^2\right] + \left(\frac{\alpha_s}{\pi}\right)^2 \Bigg[-\frac{1091}{324} - \frac{1}{27}\pi^2 - \frac{1}{9}\zeta_3 \\
& + \left(\frac{1591}{108} + 3 L_{\mu t} - \frac{1}{3}\pi^2 + \frac{4}{9}\pi^2 \ln 2 - \frac{55}{18} L_{t\tilde{q}} - \frac{5}{6} L_{t\tilde{q}}^2\right) L_{tS} \\
& + \left(-\frac{19}{18} - \frac{3}{2} L_{\mu t} + \frac{5}{3} L_{t\tilde{q}}\right) L_{tS}^2 - \frac{53}{18} L_{tS}^3 \\
& + \left(-\frac{475}{108} + \frac{5}{9}\pi^2\right) L_{t\tilde{q}} + \frac{25}{36} L_{t\tilde{q}}^2 + \frac{5}{18} L_{t\tilde{q}}^3 \\
& + \mathcal{O}\left(\frac{M_S^2}{M_{\tilde{q}}^2}\right) \Bigg] \Bigg\} ,
\end{aligned}
$$

$$
L_{tS} = \ln \frac{M_t^2}{M_{SUSY}^2}, \qquad L_{\mu t} = \ln \frac{\mu^2}{M_t^2}, \qquad L_{t\tilde{q}} = \ln \frac{Mt^2}{M_{\tilde{q}}^2}
$$

Checks

- agreement with literature
  - exact 2-loop [FeynHiggs; Degrassi, Slavich, Zwirner '01]
  - 3-loop LL and NLL [Martin '07]
- every part of the calculation has been done twice
- calculation performed in general covariant gauge
  gauge parameter independent result
- calculated also the case of unbroken SUSY
  loop corrections to M_h vanish

Numerics



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800
1000
1200
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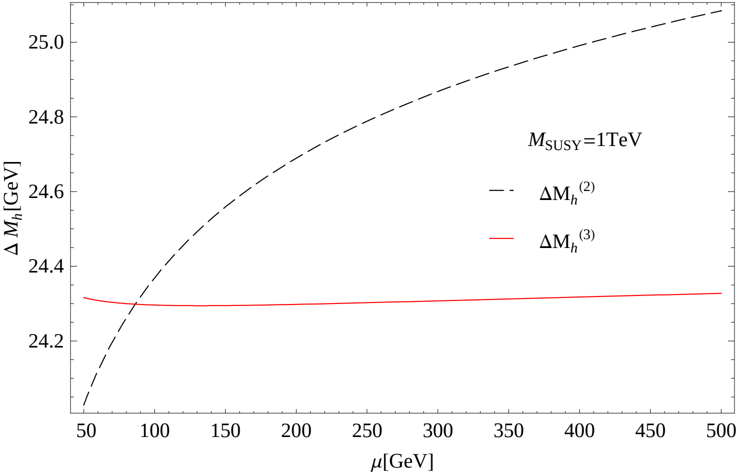
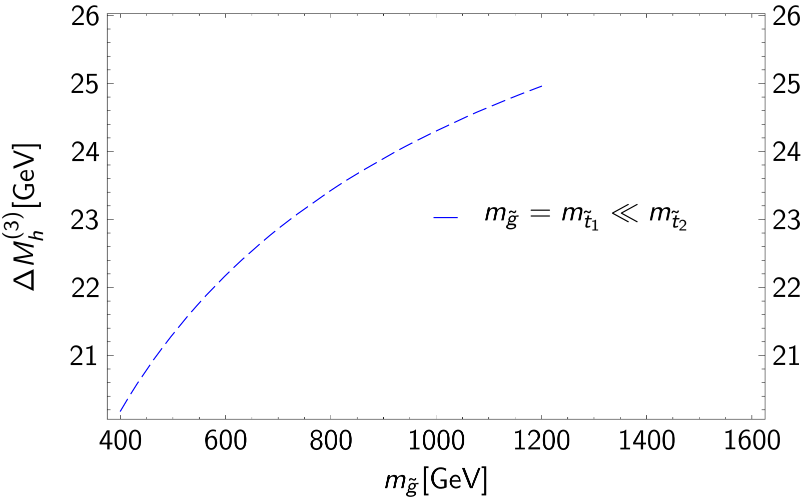
1600
$\Delta M_h^{(n)}$[GeV]
M_{SUSY}[GeV]
$\mathcal{O}(\alpha_t)$
$\mathcal{O}(\alpha_t \alpha_s)$
$\mathcal{O}(\alpha_t \alpha_s^2)$

Reduced Scale Dependency



Non-Degenerate Spectrum (preliminary)



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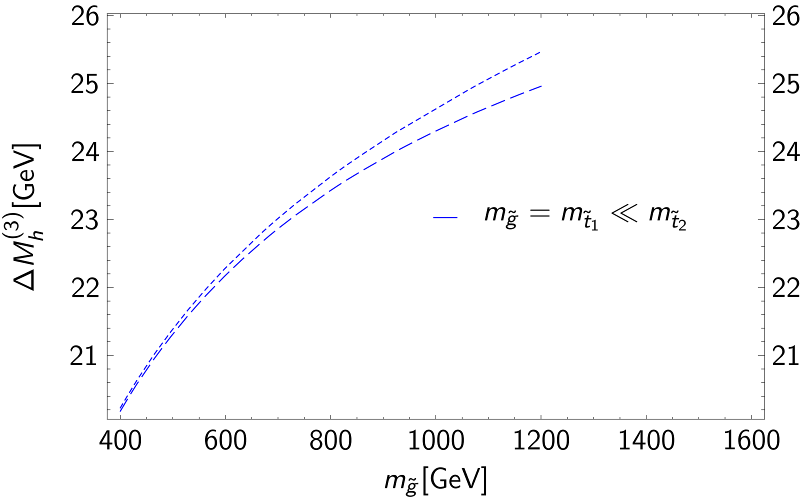

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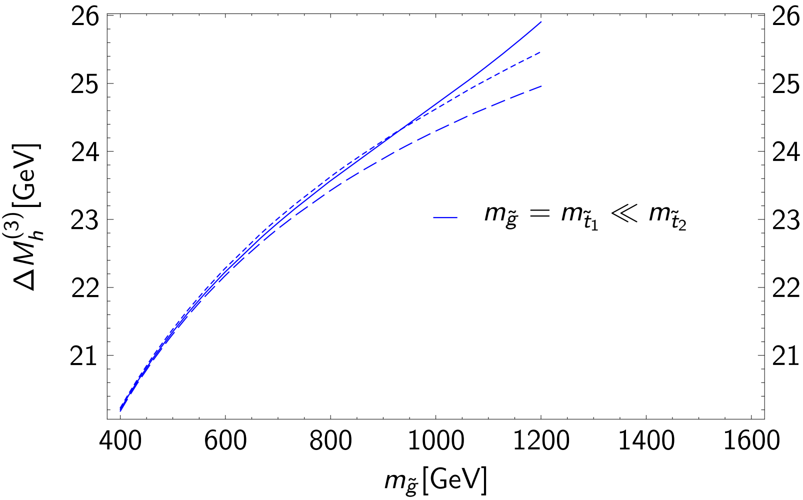

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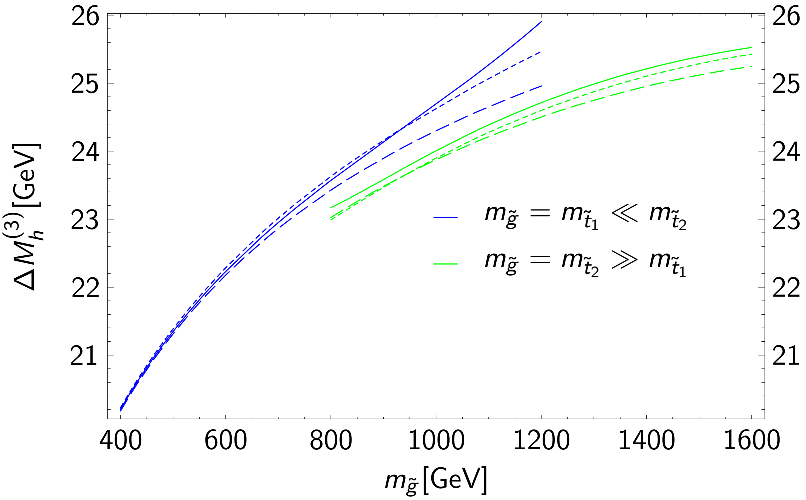

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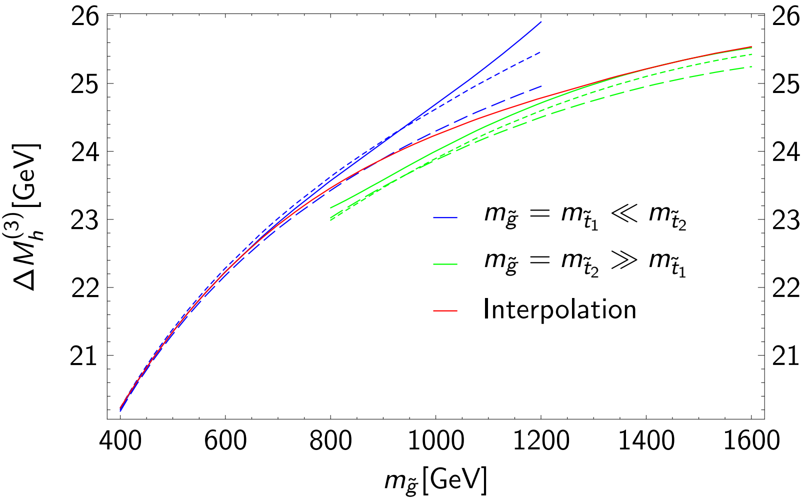

Benchmark Point SPS1a'

$$600\,\text{GeV} \approx m_{\tilde{g}} \approx m_{\tilde{q}} \approx m_{\tilde{t}_2} \gg m_{\tilde{t}_1} = 366.5\,\text{GeV} \gg m_t\,,$$

$$\begin{aligned}
\tan\beta &= 10\,,\\
M_A &= 372\,\text{GeV}\,,\\
\mu_H &= 396\,\text{GeV}\,,\\
A_t &= -565.1\,\text{GeV}\,,
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\tan\beta &= 10\,,\\
M_A &= 372\,\text{GeV}\,,\\
\mu_H &= 396\,\text{GeV}\,,\\
A_t &= -565.1\,\text{GeV}\,,\\
&\Rightarrow\\
M_h^{(0)} &= 89.3\,\text{GeV}\,,\\
M_h^{(1)} &= 115.0\,\text{GeV}\,,\\
M_h^{(2)} &= 113.3\,\text{GeV}\,,\\
M_h^{(3)} &= 114.1\,\text{GeV}\,.
\end{aligned}$$

Conclusions

- three-loop calculation of M_h in the MSSM, effect of about 500MeV
  - relevant for LHC and ILC
- renormalisation scale dependence drastically improved
- different hierarchies in progress