Bose-Einstein Condensation of Dark Matter Axions

Pierre Sikivie

11th Patras Workshop on Axions, WIMPs and WISPs

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Collaborators

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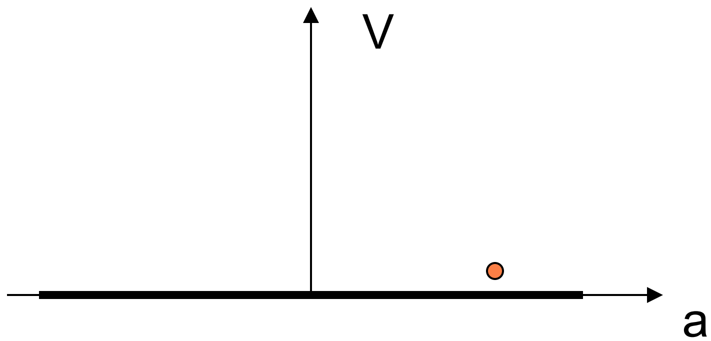
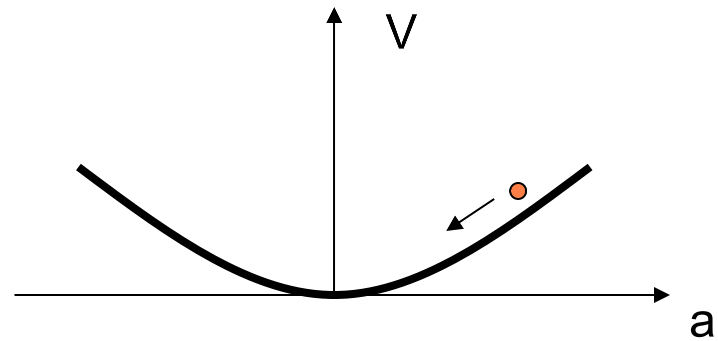
Outline

- Axion dark matter
- Cosmic axion Bose-Einstein condensation
- Evidence for axion dark matter from the study of caustics

Axion production by vacuum realignment



$T \geq 1\,\mathrm{GeV}$



$T \leq 1\,\mathrm{GeV}$

$$n_a(t_1) \simeq \frac{1}{2} m_a(t_1)\, a(t_1)^2 \simeq \frac{1}{2t_1} f_a^{\,2}\, \alpha(t_1)^2$$

initial misalignment angle

$$\rho_a(t_0) \simeq m_a\, n_a(t_1) \left(\frac{R_1}{R_0}\right)^3 \propto m_a^{-\frac{7}{6}}$$

J. Preskill, F. Wilczek + M.Wise; L. Abbott + P.S.; M. Dine + W. Fischler 1983

Cold axion properties

- number density

$$n(t) \sim \frac{4\cdot 10^{47}}{\text{cm}^3}\left(\frac{f_a}{10^{12}\ \text{GeV}}\right)^{\frac{5}{3}}\left(\frac{a(t_1)}{a(t)}\right)^3$$

- velocity dispersion

$$\delta \text{v}(t) \sim \frac{1}{m_a\, t_1}\,\frac{a(t_1)}{a(t)}$$

if decoupled

- phase space density

$$\text{N} \sim n(t)\,\frac{(2\pi)^3}{\frac{4\pi}{3}(m_a\,\delta \text{v})^3} \sim 10^{61}\left(\frac{f_a}{10^{12}\ \text{GeV}}\right)^{\frac{8}{3}}$$

Bose-Einstein Condensation

if identical bosonic particles
are highly condensed in phase space
and their total number is conserved
and they thermalize

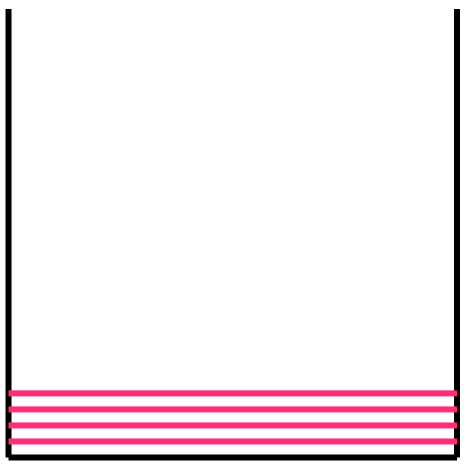
then most of them go to the lowest energy
available state

why do they do that?

by yielding their energy to the non-condensed particles, the total entropy is increased.

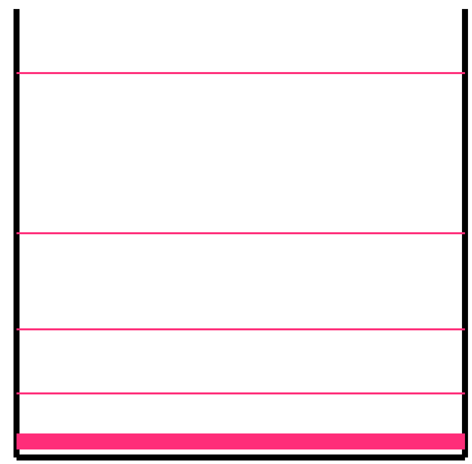
the axions thermalize and form a BEC after a time τ

$t < \tau$



the axion fluid obeys
classical field equations,
behaves like CDM

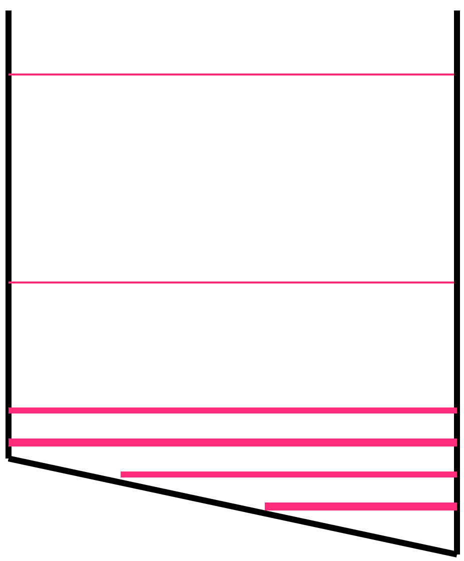
$t > \tau$



the axion fluid does not obey
classical field equations,
does not behave like CDM

the axion BEC rethermalizes

$t < \tau$



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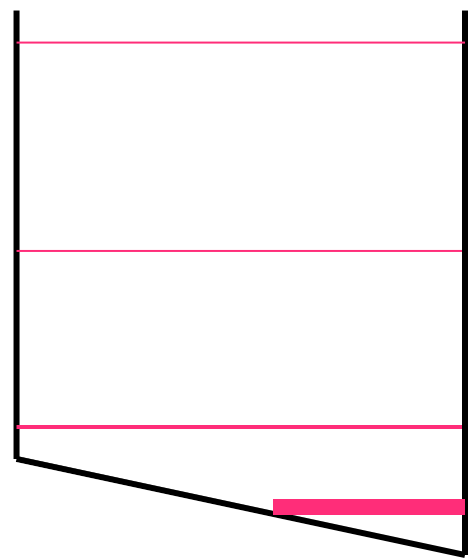

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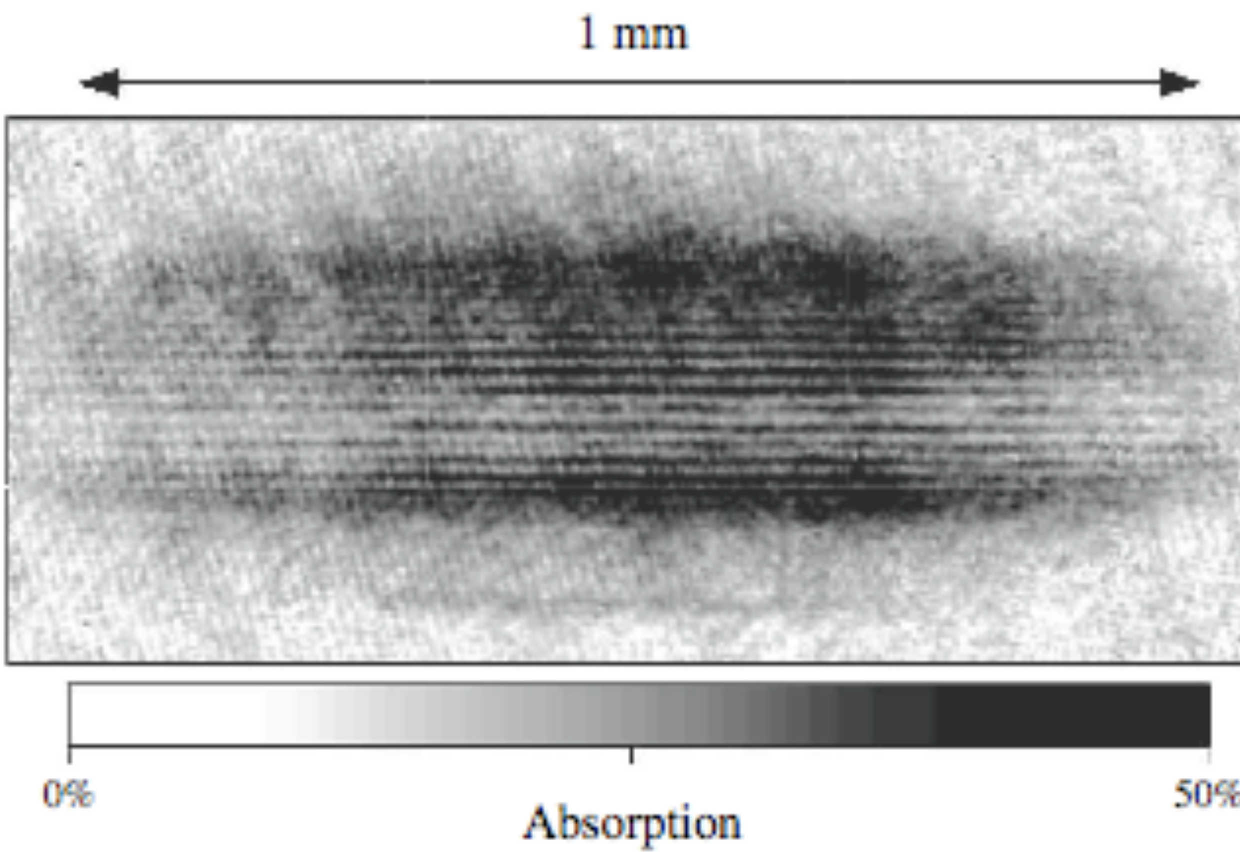

from M.R. Andrews, C.G. Townsend, H.-J. Miesner, D.S. Durfee, D.M. Kurn and W. Ketterle, Science 275 (1997) 637.

A classical scalar field behaves like CDM

in the non-relativistic limit

on distance scales much larger than the wavelength

$$\partial_t^2 \phi - \nabla^2 \phi - \omega_0^2 \phi + \frac{\lambda}{3!} \phi^3 = 0$$

$$\phi(\vec{x}, t) = \sqrt{\frac{2}{\omega_0}} Re[e^{-i\omega_0 t} \psi(\vec{x}, t)]$$

with $\partial_t \psi << \omega_0 \psi$

N. Banik, A. Christopherson, P.S., E. Todarello, 1504.05968

$$i\partial_t \psi = -\frac{1}{2m}\nabla^2\psi + V(\vec{x},t)\psi$$

$$m = \omega_0$$

$$V(\vec{x},t) = -\frac{\lambda}{8m^2}|\psi(\vec{x},t)|^2 + m\Phi(\vec{x},t)$$

$$\nabla^2\Phi(\vec{x},t) = 4\pi G m|\psi(\vec{x},t)|^2$$

when gravity is included

$$\psi(\vec{x},t) = \sqrt{n(\vec{x},t)}\; e^{i\beta(\vec{x},t)}$$

$$\vec{v}(\vec{x},t) = \frac{1}{m}\vec{\nabla}\beta(\vec{x},t)$$

$$\partial_t n + \vec{\nabla}\cdot(n\vec{v}) = 0$$

$$\partial_t \vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\vec{\nabla}\Phi - \vec{\nabla}q$$

for $\lambda = 0$

"quantum pressure"

$$q = -\frac{1}{2m^2}\frac{\nabla^2\sqrt{n}}{\sqrt{n}}$$

In the expanding universe, the Fourier components $\delta(\vec{k},t)$ of the relative overdensity grow according to:

$$\partial_t^2\delta + 2H(t)\partial_t\delta - 4\pi G\rho\delta + \frac{k^4}{4m^2a(t)^4}\delta = 0$$

There is a Jeans length

M. Khlopov, B. Malomed, Y. Zel'dovich 1985
M. Bianchi, D. Grasso, R. Ruffini, 1990

$$\ell_J = (16\pi G\rho m^2)^{-\frac{1}{4}}$$
$$= 1.01\ 10^{14}\ \mathrm{cm}\left(\frac{10^{-5}\ \mathrm{eV}}{m}\right)^{\frac{1}{2}}\left(\frac{10^{-29}\mathrm{gr/cm}^3}{\rho}\right)^{\frac{1}{4}}$$

Ultra-light axion-like particles

$$m \sim 10^{-21} \text{ to } 10^{-24} \text{ eV}$$

have been proposed

$$\ell = \frac{1}{\delta p} = \frac{1}{m\delta v} \sim 10^{22} \text{ cm}$$

when $m \sim 10^{-24}$ eV and $\delta v \sim 10^{-3}$

S.-J. Sin 1994

J. Goodman 2000

W. Hu, R. Barkana & A. Gruzinov 2000

and many others

see talk by D. Grin

A quantum scalar field differs from CDM on time scales longer than its thermal relaxation time $\tau \sim \frac{1}{\Gamma}$

$$\phi(\vec{x},t) = \sum_{\vec{n}} \frac{1}{\sqrt{2\omega_{\vec{n}} V}} [a_{\vec{n}}(t) e^{i\vec{p}_{\vec{n}}\cdot\vec{x}} + a^{\dagger}_{\vec{n}}(t) e^{-i\vec{p}_{\vec{n}}\cdot\vec{x}}]$$

$$\vec{p}_{\vec{n}} = \frac{2\pi\vec{n}}{L} \qquad V = L^3$$

$$[a_{\vec{n}}(t), a^{\dagger}_{\vec{n}'}(t)] = \delta_{\vec{n},\vec{n}'}$$

Quantum axion field dynamics

$$H = \sum_j \omega_j a_j^\dagger a_j \quad + \quad \sum_{ijkl} \frac{1}{4} \Lambda_{kl}^{ij} a_k^\dagger a_l^\dagger a_i a_j$$

O. Erken et al., PRD 85 (2012) 063520

From $\frac{1}{4!}\lambda\phi^4$ self-interactions

$$\Lambda_\lambda \,{}^{\vec{p}_3,\vec{p}_4}_{\vec{p}_1,\vec{p}_2} = -\frac{\lambda}{4m^2V}\,\delta_{\vec{p}_1+\vec{p}_2,\vec{p}_3+\vec{p}_4}$$

From gravitational self-interactions

$$\Lambda_g \,{}^{\vec{p}_3,\vec{p}_4}_{\vec{p}_1,\vec{p}_2} = -\frac{4\pi G m^2}{V}\,\delta_{\vec{p}_1+\vec{p}_2,\vec{p}_3+\vec{p}_4}\left(\frac{1}{|\vec{p}_1-\vec{p}_3|^2}+\frac{1}{|\vec{p}_1-\vec{p}_4|^2}\right)$$

$$\mathcal{N}_j = a_j^\dagger a_j$$

In the "particle kinetic" regime $\Gamma << \delta E$

$$\frac{d}{dt}\mathcal{N}_l = \sum_{ijk} \frac{1}{2} |\Lambda_{ij}^{kl}|^2 [\mathcal{N}_i \mathcal{N}_j (\mathcal{N}_l + 1)(\mathcal{N}_k + 1) - \mathcal{N}_l \mathcal{N}_k (\mathcal{N}_i + 1)(\mathcal{N}_j + 1)] \times$$
$$\times 2\pi\delta(\omega_i + \omega_j - \omega_k - \omega_l)$$

On time scales of order τ the system moves toward the Bose-Einstein distribution

$$\mathcal{N}_j = \frac{1}{e^{\beta(\epsilon_j - \mu)} - 1}$$

$$\beta = \frac{1}{k_B T}$$

$\mu =$ chemical potential

For the classical theory

$$\phi(\vec{x},t) = \sum_{\vec{n}} \frac{1}{\sqrt{2\omega_{\vec{n}} V}} [A_{\vec{n}}(t) e^{i\vec{p}_{\vec{n}}\cdot\vec{x}} + A_{\vec{n}}(t)^* e^{-i\vec{p}_{\vec{n}}\cdot\vec{x}}]$$

$A_{\vec{n}}(t)$ are c-numbers now

$$N_j(t) = A_j(t)^* A_j(t)$$

In the particle kinetic regime

$$\frac{d}{dt} N_l = \sum_{k,i,j} \frac{1}{2} |\Lambda_{ij}^{kl}|^2 \, [N_i N_j N_l + N_i N_j N_k - N_l N_k N_i - N_l N_k N_j] \times$$
$$\times \; 2\pi\delta(\omega_k + \omega_l - \omega_i - \omega_j)$$

On time scales of order the thermal relaxation time τ the classical system moves towards the Boltzmann distribution

$$N_l \omega_l = k_b T$$

$$\tau = ?$$

Axion field dynamics

$$H = \sum_j \omega_j a_j^\dagger a_j \quad + \quad \sum_{ijkl} \frac{1}{4} \Lambda_{kl}^{ij} a_k^\dagger a_l^\dagger a_i a_j$$

O. Erken et al.,
PRD 85 (2012)
063520

From $\frac{1}{4!}\lambda\phi^4$ self-interactions

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In the "particle kinetic" regime

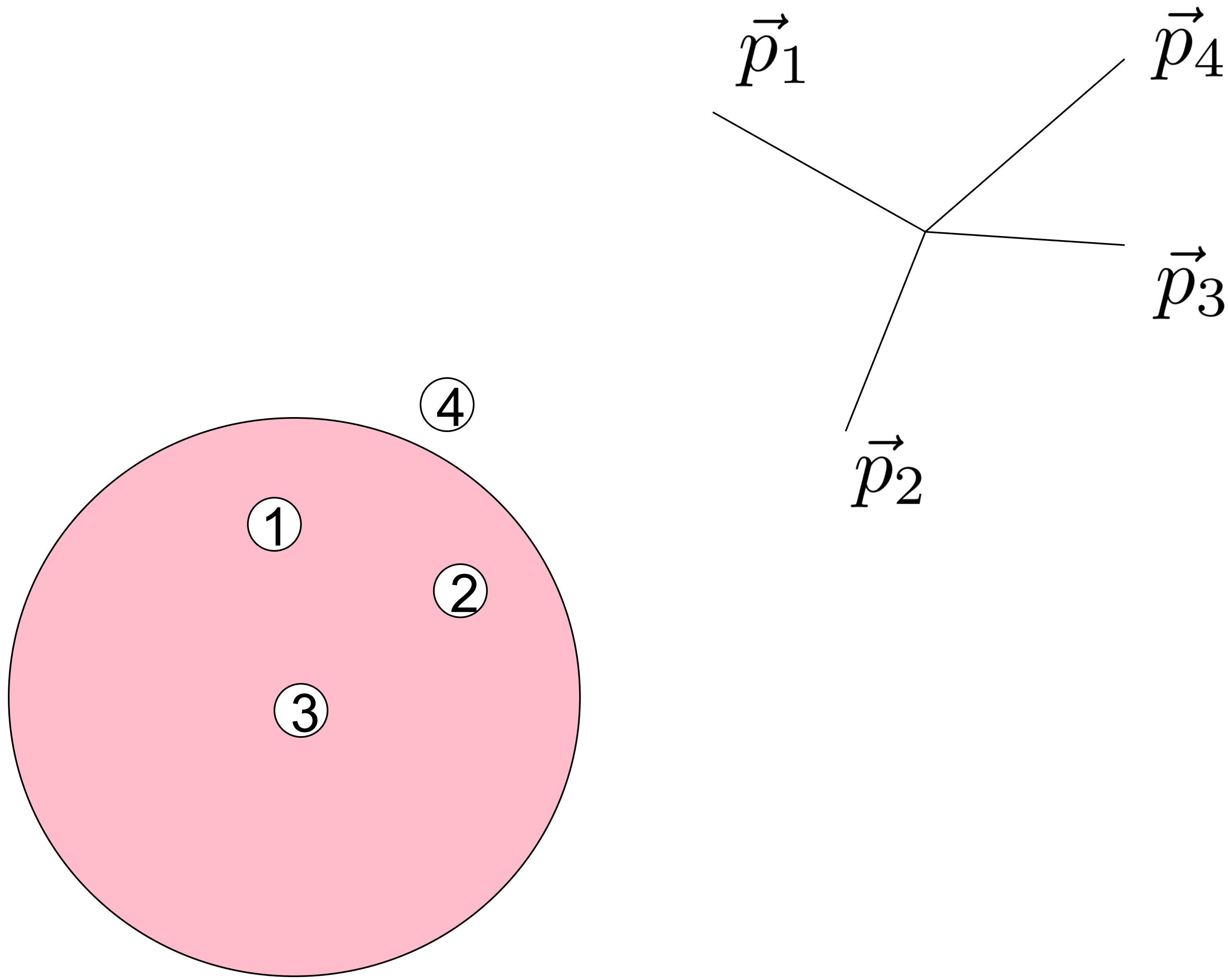
$$\Gamma << \delta E$$

$$\frac{d}{dt}\mathcal{N}_l = \sum_{ijk} \frac{1}{2}|\Lambda^{kl}_{ij}|^2[\mathcal{N}_i\mathcal{N}_j(\mathcal{N}_l+1)(\mathcal{N}_k+1) - \mathcal{N}_l\mathcal{N}_k(\mathcal{N}_i+1)(\mathcal{N}_j+\times 1)]$$
$$\times 2\pi\delta(\omega_1+\omega_2-\omega_3-\omega_4)$$

implies $\Gamma \sim n\ \sigma\ \delta v\ \mathcal{N}$

When $\mathcal{N}_1\ ,\ \mathcal{N}_2\ ,\ \mathcal{N}_3 >> \mathcal{N}_4$

$$\frac{d}{dt}\mathcal{N}_3\ \propto\ |\Lambda|^2\ \mathcal{N}_3\ \mathcal{N}_1\mathcal{N}_2$$



4
1
2
3
$\vec{p}_1$
$\vec{p}_4$
$\vec{p}_3$
$\vec{p}_2$

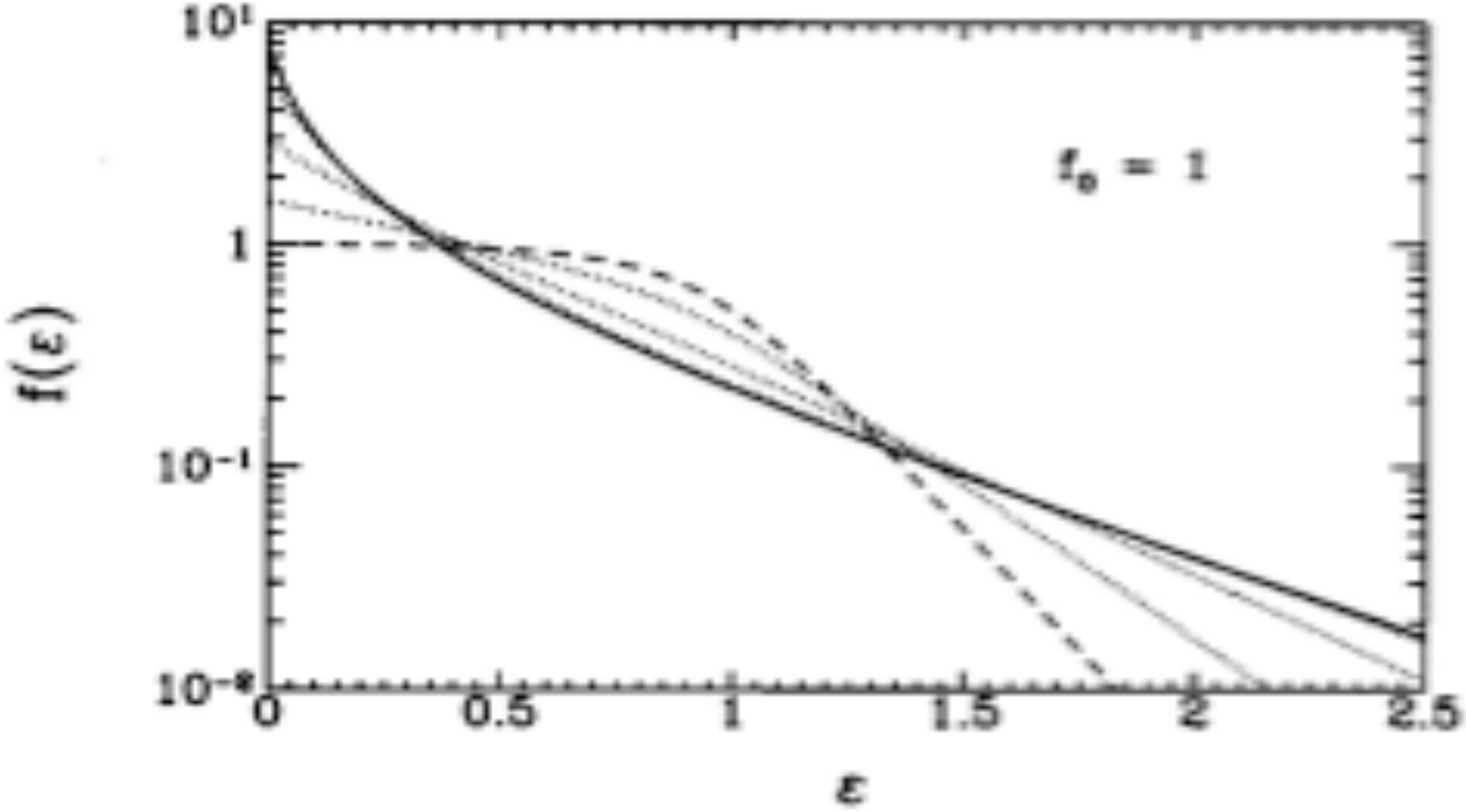


FIG. 4. Snapshots of the distribution function in the noncondensate case. The dashed line corresponds to the initial distribution; the equilibrium distribution is presented by the solid curve; dotted curves correspond to the results of numerical integration at different moments of time $\tau = 1,5,10,20$.

D. Semikoz & I. Tkachev, PRD 55 (1997) 489D.

After t_1, axions thermalize in the "condensed" regime

$$\Gamma >> \delta E$$

$\times$

$$\frac{d}{dt}\mathcal{N}_l = i \sum_{ijk} \frac{1}{2}(\Lambda_{ij}^{kl} a_i^\dagger a_j^\dagger a_k a_l - h.c.)$$

implies $\Gamma \sim \frac{1}{4} n \lambda m^{-2}$ for $\lambda \phi^4$

and $\Gamma \sim 4\pi G n m^2 \ell^2$ for self-gravity

$(\ell \equiv 1/\delta p)$

Toy model thermalizing in the condensed regime:

$$H = \sum_{j=1}^{5} \omega_j a_j^\dagger a_j + \sum_{ijkl} \frac{1}{4} \Lambda_{kl}^{ij} a_k^\dagger a_l^\dagger a_i a_j$$

with

$$\omega_j = j\omega_1$$

$$\Lambda_{kl}^{ij} = 0 \quad \text{unless} \quad k + l = i + j$$

i.e. $\omega_2 = 2\omega_1 \quad , \quad \omega_3 = 3\omega_1 \quad , \quad \omega_4 = 4\omega_1 \quad , \quad \omega_5 = 5\omega_1$

$$\Lambda_{23}^{14} \neq 0 \quad , \quad \Lambda_{24}^{15} \neq 0 \quad , \quad \Lambda_{34}^{25} \neq 0$$

50 quanta among 5 states

$$|\Psi(t)> \;= \sum_{\{\mathcal{N}\}} \Psi(\{\mathcal{N}\},t)\;|\{\mathcal{N}\}>$$

316 251 system states $\{\mathcal{N}\} = (\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3, \mathcal{N}_4, \mathcal{N}_5)$

Start with $\{\mathcal{N}\} = (20, 5, 15, 5, 5)$

Number of particles $N = \sum_j \mathcal{N}_j = 50$

Total energy $E = \sum_j \mathcal{N}_j \omega_j = 120\ \omega_1$

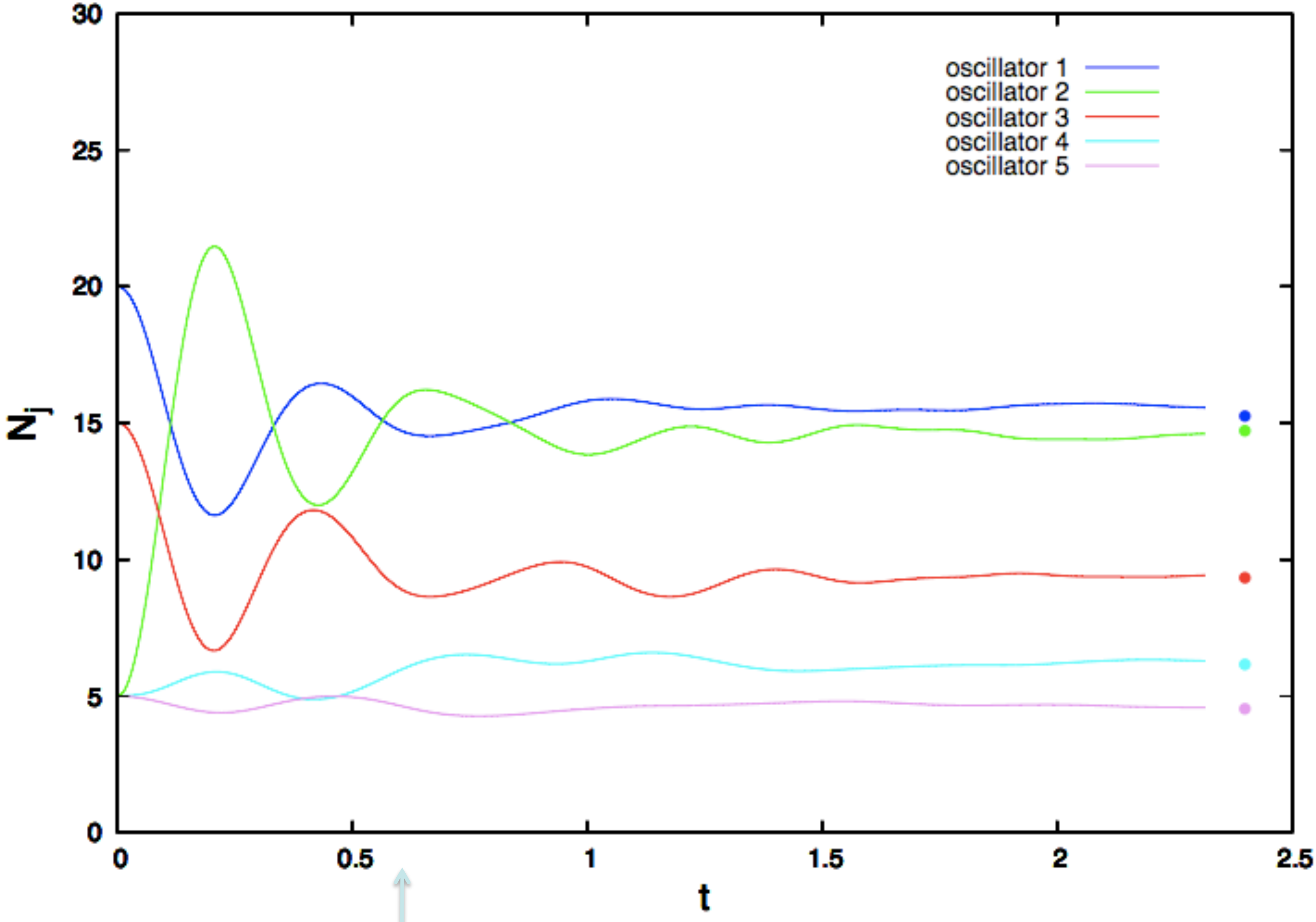
Thermal averages $\{\bar{\mathcal{N}}\} = (15.25, 14.72, 9.33, 6.16, 4.53)$

Integrate $$i\frac{\partial}{\partial t}|\Psi(t)>=H\ |\Psi(t)>$$

Calculate $$<\mathcal{N}_j(t)>=\sum_{\{\mathcal{N}\}}\mathcal{N}_j\ |\Psi(\{\mathcal{N}\},t)|^2$$

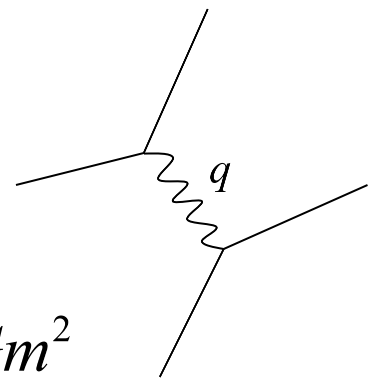
Do the $<\mathcal{N}_j(t)>$ approach the $\bar{\mathcal{N}}_j$ on the predicted time scale?

$$\Gamma\sim\sqrt{I}\ \mathcal{N}\ \Lambda=\ 1.7=\frac{1}{0.6}$$


oscillator 1
oscillator 2
oscillator 3
oscillator 4
oscillator 5
N_j
t
0
5
10
15
20
25
30
0
0.5
1
1.5
2
2.5

Thermalization occurs due to gravitational interactions

PS + Q. Yang, PRL 103 (2009) 111301



$$\frac{Gm^2}{q^2}$$

$$\Gamma_g \sim 4\pi G n m^2 l^2 \quad \text{with } l = (m\,\delta\mathrm{v})^{-1}$$

$$\sim 5 \cdot 10^{-7} H(t_1) \left(\frac{f}{10^{12}\ \mathrm{GeV}} \right)^{\frac{2}{3}}$$

at time t_1

$$\Gamma_g(t) / H(t) \propto t\, a(t)^{-1} \propto a(t)$$

Gravitational interactions thermalize the axions and cause them to form a BEC when the photon temperature

$$T_\gamma \sim 500 \text{ eV} \left(\frac{f}{10^{12} \text{ GeV}}\right)^{\frac{1}{2}}$$

After that

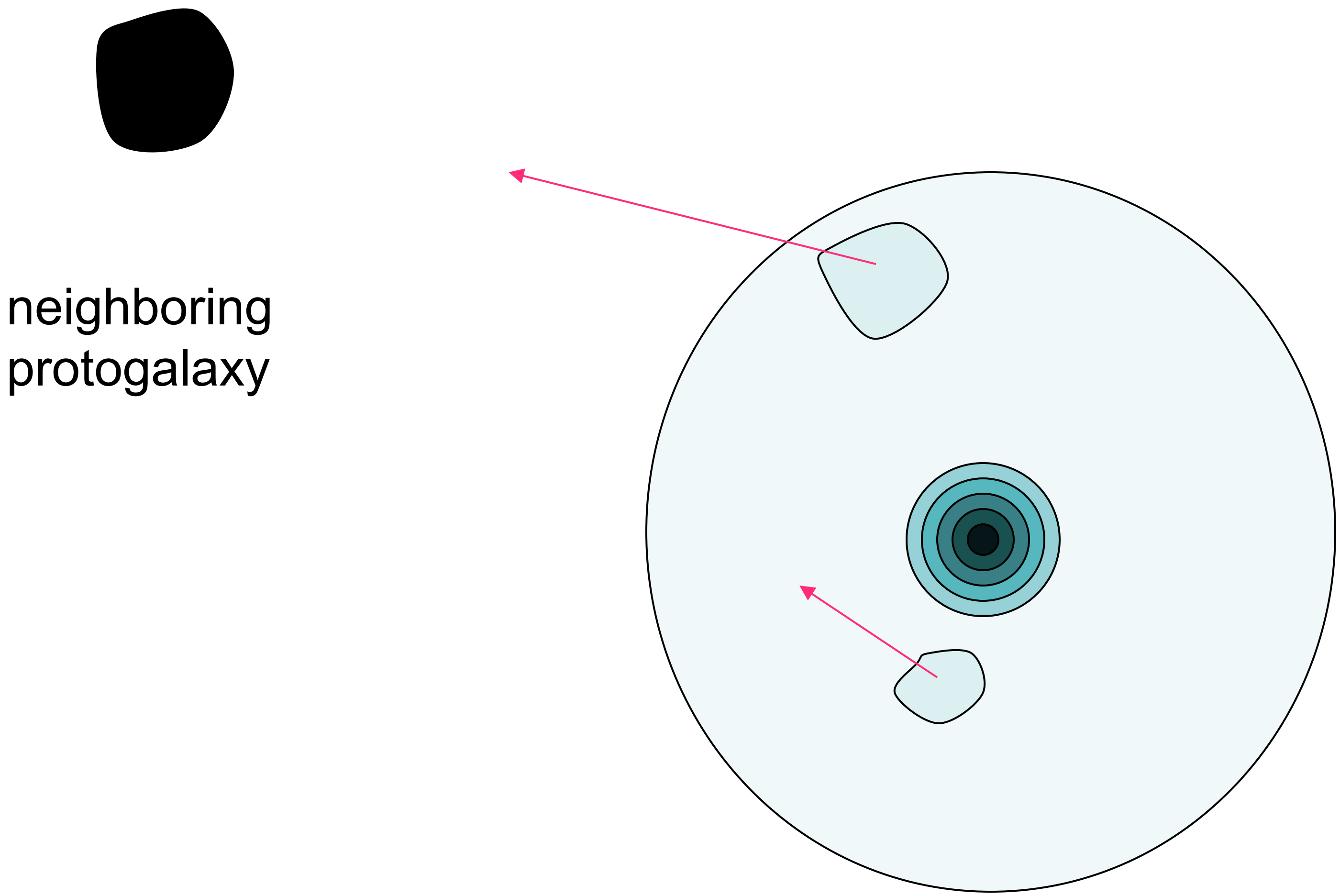
$$\delta \mathrm{v} \sim \frac{1}{m\,t}$$

$$\Gamma_g(t) / H(t) \propto t^3 \, a(t)^{-3}$$

Axion BEC thermalization has also been discussed by

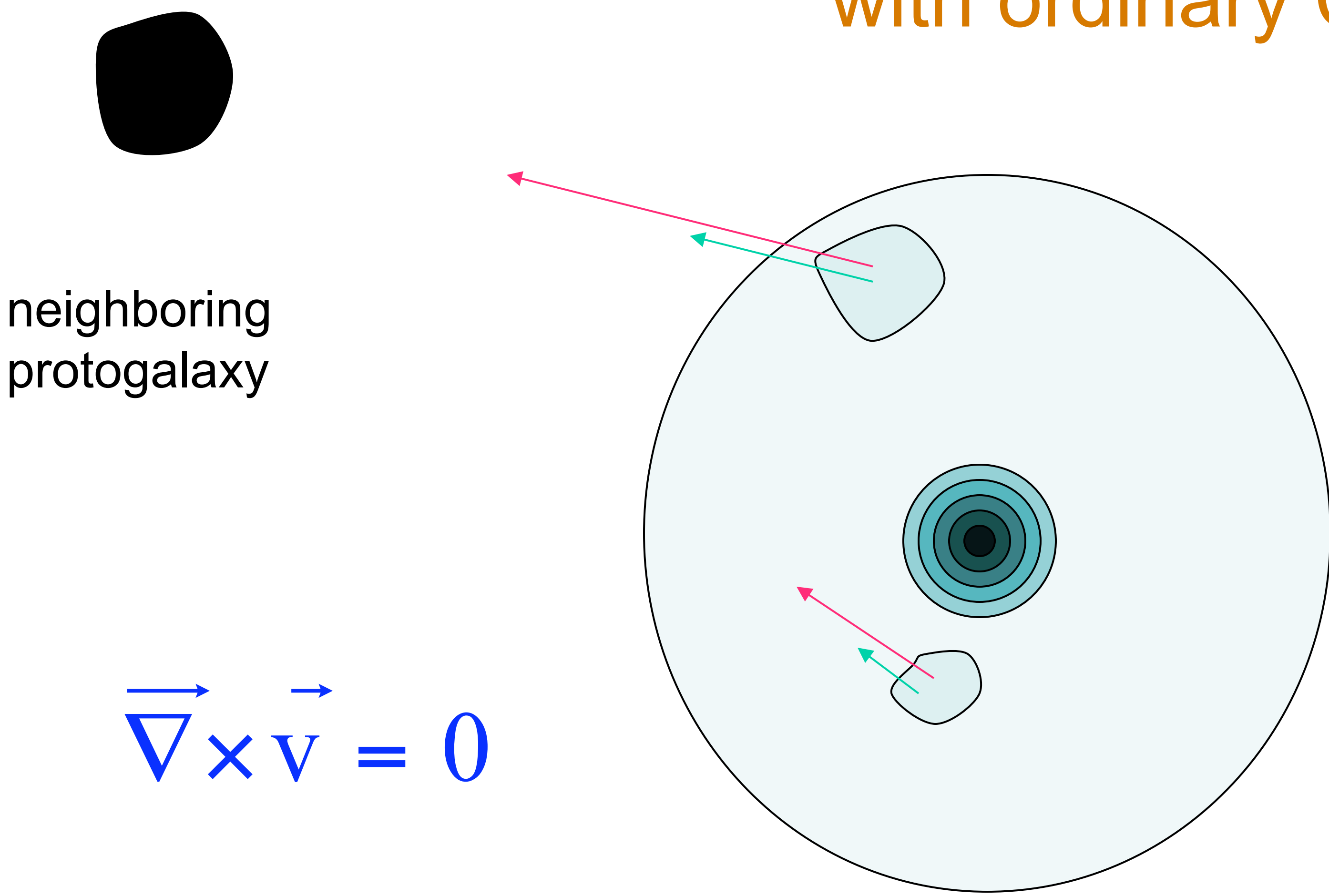
- Saikawa and Yamaguchi, Phys. Rev. D87 (2015) 085010
- S. Davidson and Elmer, JCAP 1312 (2013) 034
- J. Berges and J. Jaeckel, Phys. Rev. D91 (2015) 025020
- A. Guth, M.P. Hertzberg and C. Prescod-Weinstein, arXiv:1412.5930

Tidal torque theory



Stromberg 1934; Hoyle 1947; Peebles 1969, 1971

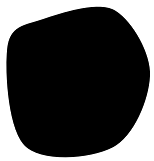
Tidal torque theory with ordinary CDM

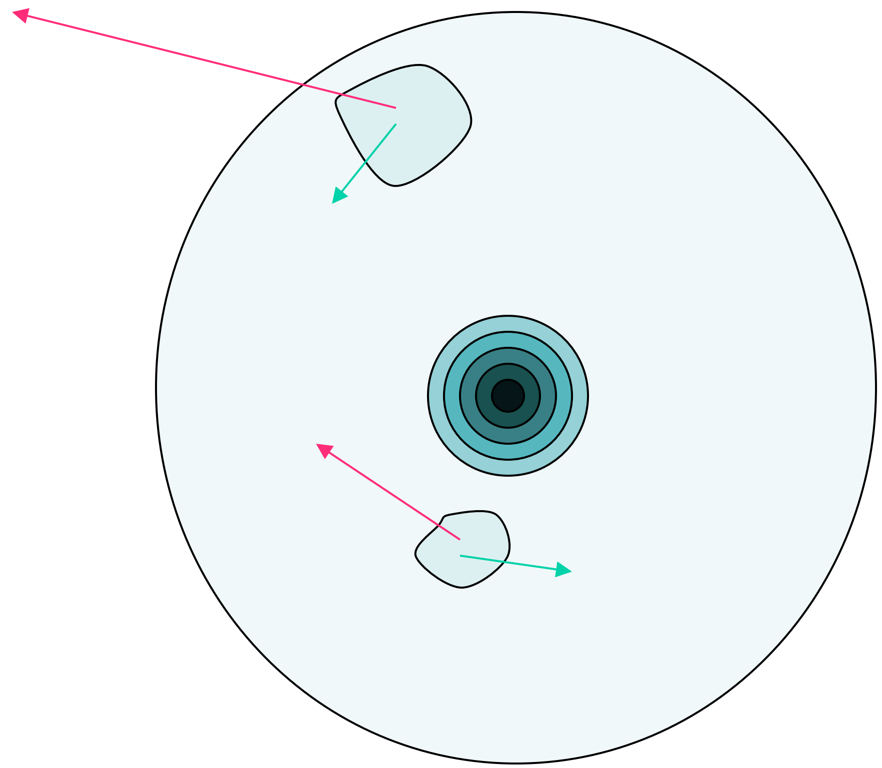


$$\vec{\nabla} \times \vec{v} = 0$$

the velocity field remains irrotational

Tidal torque theory with axion BEC





$$\vec{\nabla} \times \vec{v} \neq 0$$

in their lowest energy available state, the axions fall in with net overall rotation

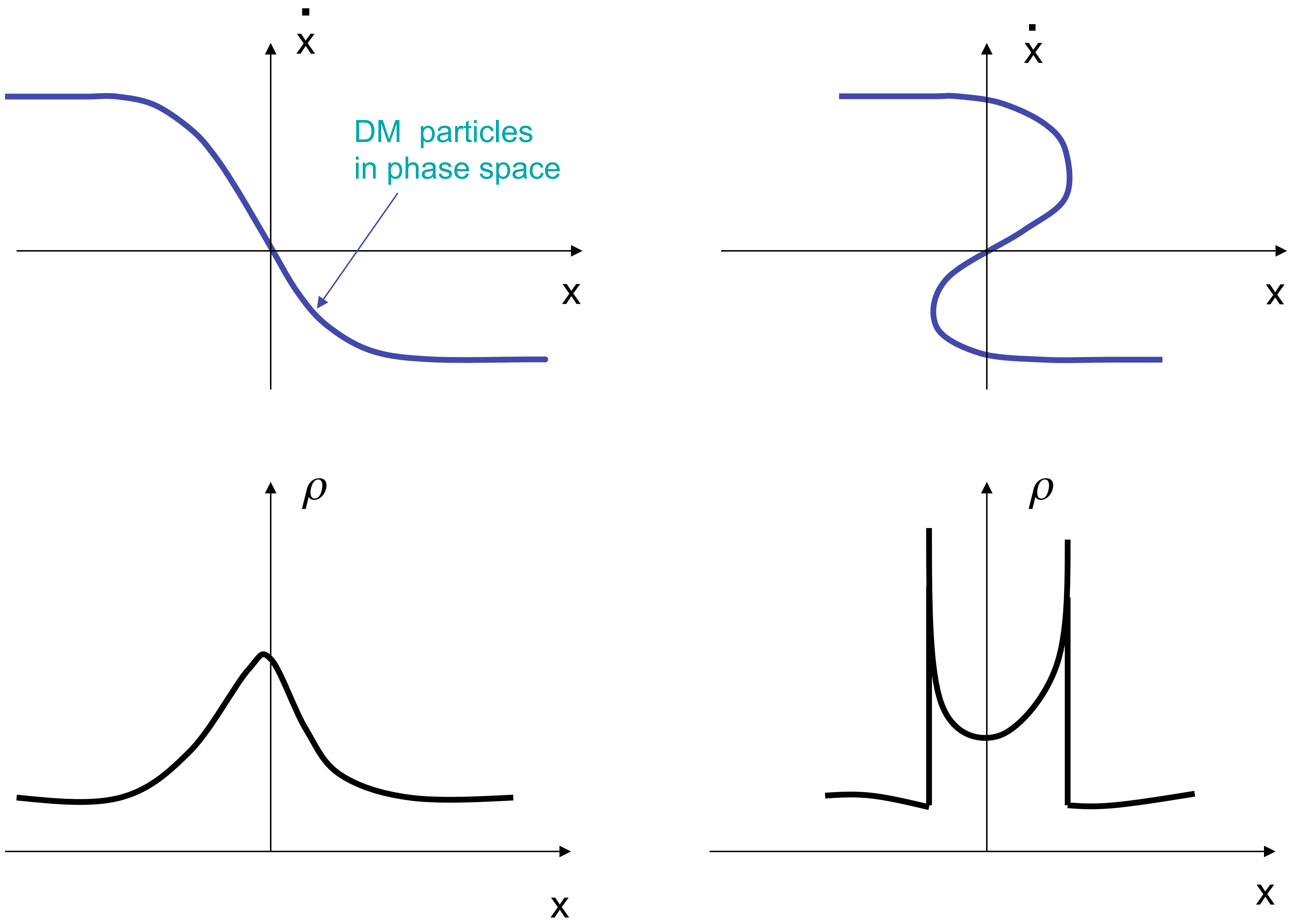
Axions rethermalize before falling onto galactic halos and go to their lowest energy state consistent with the total angular momentum they acquired from tidal torquing

provided $$4\pi G n m^2 \ell > m\dot{v}$$

i.e. $$nm > \frac{1}{30}\,\rho_{\rm DM}$$

Axion fraction of dark matter is more than of order 3%.

DM forms caustics in the non-linear regime



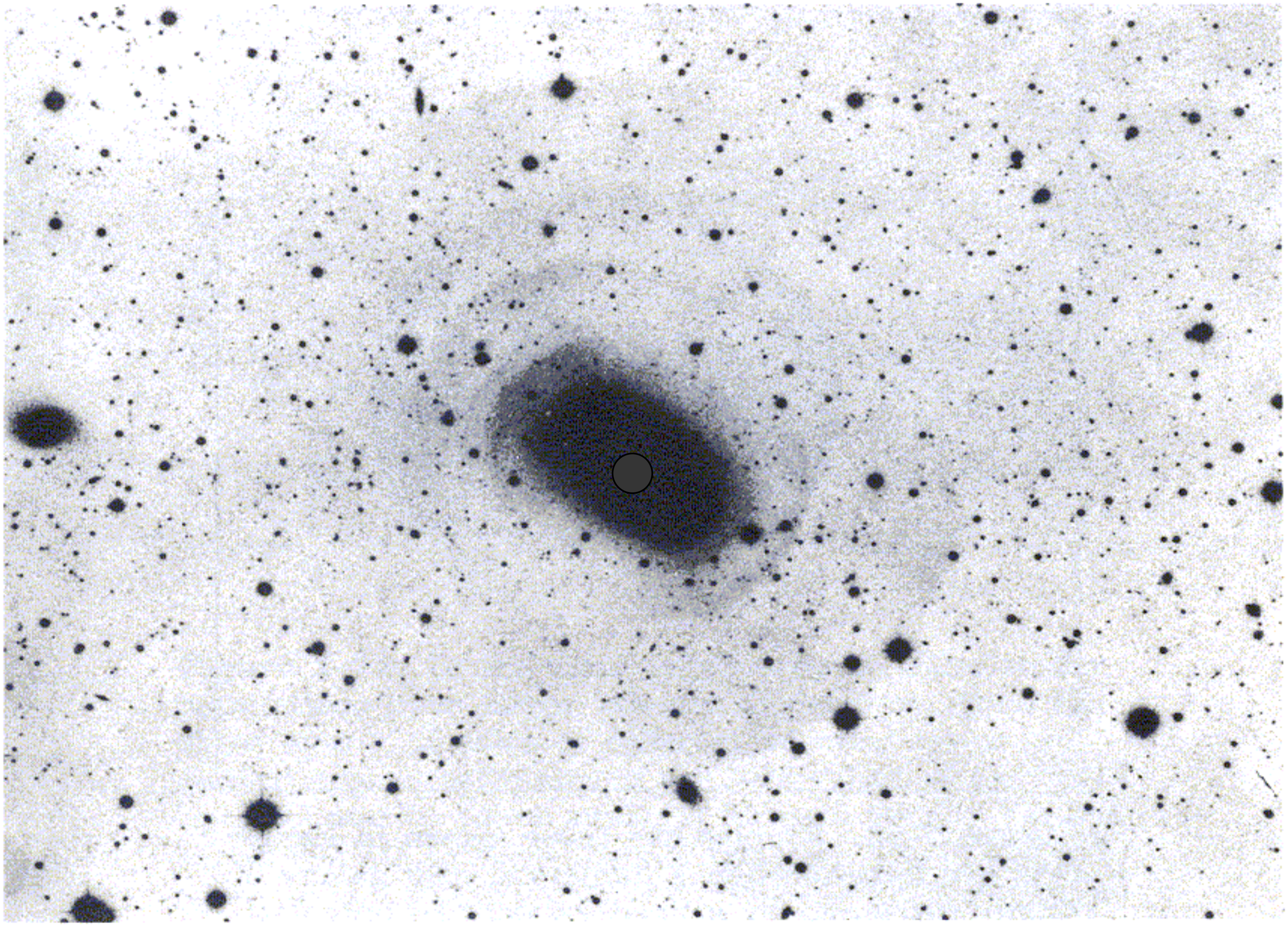


Figure 7-22. The giant elliptical galaxy NGC 3923 is surrounded by faint ripples of brightness. Courtesy of D. F. Malin and the Anglo-Australian Telescope Board.

(from Binney and Tremaine's book)

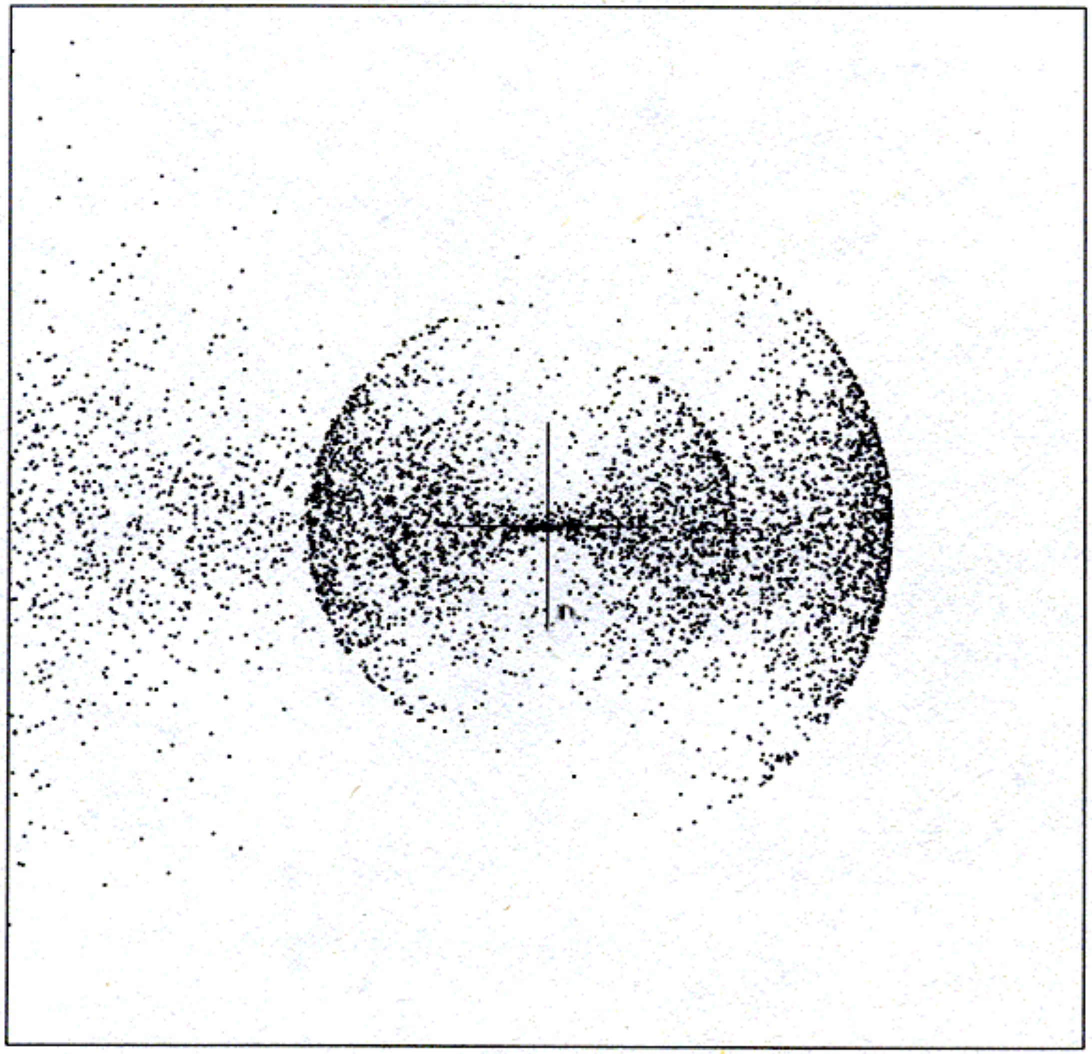


Figure 7-23. Ripples like those shown in Figure 7-22 are formed when a numerical disk galaxy is tidally disrupted by a fixed galaxy-like potential. (See Hernquist & Quinn 1987.)

Galactic halos have inner caustics as well as outer caustics.

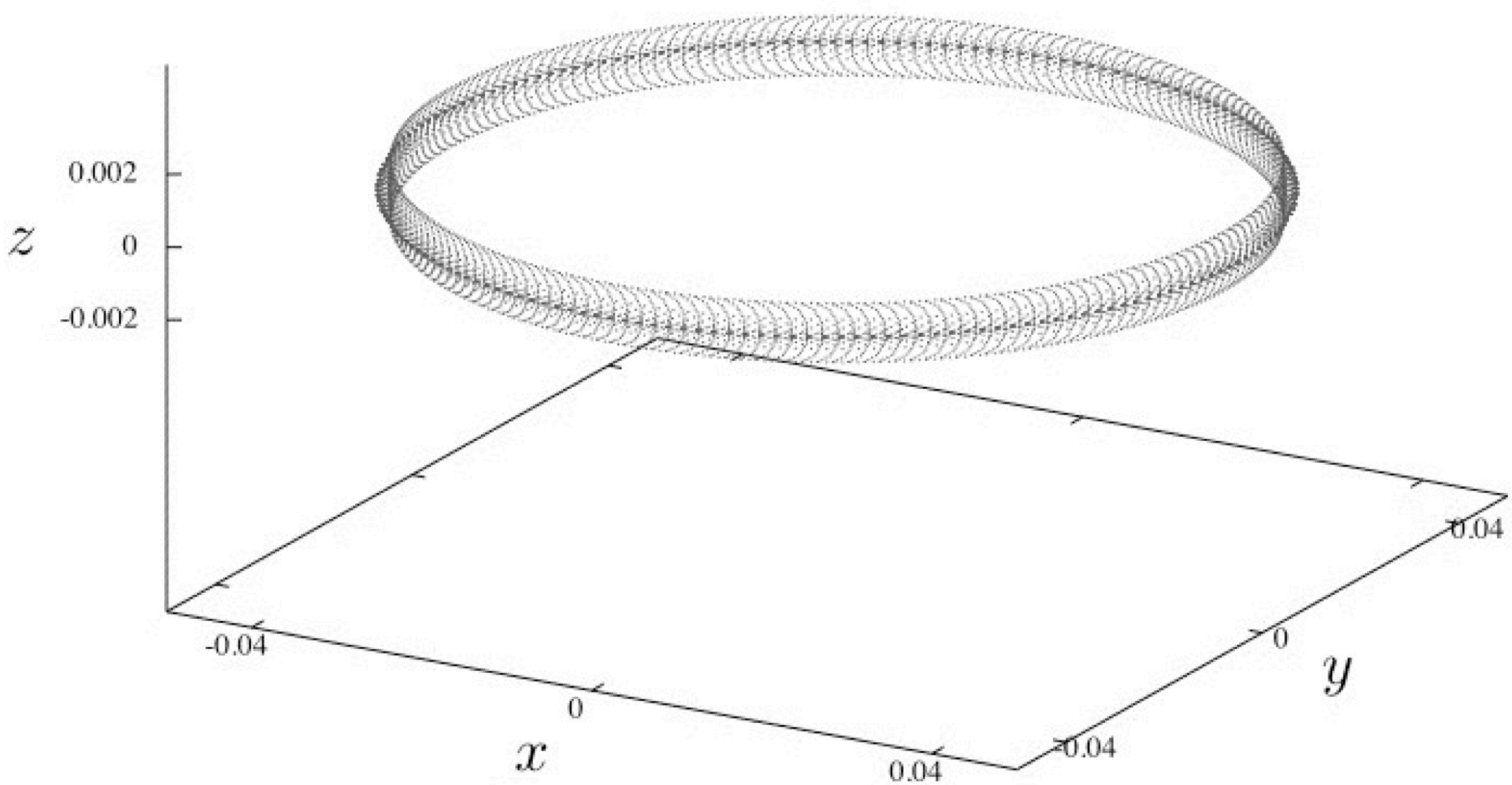
If the initial velocity field is dominated by net overall rotation, the inner caustic is a ‘tricusp ring’.

If the initial velocity field is irrotational, the inner caustic has a ‘tent-like’ structure.

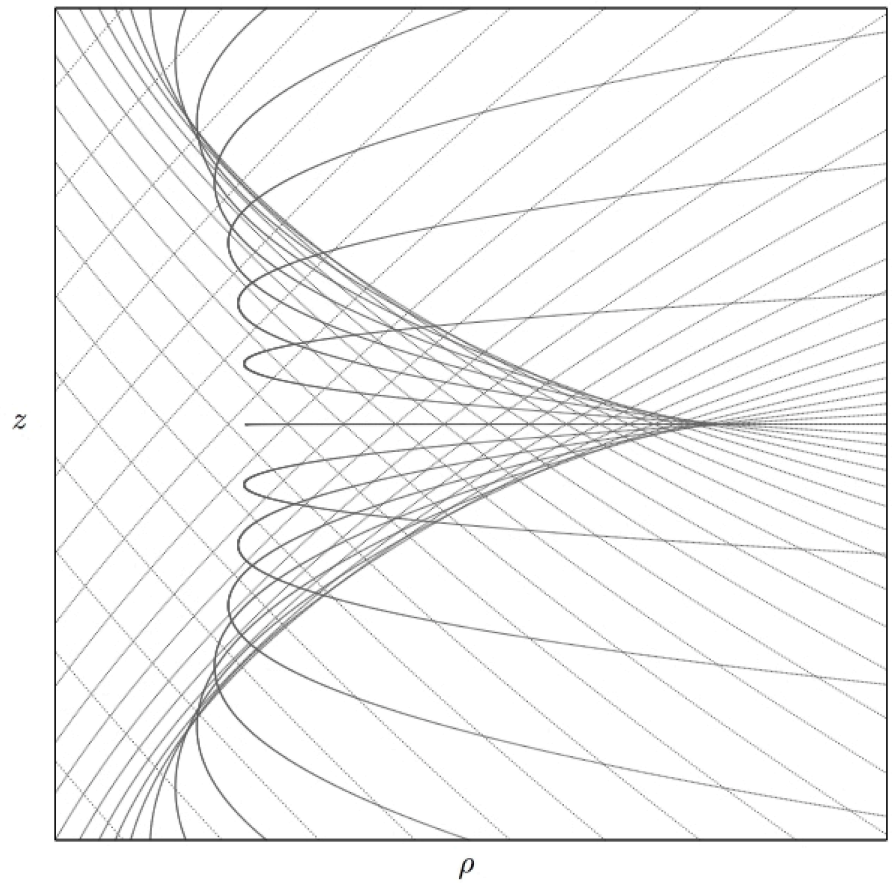
(Arvind Natarajan and PS, 2005).

simulations by Arvind Natarajan

in case of net overall rotation



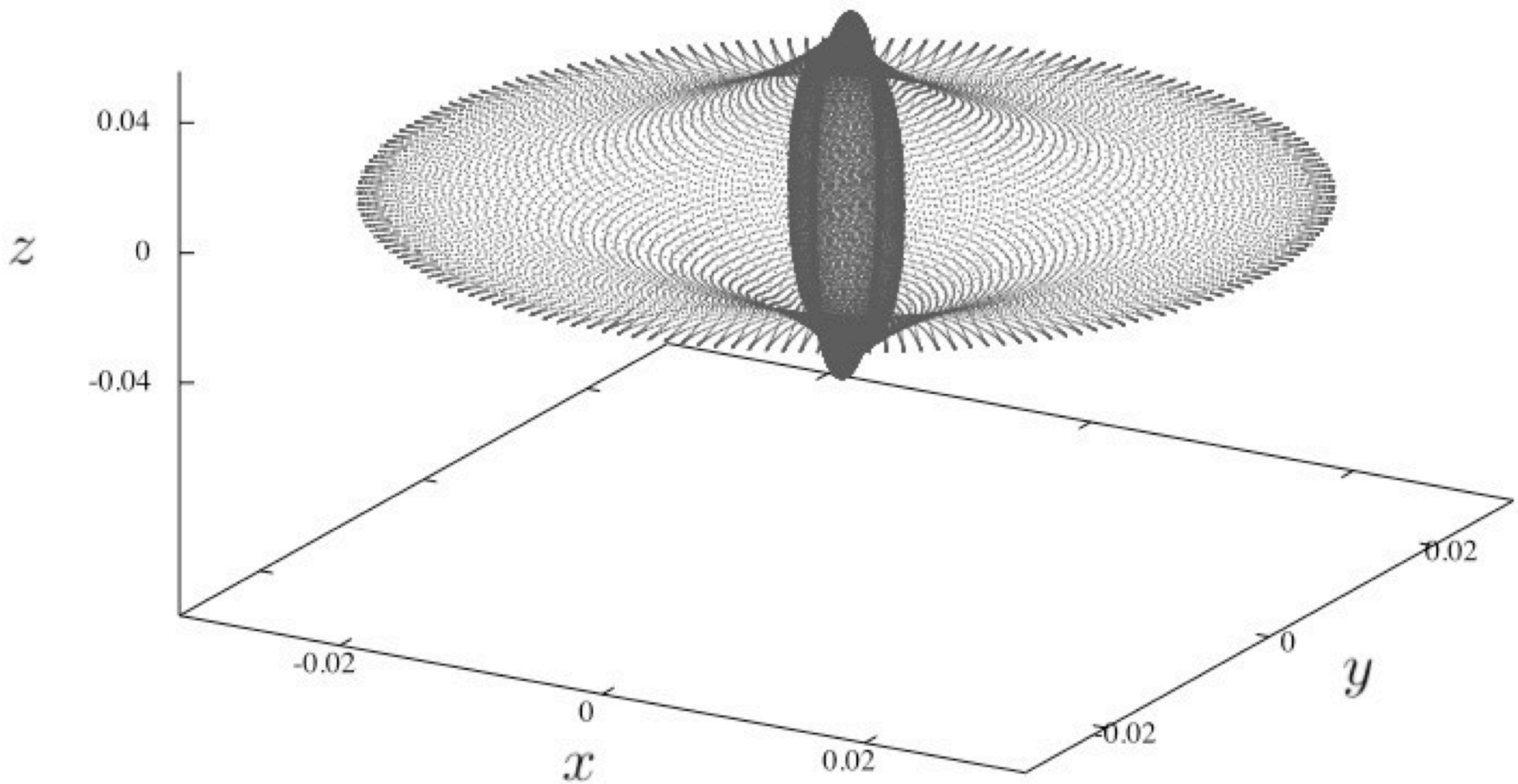
The caustic ring cross-section



D_{-4}

an elliptic umbilic catastrophe

in case of irrotational flow



On the basis of the self-similar infall model (Filmore and Goldreich, Bertschinger) with angular momentum (Tkachev, Wang + PS), the caustic rings were predicted to be

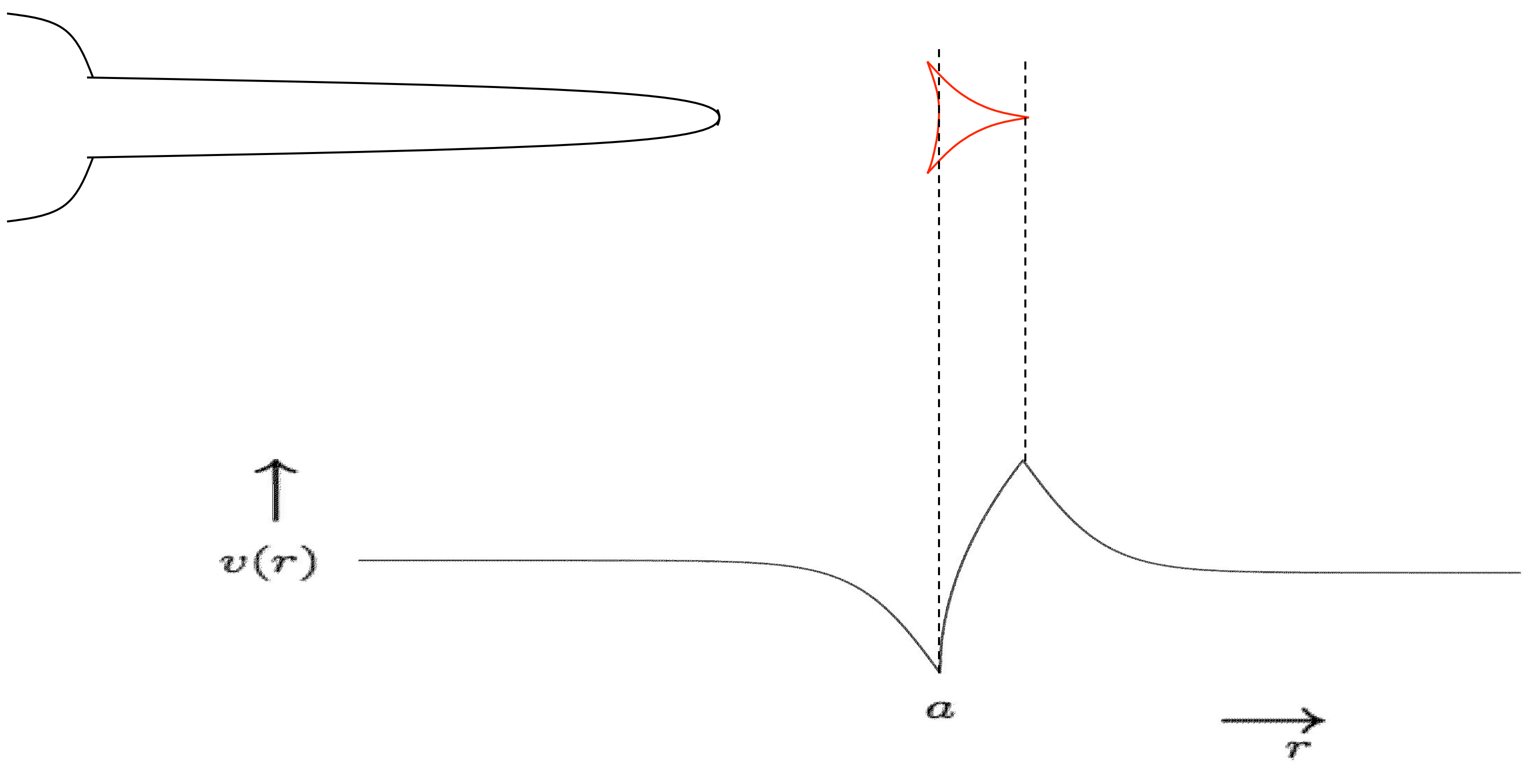
in the galactic plane

with radii $(n = 1, 2, 3 \ldots)$

$$a_n = \frac{40\,\mathrm{kpc}}{n}\left(\frac{v_{rot}}{220\,\mathrm{km/s}}\right)\left(\frac{j_{max}}{0.18}\right)$$

$j_{max} \cong 0.18$ was expected for the Milky Way halo from the effect of angular momentum on the inner rotation curve.

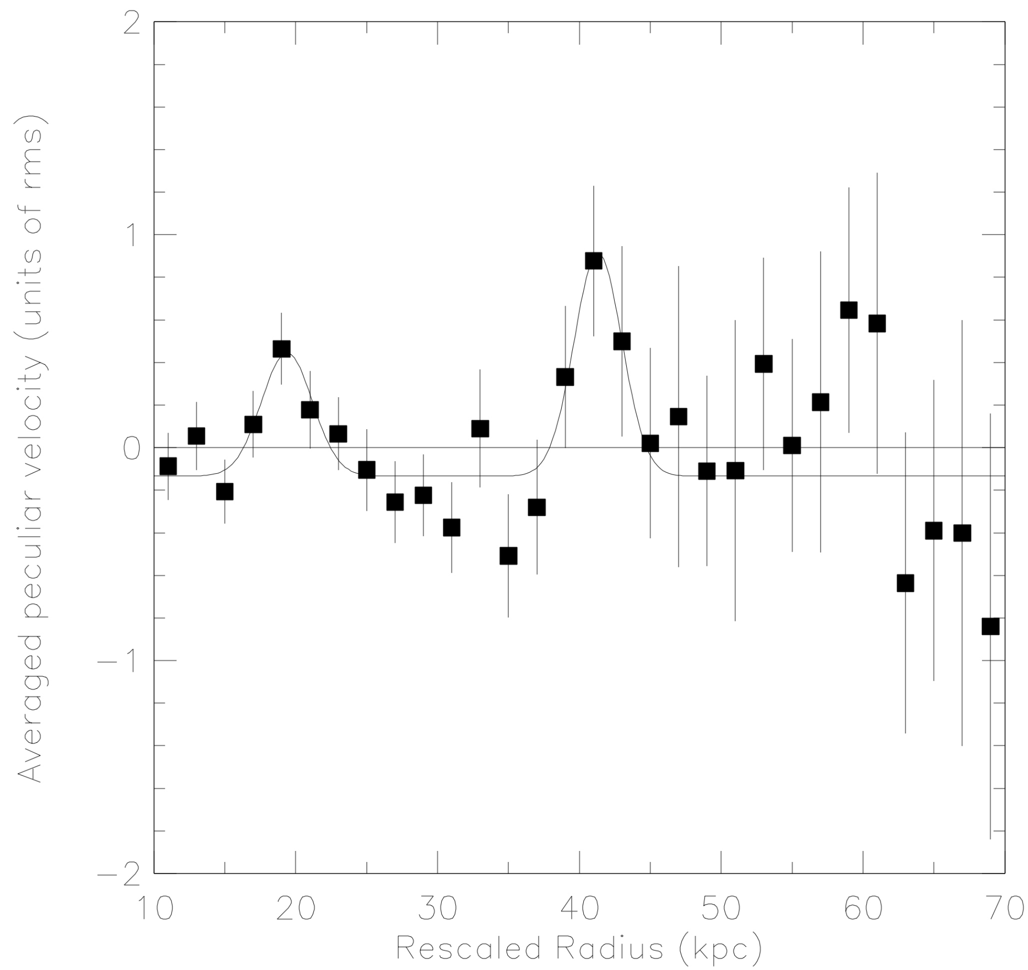
Effect of a caustic ring of dark matter upon the galactic rotation curve



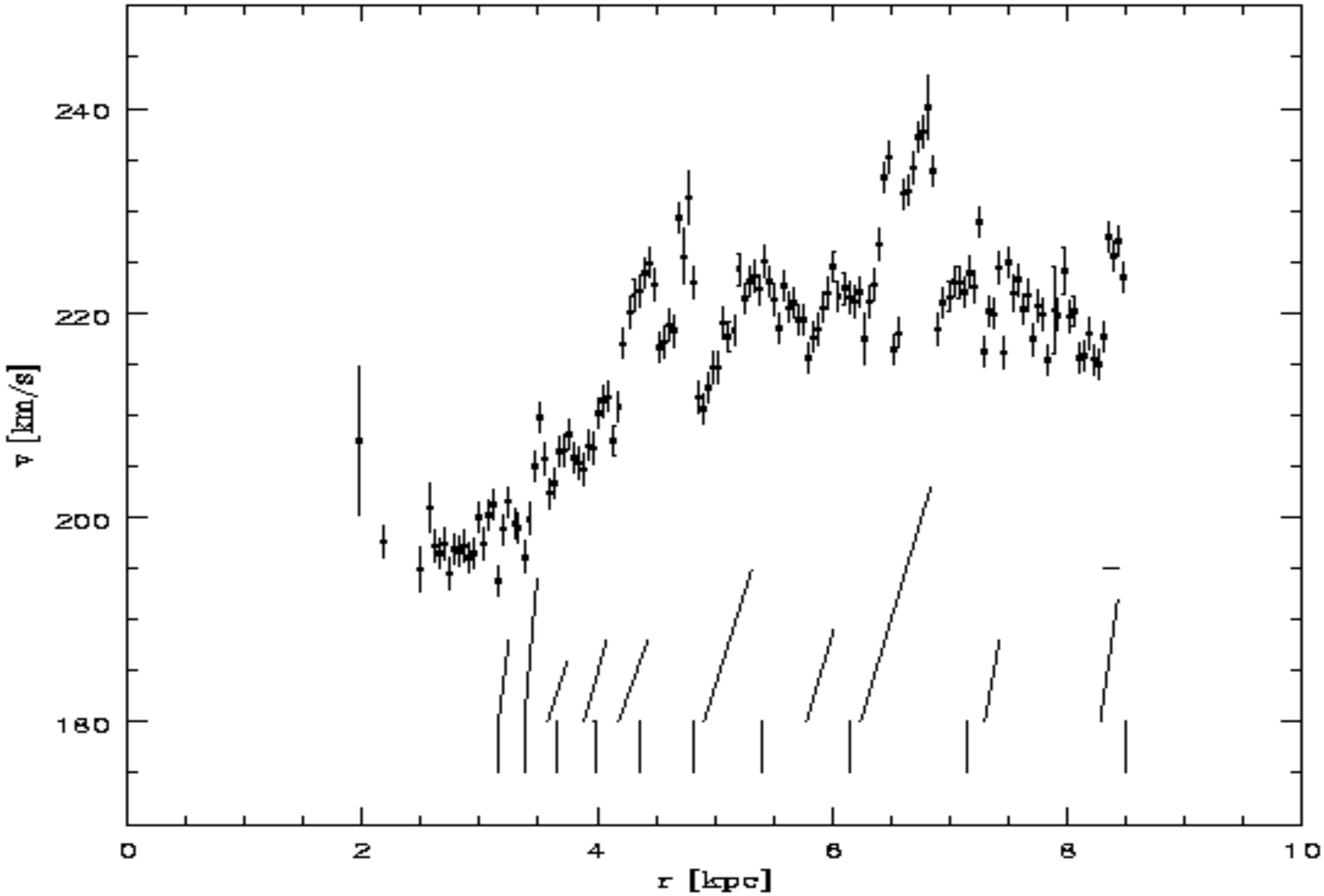
Composite rotation curve

(W. Kinney and PS, astro-ph/9906049)

- combining data on 32 well measured extended external rotation curves
- scaled to our own galaxy

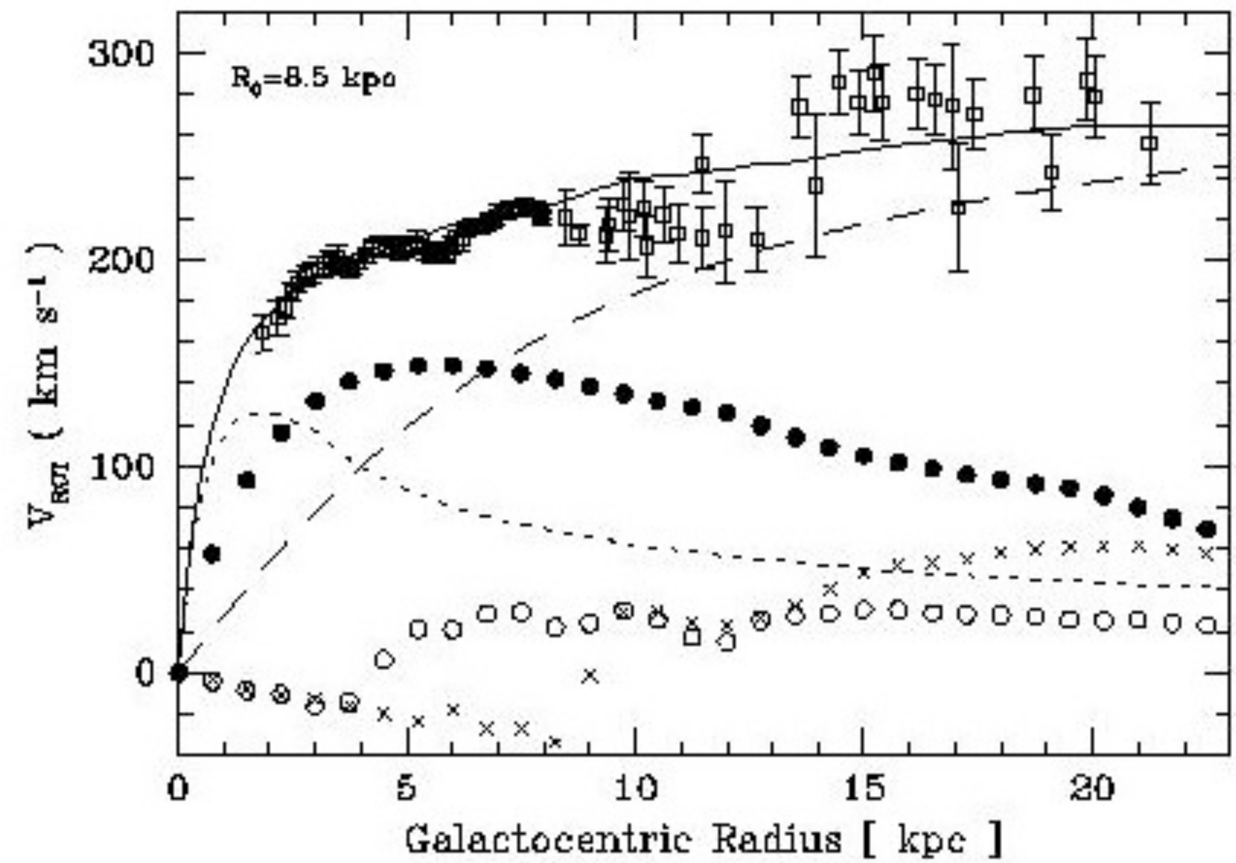


Inner Galactic rotation curve



from Massachusetts-Stony Brook North Galactic Pane CO Survey (Clemens, 1985)

Outer Galactic rotation curve



R.P. Olling and M.R. Merrifield, MNRAS 311 (2000) 361

Monoceros Ring of stars

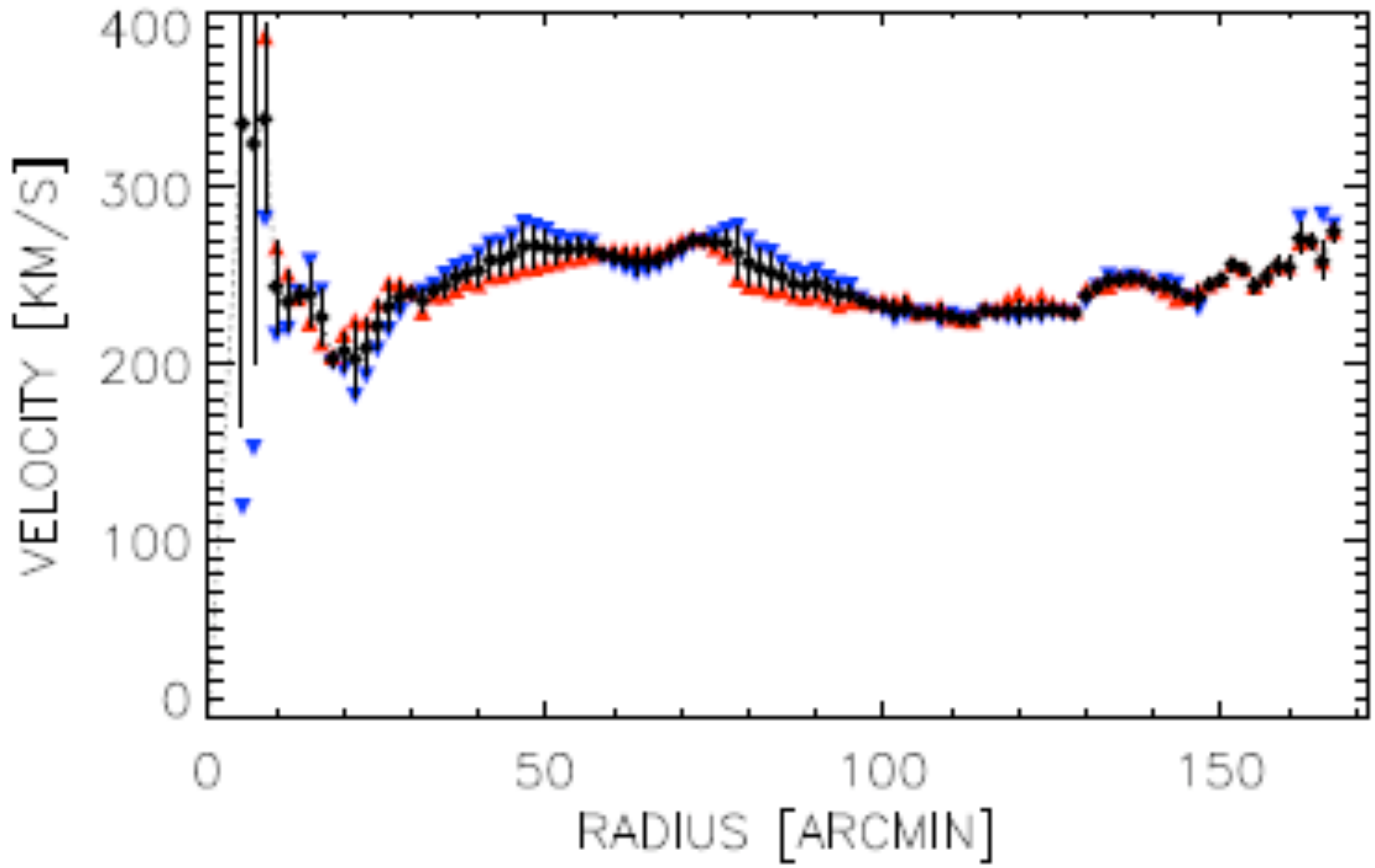
H. Newberg et al. 2002; B. Yanny et al., 2003; R.A. Ibata et al., 2003;
H.J. Rocha-Pinto et al, 2003; J.D. Crane et al., 2003; N.F. Martin et al., 2005

in the Galactic plane
at galactocentric distance $r \simeq 20\,\mathrm{kpc}$
appears circular, actually seen for $100^0 < l < 270^0$
scale height of order 1 kpc
velocity dispersion of order 20 km/s

may be caused by the n = 2 caustic ring of dark matter (A. Natarajan and P.S. '07)

Rotation curve of Andromeda Galaxy

from L. Chemin, C. Carignan & T. Foster, arXiv: 0909.3846



10 arcmin = 2.2 kpc

The caustic ring halo model assumes

L. Duffy & PS
PRD78 (2008) 063508

- net overall rotation
- axial symmetry
- self-similarity

The specific angular momentum distribution on the turnaround sphere

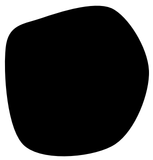
$$\vec{\ell}(\hat{n},t) = j_{\max} \; \hat{n} \times (\hat{z} \times \hat{n}) \; \frac{R(t)^2}{t}$$

$$R(t) \propto t^{\frac{2}{3}+\frac{2}{9\varepsilon}}$$

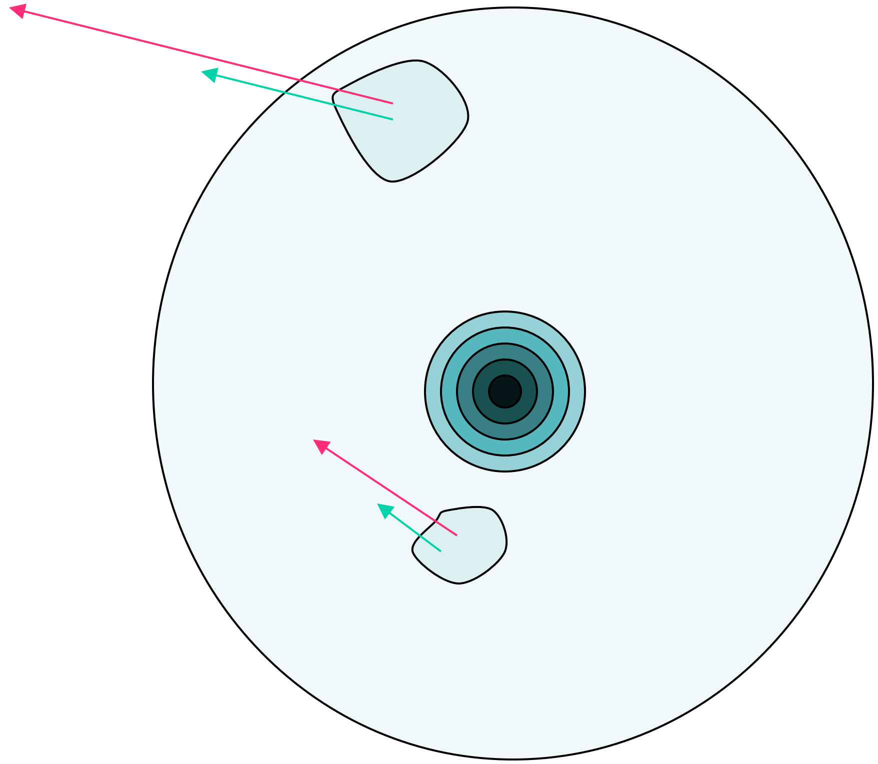
$$0.25 < \varepsilon < 0.35$$

Is it plausible in the context of tidal torque theory?

Tidal torque theory with ordinary CDM



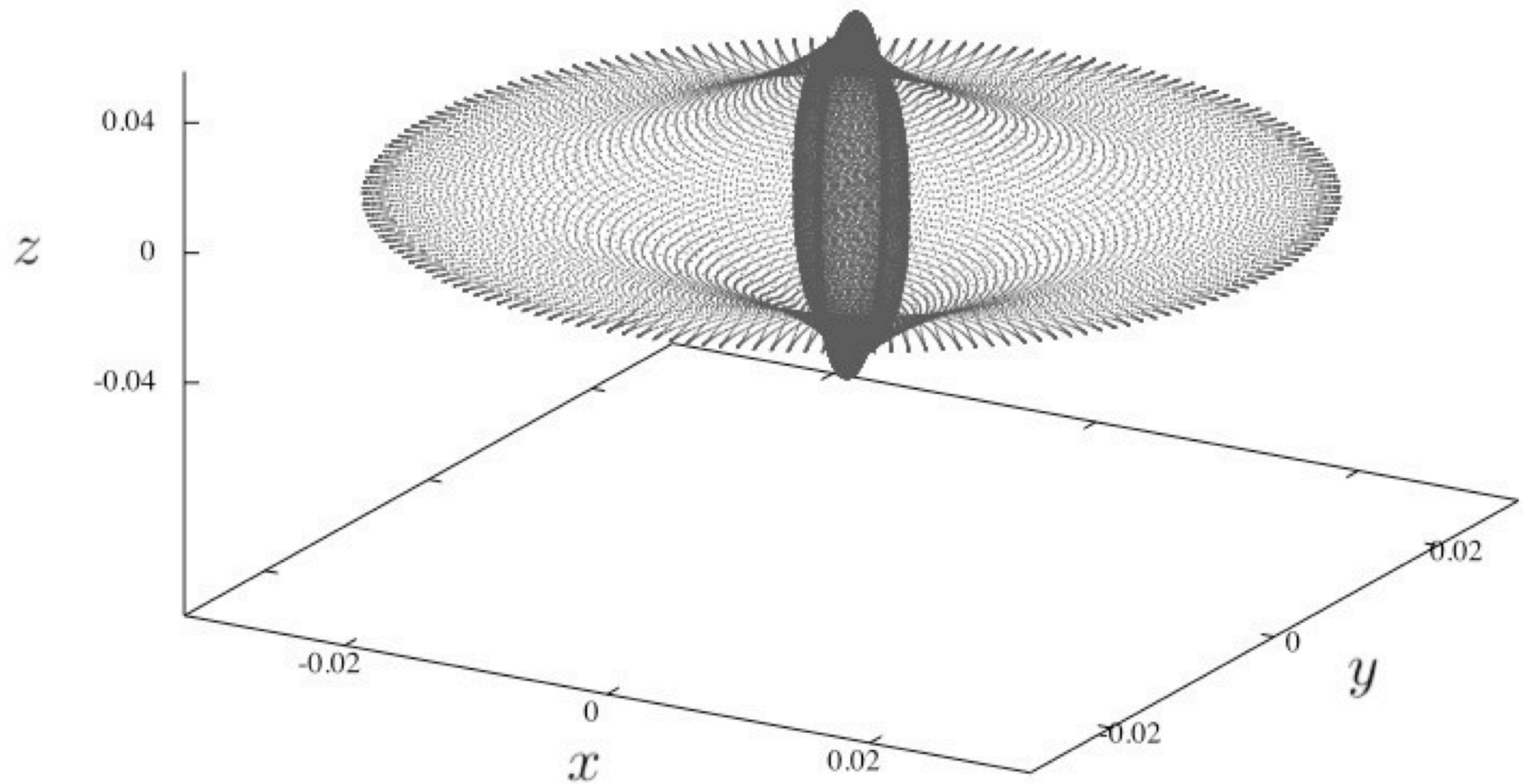
neighboring protogalaxy



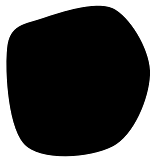
$$\vec{\nabla} \times \vec{v} = 0$$

the velocity field remains irrotational

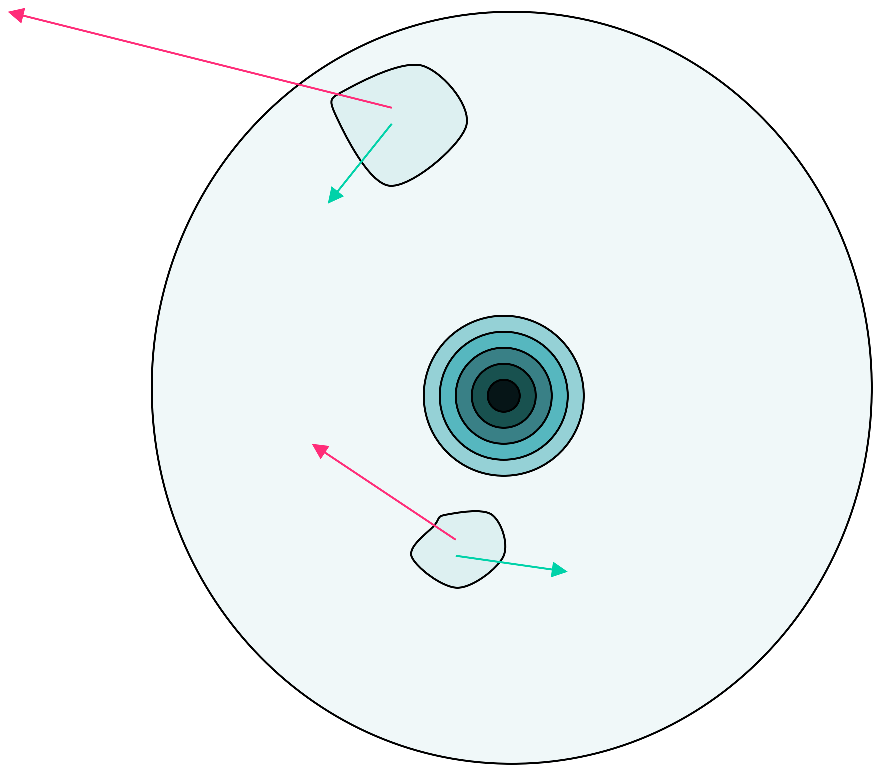
in case of irrotational flow



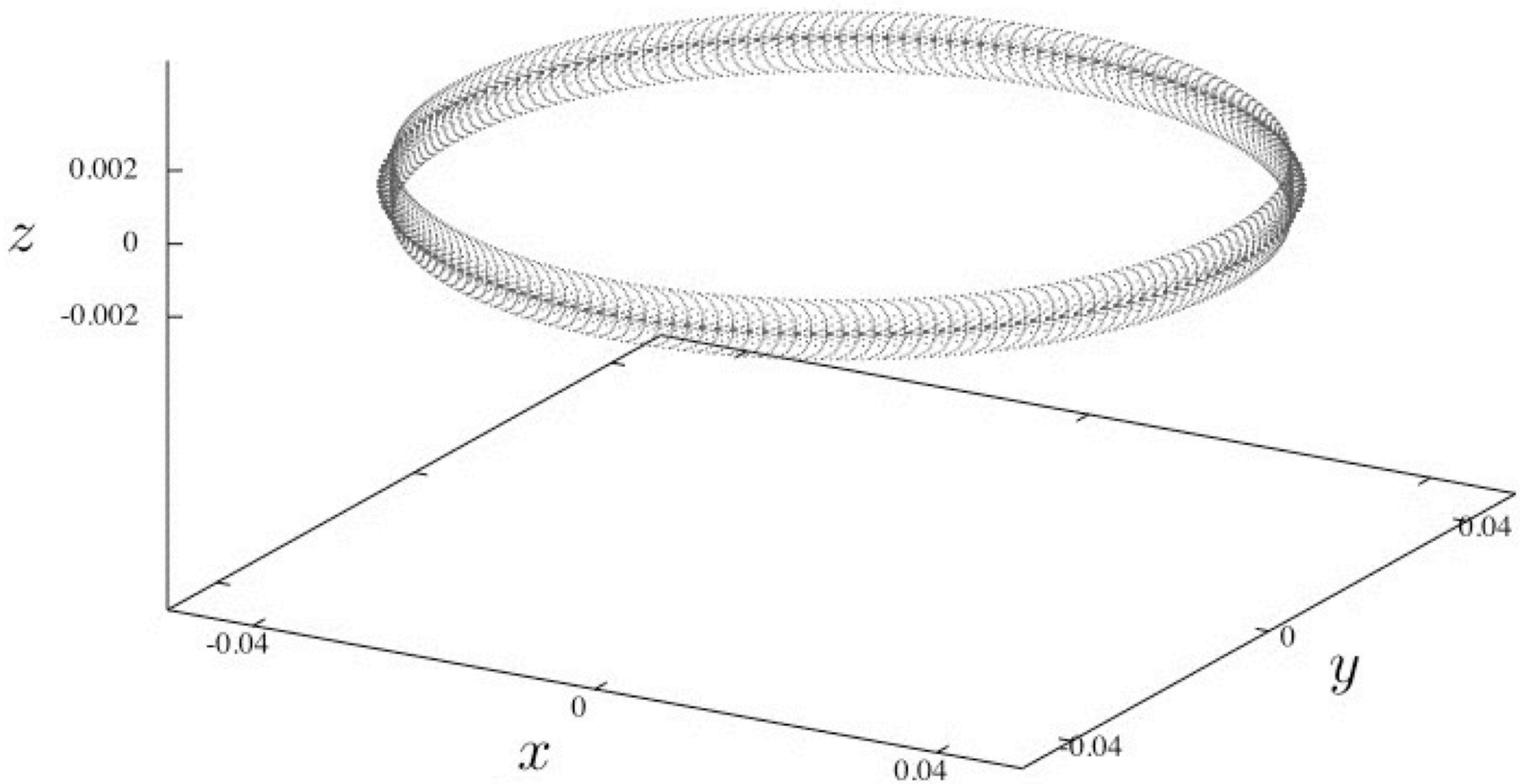
Tidal torque theory with axion BEC



$$\vec{\nabla} \times \vec{v} \neq 0$$



net overall rotation is obtained because, in the lowest energy state, all axions fall with the same angular momentum in case of net overall rotation



The specific angular momentum distribution on the turnaround sphere

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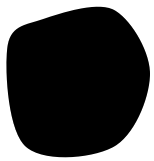
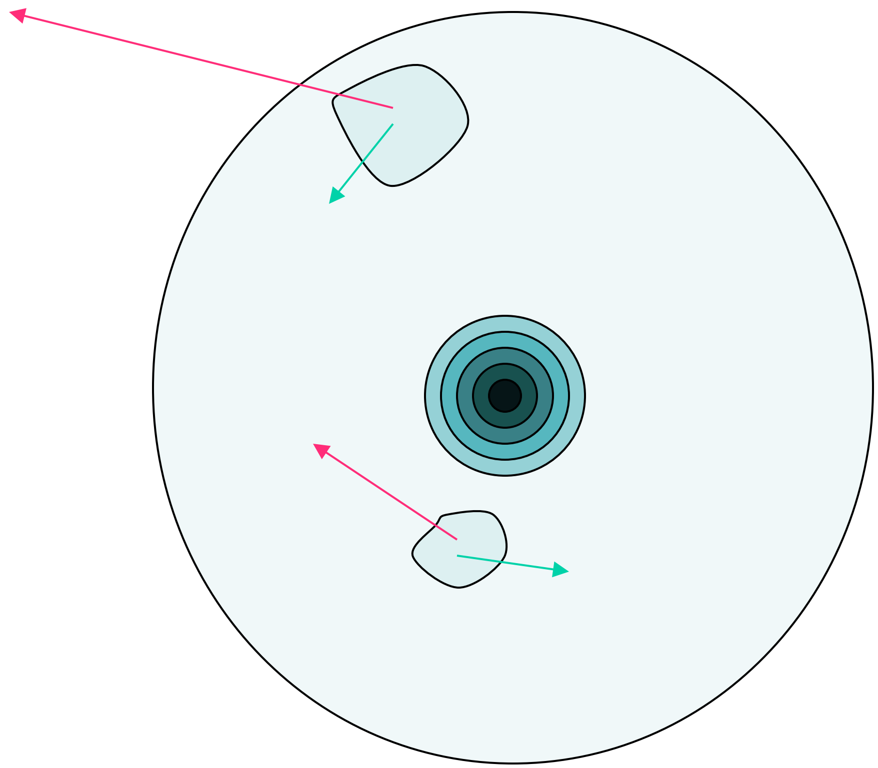
$$0.25 < \varepsilon < 0.35$$

Is it plausible in the context of tidal torque theory?

Tidal torque theory with axion BEC





$$\vec{\nabla} \times \vec{v} \neq 0$$

net overall rotation is obtained because, in the lowest energy state, all axions fall with the same angular momentum

Magnitude of angular momentum

$$\lambda = \frac{L\,|\,E\,|^{\frac{1}{2}}}{G\,M^{\frac{5}{2}}} = \sqrt{\frac{6}{5-3\varepsilon}}\;\frac{8}{10+3\varepsilon}\;\frac{1}{\pi}\;j_{\max}$$

$$\lambda \approx 0.05 \qquad\qquad j_{\max} \simeq 0.18$$

G. Efstathiou et al. 1979, 1987

from caustic rings

fits perfectly ($0.25 < \varepsilon < 0.35$)

The specific angular momentum distribution on the turnaround sphere

$$\vec{\ell}(\hat{n},t) = j_{\max} \; \hat{n}\times(\hat{z}\times\hat{n}) \; \frac{R(t)^2}{t}$$

$$R(t) \propto t^{\frac{2}{3}+\frac{2}{9\varepsilon}}$$

$$0.25 < \varepsilon < 0.35$$

Is it plausible in the context of tidal torque theory?

Self-Similarity

$$\vec{\tau}(t) = \int_{V(t)} d^3 r \;\; \delta\rho(\vec{r},t)\;\; \vec{r} \times (-\vec{\nabla}\phi(\vec{r},t))$$

a comoving volume

$$\vec{r} = a(t)\,\vec{x}$$

$$\phi(\vec{r} = a(t)\,\vec{x},\; t) = \phi(\vec{x})$$

$$\delta(\vec{r},t) \equiv \frac{\delta\rho(\vec{r},t)}{\rho_0(t)}$$

$$\delta(\vec{r} = a(t)\,\vec{x},\; t) = a(t)\,\delta(\vec{x})$$

$$\vec{\tau}(t) = \rho_0(t)\,a(t)^4 \int_V d^3 x \;\; \delta(\vec{x})\;\; \vec{x} \times (-\vec{\nabla}_x\,\phi(\vec{x}))$$

Self-Similarity (yes!)

$$\vec{\tau}(t) \propto \hat{z}\, a(t) \propto \hat{z}\ t^{\frac{2}{3}}$$

$$\vec{L}(t) \propto \hat{z}\ t^{\frac{5}{3}}$$

time-independent axis of rotation

$$\vec{\ell}(\hat{n},t) \propto \frac{R(t)^2}{t} \propto t^{\frac{1}{3}+\frac{4}{9\varepsilon}} = t^{\frac{5}{3}}$$

provided $\varepsilon = 0.33$

Conclusions

- Axions form a Bose-Einstein condensate as a result of their gravitational self-interactions
- When re-thermalizing, axion BEC behaves differently from WIMP dark matter
- The evidence for caustic rings of dark matter implies that the dark matter is axions, at least in part