Fermion representations in GUTs: on the quest for family unification

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Main idea

A very simple, yet very powerful constraint on new models

There are **well known Grand Unified Theories** (GUTs) based on the simple groups SU(5), SO(10), E6

What is their fermion content ??

$$\left\{\begin{array}{l}\overline{\mathbf{5}} \rightarrow d^c + L \\ \mathbf{10} \rightarrow Q + u^c + e^c\end{array}\right.$$

$$\mathbf{16} \rightarrow Q + u^c + d^c + L + e^c + N^c$$

$$\mathbf{27} \rightarrow Q + u^c + d^c + L + e^c + N^c + \left[\left(\mathbf{3},\mathbf{1},-\tfrac{1}{3}\right) + \left(\overline{\mathbf{3}},\mathbf{1},\tfrac{1}{3}\right)\right] + \left[\left(\mathbf{1},\mathbf{2},-\tfrac{1}{2}\right) + \left(\mathbf{1},\mathbf{2},\tfrac{1}{2}\right)\right]$$

In every case we have the SM fermions plus (maybe) **vector fermions**

One should only introduce more vector fermions, not chiral ones: these would only get an EW scale mass and so should have been seen already

!!

Beyond the standard scenarios

Scanning thoroughly the possibilities

Are there **other combinations of GUT representations (1 or more)** which yield the SM fermions (+ vector particles)?

??

We do not want more fermions remaining massless until the EW scale is broken

In order to check this, one should make a <u>triple scan</u> over ...

Groups

Any simple one which has complex representations

Representations

Use one or more GUT group fermion representations

Embeddings

There is more than one way to embed the SM group in the GUT group

{ Non-trivial and often overlooked in the literature }

Why bother with this?

Non-standard fermion representations may have **important consequences**.
Two examples ...

1. Gauge coupling unification

Usually $g_1 = \sqrt{5/3}g', g_2 = g, g_3 = g_s$

but

This is a consequence of putting d^c and L in the $\overline{\mathbf{5}}$ of SU(5)

Recall:

$$Y = n \times \mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}\right)$$

$$T_{3L} = \mathrm{diag}\left(0, 0, 0, \frac{1}{2}, -\frac{1}{2}\right)$$

$$\mathrm{Tr}\left(Y^2\right) = \mathrm{Tr}\left(T_{3L}^2\right) \Rightarrow |n| = \sqrt{3/5}$$

Imagine then an **SU(7) model** where

$$\mathbf{7} \rightarrow \underbrace{\left(\overline{\mathbf{3}}, \mathbf{1}, \tfrac{1}{3}n\right)}_{\text{One color (anti)triplet}} + \underbrace{\left(\mathbf{1}, \mathbf{2}, -\tfrac{1}{2}n\right) + \left(\mathbf{1}, \mathbf{2}, 0\right)}_{\text{Two SU(2) doublets}}$$

so

The correctly normalized SU(2) generators are $\frac{1}{2\sqrt{2}}\sigma^a$ instead of $\frac{1}{2}\sigma^a$

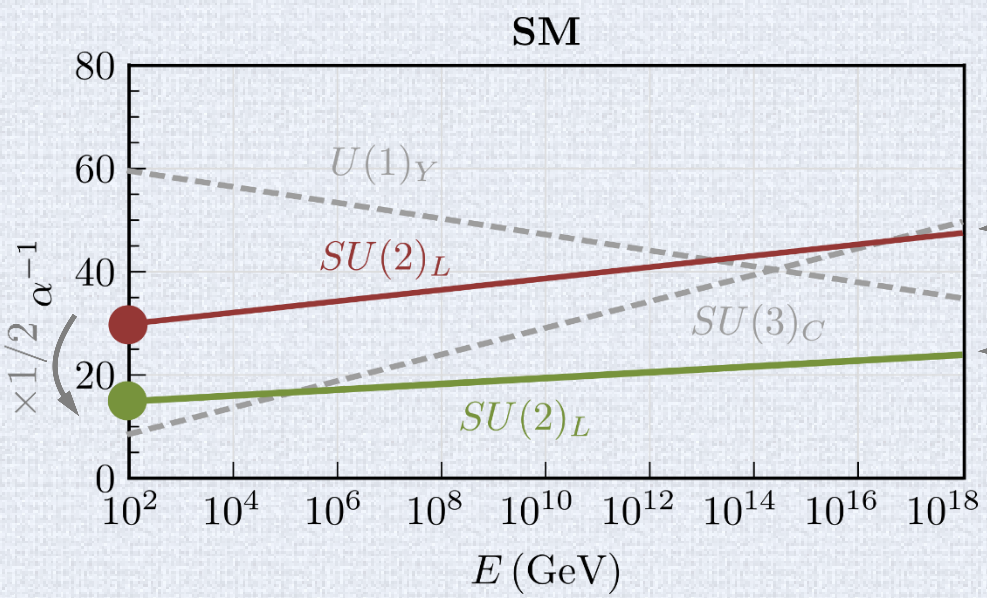
and

$$g_1 = \sqrt{5/3}g', g_2 = \boldsymbol{\sqrt{2}}g, g_3 = g_s$$

Why bother with this?

Non-standard fermion representations may have **important consequences**.
Two examples ...

1. Gauge coupling unification



Just to give an example!

Wrong

Correct line for the SU(2) gauge coupling constant in this simple SU(7) GUT

Why bother with this?

2. Explain flavor

"Standard" GUTs can relate the different SM Yukawa matrices,

(e.g.) $$Y_{ij}\,\mathbf{16}_i\,\mathbf{16}_j\,\mathbf{10} = Y_{ij}\,Q_i u^c_j H + Y_{ij}\,Q_i d^c_j H + Y_{ij}\,L_i e^c_j H + Y_{ij}\,L_i N^c_j H$$

but

Y is still a free 3 by 3 matrix. Nature's replication of the families and the associated structure are not explained

1. The problem: fermions are placed on three copies of the same representation ($\mathbf{16}$)

2. The (obvious) solution: distribute the families by different GUT representations (the gauge group then discriminates the families)

3. Ideally (*family unification*): put (really) all SM fermions in one GUT representation $\boldsymbol{R}$:

$$y\boldsymbol{RRS}$$

One **number** controlling the whole the Yukawa sector

!!

4. The (new) problem: recall that we can't have more chiral fermions in $\boldsymbol{R}$, so what are the possibilities? This is a simple but very powerful constraint! So let us be humble and allow for more than one GUT representation

A small example with SU(5)

- We know that three copies of $\overline{\mathbf{5}}+\mathbf{10}$ works.
- It is obvious that adding real or conjugate pairs of complex representations will also work.
- Are there **other non-trivial combinations of SU(5) representations smaller than 36** which contain the SM fermions + vector fermions only?

$$c_1\,(\mathbf{5})+c_2\,(\mathbf{10})+c_3\,(\mathbf{15})+c_4\,(\mathbf{35})$$

- Let's check it, assuming the **standard hypercharge normalization**:

$$\mathbf{5}\rightarrow\left(\mathbf{3},\mathbf{1},-\frac{1}{3}\right)+\left(\mathbf{1},\mathbf{2},\frac{1}{2}\right)$$

$$\mathbf{10}\rightarrow\left(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3}\right)+\left(\mathbf{3},\mathbf{2},\frac{1}{6}\right)+(\mathbf{1},\mathbf{1},1)$$

$$\mathbf{15}\rightarrow\left(\overline{\mathbf{6}},\mathbf{1},-\frac{2}{3}\right)+\left(\mathbf{3},\mathbf{2},\frac{1}{6}\right)+(\mathbf{1},\mathbf{3},1)$$

$$\mathbf{35}\rightarrow(\overline{\mathbf{10}},\mathbf{1},1)+\left(\mathbf{6},\mathbf{2},\frac{1}{6}\right)+\left(\overline{\mathbf{3}},\mathbf{3},-\frac{2}{3}\right)+\left(\mathbf{1},\mathbf{4},-\frac{3}{2}\right)$$

$$\begin{array}{c} \\ \begin{pmatrix}3\\3\\0\\0\\3\\3\\0\\0\\0\\0\\3\end{pmatrix}\end{array} = \begin{array}{cc} \begin{array}{cccc}\mathbf{5} & \mathbf{10} & \mathbf{15} & \mathbf{35}\end{array} & \\ \underbrace{\begin{pmatrix}0&1&0&0\\-1&0&0&0\\0&0&-1&0\\0&0&0&1\\-1&0&0&0\\0&1&1&0\\0&0&0&1\\0&0&-1&0\\0&0&0&1\\0&0&0&1\\0&1&0&0\end{pmatrix}}_{\mathcal{M}} & \begin{array}{l}\leftarrow(\mathbf{1},\mathbf{1},1)\\ \leftarrow(\overline{\mathbf{3}},\mathbf{1},\frac{1}{3})\\ \cdots\\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array}\end{array} \cdot \begin{pmatrix}c_1\\c_2\\c_3\\c_4\end{pmatrix}$$

Answer: NO

Solution:

$$(c_1,c_2,c_3,c_4) = (-3,3,0,0)$$

There is no other since the nullspace of $\mathcal{M}$ is trivial

SU(5): generalizing the calculation

Bigger representations and different hypercharge normalizations

1 The previous example was quite simple but interestingly it is very easy to include (much) **bigger representations**

Use the **SUSYNO program** (a package for `Mathematica`) to do it automatically

2 On the other hand, we assumed the **usual hypercharge normalization**. We can **drop this assumption**

For SU(5) this is not a big problem. But it can be messy for bigger ones (related to the problem of scanning over **different embeddings of the SM group**)

When we do this generalization, we find that the answer is the same as before:

In SU(5), the only solution is the well know one (plus trivial variations)

(considering representations up to size 1.000.000)

About Susyno

RF, Comp.Phys.Com. 183 (2012) 2298

Performing and automating group theory calculations

Ignore the name*

*SUSY related

Susyno contains various group theory functions ready to be used

Tutorial: Group theory with Susyno

0. Getting started

Susyno is a Mathematica package which can make various calculations related to Lie groups and the permutation group S_n (even though the main aim of the program is a different one, a substantial part of the code is group theory related). This page shows some examples.

To get started, the program needs to be installed: it can be download here. Decompressing the downloaded zip file will generate a folder *Susyno*. This folder should be placed in a directory visible to Mathematica (typing `$Path` in Mathematica shows a list of the possible locations). A good choice is to place it (the whole *Susyno* folder and not just its contents!) in

(Mathematica base directory)/AddOns/Applications	Linux, Mac OS
(Mathematica base directory)\AddOns\Applications	Windows

Once this is done the package is installed. To load Susyno, type in the front end

```
<<Susyno`
```

The program's reference is

"Renato M. Fonseca, *Calculating the renormalisation group equations of a SUSY model with Susyno*, Computer Physics Communications 183 (2012) 2298"

For help, there is an easy-to-use built-in documentation for each of the functions described in the following text (and other ones as well); it becomes accessible from within Mathematica once the package is installed. For questions, comments or bug reports, please contact me at

renato.fonseca@ific.uv.es

renatofonseca.net/susyno/group_theory_tutorial.php

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1. Working with representations of a Lie group

To **indicate a group**, just use it's name:

```
U1, SU2, SU3, SU5, SO10, E6, E8, G2, ...
```

If a group is the product of $U(1)$'s and/or simple groups, a list of the factor groups should be given. For example, the groups $SU(3)\times SU(2)\times U(1)$, $SU(5)\times SU(5)$ and $SO(10)\times U(1)$ are written as follows:

```
{SU3,SU2,U1}
{SU5,SU5}
{SO10,U1}
```

In fact, even groups with a single factor might have the brackets {} around. For example, one might write $SU(2)$ as

```
{SU2}
```

To **indicate a representation** of each factor group one needs to write its Dynkin coefficients (unless it is a $U(1)$, in which case the charge [i.e., one number] is enough). These are a list of n non-negative integers, where n is the group's rank. In turn, the group's rank corresponds to the maximal number of elements of the group's algebra which can be made simultaneously diagonal, and in practice this can be obtained with Susyno by typing

```
Length[<group>]
```

such as

```
Length[SU5]
```

Out[1]= 4

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2. Dimension, Casimir, Dynkin index and triangular anomaly

Given a representation R, its dimension $d(R)$, Casimir $C(R)$ and Dynkin index $T(R)$ can be calculated with the functions **DimR**, **Casimir** and **DynkinIndex**:

```
DimR[SO10, {0, 0, 0, 0, 1}]
Casimir[SO10, {0, 0, 0, 0, 1}]
DynkinIndex[SO10, {0, 0, 0, 0, 1}]
```

Out[8]= 16

Out[9]= $\frac{45}{8}$

Out[10]= 2

(Recall that $C(R)$ and $T(R)$ are defined by the relations $C(R)Id = \sum_a T^a T^a$ and $T(R)\delta^{ab} = \mathrm{Tr}\left(T^a T^b\right)$ where T^a are the representation matrices of the algebra generators.) These functions actually accept symbolic Dynkin coefficients. For example, the $SU(2)$ representation $\{d-1\}$ has dimension d and its Casimir is $\frac{1}{4}(d-1)(d+1)$:

```
DimR[SU2, {d - 1}]
Casimir[SU2, {d - 1}]
```

Out[11]= d

Out[12]= $\frac{1}{4}(-1+d)(1+d)$

Gauge triangular anomalies are known to be associated to the quantity $\mathrm{Tr}\left(\left\{T^a, T^b\right\} T^c\right) \equiv \kappa(R) d^{abc}$ where the symmetric tensor d^{abc} can be taken to be fixed (for a given group) while $\kappa(R)$ depends on the representation. It should be noted that the only groups for which there might be anomalies are those with $SU(n>2)$ and/or with $U(1)$ factors, so at least one of the indices a, b and c must refer to one of these groups.

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3. Products of representations

The decomposition of products of representations can be achieved with **ReduceRepProduct**:

```
tripletSU3 = {1, 0};
ReduceRepProduct[SU3, {tripletSU3, tripletSU3, tripletSU3}]
```

Out[28]= {{{3, 0}, 1}, {{1, 1}, 2}, {{0, 0}, 1}}

In this case, the output is saying that $\mathbf{3}\times\mathbf{3}\times\mathbf{3}$ in $SU(\mathbf{3})$ (note that {1,0} is the $\mathbf{3}$) contains the representation {3,0} once, {1,1} twice, and {0,0} once. If desired, the option UseName -> True can be used to convert these Dynkin coefficients into the names of the representations:

```
ReduceRepProduct[SU3, {tripletSU3, tripletSU3, tripletSU3}, UseName -> True]
```

Out[29]= {{**10**, 1}, {**8**, 2}, {**1**, 1}}

The group can have various factors, for example:

```
uc = {-2/3 , {0}, {0, 1}};
Q = {1/6 , {1}, {1, 0}};
H = {1/2 , {1}, {0, 0}};
ReduceRepProduct[{U1, SU2, SU3}, {uc, Q, H}, UseName -> True]
```

Out[33]= {{$0\otimes\mathbf{3}\otimes\mathbf{8}$, 1}, {$0\otimes\mathbf{3}\otimes\mathbf{1}$, 1}, {$0\otimes\mathbf{1}\otimes\mathbf{8}$, 1}, {$0\otimes\mathbf{1}\otimes\mathbf{1}$, 1}}

Also, there is no limit to the number of representations being multiplied:

```
rep10 = {1, 0, 0, 0, 0};
ReduceRepProduct[SO10, {rep10, rep10, rep10, rep10, rep10}, UseName -> True]
```

Out[35]= {{**1782**, 1}, {**4608**, 4}, {**210'**, 10}, {**4410**, 5}, {**4312**, 6}, {**320**, 20}, {**10**, 15}, {**2970**, 5}, {**1728**, 4}, {**120**, 10}, {$\overline{\mathbf{126}}$, 1}, {**126**, 1}}

Sometimes, the product of representation being calculated contains repeated representations, such as the $\mathbf{3}\times\mathbf{3}\times\mathbf{3}$ example above. In those cases, there is a permutation symmetry involved: for example, it is well known that the singlet in $\mathbf{3}\times\mathbf{3}\times\mathbf{3}$ is completely anti-symmetric. To calculate

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5. Group invariant combinations of representations

Often one needs to know how to contract, in a group invariant way, the components of a product of representations. The simplest example would be two $SU(2)$ doublets --- let us call them $D=(D_1,D_2)^T$ and $D'=(D'_1,D'_2)^T$: it is well known that the combination $D_1D'_2-D_2D'_1$ is left invariant under the action of the $SU(2)$ group. To calculate group invariant combinations with Susyno, use the **Invariants** function:

```
Invariants[SU2,{{1},{1}}] (*product of two SU(2) doublets*)
```

Out[43]= {a[2] b[1] - a[1] b[2]}

The syntax is Invariants[<group>,{<rep1>, <rep2>, ...}], with an arbitrary number of representations. In the output, the program considers that the components of <rep1> are named a[1], a[2], etc., that those of <rep2> are b[1], b[2], etc., and so forth.

Notice that the output above is not just a[2] b[1] - a[1] b[2]; this expression is surrounded by curly brackets. The reason is, in general, there might more than one independent way of contracting the representations in an invariant way. To illustrate this point, consider the product of four $SU(2)$ doublets, which is known to have two independent invariants (this statement can be confirmed with the ReduceRepProduct function, by counting the number of singlets in the product of four doublets):

```
Invariants[SU2, {{1}, {1}, {1}, {1}}] (*product of four SU(2) doublets*)
```

Out[44]=
$$\Big\{-a[2]\,b[2]\,c[1]\,d[1]+a[1]\,b[2]\,c[2]\,d[1]+a[2]\,b[1]\,c[1]\,d[2]-a[1]\,b[1]\,c[2]\,d[2],\ \frac{a[2]\,b[2]\,c[1]\,d[1]}{\sqrt{3}}-\frac{2\,a[2]\,b[1]\,c[2]\,d[1]}{\sqrt{3}}+\frac{a[1]\,b[2]\,c[2]\,d[1]}{\sqrt{3}}+\frac{a[2]\,b[1]\,c[1]\,d[2]}{\sqrt{3}}-\frac{2\,a[1]\,b[2]\,c[1]\,d[2]}{\sqrt{3}}+\frac{a[1]\,b[1]\,c[2]\,d[2]}{\sqrt{3}}\Big\}$$

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7. Symmetry breaking: branching rules and more

Concerning symmetry breaking, one usually wants to know how a given representation of a group breaks into irreducible representations of some subgroup. This can be calculated with **DecomposeRep**, which requires 3 elements from the user:

1. the group G;
2. the subgroup $H \subset G$;
3. information on how H is embedded in G — the so-called **projection matrix**, to be specific;
4. the representation R of G to be decomposed.

(1), (2) and (4) are trivial to provide, while (3) might require a little bit of work, but not much (more on this later). In any case, the user needs to figure out (3) only once for a given symmetry breaking pattern $G \to H$.

Consider the simple case where $SU(3)$ breaks into $SU(2) \times U(1)$. The projection matrix in this case can be chosen to be

$$\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$$

We may then go ahead and see, for example, how does the triplet of $SU(3)$ decomposes:

```
group={SU3};
rep={{1,0}}; (* 3 of SU(3) *)
subgroup={SU2,U1};
prjMat = {{1, 0}, {1, 2}};

DecomposeRep[group,rep,subgroup,prjMat,UseName->True]
```

Out[58]= $\{\mathbf{2} \otimes 1,\ \mathbf{1} \otimes -2\}$

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What about SO(10) GUTs?

Scanning over **different embeddings of the SM group** becomes more complicated than in SU(5)

In any case, one can still handle the problem

Without entering into too many details, the **non-abelian part of the SM** group might be embedded in more than one way in SO(10)

and

The **SM hypercharge group can be formed from any linear combination of two U(1)**'s contained in SO(10)

Result of the scan

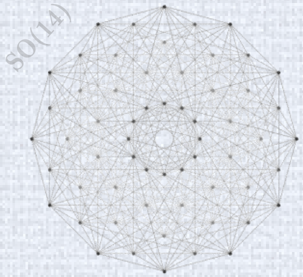
The standard solution ($3\times\mathbf{16}$) is not unique

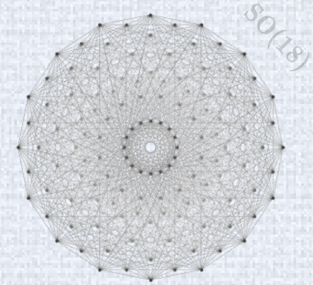
but ... "almost"

The other ones require many more fields (and huge representations!)

It is actually possible not to use the spinor representation

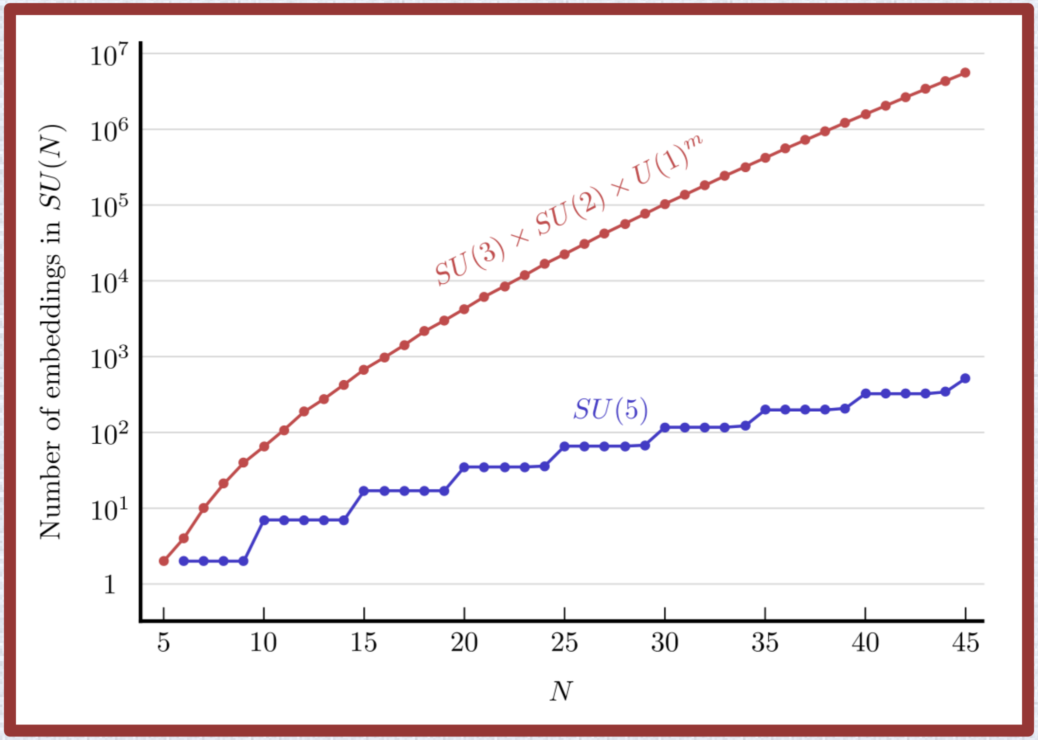
(more details on the paper)





Bigger groups

The bigger the group ... the harder it is to analyze it



1 The number of embeddings **grows exponentially** with the size of the GUT group

2 It might not be enough to consider the **embeddings of SU(5) in a GUT group**

List of interesting results*

[Reminder] Just from forbidding extra chiral fermions!

1 The only **non-SU(N) simple groups** with any solution at all are **SO(10)** and **E(6)**

2 In the case of **SU(5) and E(6)** the only solutions are the well known ones (plus trivial variations)

3 **SU(N>5)** and **SO(10)** appear to have infinite solutions

4 For the **SU(15>N>5)** groups, the only valid branching rule is $\boldsymbol{N} \rightarrow d^c + L + (N-5)N^c$

5 For **SU(N>14)** two more branching rules lead to valid solutions ....

6 $g_1 = \sqrt{5/3}g', g_2 = g, g_3 = g_s$ This standard **relation is universal** (one cannot get a different one)

[Next slide]

*Keeping in mind that the scan is necessarily finite (even though very large representations were considered)...

List of interesting results*

[Reminder] Just from forbidding extra chiral fermions!

5 For **SU(N>14)** two more branching rules lead to valid solutions ....

[Next slide]

Family Unification with SU(19)

$$\left(\begin{array}{c}\textbf{Notation:}\\ \boldsymbol{F} = \text{fundamental representation of SU(N)}\\ \boldsymbol{K} = \text{representation corresponding to the}\\ \text{anti-symmetric part of } \boldsymbol{F}\times\boldsymbol{F}\end{array}\right)$$

"Standard" branching rules from a $SU(N) \to SU(5)$ viewpoint:

$$F \to \mathbf{5} + (N-5)\,\mathbf{1}$$

$$K \to \mathbf{10} + (N-5)\,\mathbf{5} + \frac{(N-5)\,(N-6)}{2}\mathbf{1}$$

Family Unification with SU(19)

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$$F \rightarrow \mathbf{5} + (N-5)\,\mathbf{1}$$

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For $N \geq 15$ the following **new branching rule** is possible as well:

$$\overline{F} \rightarrow \overline{\mathbf{5}} + \mathbf{10} + (N-15)\,\mathbf{1}$$

$$\overline{K} \rightarrow \mathbf{45} + \overline{\mathbf{45}} + (N-15)\,\overline{\mathbf{5}} + \mathbf{5} + (N-15)\,\mathbf{10} + \overline{\mathbf{10}} + \frac{(N-15)(N-16)}{2}\mathbf{1}$$

Family Unification with SU(19)

Notation:
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"Standard" branching rules from a $SU(N)\rightarrow SU(5)$ viewpoint:

$$F\rightarrow \mathbf{5}+(N-5)\,\mathbf{1}$$

$$K\rightarrow \mathbf{10}+(N-5)\,\mathbf{5}+\frac{(N-5)\,(N-6)}{2}\mathbf{1}$$

For $N\geq 15$ the following **new branching rule** is possible as well:

$$\overline{F}\rightarrow \overline{\mathbf{5}}+\mathbf{10}+(N-15)\,\mathbf{1}$$

$$\boxed{\overline{K}\rightarrow \mathbf{45}+\overline{\mathbf{45}}+(N-15)\,\overline{\mathbf{5}}+\mathbf{5}+(N-15)\,\mathbf{10}+\overline{\mathbf{10}}+\frac{(N-15)\,(N-16)}{2}\mathbf{1}}$$

Family Unification with SU(19)

Notation:
F = fundamental representation of SU(N)
K = representation corresponding to the anti-symmetric part of $\boldsymbol{F} \times \boldsymbol{F}$

"Standard" branching rules from a $SU(N) \to SU(5)$ viewpoint:

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$$\boxed{\overline{K} \to \mathbf{45} + \overline{\mathbf{45}} + (N-15)\,\overline{\mathbf{5}} + \mathbf{5} + (N-15)\,\mathbf{10} + \overline{\mathbf{10}} + \frac{(N-15)\,(N-16)}{2}\mathbf{1}}$$

!

Meaning that for $\boldsymbol{SU(16+N')}$ $\overline{\boldsymbol{K}}$ contains exactly $\boldsymbol{N'}$ SM families plus vector fermions only

!

From a $SU(19) \to SO(10)$ viewpoint,
$$\mathbf{171} \to 3\,(\mathbf{16}) + \mathbf{120} + 3\,(\mathbf{1})$$

!

However, there is a **gauge anomaly**

Summary

1

GUTs have been around for 40 years, yet they do not explain the flavor problem

2

To do so requires putting the SM families in different GUT representations
(or, ideally, in a single one)

?

3

Is this possible? One cannot introduce new chiral fermions!
(There was no systematic answer to this question)

!

4

Yes, it is possible: family unification can be achieved with the **171** of SU(19)

Thank you

SO(10): other solutions

Note that we do know one solution: $3 \times \mathbf{16}$

All we have to do is consider non-trivial combinations of representations which we can add to this solution and which do not introduce more chiral fermions

(True for a fixed embedding of the SM group in SO(10))

The smallest such nontrivial combinations:

$$\begin{aligned} n_1: \quad & -\mathbf{126}-\mathbf{144}-\mathbf{1200}+\mathbf{2772}+\mathbf{3696}-\mathbf{4950}-\mathbf{6930}'+\mathbf{7920}+\mathbf{8064}+\mathbf{11088}-\mathbf{15120} \\ & -\mathbf{17280}+\mathbf{17325}+\mathbf{30800}-\mathbf{34992}-\mathbf{38016}+\mathbf{48114}+\mathbf{49280}\,, \\ n_2: \quad & -\mathbf{16}+\mathbf{126}-\mathbf{560}+\mathbf{8064}+\mathbf{20592}-\mathbf{20790}+\mathbf{23760}+\mathbf{25200}-\mathbf{29568}-\mathbf{48114}-\mathbf{50050} \\ & -\mathbf{90090}-\mathbf{102960}-\mathbf{124800}-\mathbf{128700}-\mathbf{144144}+\mathbf{164736}+\mathbf{196560}-\mathbf{199017}\,. \end{aligned}$$

Huge representations are needed!