

Extending the Matrix Element Method beyond the Born approximation: Calculating event weights at next-to-leading order accuracy.

(arXiv:1506.08798 / JHEP09(2015)083)

Till Martini[•] & Peter Uwer

Humboldt-Universität zu Berlin, Institut für Physik
Phänomenologie der Elementarteilchenphysik

Physics at the LHC and beyond - DESY Theory Workshop
1.10.15

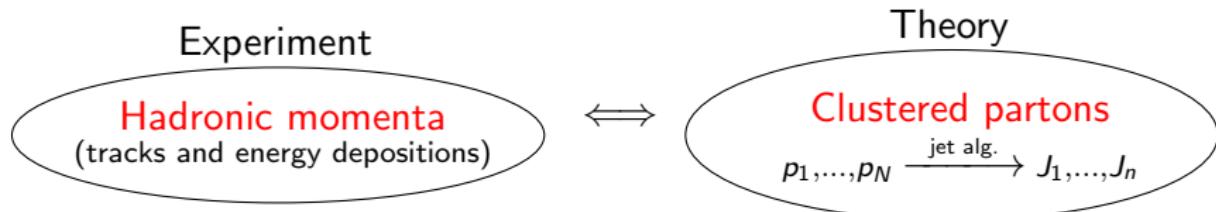


Bundesministerium
für Bildung
und Forschung



Jet Event Weight: Definition & Application

Jet



Jet event

Observed (jet-) momenta J_1, \dots, J_n from $P_A + P_B \rightarrow J_1 + \dots + J_n + X$

Jet event weight

Interpreting differential cross section as probability density to measure a specific event: $\rho \propto \frac{d\sigma_{AB \rightarrow n}}{d^4 J_1 \dots d^4 J_n}$

- ▶ Construct more inclusive (jet-) observables
- ▶ Generate unweighted events according to ρ
- ▶ Used in likelihood analysis methods (Matrix Element Method)

Matrix Element Method (MEM) in a nutshell [Kondo '88,'91]

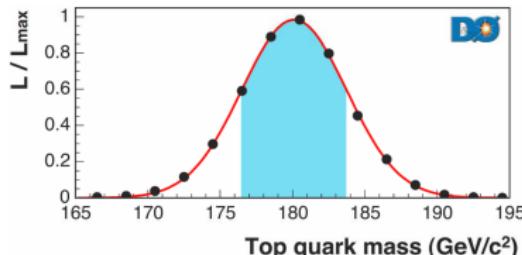
Extraction of model parameters Ω from data by maximizing a likelihood \propto differential cross section ($\propto |\mathcal{M}^{\text{LO}}|^2$):

Likelihood for Ω for set of events e.g. $\vec{x}_i = (J_1, \dots, J_n)_i$

$$\mathcal{L}^{\text{LO}}(\Omega) = \prod_i \frac{1}{\sigma^{\text{LO}}(\Omega)} \int d\vec{y} \frac{d\sigma^{\text{LO}}(\Omega)}{d\vec{y}} \underbrace{W(\vec{x}_i, \vec{y})}_{\substack{\text{transfer function,} \\ \text{here: } = \delta(\vec{x}_i - \vec{y})}} = \prod_i \frac{1}{\sigma^{\text{LO}}(\Omega)} \frac{d\sigma^{\text{LO}}(\Omega)}{d\vec{x}_i}$$

Maximizing wrt Ω yields estimator $\hat{\Omega}$: $\mathcal{L}^{\text{LO}}(\hat{\Omega}) = \sup_{\Omega} \mathcal{L}^{\text{LO}}(\Omega)$

All information from event used \implies most efficient estimator!



e.g. top mass measurement at Tevatron
[D0: Nature 429, 638], [CDF: PRD 50, 2966 (1994)] based on $O(40)$ events!

Likelihood at NLO and 3 problems

$$\mathcal{L}^{\text{NLO}}(\Omega) = \prod_i \frac{1}{\sigma_{n\text{-jet}}^{\text{NLO}}(\Omega)} \left(\frac{d\sigma_{n \rightarrow n\text{-jet}}^{\text{NLO,B+V}}(\Omega)}{dJ_1 \dots dJ_n} + \frac{d\sigma_{n+1 \rightarrow n\text{-jet}}^{\text{NLO,R}}(\Omega)}{dJ_1 \dots dJ_n} \right) \Big|_{\text{event } i}$$

Born+virtual and real contribution separately IR divergent

NEED: Point-wise cancelation

→ both contributions must be evaluated for same jet momenta

Real (one recombination $J_i = \tilde{J}_i(p_1, \dots, p_{n+1})$)

$$\frac{\sigma_{n+1 \rightarrow n\text{-jet}}^{\text{NLO,R}}(\Omega)}{dJ_1 \dots dJ_n} = \int dR_{n+1} \frac{d\sigma_{\text{R}}^{\text{NLO}}(\Omega)}{dR_{n+1}} \prod_{i=1}^n \delta(\tilde{J}_i(p_1, \dots, p_{n+1}) - J_i)$$

Integration over **δ -function** numerically not feasible!

NEED: factorisation of phase space

→ Integration trivial:

$$dR_{n+1}(p_1, \dots, p_{n+1}) = dR_n(\tilde{J}_1, \dots, \tilde{J}_n) dR_{\text{unres}}(\Phi)$$
$$\Rightarrow dR_{n+1}(p_1, \dots, p_{n+1}) \prod_{i=1}^n \delta(\tilde{J}_i - J_i) = dR_{\text{unres}}(\Phi) \Big|_{\tilde{J}_i = J_i}$$

$dR_{\text{unres}}(\Phi)$ generates **only** partonic configurations that result in given jet event (inverted jet algorithm)

Born+virtual (no recombination $J_i = p_i = \tilde{J}_i$)

$$\frac{\sigma_{n \rightarrow n\text{-jet}}^{NLO,B+V}(\Omega)}{dJ_1 \dots dJ_n} = \int dR_n \frac{d\sigma_{B+V}^{NLO}(\Omega)}{dR_n} \prod_{i=1}^n \delta(p_i - J_i) = \frac{\sigma_{B+V}^{NLO}(\Omega)}{dJ_1 \dots dJ_n}$$

Born+virtual matrix elements **only** defined for Born kinematics

NEED: clustered jets obeying Born kinematics

→ on-shell condition and momentum conservation:

$$\tilde{J}_i^2 = m_i^2 \quad \text{and} \quad p_1 + \dots + p_{n+1} = \tilde{J}_1 + \dots + \tilde{J}_n$$

not possible with $2 \rightarrow 1$ clustering/recombination

$3 \rightarrow 2$ clustering [Catani,Seymour '97], [Catani, Dittmaier,Seymour,Trocsanyi '02]

$3 \rightarrow 2$ clusterings from dipole subtraction method
meet **all 3** requirements at the same time



Using $3 \rightarrow 2$ jet algorithm instead of $2 \rightarrow 1$
solves the 3 problems

Jet event weight at NLO

Phase space factorisation allows to define an event weight
(differential jet cross section) at NLO

$$\frac{d\sigma_{n\text{-jet}}^{\text{NLO}}(\Omega)}{dJ_1 \dots dJ_n} = \frac{d\sigma_{\text{B+V}}^{\text{NLO}}(\Omega)}{dJ_1 \dots dJ_n} + \underbrace{\int dR_{\text{unres}}(\Phi)}_{\text{3dim integration}} \frac{d\sigma_{\text{R}}^{\text{NLO}}(\Omega)}{dp_1 \dots dp_{n+1}}$$

Mutual cancelation of IR-divergences from the **virtual** and the **real** part has to be carried out by a suitable method (e.g. phase space slicing)

Validation

Calculate differential jet distributions in NLO accuracy using traditional approach (parton level MC + 3 → 2 jet alg.)

1. Compare with distributions obtained from $\frac{d\sigma_{n\text{-jet}}^{\text{NLO}}(\Omega)}{dJ_1 \dots dJ_n}$
2. Compare with histogrammed unweighted events generated according to $\rho = \frac{d\sigma_{n\text{-jet}}^{\text{NLO}}(\Omega)}{dJ_1 \dots dJ_n}$

Sample processes:

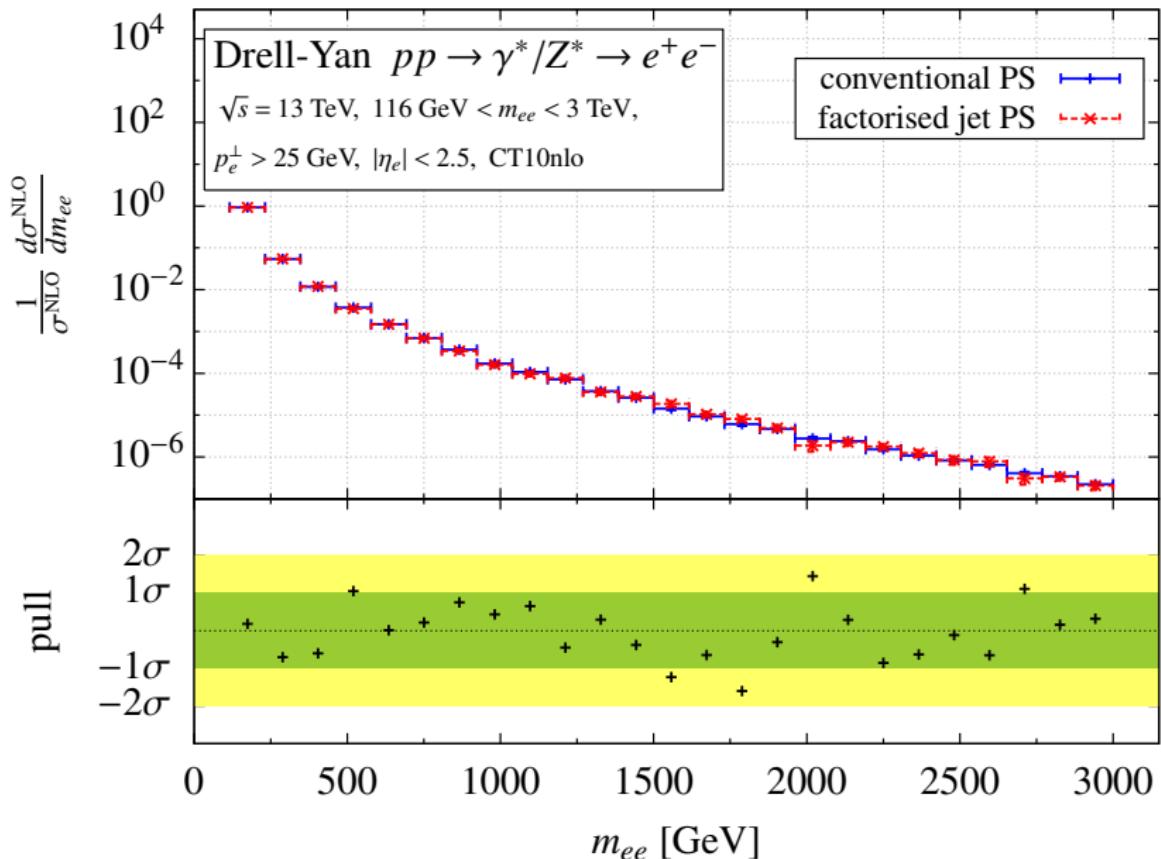
Drell-Yan $pp \rightarrow e^+e^-$
hadrons in initial state

top pair production $e^+e^- \rightarrow t\bar{t}$ (no decay)
massive colored particles in final state

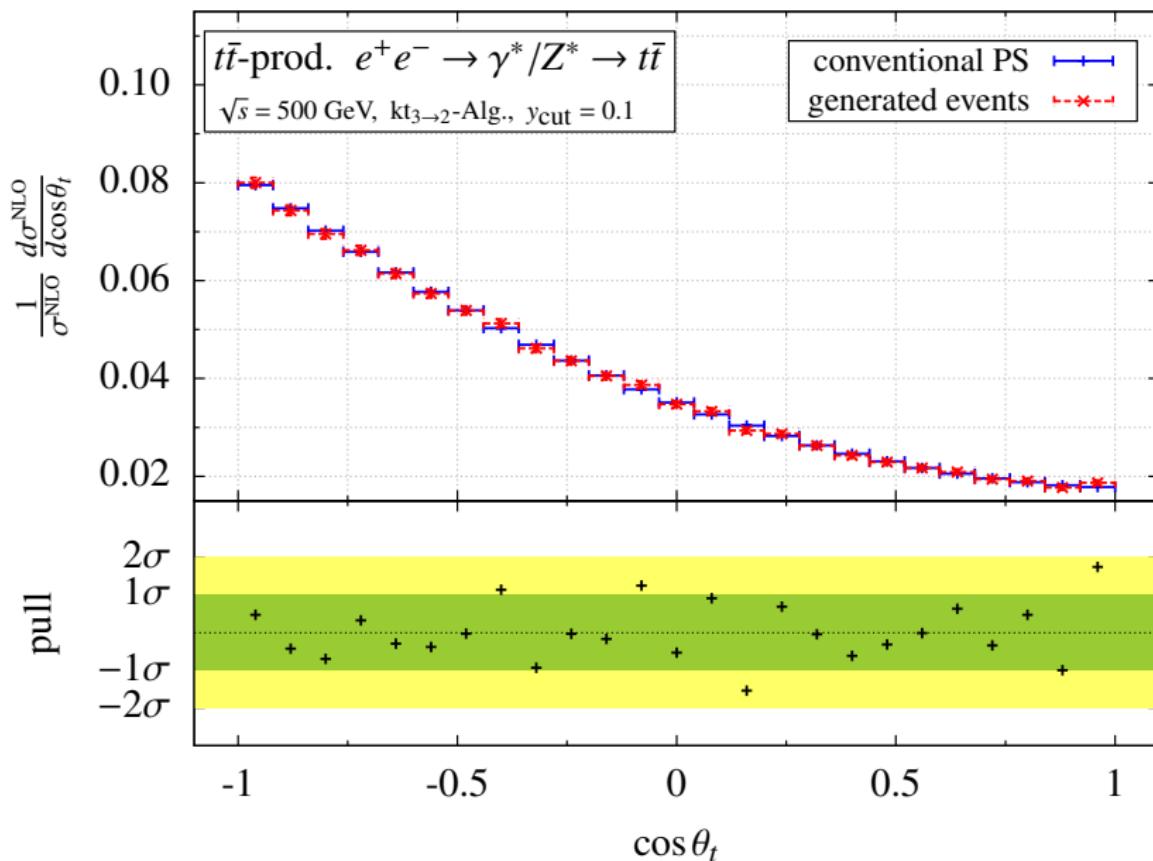
Veto on additional jet emission, no resolved additional jet!

Although simple cover all essential aspects

Validation 1: Phase space generation ($pp \rightarrow e^+e^-$)



Validation 2: Unweighted events ($e^+e^- \rightarrow t\bar{t}$)



Matrix Element Method at NLO

(example: $e^+e^- \rightarrow t\bar{t}$ without additional jet)

Toy experiment: Generated sample of N unweighted NLO $t\bar{t}$ events

$$\vec{x}_i = (\cos\theta_t, \phi_t, \cos\theta_{\bar{t}}, \phi_{\bar{t}})_i \quad \text{with } \Omega = m_t = m_t^{\text{true}} = 174 \text{ GeV}$$

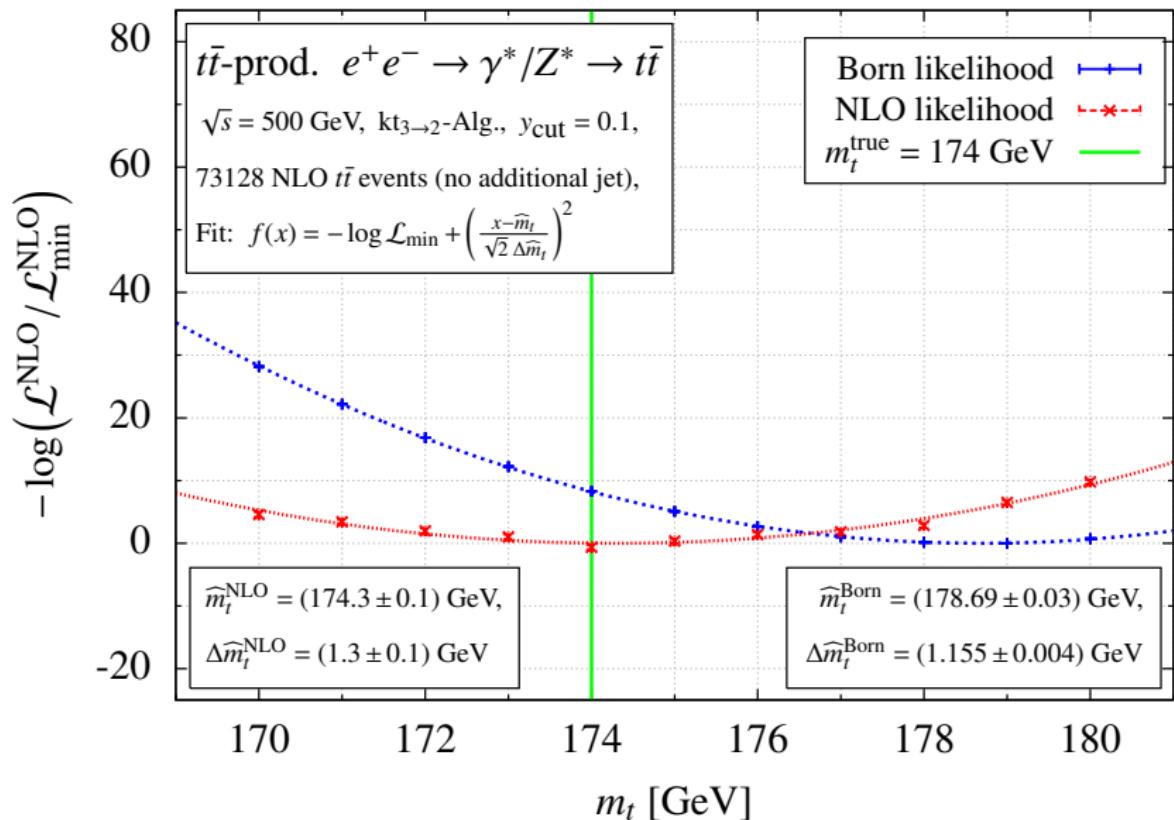
NLO likelihood function for event sample

$$\mathcal{L}^{\text{NLO}}(m_t) = \prod_i^N \mathcal{L}^{\text{NLO}}(\vec{x}_i | m_t) = \left(\frac{\beta_t}{32\pi^2 \sigma_{t\bar{t}}^{\text{NLO}}(m_t)} \right)^N \prod_i^N \frac{d\sigma_{t\bar{t}}^{\text{NLO}}(m_t)}{dJ_t dJ_{\bar{t}}} \Big|_{\text{event } i}$$

Find minimum of negative logarithm of NLO likelihood
("Log-Likelihood") to obtain estimator \hat{m}_t for top mass

$$-\log \mathcal{L}^{\text{NLO}}(\hat{m}_t) = \inf_{m_t} \left(-\log \mathcal{L}^{\text{NLO}}(m_t) \right)$$

MEM at NLO: top-quark mass extraction via parabola fit

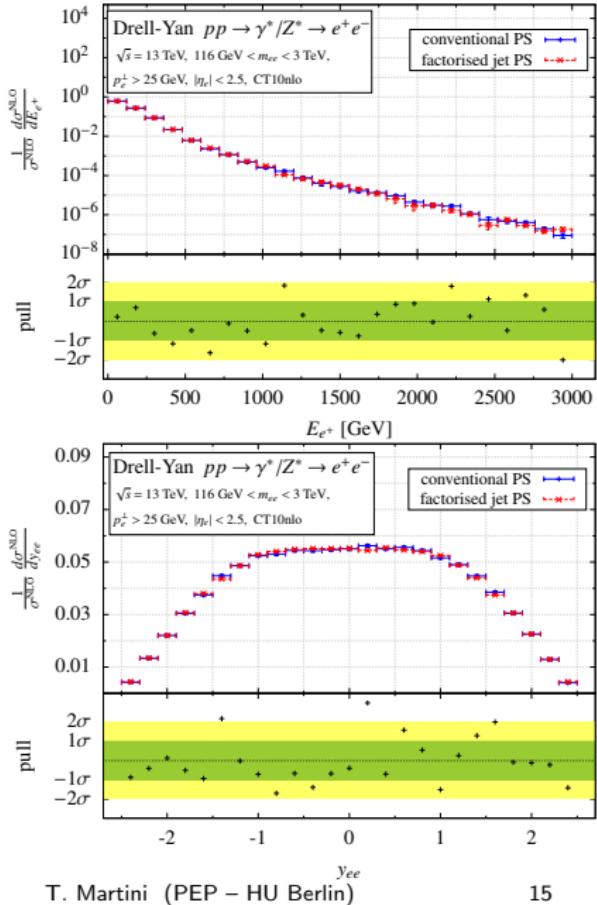
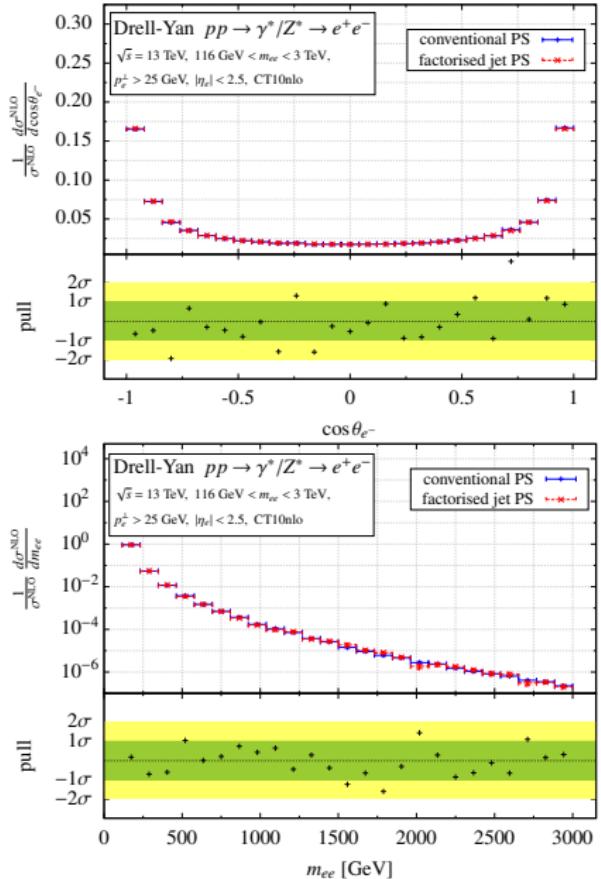


Conclusion

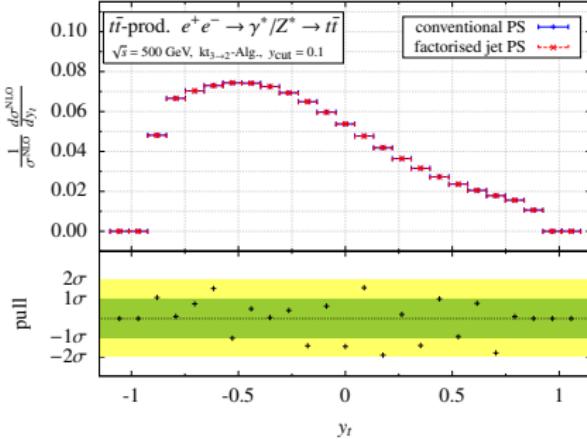
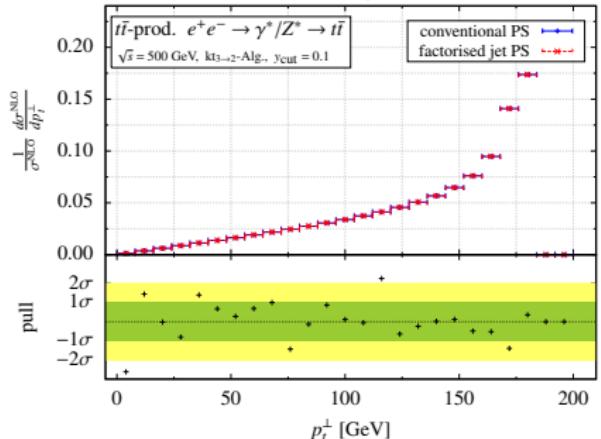
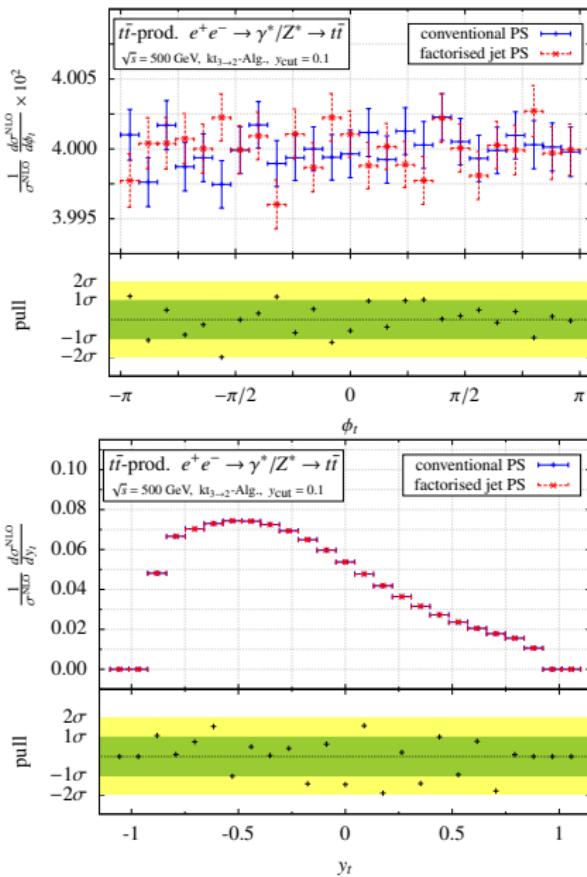
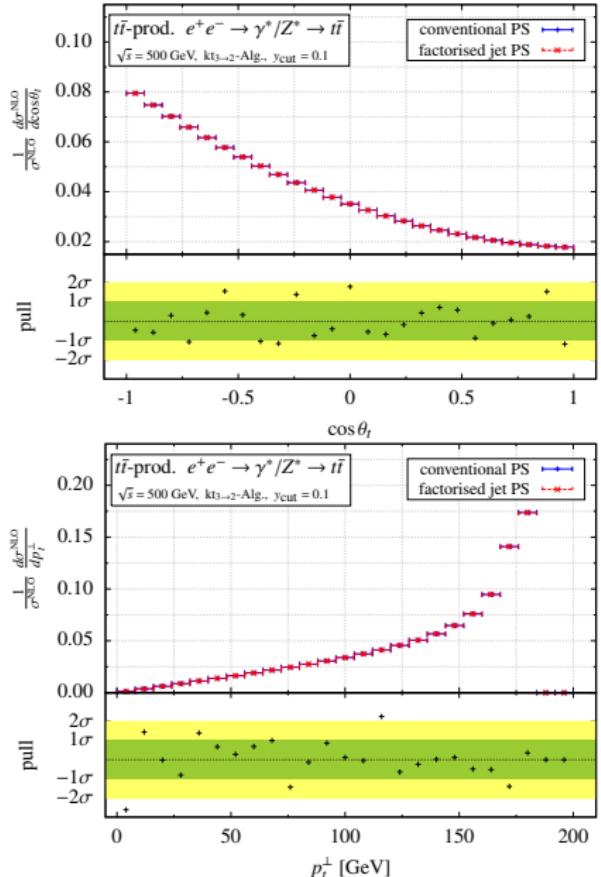
- ▶ $3 \rightarrow 2$ jet clustering algorithm:
 - ▶ Unique mapping of real corrections onto Born kinematics
- ▶ Evaluation of event weights for jet events in NLO accuracy
- ▶ Generation of unweighted events at NLO
- ▶ Application of MEM at NLO
 - ▶ Extraction of m_t with NLO likelihood from NLO $t\bar{t}$ events:
Perfect agreement with input value!
 - ▶ Extraction with Born likelihood: **Large deviation from input value possible!** (despite small NLO corrections)
 - ▶ Renormalization scheme well-defined in MEM at NLO

Outlook: MEM at NLO for top-pair, single top, ... at LHC

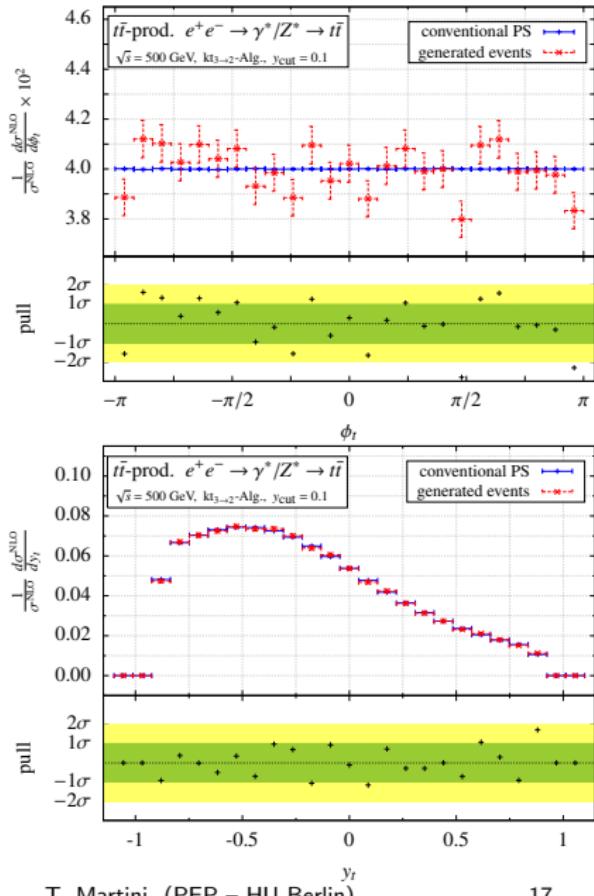
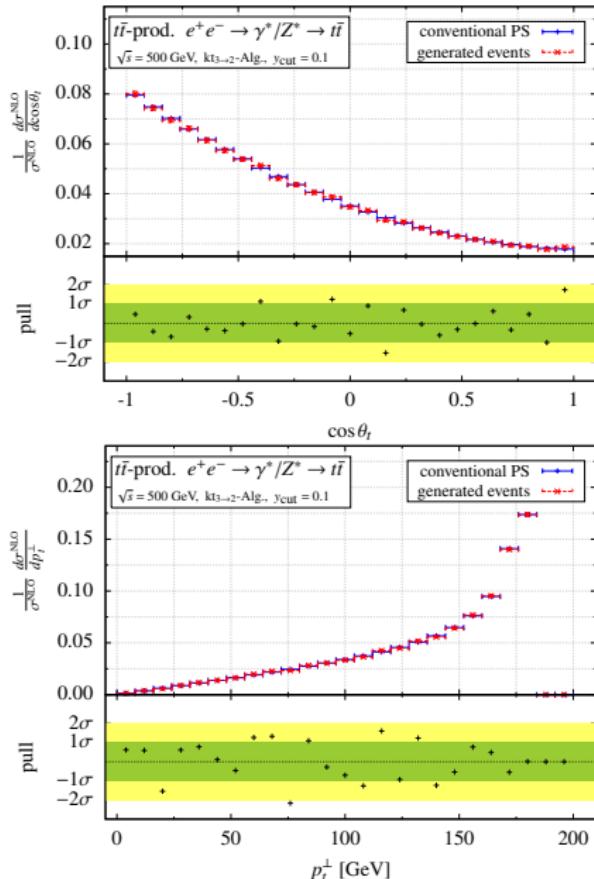
BackUp: Valid. 1: Phase space generation ($pp \rightarrow e^+e^-$)



BackUp: Valid. 1: Phase space generation ($e^+e^- \rightarrow t\bar{t}$)



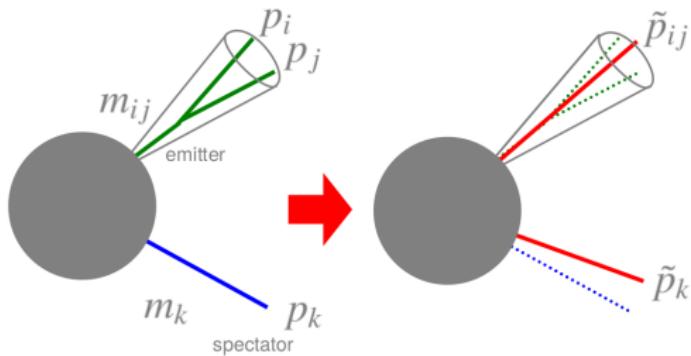
BackUp: Valid. 2: Unweighted events ($e^+e^- \rightarrow t\bar{t}$)



BackUp: Modified clustering

[Catani,Seymour '97], [Catani, Dittmaier,Seymour,Trocsanyi '02]

$$(\textcolor{green}{p}_i, \textcolor{green}{p}_j, \textcolor{blue}{p}_k) \rightarrow (\tilde{p}_{ij}, \tilde{p}_k) \equiv (\textcolor{red}{J}_{ij}, \textcolor{red}{J}_k)$$



$$\textcolor{red}{J}_{ij} + \textcolor{red}{J}_k = \textcolor{green}{p}_i + \textcolor{green}{p}_j + \textcolor{blue}{p}_k$$

$$\text{and } \textcolor{red}{J}_{ij}^2 = \textcolor{green}{m}_{ij}, \textcolor{red}{J}_k = \textcolor{blue}{m}_k$$

Phase space factorises:

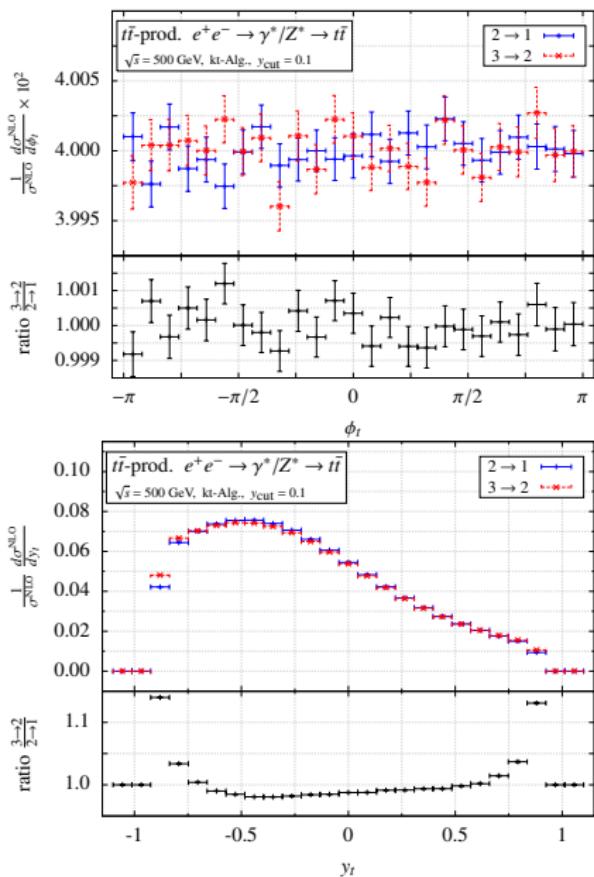
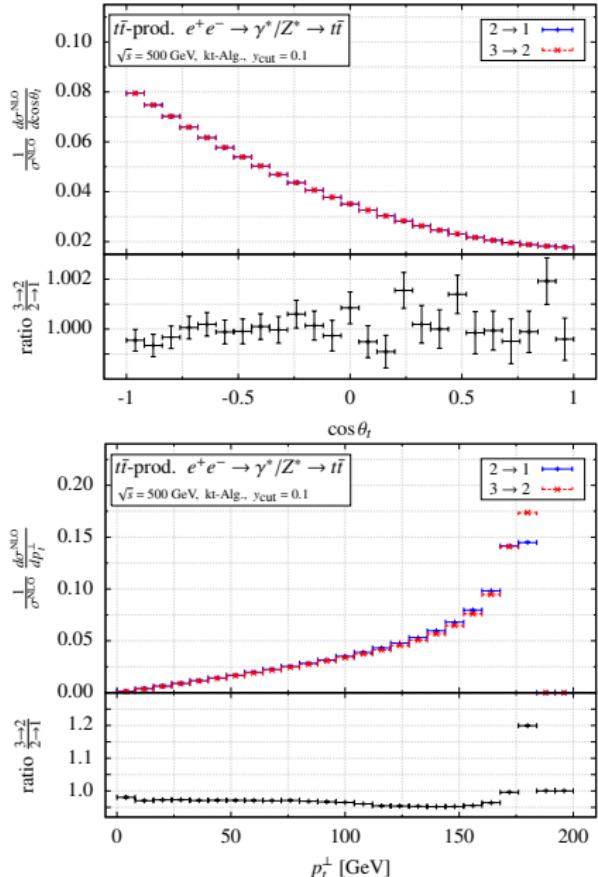
$$dR_{n+1}(\textcolor{green}{p}_i, \textcolor{green}{p}_j, \textcolor{blue}{p}_k) = dR_n(\textcolor{red}{J}_{ij}, \textcolor{red}{J}_k) dR_{\text{unres}}(\Phi)$$

$dR_{\text{unres}}(\Phi)$ generates all $\textcolor{green}{p}_i, \textcolor{green}{p}_j, \textcolor{blue}{p}_k$ with $(\textcolor{green}{p}_i, \textcolor{green}{p}_j, \textcolor{blue}{p}_k) \xrightarrow{!} (\textcolor{red}{J}_{ij}, \textcolor{red}{J}_k)$

BackUp: $3 \rightarrow 2$ as an augmented $2 \rightarrow 1$

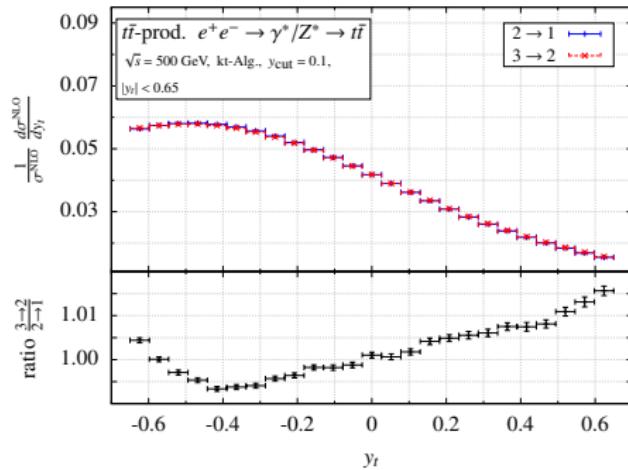
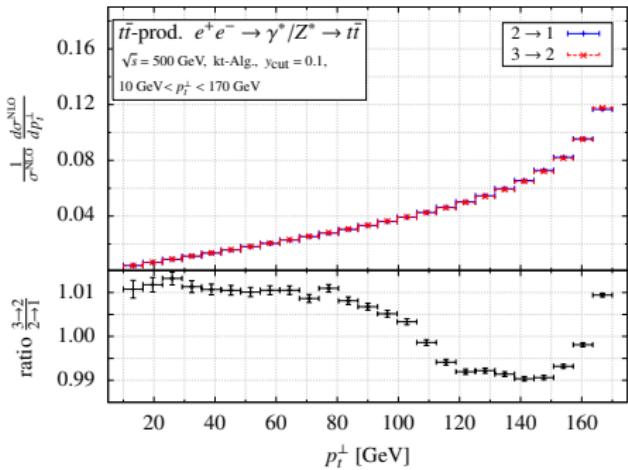
- ▶ Use resolution criterium of the $2 \rightarrow 1$ algorithm to pick final state particle to be clustered with final state particle or beam (“**emitter**”)
- ▶ Choose final state particle or beam as “**spectator**”
- ▶ 4 different types of mappings
 $(\text{emitter}, \text{spectator}) = (\text{final}, \text{final}), (\text{final}, \text{initial}), (\text{initial}, \text{initial}), (\text{initial}, \text{final})$
- ▶ Respective clusterings for massless and massive particles already worked out in Catani-Seymour dipole subtraction method [Catani,Seymour '97], [Catani, Dittmaier,Seymour, Trocsanyi '02] (let's use those!)

BackUp: Impact of $3 \rightarrow 2$ wrt $2 \rightarrow 1$ for $e^+e^- \rightarrow t\bar{t}$



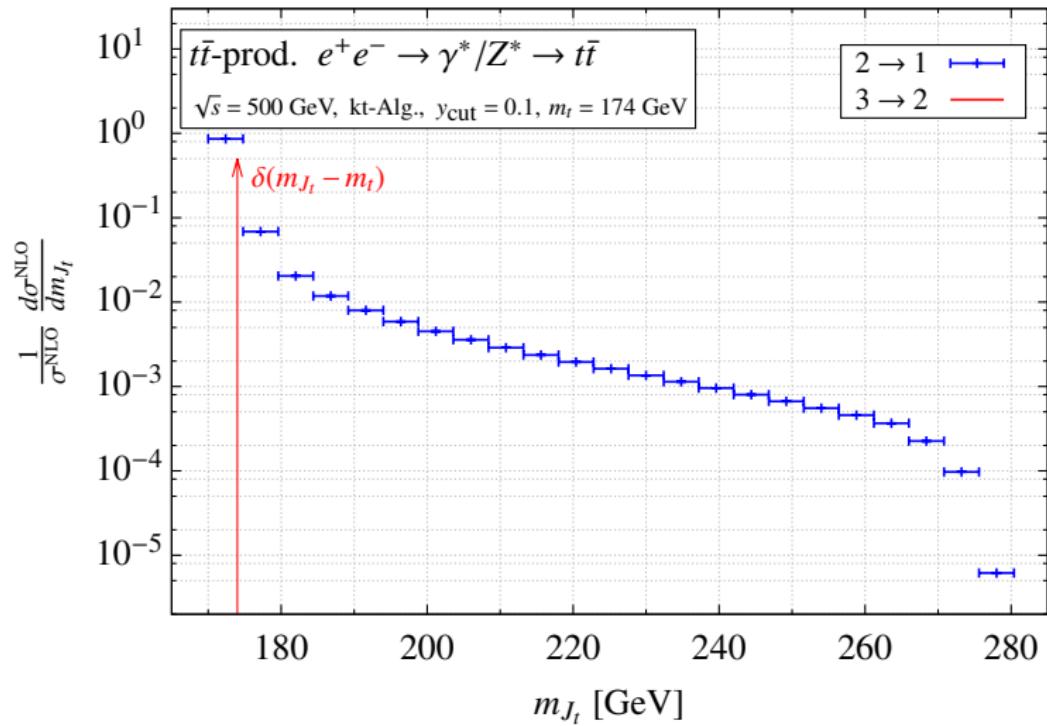
BackUp: Impact of $3 \rightarrow 2$ wrt $2 \rightarrow 1$ for $e^+e^- \rightarrow t\bar{t}$

p_t^\perp and y_t distributions with cuts to avoid phase space boundaries

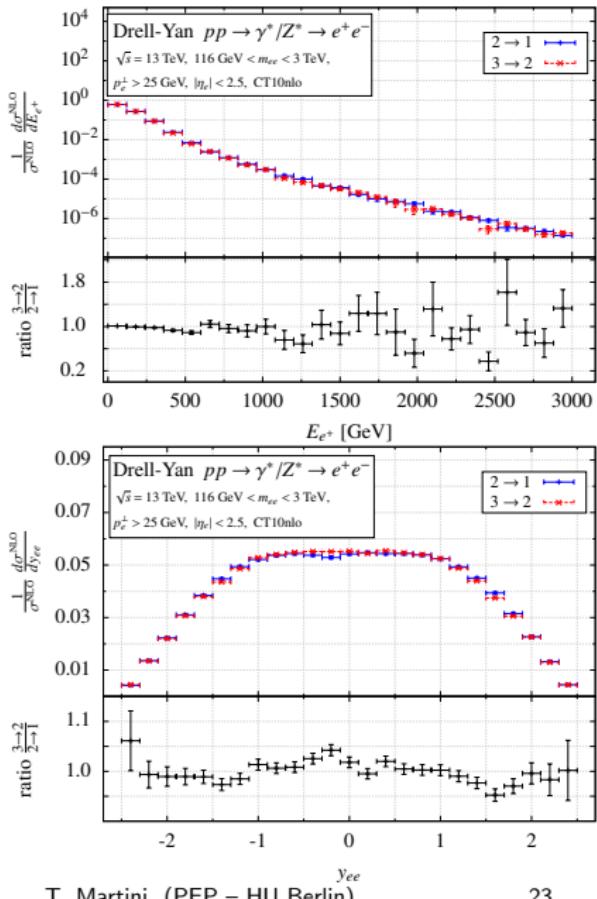
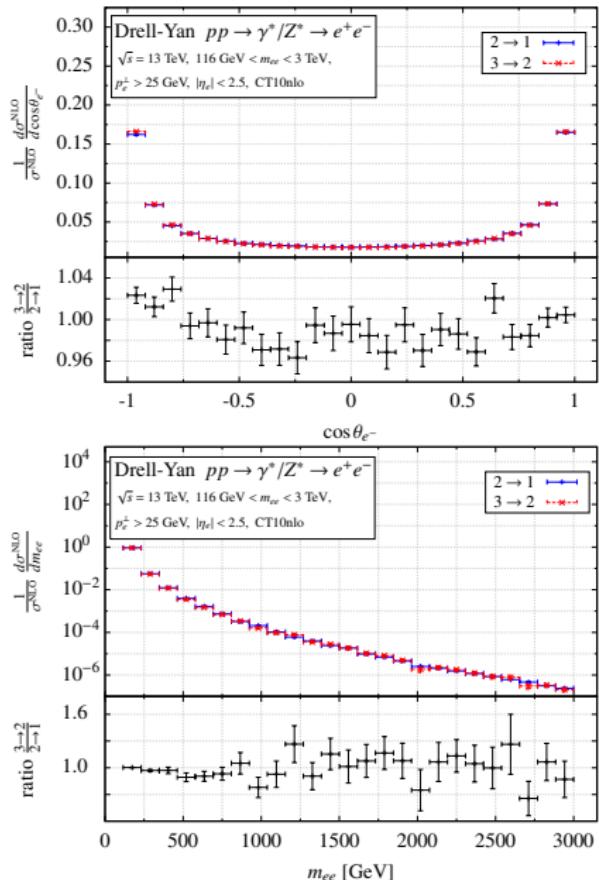


BackUp: Impact of $3 \rightarrow 2$ wrt $2 \rightarrow 1$ for $e^+e^- \rightarrow t\bar{t}$

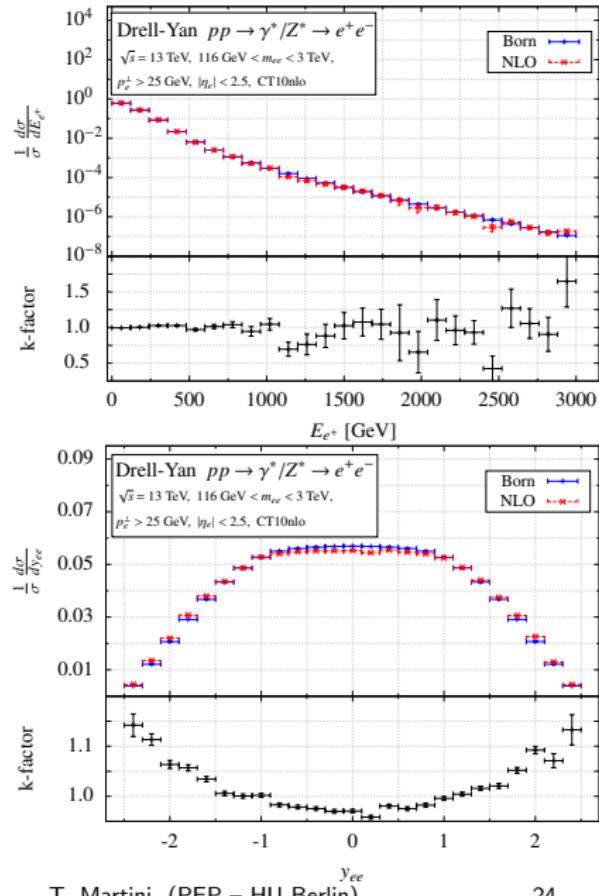
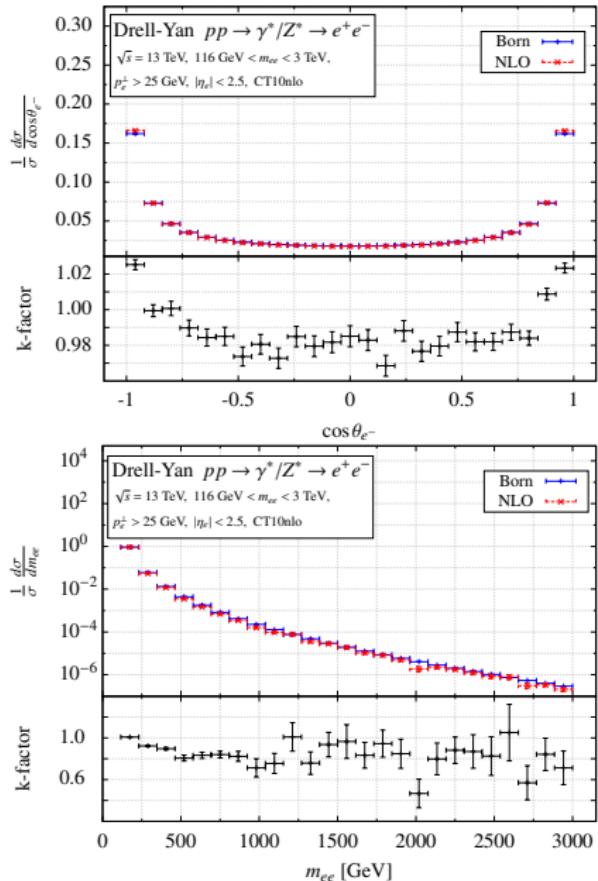
Mass distribution of top jet from $2 \rightarrow 1$ clustering



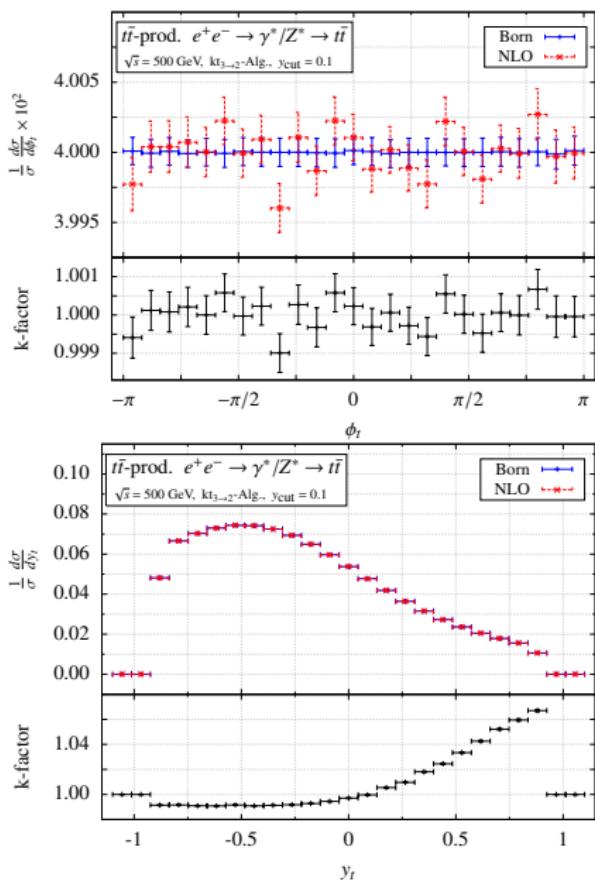
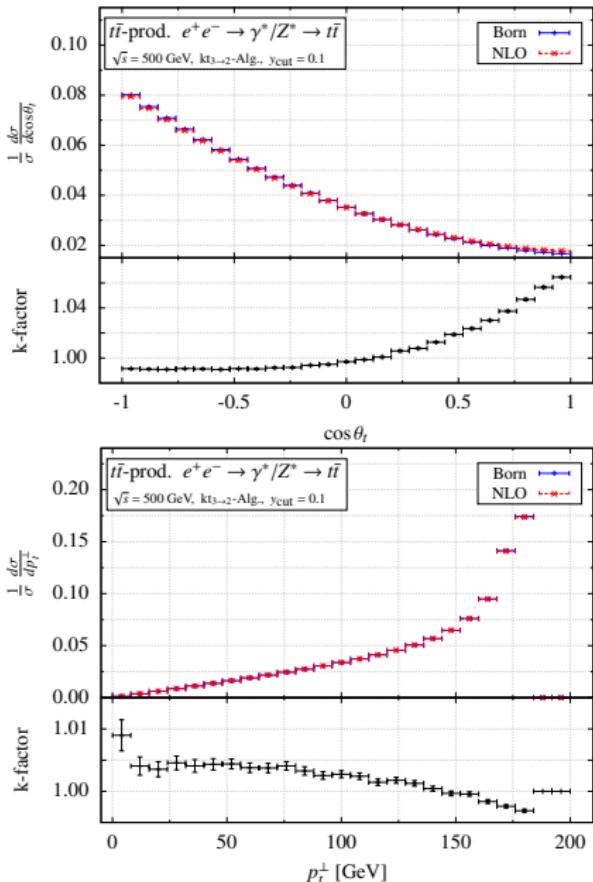
BackUp: Impact of $3 \rightarrow 2$ wrt $2 \rightarrow 1$ for $pp \rightarrow e^+e^-$



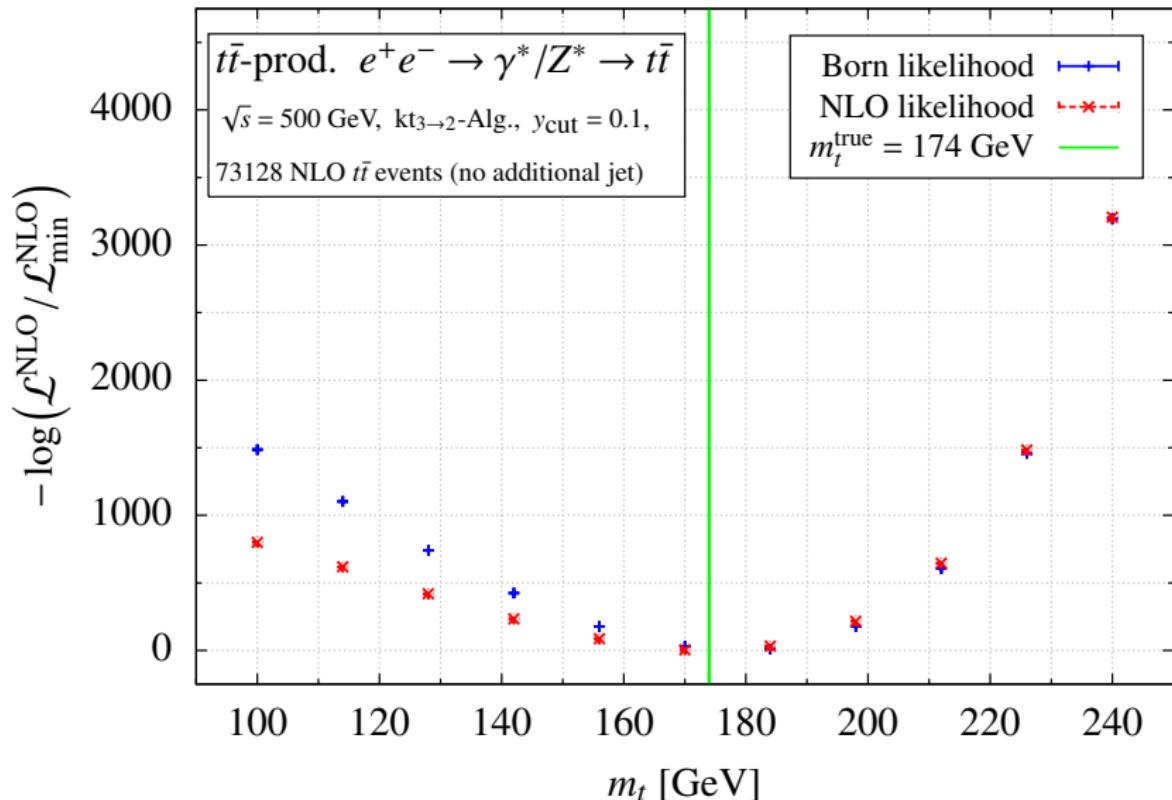
BackUp: Impact of NLO: k-factors for $pp \rightarrow e^+e^-$



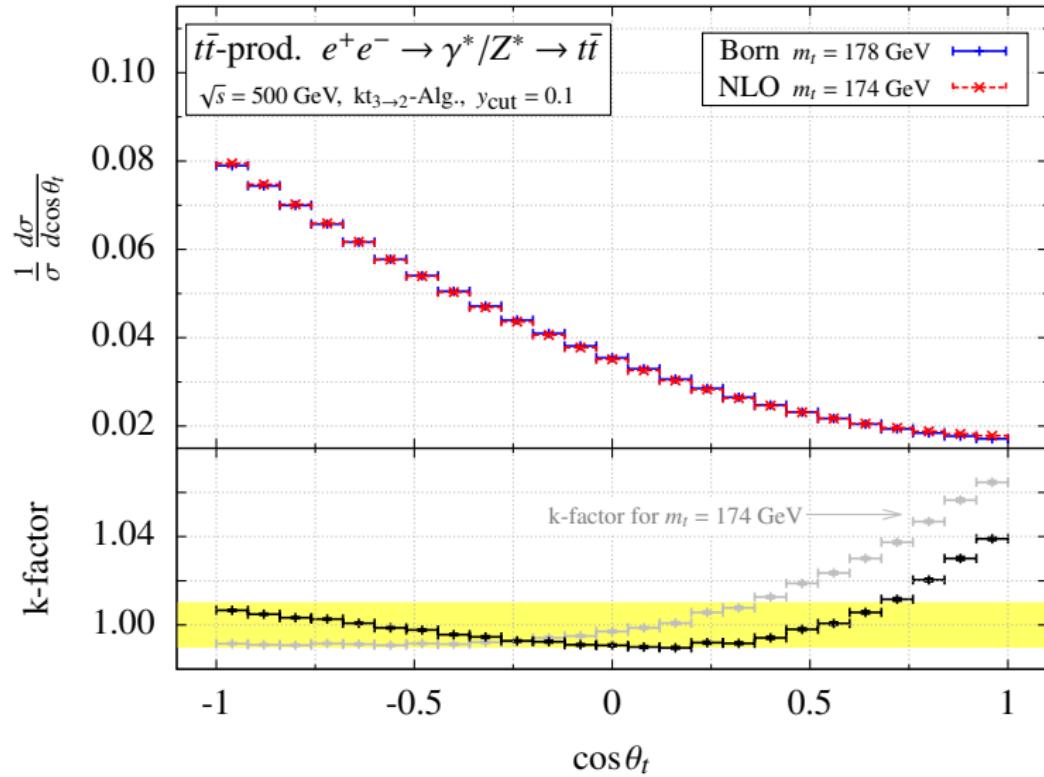
BackUp: Impact of NLO: k-factors for $e^+e^- \rightarrow t\bar{t}$



BackUp: MEM at NLO: Log-likelihood as a function of m_t

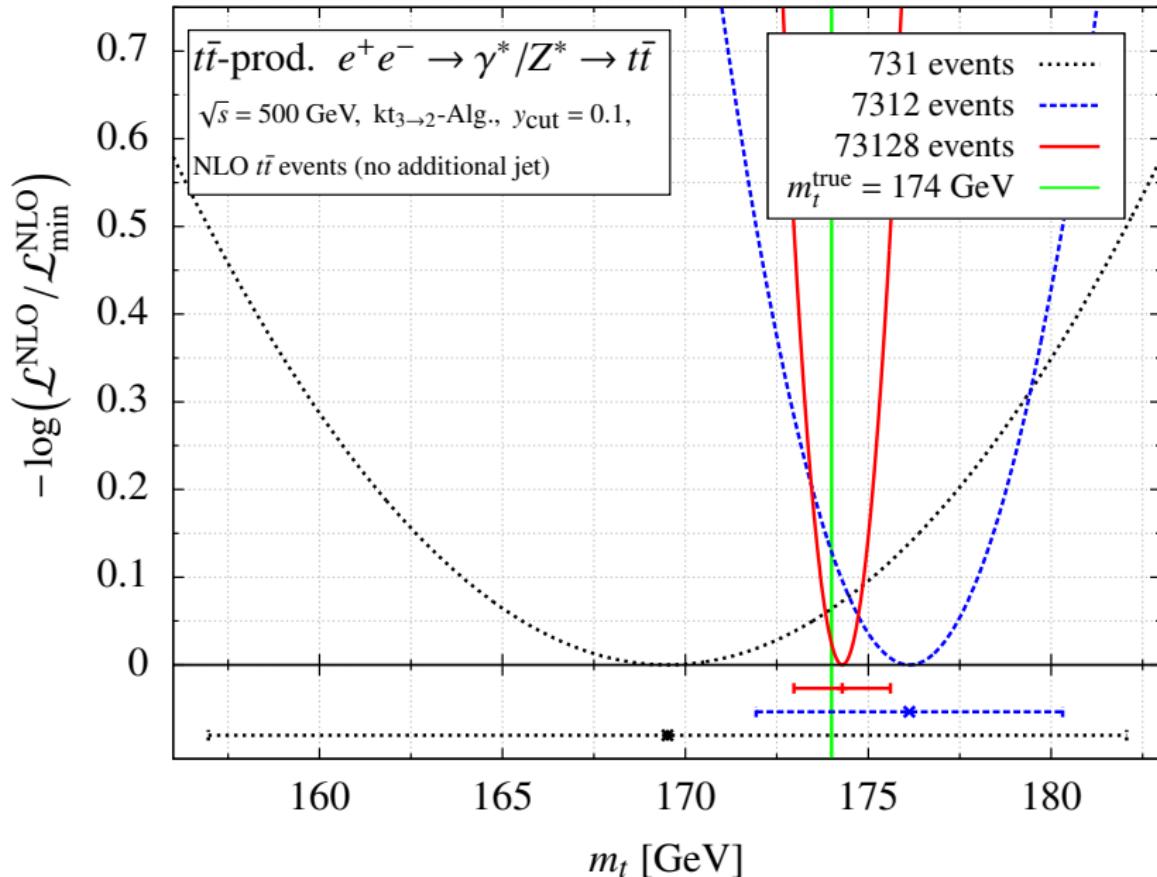


BackUp: NLO $m_t = 174$ GeV vs. Born $m_t = 178$ GeV



NLO corrections small but large effects in MEM anyway!

BackUp: Consistency of MEM at NLO



BackUp: Consistency of MEM at NLO

