F-Theory Compactifications with Discrete Selection Rules

joint work with L. Lin, O. Till and T. Weigand: arXiv:1508.00162
&
E. Palti, O. Till and T. Weigand: arXiv:1408.6831 & arXiv:1410.7814

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- Type IIB string theory with varying axion-dilaton:

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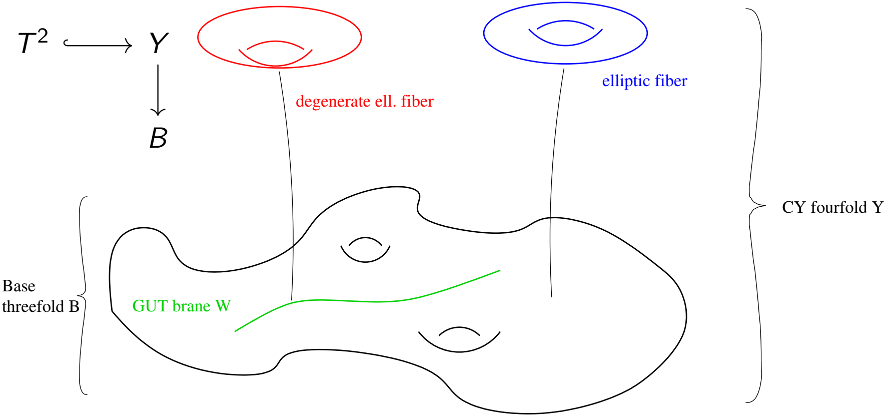
- F-theory basically 'book-keeping' device to describe vacua of IIB;
- From duality via M-theory and assumptions on non-compact 4d space, we find that compact space Y_4 has to be elliptically fibred CY_4;

Picture of elliptically fibred CY_4



by J.K.

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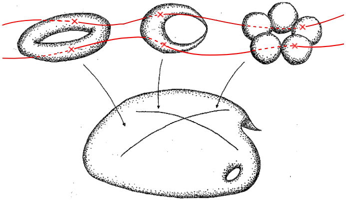

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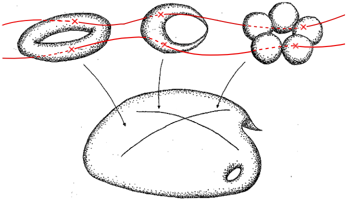

- Reduction of M-theory 3-form leads to:
  - Coulomb branch $U(1)$'s from fibral divisors;
  - $U(1)$'s from sections;

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$$\mathrm{w}^2 + b_0 \mathrm{wu}^2 + b_1 \mathrm{uvw} + b_2 \mathrm{v}^2\mathrm{w} + c_0 \mathrm{u}^4 + c_1 \mathrm{u}^3\mathrm{v} + c_2 \mathrm{u}^2\mathrm{v}^2 + c_3 \mathrm{uv}^3 = 0$$

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- Curve has two rational points:

  $$\mathrm{Sec}_0 : [0 : 1 : 0] \,, \qquad \mathrm{Sec}_1 : [0 : 1 : -b_2];$$

  Hence, describes torus fibrations with two sections; Such fibration gives $U(1)$ in F-theory (Note: two $U(1)$'s in M-theory);

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- If we add $c_4\, v^4$, above rational points become

  $$[0:1:-\tfrac{1}{2}(b_2 - \sqrt{b_2^2 - 4\,c_4})]\,, \qquad [0:1:-\tfrac{1}{2}(b_2 + \sqrt{b_2^2 - 4\,c_4})];$$

  Not rational any more, hence, no sections but bi-section;

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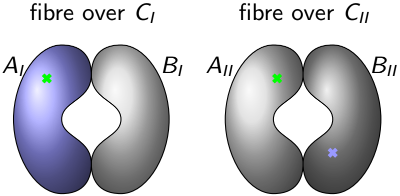

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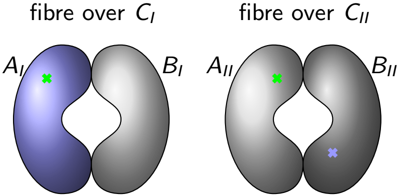
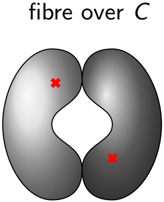


- Bi-section-model: only one distinct degeneration/factorisation (over C)



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- Pictures generalises, i.e. in case of ter-section and quadr-section we lose/Higgs state with charge three and four, respectively; $\leftarrow$ see also talk by Oehlmann;

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- Note: For generic $SU(5)$ only one type of $\mathbf{10}$-curve, one type of $\mathbf{5}$-curve and two different Yukawa's;

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  - $\Rightarrow$ Get same number of states as in $\mathbb{Z}_2$-case; Furthermore $\mathbb{Z}_2$-charges are $U(1)$-charges mod 2 (up to subtleties involving the center of $SU(5)$);

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A twist to the story I: From F to M I

- As mention, in M-theory every section gives $U(1)$;
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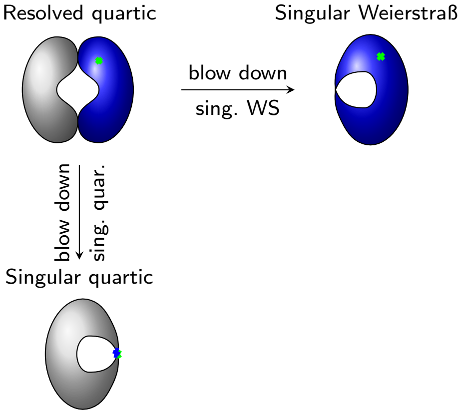
- With additional $U(1)$ there is distinction between two $\mathbb{P}^1$'s over C_I locus;
  - $\Rightarrow$ There are actually two different Higgsings;



	A_I	B_I	A_{II}	B_{II}
U	-1	2	0	1
S	1	0	1	0
$S+U$	0	2	1	1
$S-U$	-2	2	-1	1

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- Note: In this case Higgs is KK mode; Massless due to discrete Wilson line along S^1;

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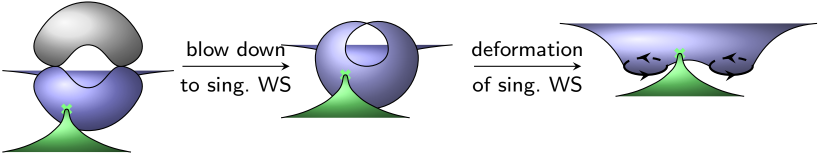
- Indeed, under transition divisor corresponding to section S develops boundary with $\partial\Sigma_S = 2\Gamma$; Hence, only 2Γ is exact in homology;



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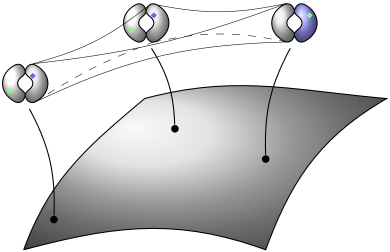
- Dual to above torsion in homology is $\mathrm{Tor} H_2(\mathbb{Z})$; Generated by $[B_{II}]$ because prior to transition $2[B_{II}] = [B_I] \Rightarrow \exists\Sigma : \partial\Sigma = 2[B_{II}] - [B_I]$ which becomes $\partial\Sigma' = 2[B_{II}]$ after transition;



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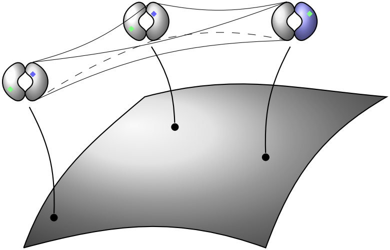

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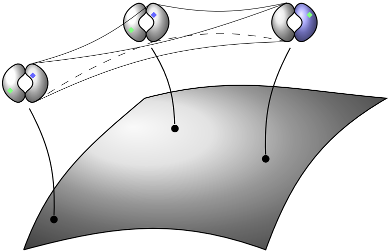

- M2 and M5 branes wrapped on torsion cycles give electro-magnetic states of discrete gauge theory;
- Subtlety: For this transition to take place not only B_I has to shrink but B_{II} too; However, deformation only resolves sing. over C_I;

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- Chirality induced by fluxes is

$$\int_{C_R} G_4$$

where C_R is matter surfaces w/ matter in rep. R; consists of linear combinations of blow-up $\mathbb{P}^1$'s fibred over enhancement curve $\mathcal{C}_R$;

Fluxes for $SU(5) \times \mathbb{Z}_2$ model

- For (generic) vertical part there are two solutions to all constraints:

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- Extend this analysis beyond topological indices, i.e. number of vector-like pairs;

Thank you for your attention!