DESY Theory Workshop 2015

The Dynamics Of Electroweak Relaxation

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based on `arXiv:1508.03321` with
Jörg Jäckel and Viraf M. Mehta

The Hierarchy Problem

The Higgs mass parameter in the SM is sensitive to the UV.

Can address this

- dynamically by allowing for new physics at the weak scale,
  most famously SUSY.

- by accepting tuning and explaining it anthropically in the context of a multiverse.

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Theoretical challenges

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Theoretical challenges

A new hope? [P. Graham, D. Kaplan, S. Rajendran 2015] [*inspired by* Abbott 1985]

- generate the weak scale dynamically in a technically natural setup without the need for new physics at the weak scale.
- **“Cosmological Relaxation of the Electroweak Scale”**

Introducing EW Relaxation

Lagrangian: [P. Graham, D. Kaplan, S. Rajendran 2015]

$$V = -\frac{1}{2}(M^2 + g\phi)|h|^2 - c_1 g M^2 \phi + \frac{c_2}{2} g^2 \phi^2 + \frac{\lambda}{4} h^4 - \Lambda^4 \cos\left(\frac{\phi}{f}\right)$$

- ϕ is the QCD axion or a different axion-like field, from now on "relaxion".
- Here h is the Higgs doublet.
- M is the UV cutoff.
- The axionic shift symmetry is broken by the coupling $g\phi h^2$.
- Break the shift symmetry weakly for $g/M \ll 1$.
- A **technically natural** model then also requires terms $gM^2\phi,\ g^2\phi^2$.

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Mechanism: [P. Graham, D. Kaplan, S. Rajendran 2015]

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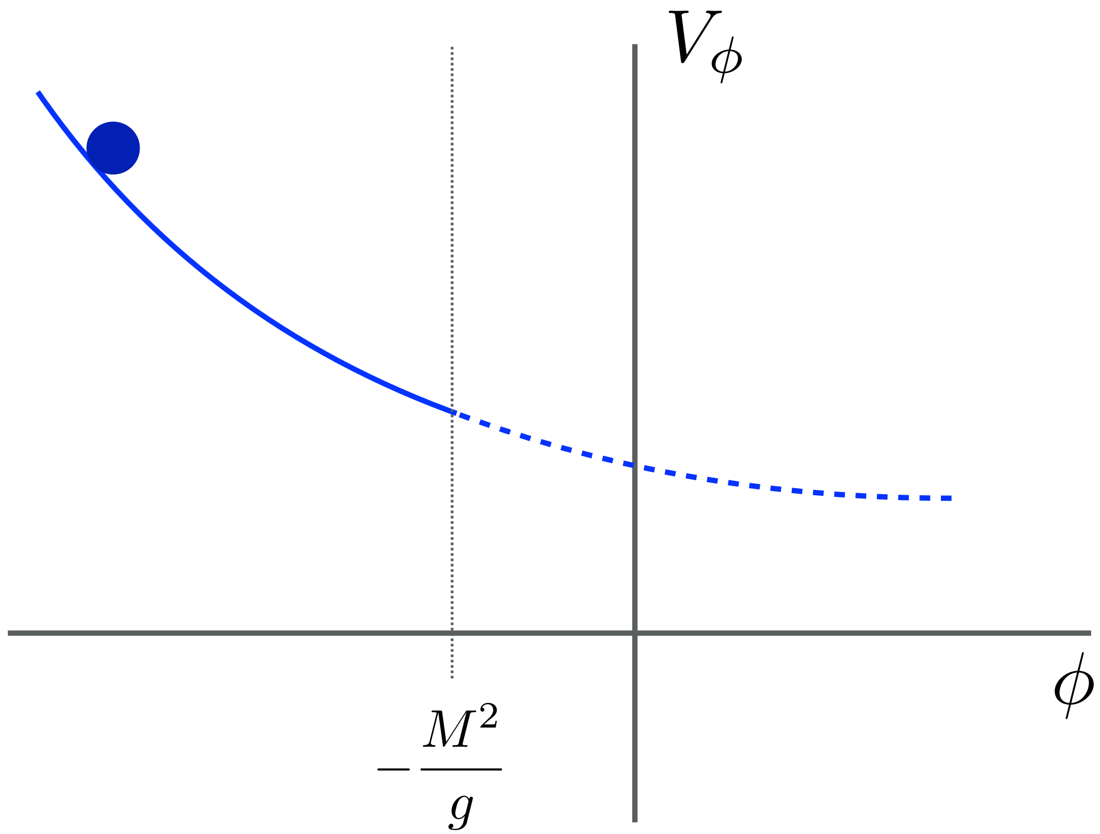
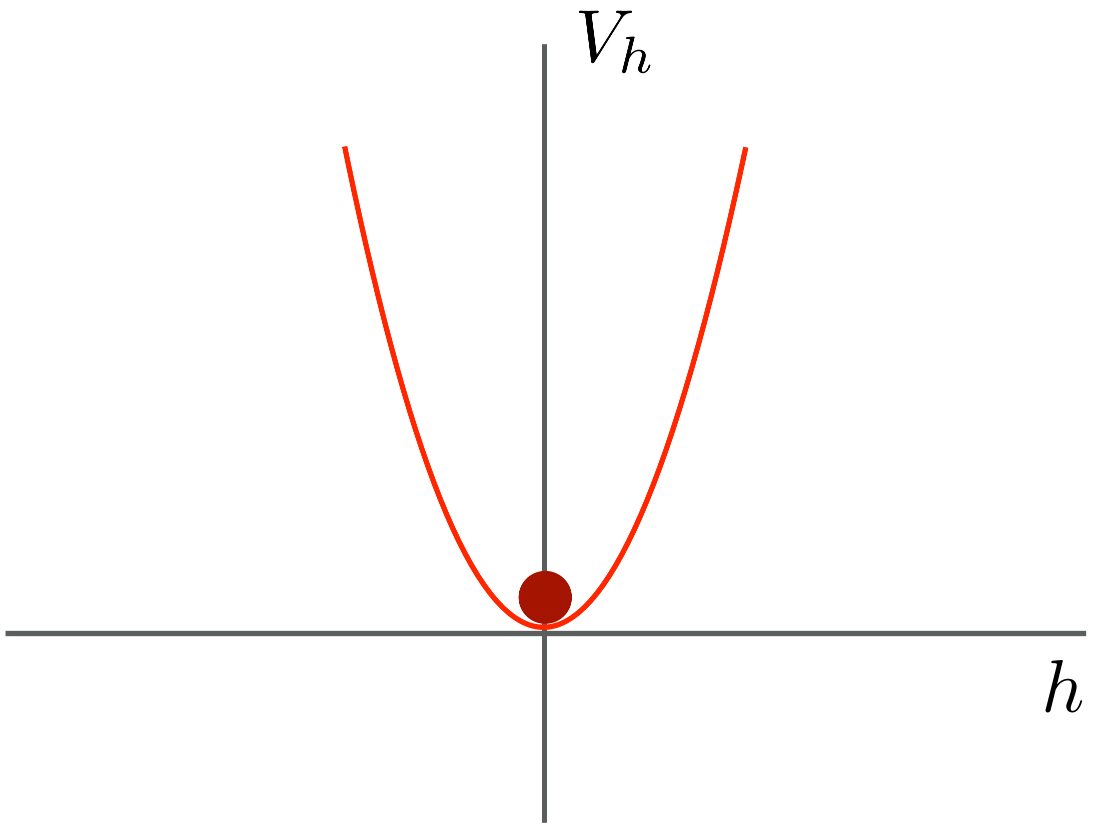
- Initially, ϕ takes a large negative value & the Higgs has a positive mass squared.





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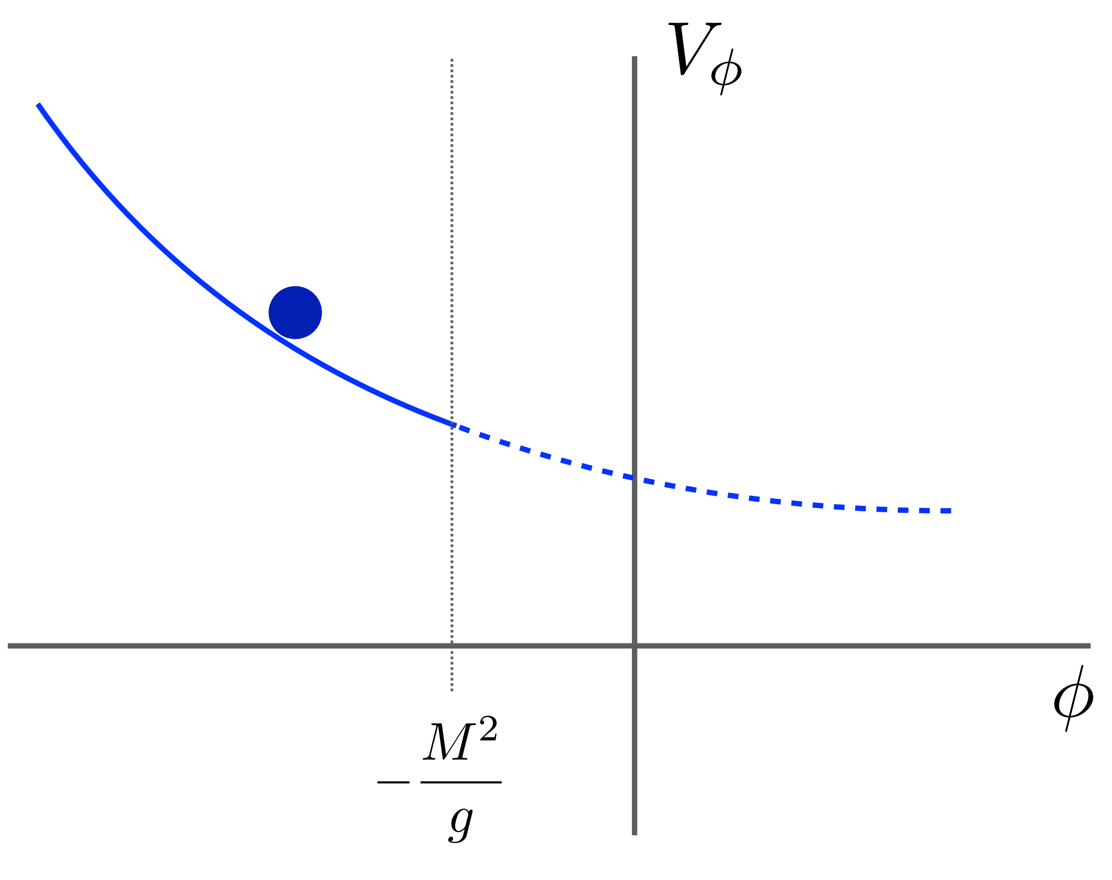
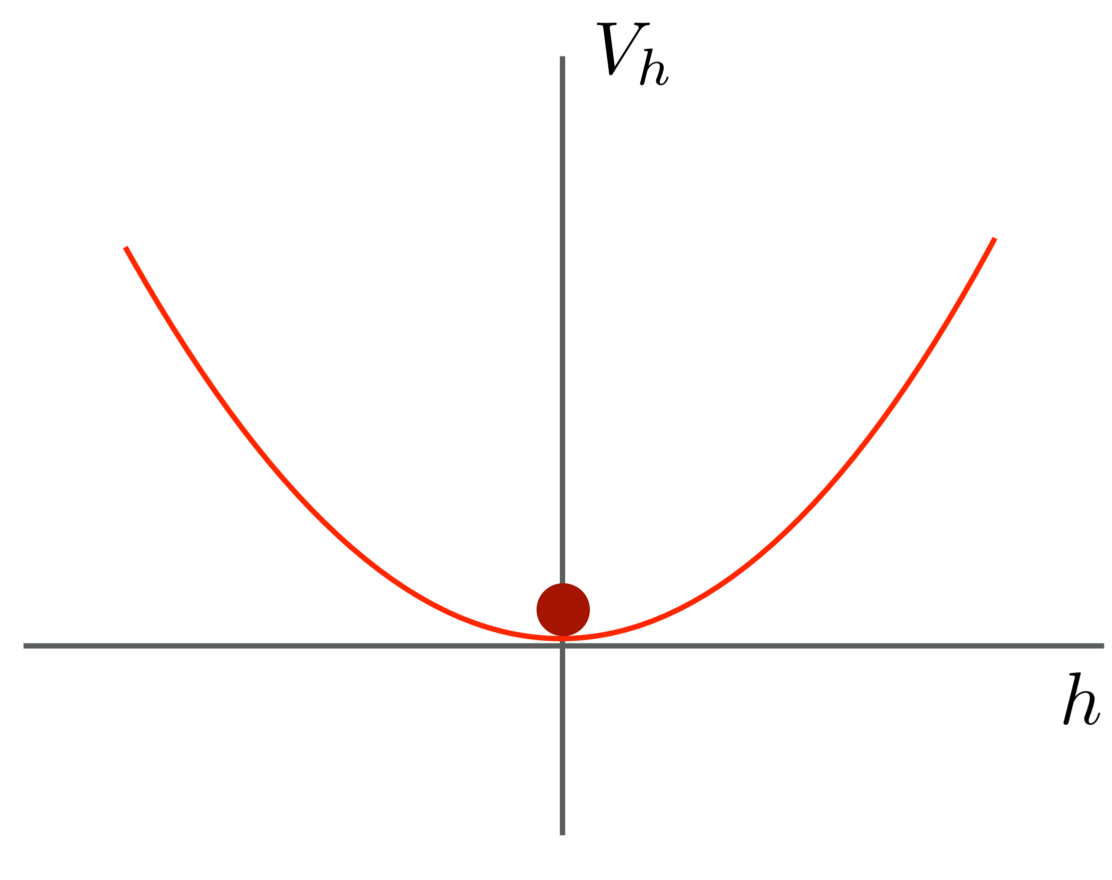
- The Higgs mass decreases as ϕ rolls down its potential.





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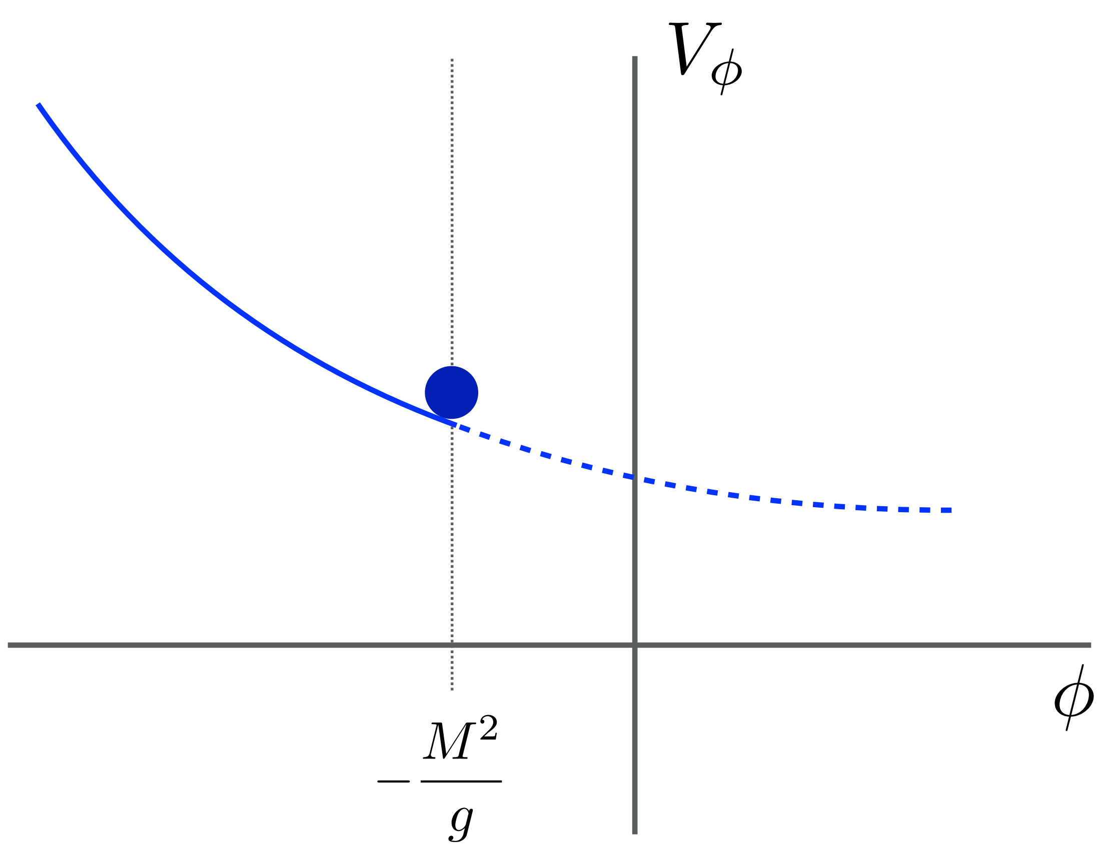
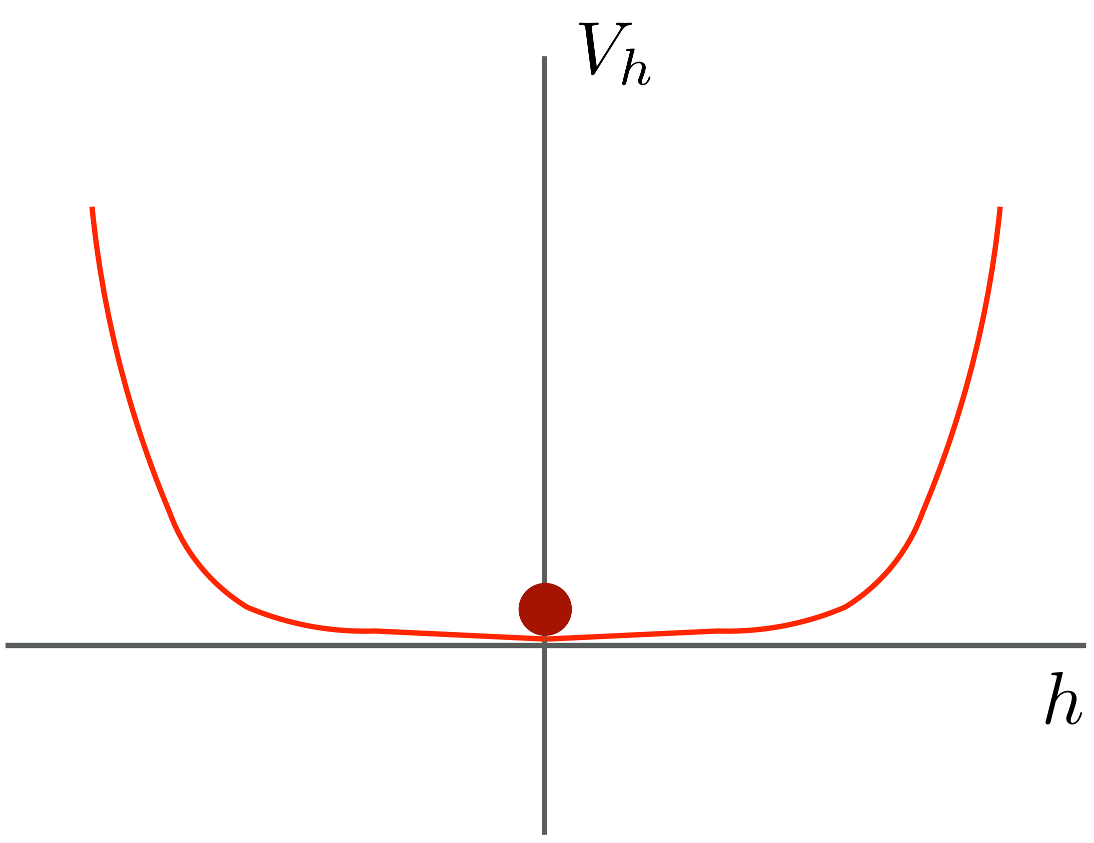
- The Higgs mass vanishes at $\phi = -M^2/g$ & becomes tachyonic for larger ϕ.





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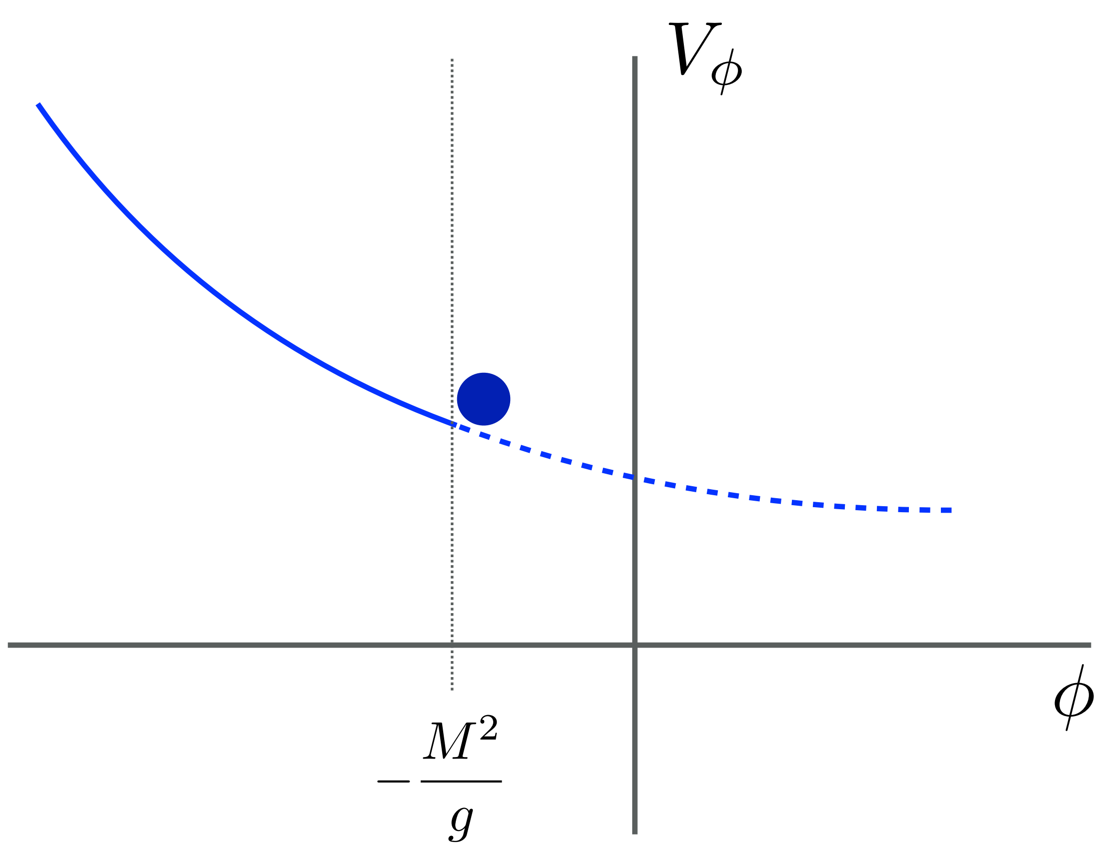
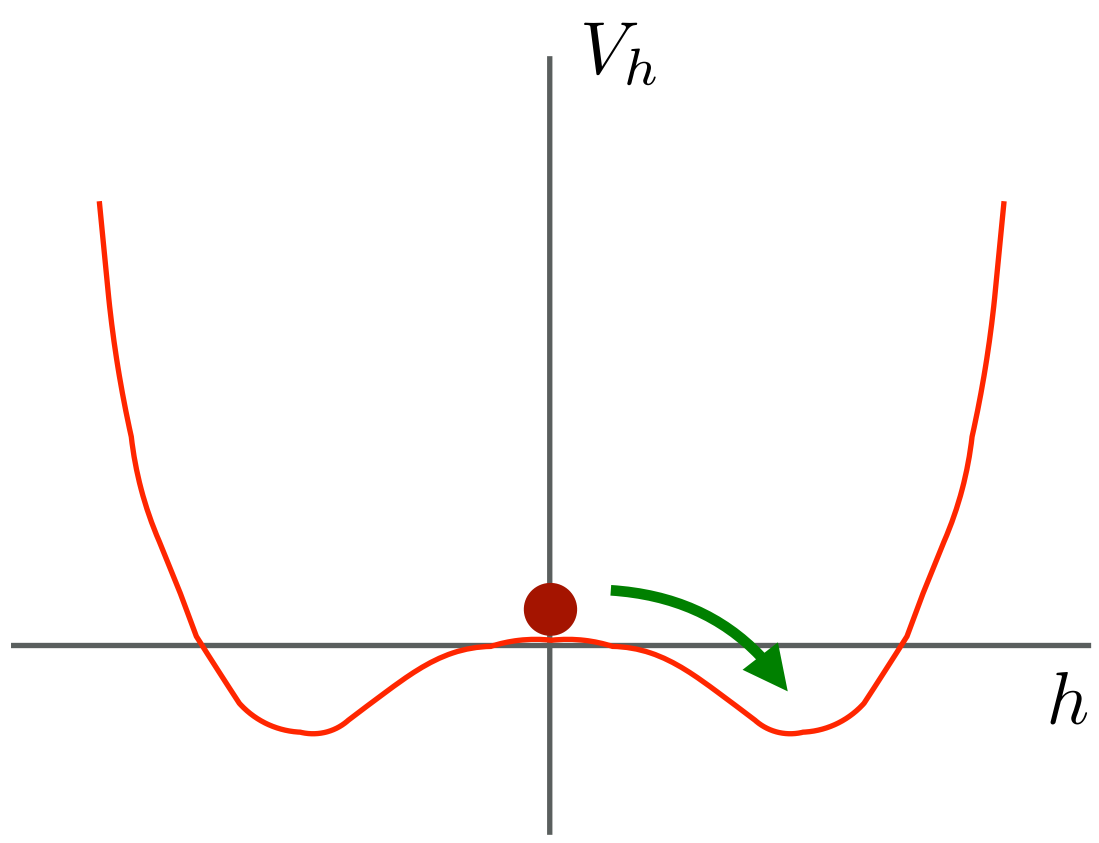
- The Higgs will fall into the new minimum and EW symmetry is broken.
- In the case of QCD $\Lambda^4 \sim f_\pi^2 m_\pi^2$ depends linearly on h .





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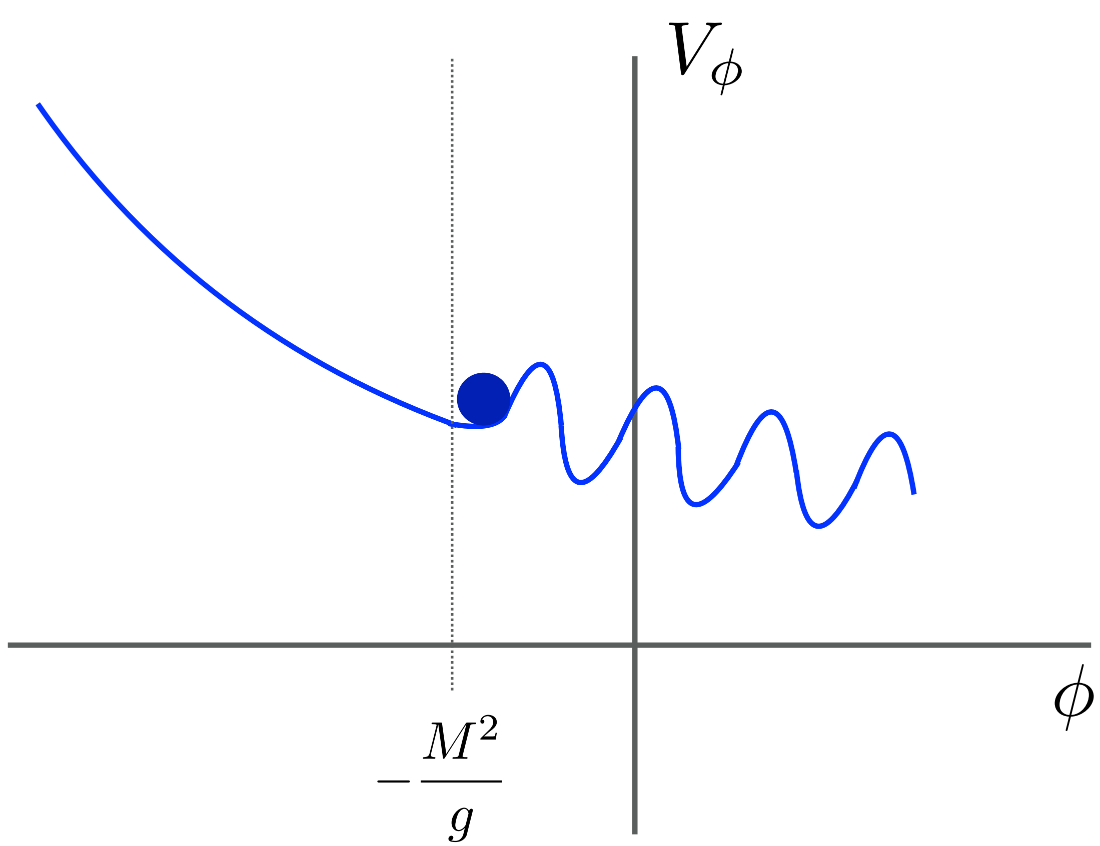
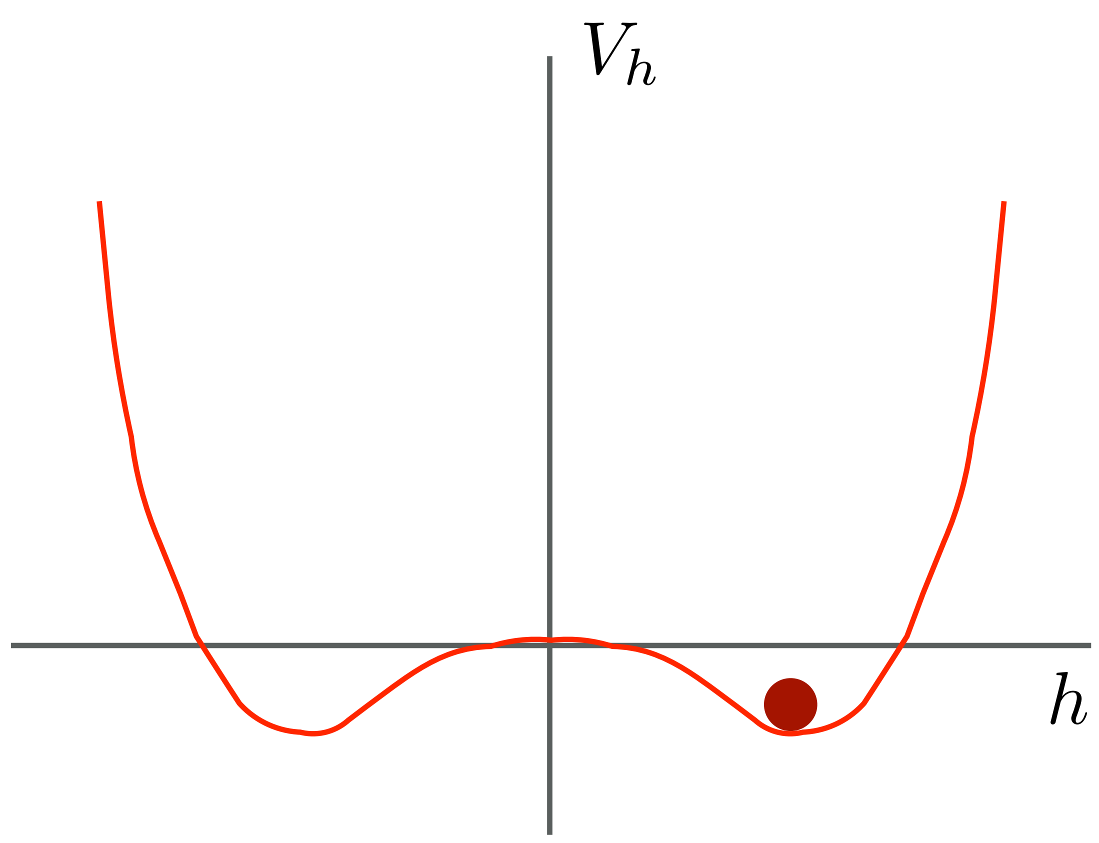
- As Λ^4 grows linearly with h, a finite Higgs vev induces cos-barriers for ϕ which eventually stop its evolution.





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- The mechanism is active during inflation:

  Need to ensure that inflation lasts long enough for ϕ to scan its field range.

  Need to ensure further consistency conditions:

  Slow-roll condition for ϕ.

  Energy density in ϕ and h subheading compared to inflaton.

  Classical rolling dominates over quantum jumps.

Strong CP problem:

- When ϕ is the QCD axion this minimal model does not solve the strong CP problem.

Introducing EW Relaxation

Stopping condition: [P. Graham, D. Kaplan, S. Rajendran 2015]

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- Rolling of ϕ stops when the negative slope of the polynomial potential is matched by the positive slope of a cos-bump.

- Ideally, we stop shortly after the Higgs mass turns tachyonic, i.e. $g\phi \sim M^2$.

- Evolution stops when $gM^2 \sim \frac{\Lambda^4}{f} \propto \frac{|h|}{f}$. **Achieve small** $v \equiv \langle h \rangle$ **for small** g .

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Many more models employing this basic idea have now been proposed:

[Kobakhidze; Espinosa, Grojean, Panico, Pomarol, Pujolàs, Servant; Hardy; Patil, Schwaller; Antipin, Redi; Batell, Giudice, McCullough; Matsedonskyi; 2015]

Outline

1. How robust is this mechanism?

- Does this mechanism always work as described above?
- How does the Higgs vev depend on the model parameters?
- What are the allowed parameter ranges to obtain $v \equiv \langle h \rangle \ll M$?
- When does this mechanism fail? Are there instabilities?

Outline

2. Progress towards solving the hierarchy problem?

- EW Relaxation: technically natural model allowing for $v \equiv \langle h \rangle \ll M$. No fine-tuning of bare $\mathcal{L}$ parameters against radiative corrections at UV scale is needed for $v \equiv \langle h \rangle \ll M$.
- Still, the model requires at least one parameter, g, to be chosen small.
- **Q: how severely does one need to tune the model parameters?**
- Whether such a tuning is OK / disastrous can only be decided in an embedding into a UV complete theory.
- In the absence of such an embedding, let us compare the necessary level of tuning to the tuning required in the SM, v^2/M^2.

Introducing EW Relaxation

The model:

$$V = -\frac{1}{2}(M^2 + g\phi)h^2 - c_1 g M^2 \phi + \frac{c_2}{2} g^2 \phi^2 + \frac{\lambda}{4} h^4 - \kappa |h| \cos\left(\frac{\phi}{f}\right)$$

- Let h be the Higgs vev already.
- Replace $\Lambda^4 \to \kappa |h|$ to make linear dependence on the Higgs vev explicit

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- Let h be the Higgs vev already.
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- It will be useful to rewrite using the field $\tilde{\phi} \equiv \phi + \frac{M^2}{g}$.

EW symmetry is unbroken for $\tilde{\phi} \leq 0$ and broken for $\tilde{\phi} > 0$.

$$V = -\frac{1}{2} g \tilde{\phi} h^2 - (c_1 + c_2) M^2 g \tilde{\phi} + \frac{c_2}{2} g^2 \tilde{\phi}^2 + const. + \frac{\lambda}{4} h^4 - \kappa |h| \cos\left(\frac{g\tilde{\phi} - M^2}{fg}\right)$$

Dynamics Of EW Relaxation

The single-field approximation:

$$V = -\frac{1}{2}g\tilde{\phi}h^2 - (c_1+c_2)M^2 g\tilde{\phi} + \frac{c_2}{2}g^2\tilde{\phi}^2 + const. + \frac{\lambda}{4}h^4 - \kappa|h|\cos\left(\frac{g\tilde{\phi} - M^2}{fg}\right)$$

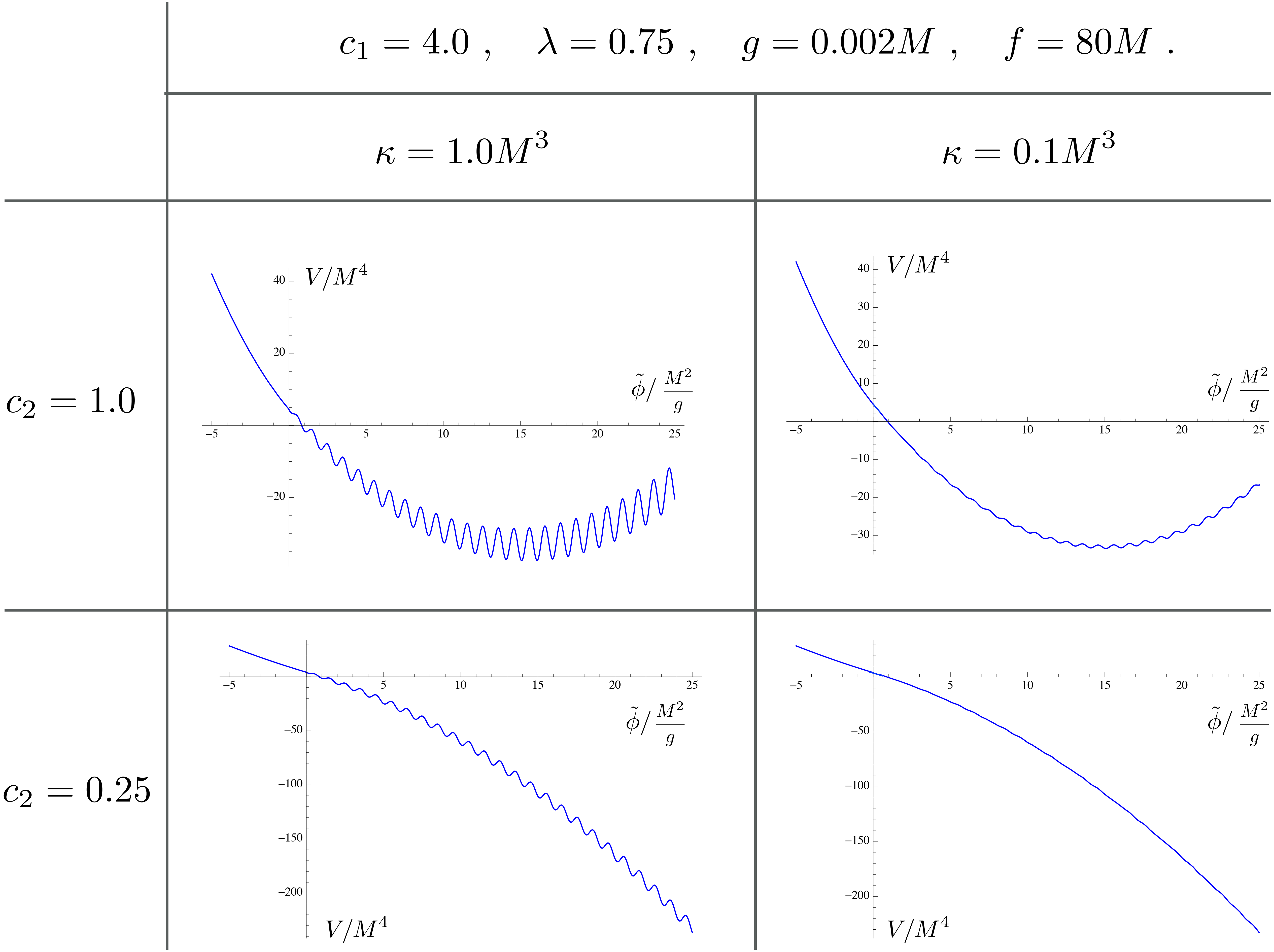
h dynamics typically faster than ϕ evolution. Write as effective one-field model.

- $\tilde{\phi} \leq 0$: Higgs vev $\langle h \rangle = 0.$

$$V_1 = \frac{1}{2}g^2 c_2 \tilde{\phi}^2 - gM^2\left(c_1+c_2\right)\tilde{\phi} + const.$$

- $\tilde{\phi} > 0$: Higgs vev $\langle h \rangle = \sqrt{g\tilde{\phi}/\lambda}$.

$$V_1 = \frac{1}{2}g^2\left(c_2 - \frac{1}{2\lambda}\right)\tilde{\phi}^2 - gM^2\left(c_1+c_2\right)\tilde{\phi} - \kappa\frac{\sqrt{g\tilde{\phi}}}{\sqrt{\lambda}}\cos\left(\frac{\tilde{\phi}+\phi_c}{f}\right) + const.$$



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$\kappa = 1.0M^3$
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V/M^4
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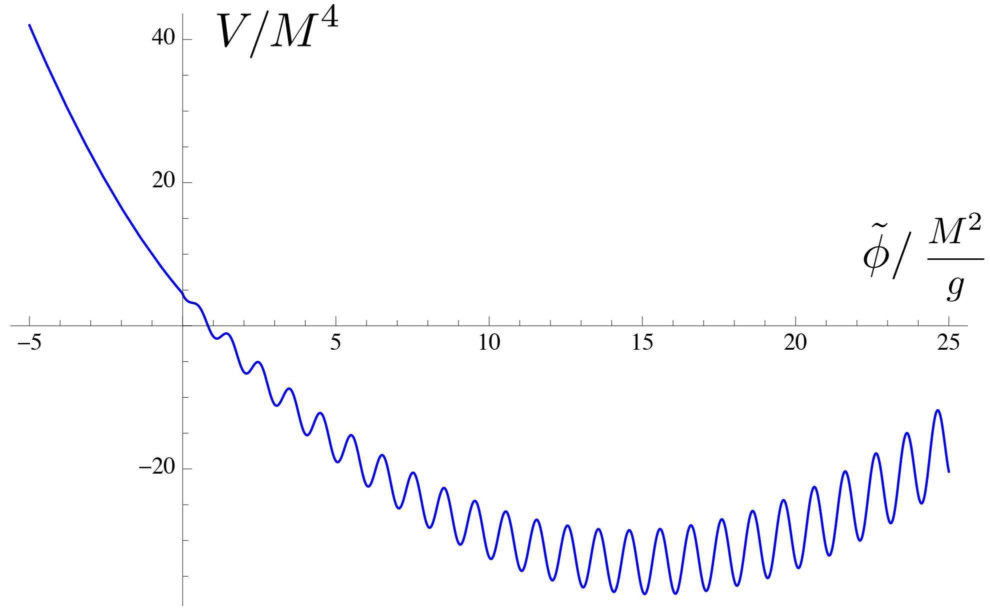

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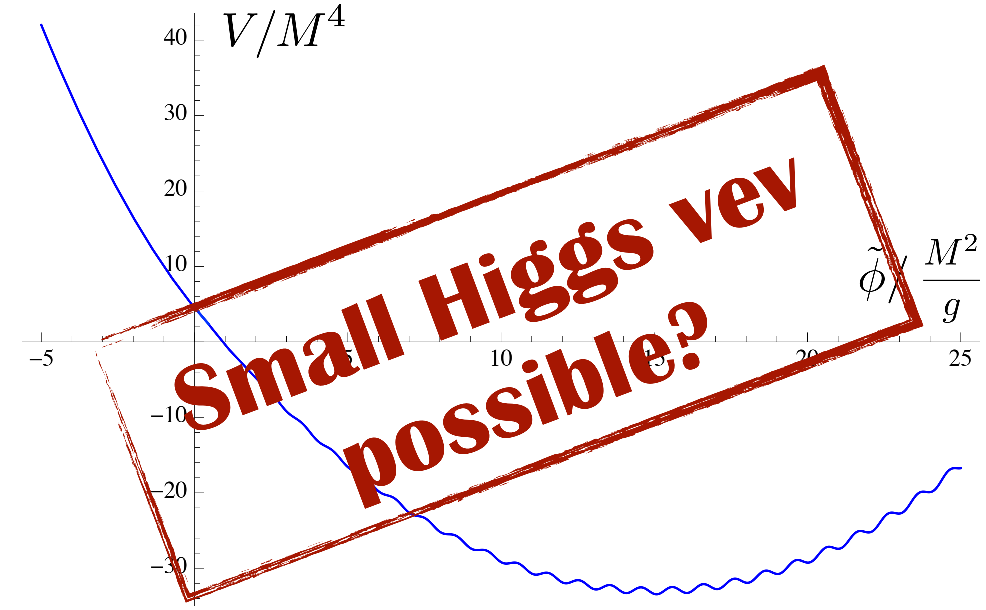

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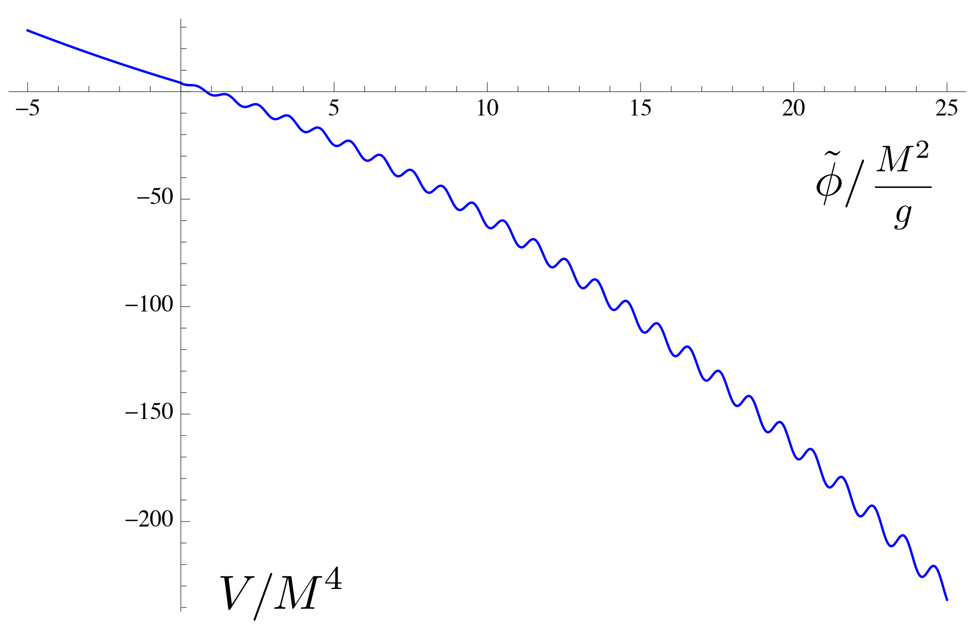

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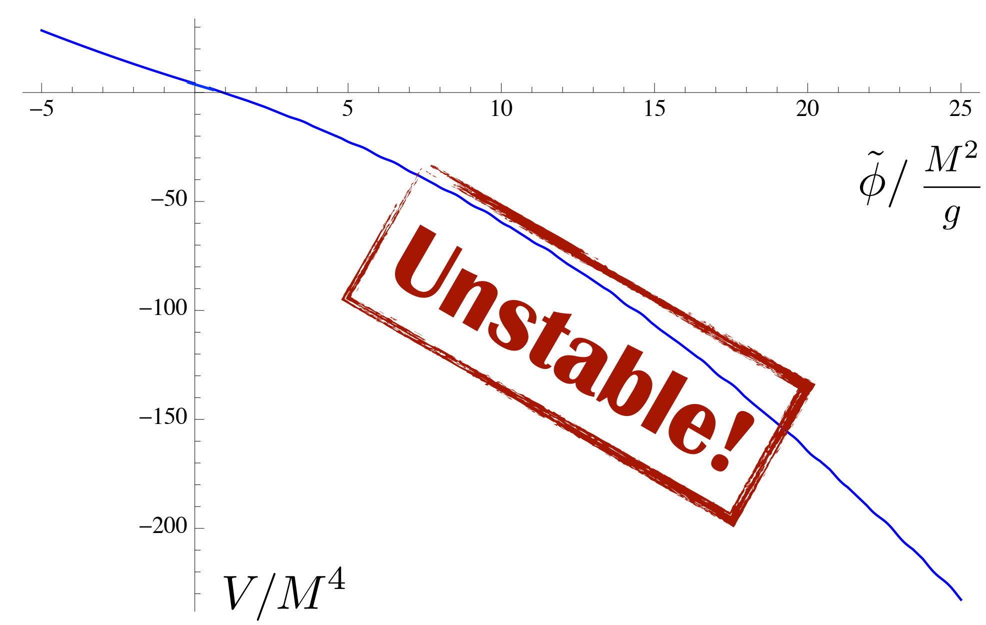

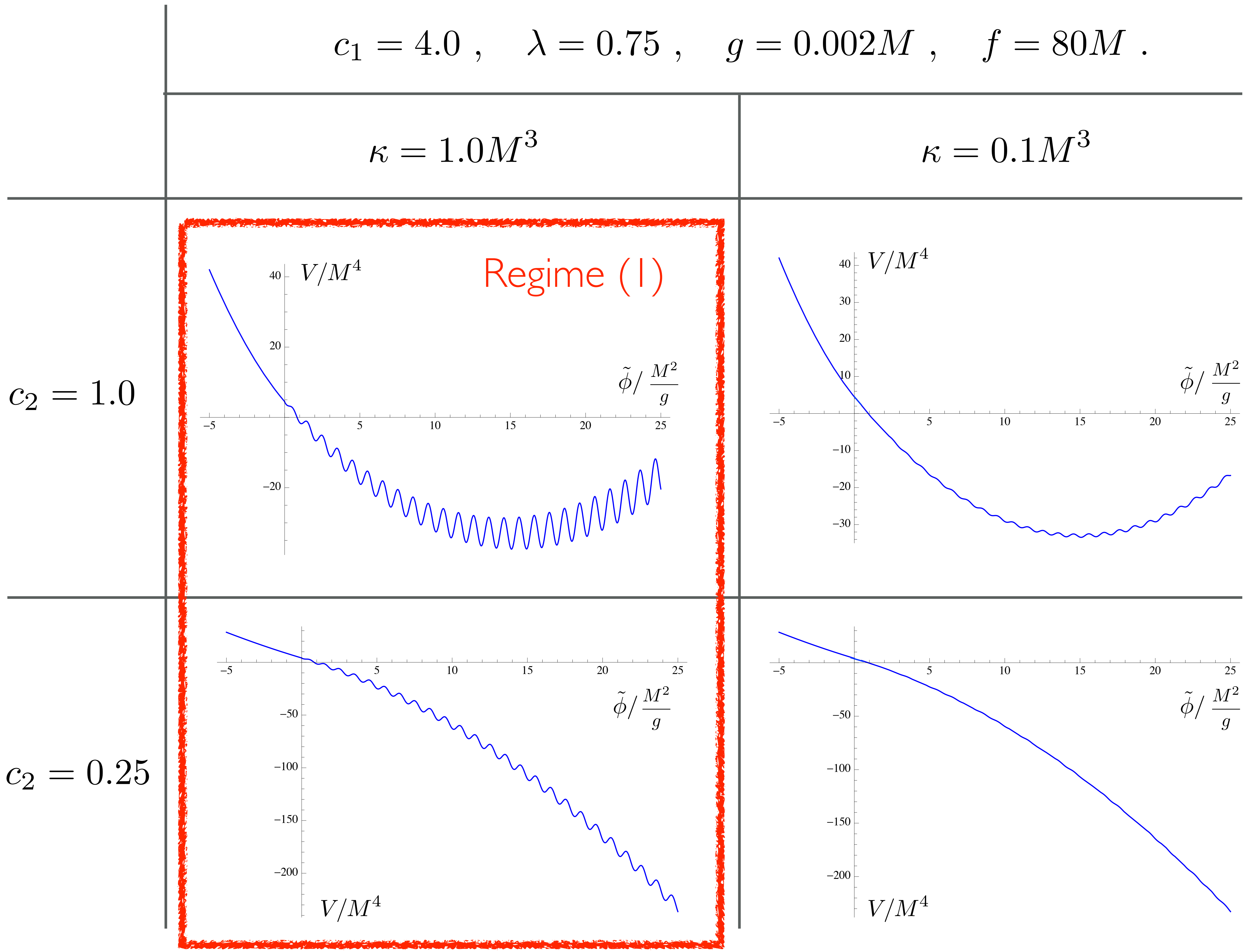

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Regime (I)
V/M^4
$\tilde{\phi}/\ \frac{M^2}{g}$

Dynamics Of EW Relaxation: Regime (1)

This occurs for sufficiently small g:

$$V_1 = \frac{1}{2} g^2 \left(c_2 - \frac{1}{2\lambda} \right) \tilde{\phi}^2 - g M^2 \left(c_1 + c_2 \right) \tilde{\phi} - \kappa \frac{\sqrt{g \tilde{\phi}}}{\sqrt{\lambda}} \cos \left(\frac{\tilde{\phi} + \phi_c}{f} \right) + const.$$

A comparison of slopes gives the result for $\langle h \rangle$:

$$\langle h \rangle \sim g \frac{f M^2}{\kappa}$$

The Higgs vev is small to the extent that g is small.

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Does the single-field approximation captures the dynamics correctly?

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Go back to the full two-field model:

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The cos-term constitutes a source term for h.
It can destabilise the Higgs well before the Higgs mass becomes tachyonic.

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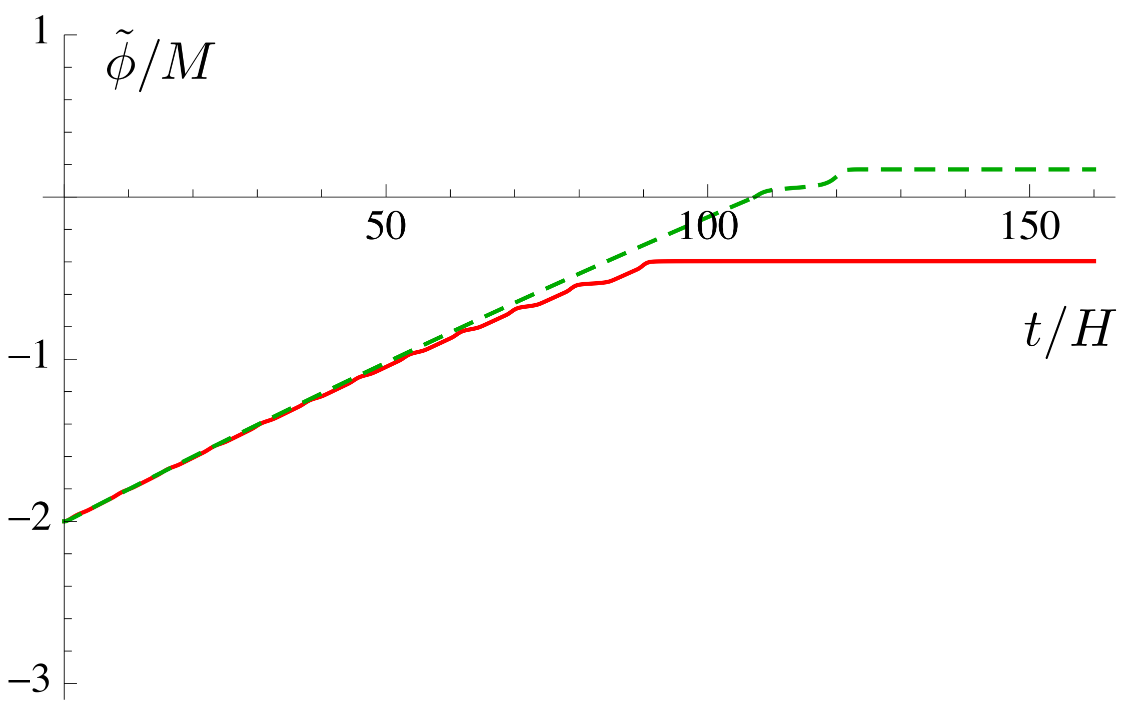
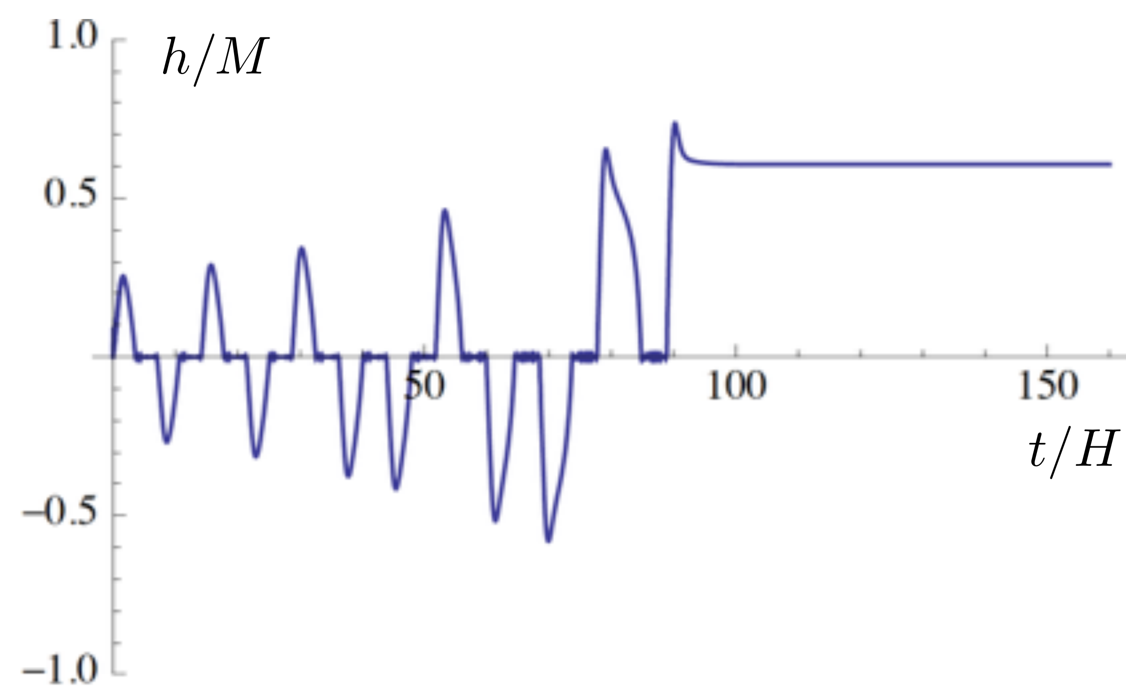


Two-field model

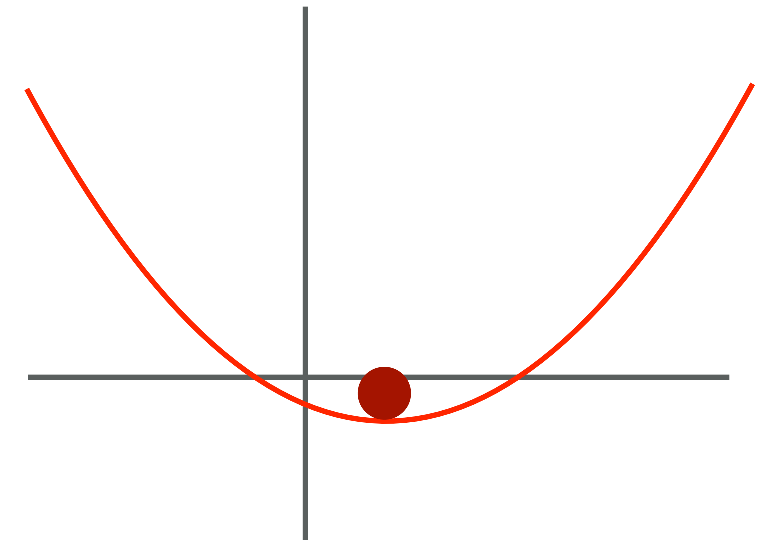
Single-field approximation



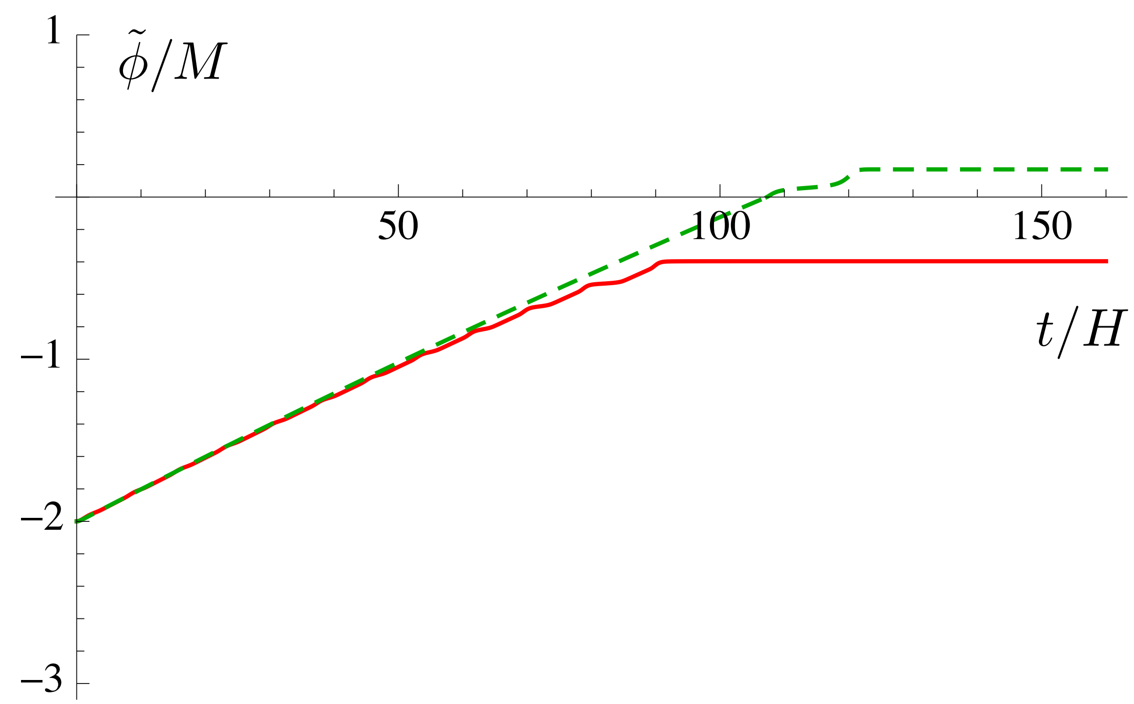
Two-field model

Dynamics Of EW Relaxation: Regime (1)

- The Higgs is trapped in a non-standard vacuum, where EW symmetry is broken by a Higgs source term.



- Nevertheless, $\langle h \rangle \sim g\dfrac{f M^2}{\kappa} \ll M$ possible.



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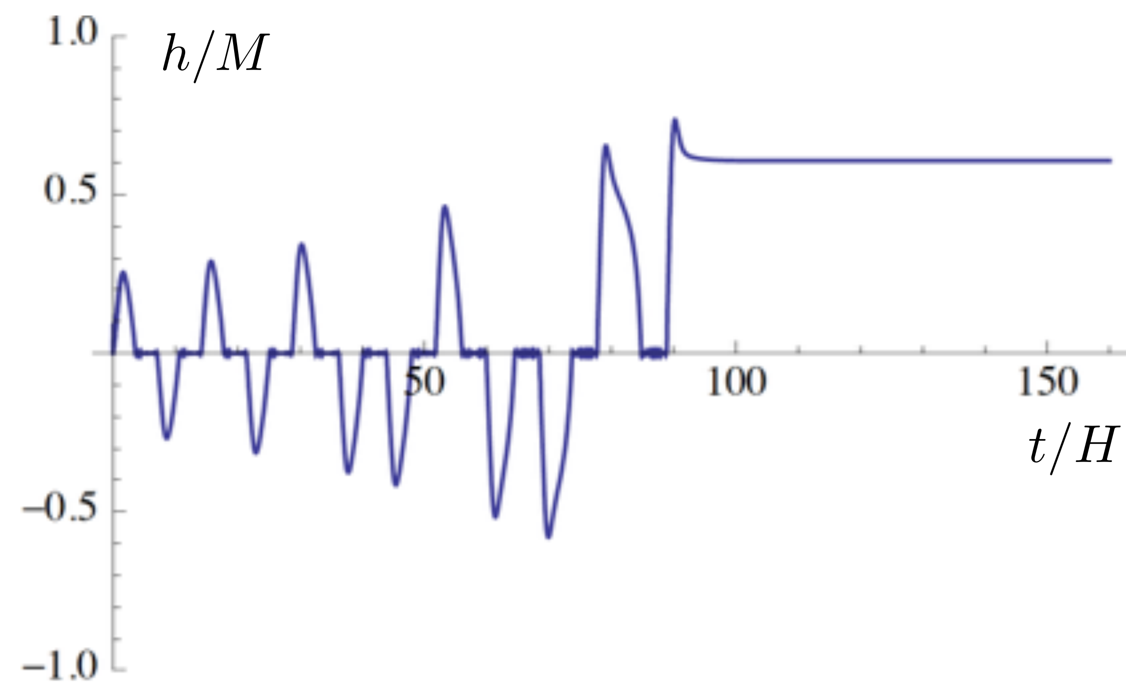

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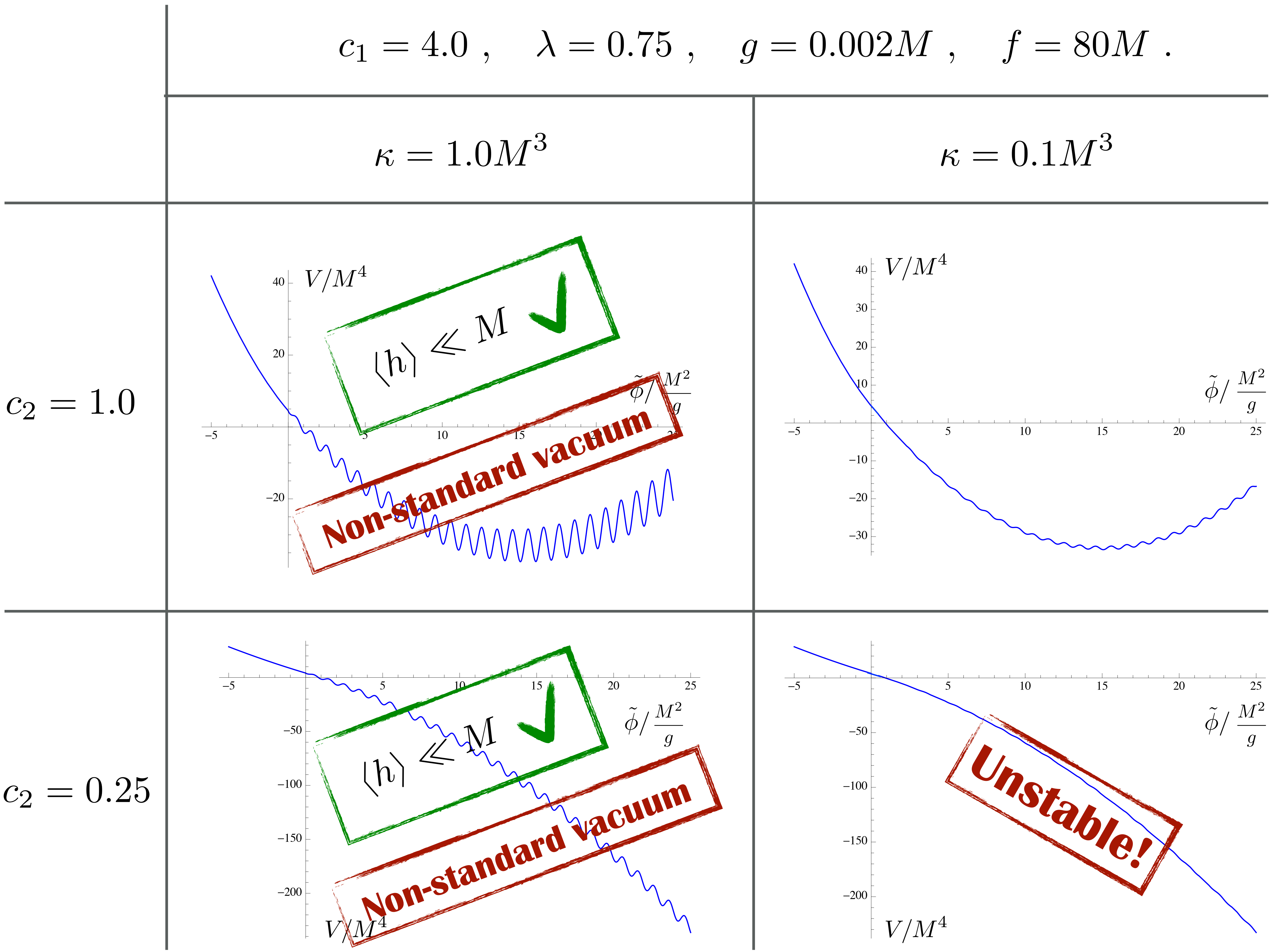

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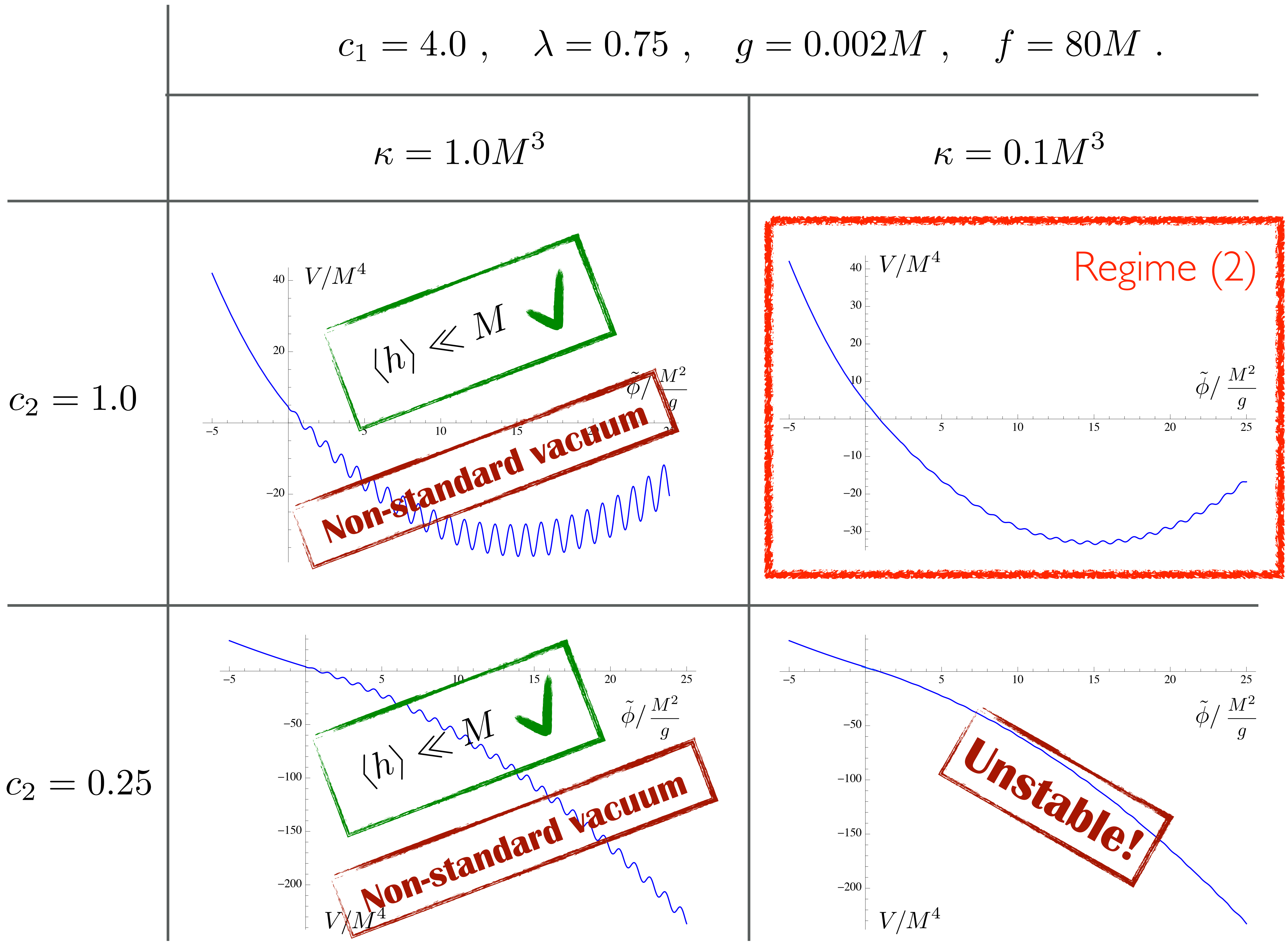
Unstable!



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Regime (2)
Unstable!

Dynamics Of EW Relaxation: Regime (2)

- In **regime (2)** the axion does not get caught by the first cos-bump and as a result we have $\tilde{\phi}_f \gg f$.

A. Case (2A): $g \ll \dfrac{\kappa}{fM}$ $\langle h \rangle \sim g\dfrac{fM^2}{\kappa} \ll M$

B. Case (2B): $g \sim \dfrac{\kappa}{fM}$ or $g > \dfrac{\kappa}{fM}$ $\langle h \rangle \sim M$ or instability reached

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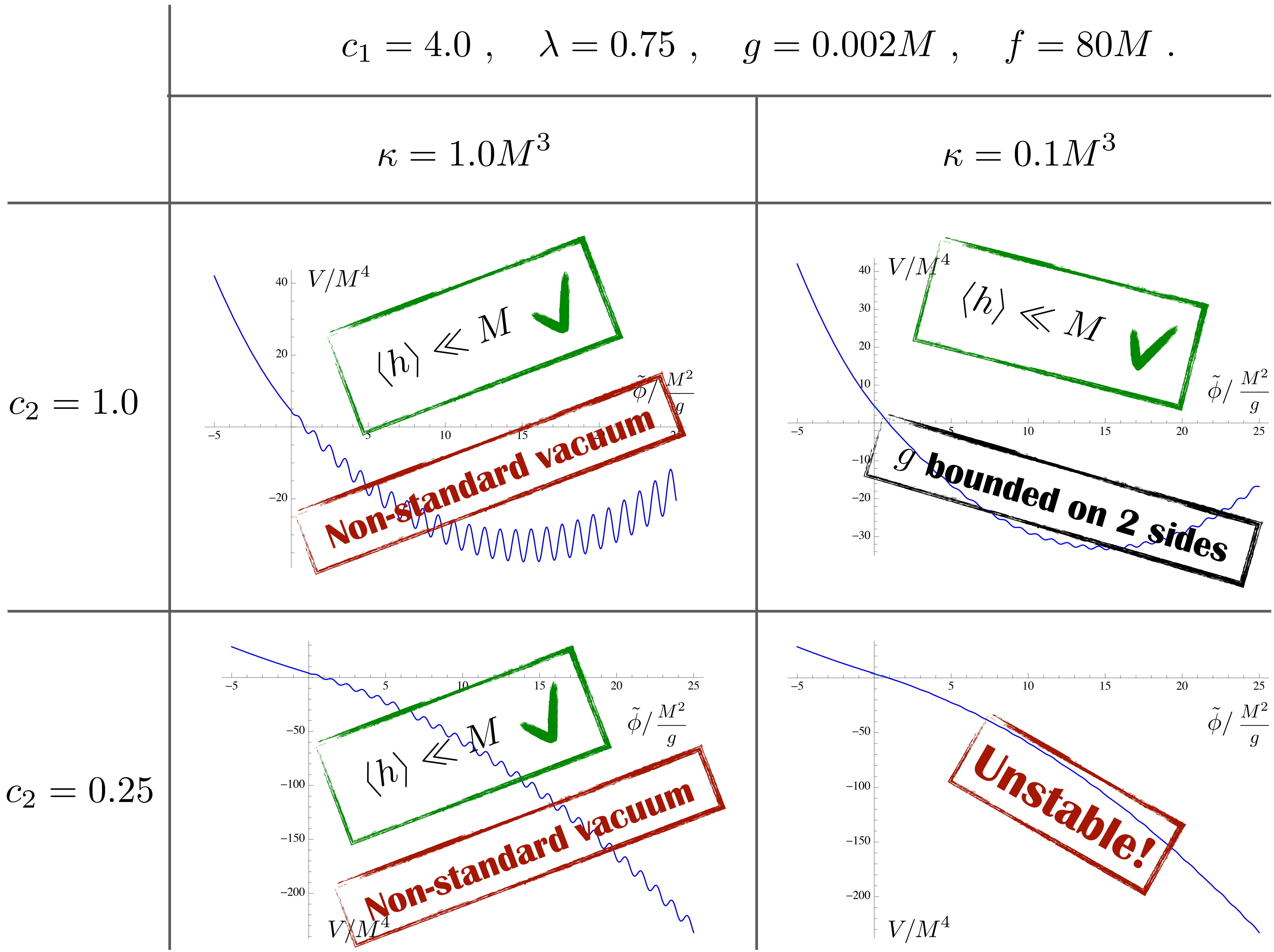
- The condition $\tilde{\phi}_f = \dfrac{\lambda}{g}\langle h \rangle^2 \gg f$ can be rewritten as a lower bound on g.

  Then g is bounded on both sides. Taking $\lambda \sim \mathcal{O}(1)$ we have

$$\frac{\kappa^2}{fM^4} \ll g \ll \frac{\kappa}{fM}$$



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V/M^4
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$\langle h \rangle \ll M$
Non-standard vacuum
g bounded on 2 sides
Unstable!

Tuning

- The relaxation model requires a small parameter g/M to ensure $v \ll M$.

  ➡ The SM with a cutoff M is tuned to a degree v^2/M^2.

- How severe is the tuning in the relaxion model?

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Have $$\frac{v^2}{M^2} \sim \frac{f^2 g^2 M^2}{\kappa^2}.$$

But the lower bound on g gives $$g \gg \frac{\kappa^2}{f M^4}.$$

Inserting we obtain $$\frac{v^2}{M^2} \sim \frac{f^2 g^2 M^2}{\kappa^2} \gg \frac{fg}{M^2}$$

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Inserting we obtain $$\frac{v^2}{M^2} \sim \frac{f^2 g^2 M^2}{\kappa^2} \gg \frac{f g}{M^2} .$$

➡ $$\epsilon \equiv \frac{g}{M} \ll \frac{1}{f/M} \frac{v^2}{M^2}$$ **Can be as severely tuned as the SM!**

Tuning

- How does the tuning evolve when increasing the cutoff?

SM:

$$\frac{v^2}{M^2} \to \frac{v^2}{M'^2} = \frac{v^2}{M^2}\left(\frac{M}{M'}\right)^2$$

Relaxion:

$$v \sim \frac{\epsilon M^3 f}{\kappa} = \frac{\epsilon' M'^3 f}{\kappa} \quad \Rightarrow \quad \epsilon' = \epsilon \left(\frac{M}{M'}\right)^3$$

- **The tuning becomes worse more quickly in the relaxion model.**

Tuning

- How does the tuning evolve when increasing the cutoff?

SM:

$$\frac{v^2}{M^2} \to \frac{v^2}{M'^2} = \frac{v^2}{M^2}\left(\frac{M}{M'}\right)^2$$

Relaxion:

$$v \sim \frac{\epsilon M^3 f}{\kappa} = \frac{\epsilon' M'^3 f}{\kappa} \quad \Rightarrow \quad \epsilon' = \epsilon \left(\frac{M}{M'}\right)^3$$

- The tuning becomes worse more quickly in the relaxion model.

- **Main advantage:** the hierarchy between v and the cutoff is controlled by the smallness of just one parameter, $\epsilon = g/M$.

- This could facilitate an embedding into a UV complete theory, e.g. string theory.

Conclusions

Mechanism of Cosmological Relaxation: [P. Graham, D. Kaplan, S. Rajendran 2015]

- makes the weak scale technically natural without new weak scale physics.

Dynamics of the original model:

- Indeed, $v \sim \langle h \rangle \ll M$ is possible for sufficiently small g.
- However, g cannot be too small to prevent h from being trapped in a non-standard vacuum with EW symmetry broken by a source term.
- Instabilities are possible.

Progress towards solving the hierarchy problem?

- Smallness of v controlled by a single parameter g. Model reminiscent of axion monodromy inflation. Possible embedding into string theory?
- Tuning of the small parameter g can be as severe as the equivalent level of tuning in the SM.