1/4 BPS Wilson loops in $AdS_5 \times S^5$: does σ-model perturbation theory meet localization?

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Wilson loops in the AdS/CFT correspondence

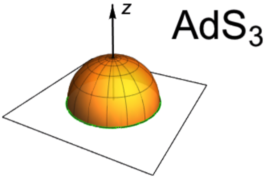
4D $\mathcal{N}=4$ Yang-Mills theory	$\longleftrightarrow$	**Type IIB string theory in $AdS_5 \times S^5$**
Wilson loop on path $\mathcal{C}$	$=$	**Partition function for string** on $\mathcal{C}$
$\langle \mathcal{W}[\mathcal{C}] \rangle$		$Z_{\text{string}}[\mathcal{C}] \equiv \int [\mathcal{D}X][\mathcal{D}\Psi]\, e^{-S_{\text{string}}}$

1/2 BPS circular Wilson loop [Erickson Semenoff Zarembo 00] [Drukker Gross 00]

- All-loop answer from SUSY localization [Pestun 07, 09].

Localization $\quad \langle \mathcal{W}(\lambda) \rangle = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) \overset{\lambda \gg 1}{\approx} \sqrt{\frac{2}{\pi}} \lambda^{-3/4} e^{\sqrt{\lambda}}$

σ-model pert. theory $\quad Z(\lambda) \overset{\lambda \gg 1}{\approx} \underbrace{\frac{1}{\sqrt{2\pi}}}_{\text{1-loop}} \times \underbrace{c\lambda^{-3/4}}_{\text{zero modes}} \times \underbrace{e^{\sqrt{\lambda}}}_{\text{classical area}}$

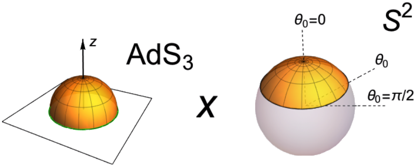


✓ Holographically ($\lambda \gg 1$) as a minimal-area surface [Drukker Gross Tseytlin 00].

✗ **No match** at **one-loop order** due to c (diffeo-ghost zero modes) [Kruczenski Tirziu 08].

Reconcile perturbation theory with localization for $\lambda \gg 1$

- Perturbatively matching a **finite vev** is difficult (must include all factors).
- Generalize to a **one-parameter** family of **1/4 BPS latitude Wilson loops** [Drukker Fiol 05] [Drukker 06].



Goals

- **Compute** $(\lambda \gg 1)$ and **normalize** $Z(\lambda, \theta_0)$ **to the circular case** $Z(\lambda, \theta_0 = 0)$ to wash out the constant c

$$Z(\lambda, \theta_0)\,/\,Z(\lambda, 0) \qquad\qquad \text{in } \sigma\text{-model pert. theory at 1-loop}$$

and **snatch** the expected **one-loop θ_0-dependence**

$$\langle \mathcal{W}(\lambda, \theta_0)\rangle \,/\, \langle \mathcal{W}(\lambda, 0)\rangle \qquad \text{from localization (Bessel f. with } \lambda \to \lambda \cos^2\theta_0).$$

- String σ-model computations are typically **plagued by divergencies**: systematic regularization scheme, particularly for **observables lacking predictions**.

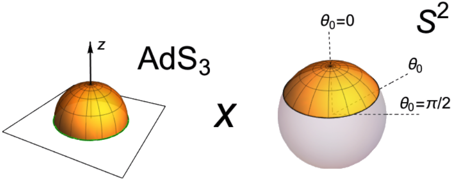
Perturbation theory for the string σ-model

- Type IIB **Green-Schwarz string action** in $AdS_5 \times S^5$, to be **gauge-fixed**

$$S = \frac{\sqrt{\lambda}}{2\pi} \int d\tau d\sigma \left[\sqrt{h} + \left(\sqrt{h} h^{ij} \delta^{IJ} - i\epsilon^{ij} s^{IJ} \right) \bar{\Psi}^I \rho_i \left(D_j \Psi \right)^J + o(\Psi^I)^2 \right]$$

- **Expand** around minimal-area surface (see [Forini, Puletti, Griguolo, Seminara, EV 15])

$$X(\tau,\sigma) = X_{\text{classical}}(\tau,\sigma)$$
$$\Psi(\tau,\sigma) = 0$$



- **Saddle-point expansion** for $\lambda \gg 1$

$$Z(\lambda,\theta_0) \equiv \int [\mathcal{D}X][\mathcal{D}\Psi]\, e^{-S} \approx \underbrace{e^{\sqrt{\lambda}(\cos\theta_0 - 1/\epsilon)}}_{\text{exp of classical action}}$$

(cutoff $\epsilon \ll 1$ will cancel out in ratio)

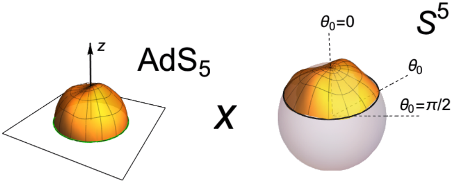
Perturbation theory for the string σ-model

- Type IIB **Green-Schwarz string action** in $AdS_5 \times S^5$, to be **gauge-fixed**

$$S = \frac{\sqrt{\lambda}}{2\pi} \int d\tau d\sigma \left[\sqrt{h} + \left(\sqrt{h} h^{ij} \delta^{IJ} - i\epsilon^{ij} s^{IJ} \right) \bar{\Psi}^I \rho_i \left(D_j \Psi \right)^J + o(\Psi^I)^2 \right]$$

- **Expand** around minimal-area surface (see [Forini, Puletti, Griguolo, Seminara, EV 15])

$$X(\tau,\sigma) = X_{\text{classical}}(\tau,\sigma) + \delta X(\tau,\sigma)$$
$$\Psi(\tau,\sigma) = 0 + \delta\Psi(\tau,\sigma)$$



- **Saddle-point expansion** for $\lambda \gg 1$

$$Z(\lambda,\theta_0) \equiv \int [\mathcal{D}X][\mathcal{D}\Psi]\, e^{-S} \approx \underbrace{e^{\sqrt{\lambda}(\cos\theta_0 - 1/\epsilon)}}_{\text{exp of classical action}} \times \underbrace{\prod_{\omega\in\mathbb{Z}} \frac{\prod_{p_1,p_2,p_3=\pm 1} \mathrm{Det}^{1/4}\left(\mathcal{O}_{p_1,p_2,p_3}\right)^2}{\mathrm{Det}^{3/2}\mathcal{O}_1 \mathrm{Det}^{3/2}\mathcal{O}_2 \mathrm{Det}^{1/2}\mathcal{O}_3}}_{\text{from Gaussian integrals over fluctuations}}$$

(cutoff $\epsilon \ll 1$ will cancel out in ratio)

One-loop partition function

- **Functional determinants** of complicated scalar-/matrix-valued operators with **hyperbolic-functions** dependence.

$$Z(\lambda,\theta_0) \approx \underbrace{e^{\sqrt{\lambda}(\cos\theta_0 - 1/\epsilon)}}_{\text{exp of classical action}} \times \underbrace{\prod_{\omega\in\mathbb{Z}} \frac{\prod_{p_1,p_2,p_3=\pm1} \mathrm{Det}^{1/4}\left(\mathcal{O}_{p_1,p_2,p_3}\right)^2}{\mathrm{Det}^{3/2}\mathcal{O}_1 \mathrm{Det}^{3/2}\mathcal{O}_2 \mathrm{Det}^{1/2}\mathcal{O}_3}}_{\text{from Gaussian integrals over fluctuations}}$$

Decouple τ,σ-dependence in $\mathcal{O}$:

τ) Surface rotational symmetry **Fourier-transforms** $\partial_\tau \to i\omega$.

Periodic b.c. along $\tau \in [0, 2\pi)$ $\longrightarrow$ overall **infinite product** $\prod_{\omega\in\mathbb{Z}}$

σ) Insert **cutoffs** $\epsilon \ll 1$ and $R \gg 1$ to make spectral problems (in σ) well-defined.

Dirichlet b.c. near *AdS* boundary (ϵ) and at a fictitious boundary inside *AdS* (R) $\longrightarrow$ **Gel'fand-Yaglom method** and derived technology for $\mathrm{Det}\mathcal{O}$ [Forman 87] [Lesch Tolksdorf 98]

Normalization and final comments

One-loop partition function in string σ-**model perturbation theory** in $AdS_5 \times S^5$ capturing the dual family of **1/4 BPS Wilson loops** in planar $\mathcal{N} = 4$ SYM.

The **normalization** gets rid of (unevaluated extra) constants. (Below, $\cos\theta_0 \equiv \tanh\sigma_0$.)

Localization $\quad \frac{\langle \mathcal{W}(\lambda,\theta_0)\rangle}{\langle \mathcal{W}(\lambda,0)\rangle} \overset{\lambda \gg 1}{\approx} \cos^{-3/2}\theta_0\, e^{\sqrt{\lambda}(\cos\theta_0 - 1)}$

σ-model pert. theory $\quad \frac{Z(\lambda,\theta_0)}{Z(\lambda,0)} \overset{\lambda \gg 1}{\approx} \underbrace{\frac{\cos^{-3/2}\theta_0}{(1+e^{-2\sigma_0})^{1/2}}}_{\text{from } \omega=0} \underbrace{\cancel{\frac{\prod_{\omega\geq 1}\left(\frac{\omega+\tanh\sigma_0}{\omega+1}\right)^4}{\prod_{\omega\geq 1}\left(\frac{\omega+\tanh\sigma_0}{\omega+1}\right)^4}}}_{\text{from } \omega\neq 0} \times e^{\sqrt{\lambda}(\cos\theta_0 - 1)}$

✓ The cancellation of bosonic/fermionic modes is a **vestige of SUSY**.

✓/✗ $\omega = 0$ modes **match** $\cos^{-3/2}\theta_0$ **up to a** θ_0**-factor**.

Order-of-limits problem (must keep ϵ in $\prod_{\omega\in\mathbb{Z}}$), smart pairing of bos/ferm modes?

Future directions: symmetry-preserving reg. scheme (better than ϵ, R), heat kernel...

Extra slides

The worldsheet geometry

Endowing the $AdS_5 \times S^5$ space with a **Lorentzian metric**

$$ds^2_{10\mathrm{D}} \equiv G_{MN} dx^M dx^N = -\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho \frac{dy_m^2}{(1+\frac{y^2}{4})^2} + \frac{dz_n^2}{(1+\frac{z^2}{4})^2},$$

$$y^2 \equiv \sum_{m=1}^{3} y_m^2, \qquad z^2 \equiv \sum_{n=1}^{5} z_n^2$$

the classical configuration is the **spacelike surface**

$$t = 0, \quad \rho = \rho(\sigma), \quad y_1 = 2\sin\tau, \quad y_2 = 2\cos\tau, \quad y_3 = 0\,,$$
$$z_1 = z_2 = 0, \quad z_3 = 2\cos\theta(\sigma), \quad z_4 = 2\sin\theta(\sigma)\sin\tau, \quad z_5 = 2\sin\theta(\sigma)\cos\tau\,.$$

The worldsheet geometry

It implements the correct boundary geometry and minimizes the area functional for

$$\sinh\rho(\sigma) = \frac{1}{\sinh\sigma}, \qquad \cosh\rho(\sigma) = \frac{1}{\tanh\sigma},$$
$$\sin\theta(\sigma) = \frac{1}{\cosh(\sigma+\sigma_0)}, \qquad \cos\theta(\sigma) = \tanh(\sigma+\sigma_0),$$
$$\cos\theta_0 \equiv \tanh\sigma_0, \qquad \tau\in[0,2\pi), \qquad \sigma\in[0,\infty).$$

The **induced worldsheet metric**

$$ds^2_{\mathrm{2D}} \equiv h_{\tau\tau}d\tau^2 + h_{\sigma\sigma}d\sigma^2 = \Omega^2(\sigma)\left(d\tau^2+d\sigma^2\right)$$

shows a conformal factor $\Omega^2(\sigma) \equiv \sinh^2\rho(\sigma) + \sin^2\theta(\sigma)$ depending on the latitude polar angle θ_0 .