Topological strings and Siegel modular forms

DESY Theory Workshop 2015

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Based on [1502.00557]

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Topological string theory

- *Topological string theory* partition function on M captures BPS-states arising from wrapped M2-branes in $H^2(M,\mathbb{Z})$

$$F_{top} = \sum_{g\geq 0} \lambda^{2g-2} F_g$$

  TST can be used to study
  - Geometrical invariants
  - Non-perturbative completions
  - ...

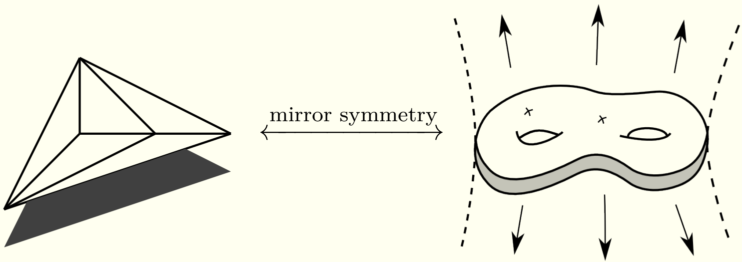
- There are *two types* of topological strings

$$\text{A-Model} \quad \xleftrightarrow{\text{mirror symmetry}} \quad \text{B-Model}$$

- *Exactly solvable* on non-compact geometries

Non-compact mirror pairs

Toric Calabi-Yau threefold | Riemann surface + non-compact directions



- Well established for genus one Riemann surfaces

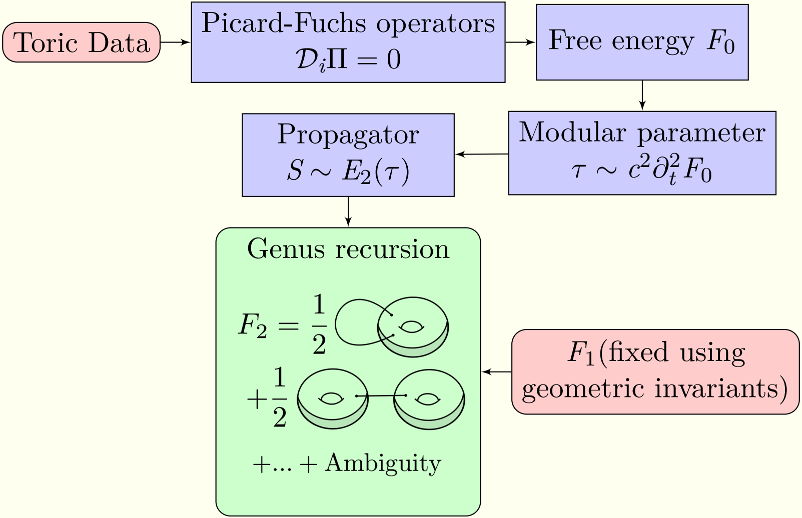
  e.g. Haghighat, Klemm, Rauch [0809.1674]

- Beautiful interplay with *(almost) modular forms*

  e.g. Aganagic, Bouchard, Klemm [hep-th/0607100]

- But *what happens for genus two mirror curves?*

Algorithm for solving genus one



Algorithm for solving genus one

Toric Data → **Picard-Fuchs operators $\mathcal{D}_i \Pi = 0$** → Free energy F_0

Propagator

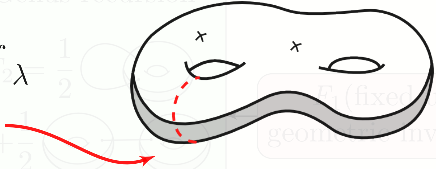
Modular parameter

Genus recursion

F_1 (fixed using geometric invariants)

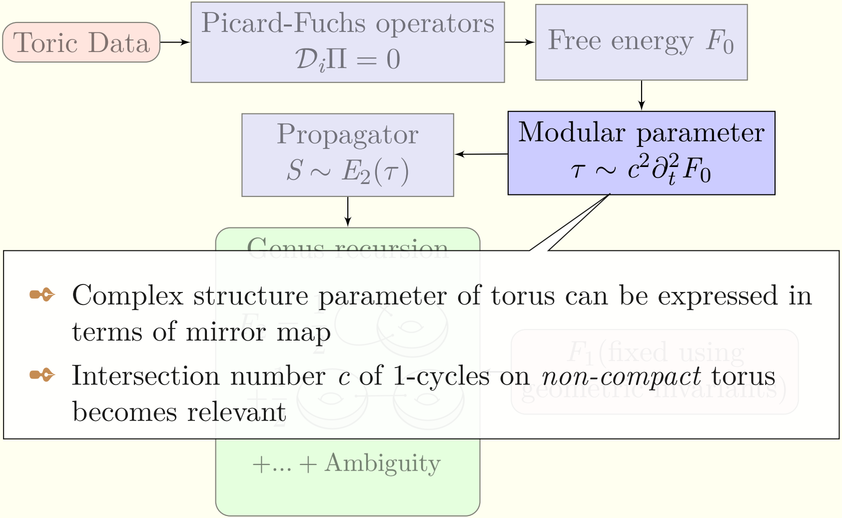
- Genus zero free energy F_0 and mirror map encoded in periods over meromorphic 1-form

$$\Pi_i = \int_{\alpha_i} \lambda$$



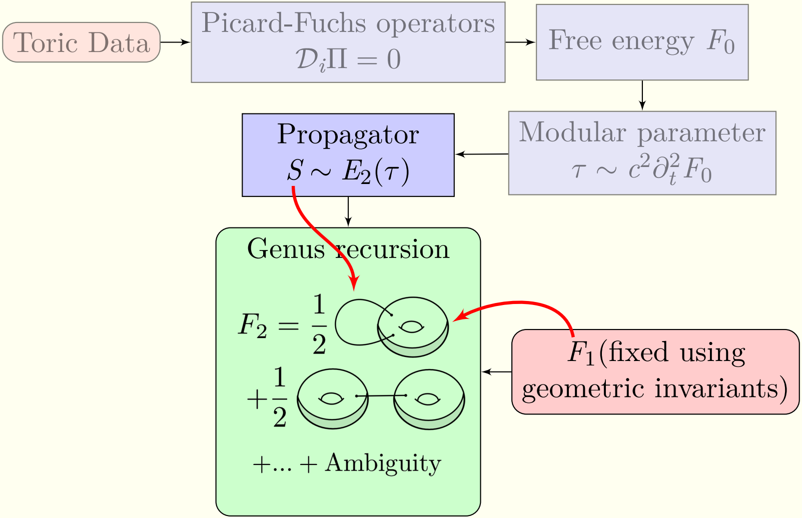
- Periods are annihilated by system of differential operators

Algorithm for solving genus one



- Complex structure parameter of torus can be expressed in terms of mirror map
- Intersection number c of 1-cycles on *non-compact* torus becomes relevant

Algorithm for solving genus one



Objects needed for genus two

For genus two the relevant objects are *Siegel modular forms*

- Complex structure parameter of genus 2 surface τ_{ij}, symmetric 2×2, $\mathrm{Im}\,\tau > 0$

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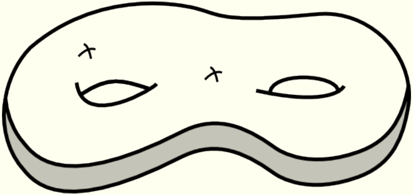
  - No weight 2 almost holomorphic Siegel modular form
  - But *vector valued almost meromorphic Siegel modular form*

$$S^{ij} \sim C_p^i C_q^j \frac{\partial}{\partial \tau_{pq}} \log \chi_{10}$$

  - χ_{10} is cusp form of weight 10 (compare $E_2 \sim \partial_\tau \log \eta^{24}$)

A note on the intersection

Actually the mirror curve itself is non-compact



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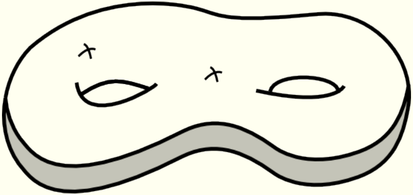


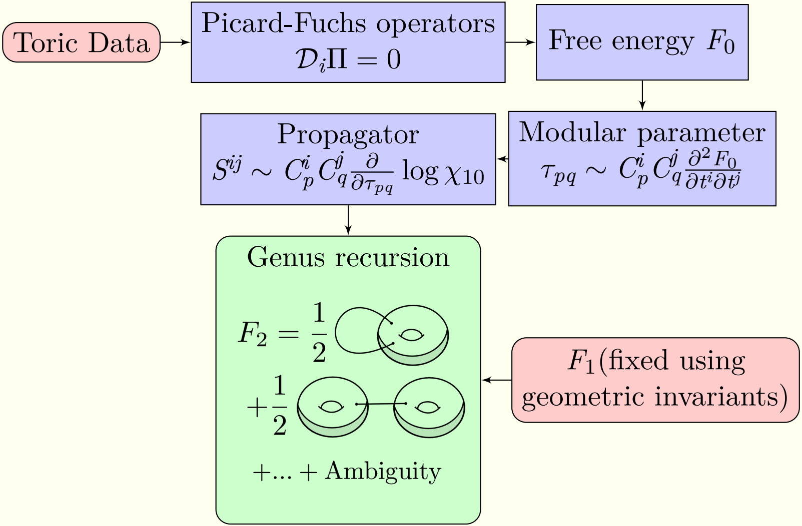
- Intersection of cycles $\neq \delta_{ij}$, non-compact homology is subtle
- But homological mirror symmetry gives intersection for free

Hosono [hep-th/0404043]

A-model
Toric data
$\widehat{\mathbb{C}^3/\mathbb{Z}_5}$

$$\left(\begin{array}{ccc|cc} 0 & 0 & 1 & -3 & 1 \\ 1 & 0 & 1 & 1 & -2 \\ 2 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ -1 & -1 & 0 & 1 & 0 \end{array}\right) \to C^i_p$$

Algorithm for solving genus two



Conclusion

- We describe a straightforward *algorithm to solve the topological string on mirror curves of genus two*
- *Siegel modular forms* become the relevant objects
- Second Eisenstein series as *propagator* gets replaced by *vector valued almost meromorphic Siegel modular form*
- This satisfies a *generalised Ramanujan identity*