A class of 2D non-Abelian gauged linear sigma models

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Physics at the LHC and beyond

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Motivation

- 4D type II supersymmetric string theories:

$$\mathbb{M}^{1,3} \quad \times \quad \underbrace{\mathcal{N} = (2,2) \text{ SCFT with } c = 9}_{\text{internal worldsheet theory}}$$

- **Aim:** Realize internal SCFT as IR RG-flow fixed point of 2D (non-Abelian) gauge theories

- **Result:** Non-trivial type II quantum moduli space arising from a strong-weak coupling duality in 2D gauge theories

Gauged linear sigma model: Essentials

- $\mathcal{N} = (2,2)$ two-dimensional gauge theory, given by Witten, 93; ...

  1. Gauge group: $G = \mathrm{U}(1)^\ell \times$ semi-simple Lie group
     Fayet-Iliopoulos term for each U(1) $\Rightarrow$ Kähler moduli t, (D-terms)
  2. Chiral superfields $\{\Phi_1, \ldots \Phi_N\}$ and their G-representation
  3. Vector R-charges: $\mathfrak{q}_V(\Phi_i)$

- Superpotential $W(\Phi_i)$ $\Rightarrow$ Complex structure moduli z, (F-terms)

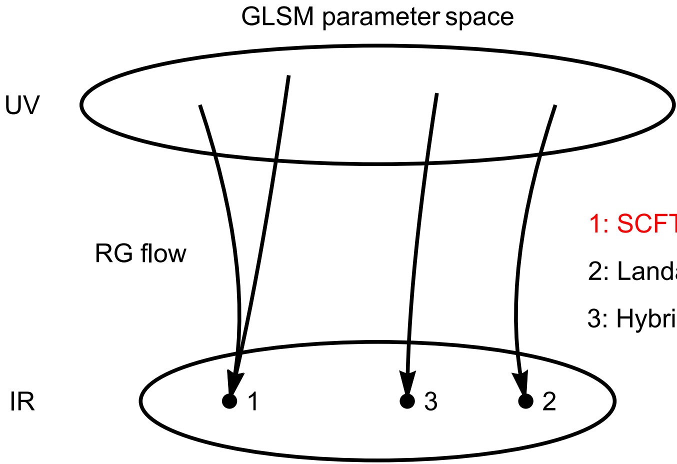
- Moduli dependent semi-classical scalar potential

$$U_{\mathrm{t,z}}(\phi_i) = |\text{D-terms}|^2 + |\text{F-terms}|^2$$

- Semi-classical IR target space $\mathcal{X}_{\mathrm{t,z}}$ = space of inequivalent vacua

$$\mathcal{X}_{\mathrm{t,z}} = \frac{U^{-1}_{\mathrm{t,z}}(0)}{G}$$

Gauged linear sigma model: Illustration



1: SCFT on CY_3

2: Landau–Ginzburg orbifold

3: Hybrid phase

Skew Symplectic Sigma Models: $SSSM_{k,m,n}$

- Class of skew symplectic sigma models $SSSM_{k,m,n}$:

1. Gauge group: $G = \frac{U(1) \times USp(2k)}{\mathbb{Z}_2}$ $\Rightarrow$ 1 Kähler parameter t
2. Matter spectrum:

chiral multiplets	G representation	multiplicity	$\mathfrak{q}_V$
$P_{[ij]}$, $1 \leq i < j \leq n$	$\mathbf{1}_{-2}$	$\binom{n}{2}$	$2 - 2\mathfrak{q}$
Q	$\mathbf{2k}_{-3}$	1	$2 - 3\mathfrak{q}$
ϕ_a, $a = 1, \ldots, m$	$\mathbf{1}_2$	m	$2\mathfrak{q}$
X_i, $i = 1, \ldots, n$	$\mathbf{2k}_1$	n	$\mathfrak{q}$

3. Superpotential: $W(P, Q, \phi, X)$

Conformal models with CY_3 target space

$$SSSM_{1,12,6} \quad \text{and} \quad SSSM_{2,9,6}$$

$SSSM_{k,m,n}$ phase $r \gg 0$: Weak coupling

- Solving D-terms = F-terms = 0 gives
  1. Not all ϕ_a vanish simultaneously: $\phi \in \mathbb{P}^{m-1}$
  2. For skew-symmetric matrix $A(\phi)$, $X = (X_1, \ldots, X_n)$

$$A(\phi) = -X^T \epsilon X \quad \text{with} \quad \epsilon = \begin{pmatrix} 0 & \mathbf{1}_{k\times k} \\ -\mathbf{1}_{k\times k} & 0 \end{pmatrix}$$

$$\Leftrightarrow \text{rk}\, A(\phi) \leq 2k \quad \text{since} \quad X \quad \text{is } 2k \times n \text{ matrix}$$

  3. For n-dimensional vector $B(\phi)$: $A(\phi) \cdot B(\phi) = 0$

Semi-classical target space variety $r \gg 0$

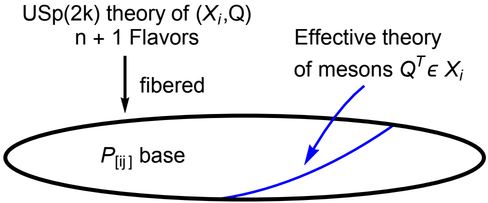
$$\mathcal{X}_{k,m,n} = \{\phi \in \mathbb{P}^{m-1} \mid \text{rk}\, A(\phi) \leq 2k \text{ and } A(\phi) \cdot B(\phi) = 0\}$$

c.f. Galkin (talk), 13

- $USp(2k)$ broken $\Rightarrow$ phase weakly coupled

$SSSM_{k,m,n}$ phase $r \ll 0$: Strong coupling

- Gauge group broken to USp$(2k)$
  $\Rightarrow$ non-Abelian strong coupling dynamics important — c.f. Hori, Knapp, 13



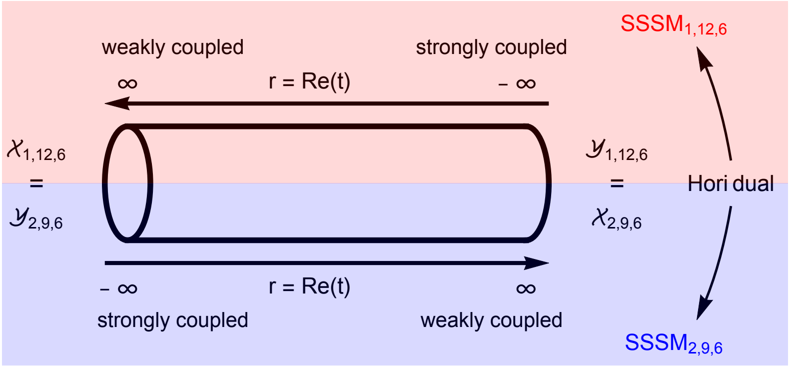
- USp$(2k)$ theory dual to a weakly coupled theory of mesons — Hori, Tong, 06; Hori, 11

Semi-classical target space variety $r \ll 0$

$$\mathcal{Y}_{k,m,n} \simeq \mathcal{X}_{\tilde{k},\tilde{m},n} \quad \text{with} \quad \tilde{k} = \frac{n}{2} - k, \quad \tilde{m} = \frac{n(n+1)}{2} - m, \quad n \text{ even}$$

- In particular: $\mathcal{Y}_{1,12,6} \simeq \mathcal{X}_{2,9,6} \qquad \mathcal{Y}_{2,9,6} \simeq \mathcal{X}_{1,12,6}$

Phase structure and duality



- Strong evidence: $Z_{S^2}(SSSM_{1,12,6}, t) = Z_{S^2}(SSSM_{2,9,6}, -t)$
- Geometric data

Miura, 13; Galkin (talk), 13; van Straten's database

$$\mathcal{X}_{1,12,6}: \quad \chi = -102, \quad \text{deg} = 33, \quad N_d = 252, 1854 \ldots$$
$$\mathcal{Y}_{1,12,6}: \quad \chi = -102, \quad \text{deg} = 21, \quad N_d = 387, 4671 \ldots$$

Summary

- We have analysed class of non-Abelian models GLSMs, $SSSM_{k,m,n}$
- $SSSM_{1,12,6}$ and $SSSM_{2,9,6}$: NLSM interpretation on CY_3 target space
- Strong and weak coupling phases

Weakly coupled: $\mathcal{X}_{1,12,6}$ Strongly coupled: $\mathcal{Y}_{1,12,6}$
Weakly coupled: $\mathcal{X}_{2,9,6}$ Strongly coupled: $\mathcal{Y}_{2,9,6}$

- Duality of $SSSM_{1,12,6}$ and $SSSM_{2,9,6}$, with interchange of strong and weak coupling phases

$$\mathcal{X}_{1,12,6} \simeq \mathcal{Y}_{2,9,6} \quad \text{and} \quad \mathcal{X}_{2,9,6} \simeq \mathcal{Y}_{1,12,6}$$

- Future work: Relation between Hori duality and phases of non-Abelian GLSMs?