# Beyond N$^2$LL' $q_T$ resummation with CuTe

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DESY Theory Workshop 2015

# Outline

# Observable

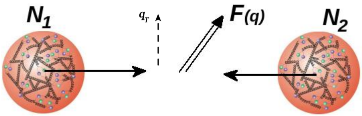
Consider:
$N_1 + N_2 \to F(q) + X$

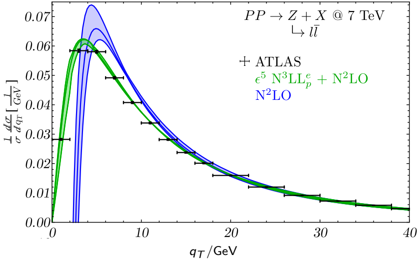
- $F = \gamma^*, Z, W, H, Z', \ldots$
- Test (B)SM to high precision.
- $d\sigma/dq_T$ in region $q_T^2 \ll M^2$. **Large log $\mathbf{q_T^2/M^2}$.**

$\Rightarrow$ Need to **resum** these. Soft/collinear origin.

$\Rightarrow$ Transverse PDFs (Beam functions).

$\Rightarrow$ $F$ recoils against initial state radiation.




[Becher, TL, Neubert, Wilhelm]$_{\mathrm{prg.}}$

# $\frac{d\sigma}{dq_T dy}$ - counting $a_s$ and $L$

$\lambda = q_T/M\,,\quad L = \log\lambda\,,\quad a_s = \frac{\alpha_s}{4\pi}\,,$ PC= power correction.

$$\frac{d\sigma}{dq_T dy} = C(a_s)\exp\left[L g_1(a_sL) + g_2(a_sL) + a_s g_3(a_sL) + a_s^2 g_4(a_sL) + \ldots\right] + \mathcal{O}(\lambda)\,.$$

When expand:

| FO \ RES | | LL | NLL | NLL' | N$^2$LL | N$^2$LL' | N$^3$LL | ... | PC | |
|---|---|---|---|---|---|---|---|---|---|---|
| LO | $a_s^0$[ | 1 | | | | | | | | ] |
| NLO | $a_s^1$[ | $L^2$ | $L^1$ | 1 | | | | | $\mathcal{O}(\lambda)$ | ] |
| N$^2$LO | $a_s^2$[ | $L^4$ | $L^3$ | $L^2$ | $L^1$ | 1 | | | $\mathcal{O}(\lambda)$ | ] |
| N$^3$LO | $a_s^3$[ | $L^6$ | $L^5$ | $L^4$ | $L^3$ | $L^2$ | $L^1$ | 1 | $\mathcal{O}(\lambda)$ | ] |
| | | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | | |

In this talk will discuss **CuTe**'s

- N$^3$LL$_p$ * resummation and
- N$^2$LO matching ($\Rightarrow$ recover PC).

* p=partial: lacking values of $\Gamma_3^i$ and $F_i^{(3,0)}$.

# Factorization formula

- [Collins, Soper, Sterman], ..., [Becher, Neubert] for $\Lambda^2_{\rm QCD} \ll q_T^2 \ll M^2$ :

$$\frac{d^2\sigma_c}{dq_T^2 dy} = \sigma_c^{(0)} \sum_{k,j} \int d^2x_T\, e^{iq_T\cdot x_T}\, \widetilde{C}_{c\leftarrow kj}(z_1,z_2,x_T^2,M^2,\mu) \otimes \phi_{k/N_1}(z_1,\mu) \otimes \phi_{j/N_2}(z_2,\mu)\,,$$

- in impact parameter $(x_T)$ space
- with **perturbative**

$$\widetilde{C}_{c\leftarrow ij}(z_1,z_2,x_T^2,M^2,\mu) = |C_c(-M^2,\mu)|^2 \bar{I}_{i\leftarrow k}(z_1,L_\perp,a_s)\bar{I}_{\bar{i}\leftarrow j}(z_2,L_\perp,a_s)e^{g_i(M,x_T,\mu)}\,,$$

- where $i(c) = q,\ g$ ; $L_\perp = \log \frac{x_T^2\mu^2}{b_0^2}$, $\quad a_s = \alpha_s(\mu)/4\pi \quad$ and

  $g_i(M,x_T,\mu) = 2h_i(L_\perp,a_s) - F_i(L_\perp,a_s)\cdot \log \frac{x_T^2 M^2}{b_0^2}$ .

  Note: $F_i(L_\perp,a_s) = \gamma^i_{B,\nu}$ in RRG framework $\to$ Markus Ebert's talk.
- Each function depends on **single** physical scale $\Rightarrow$ Safely determine pert..
- Solving **RGEs** $(\mu)$ $\Rightarrow$ **resummation** of $L = \log(x_T^2 M^2/b_0^2)$.

In $\widetilde{C}$ suppressed sum over tensor components $(i(c) = g)$ or quark charges $(i(c) = q)$.

# $x_T$ integral and scale choice

- CuTe: Combine everything and numerically evaluate $\otimes\phi$, $x_T$, $y$ integrals.

$$\text{Perform} \int d^2x_T\, e^{iq_T\cdot x_T}\,\widetilde{C}_{c\leftarrow ij} = \int_0^\infty dx_T\, x_T J_0(x_T q_T)\widetilde{C}_c(z_1,z_2,x_T^2,M^2,\mu)\,.$$

- Essentially two kind of logs: $L_M = \log M^2/\mu^2$ and $L_\perp = \log {x_T}^2\mu^2/b_0^2$.
- Aim: small $L_\perp$.
- $\mu_x = b_0/x_T$: Run into Landau pole.
- Moreover, prefer physical choice $\mu(q_T, M)$.
- $\boldsymbol{\mu = \mu_* = q_T + q_* \exp(-q_T/q_*)}$, $q_* = M_i/\exp\left(1/2\Gamma_0^i a_s\right) \Rightarrow \langle L_\perp\rangle$ small.
- Thanks to $x_T e^{g_i}$ →Gaussian peak: determines $q_*$, width $\sim 1\sqrt{a_s}$.
- **Power counting** for $\int dx_T$: $\mathbf{a_s \sim \epsilon^2}$, $\mathbf{L_M \sim \epsilon^{-2}}$, $\mathbf{L_\perp \sim \epsilon^{-1}}$
  Do up to $\epsilon^5$.

# Matching to fixed order

**Restoring power** $(qT/M)$ **suppressed contributions**.

$$\frac{d\sigma^{\rm matched}}{dq_T} = \frac{d\sigma^{\rm res}}{dq_T} + \left.\frac{d\sigma^{\rm MC}}{dq_T}\right|_{\rm MS},$$
$$\left.\frac{d\sigma^{\rm MC}}{dq_T}\right|_{\rm MS} = R_{\rm ms}\left(\frac{d\sigma^{\rm FO}}{dq_T} - \left.\frac{d\sigma^{\rm res}}{dq_T}\right|_{\text{expanded to FO}}\right)$$

with $H(-M^2,\mu_h,\mu) = U_H(\mu,\mu_h)\cdot|C_i(-M^2,\mu_h)|^2$ and

$$R_{\rm add} = R_{\rm ms}(\mu_h) = 1\,,$$
$$R_{\rm ms}(\mu_*) = H(-M^2,\mu_h,\mu_*)\cdot H^{-1}(-M^2,\mu_*,\mu_*)\,,$$
$$R_{\rm ms}(q_T) = H(-M^2,\mu_h,q_T)\cdot H^{-1}(-M^2,q_T,q_T)\,.$$

$H(-M^2,\mu,\mu)$ corresponds to the FO expansion of resummed $H(-M^2,\mu_h,\mu)$.
$R_{\rm ms} = 1 + \mathcal{O}(a_s^3)$ but **can supply Sudakov suppression** to $d\sigma^{\rm MC}$.

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FO results: V→[Gonsalves,Pawlowski,Wai], H→[Glosser,Schmid].

# CuTe: This is New

**CuTe**: handle all of this for $\gamma^*$,$W$,$Z$ and $H$.

## CuTe 1.1

- Public C$^{++}$-code.
- LHAPDF 5.
- N$^2$LL resummation.
- NLO matching.
- Power counting for very small $q_T$: $\mathcal{O}(\epsilon^2)$.

Now up to **N$^2$LO+N$^3$LL$_p$ precision**.

## CuTe 2.0

- Not yet public.
- Linked to LHAPDF 6, Cuba.
- Full N$^3$LL implementation:
  - 2-loop beam-functions
  - Unknown $\Gamma_3^i$ and $F_i^{(3,0)}$ $\Rightarrow$ N$^3$LL$_p$ (partial)
- N$^2$LO matching.
- $\mathcal{O}(\epsilon^5)$.
- Phase space cuts $y$,$p_T^l$,$\eta^l$ .

[Becher,Neubert,TL,Wilhelm]$_{\text{pgr.}}$

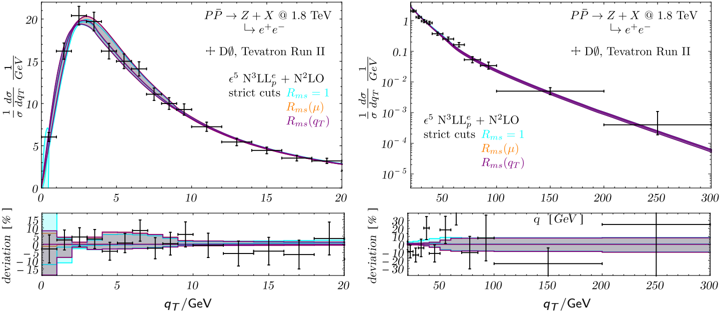
# Phenomenology

## Apply CuTe

# Settings

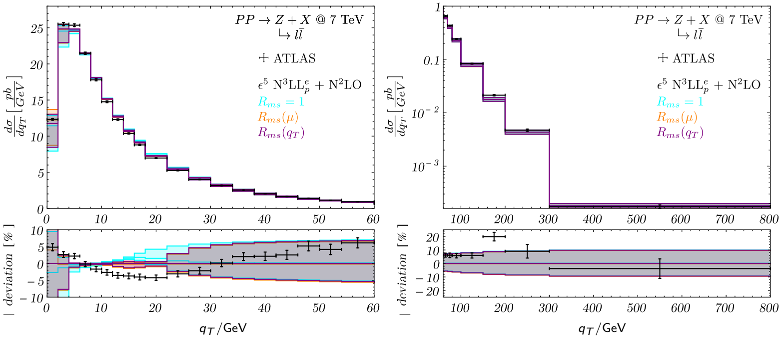
- $\epsilon^5$ N$^3$LL$_p^e$+N$^2$LO precision.
- $\mu = q_T + q_* \exp(-q_T/q_*)$, $q_* = M_i / \exp\left(1/2\Gamma_0^i a_s\right)$.
  $q_*^Z \sim 2GeV$, $q_*^H \sim 8GeV$.
- Uncertainty bands: vary $\mu$ by factor 2.
- Flavor scheme according to pdf set.
- NNPDF 3.0.
- Smooth transit to pure fixed order between $\frac{M_i}{2}$ and $\frac{3M_i}{4}$.
- Decay leptons: Narrow width approx.; cuts as provided by experiment.
- $\Gamma_3$ Pade approximation, $F^{(3,0)} \to F^{(2,0)}$.

# Matched vs data - Z D0



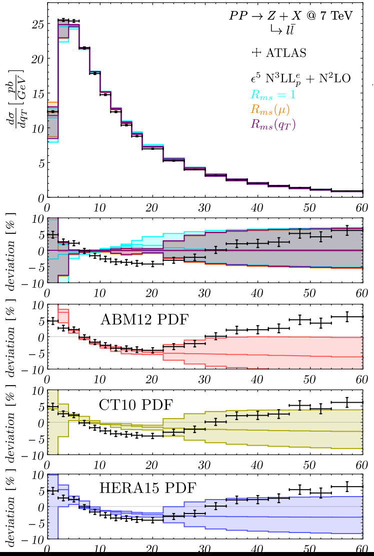

- Very small matching scheme dependence.
- Very good agreement with data: [Becher,TL,Neubert,Wilhelm]$_{\text{pgr.}}$
  - D0, hep-ex/9909020 $Z$ at 1.8TeV.
  - Cuts for $d\sigma_{\text{fiducial}}/dq_T$: $60 < M_{ll}/GeV < 120$,
  - $p_{T,l} > 25\text{GeV}$, $|\eta_{l,1}| < 1.1$, $|\eta_{l,2}| < 2.4$, excluding $1.11 < |\eta_{l,2}| < 1.5$
- Symmetrized lepton cuts included in theory prediction: Normalization and shift peak to the right.

# Matched vs data - Z ATLAS



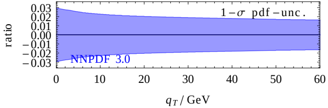

- Very small matching scheme dependence. [Becher,TL,Neubert,Wilhelm]$_{\text{pgr.}}$
- Good agreement with data:
  - ATLAS hep-ex/1406.3660 $Z/\gamma^*$ at 7TeV.
  - Cuts for $d\sigma_{\text{fiducial}}/dq_T$: $66 < M_{ll}/GeV < 116$,
  - $p_{T,l} > 20\text{GeV}$, $|\eta_l| < 2.4$, excluding $1.37 < |\eta_l| < 1.52$
- Lepton cuts included in theory prediction: Normalization and shift peak to the right.
- $\mu$ uncertainty band only.

# PDF uncertainties




Further N$^2$LO pdf sets
$\mu$ variation for each

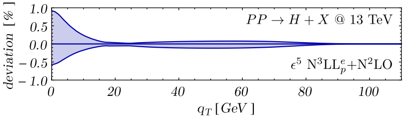
Pdf-member variation of N$^2$LO NNPDF 3.0:

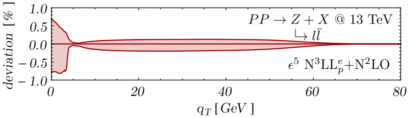



- Sizeable pdf uncertainty.
- Difference between sets not fully reflected by 1-$\sigma$ pdf uncertainty.

# Estimated impact of unknown $\Gamma_3$, $F^{(3,0)}$

- Vary unknown $\Gamma_3$, $F^{(3,0)}$ between $[-2, 2]$ def. values:
  $\Gamma_3$ Pade approx.; $F^{(3,0)} \to F^{(2,0)}$.



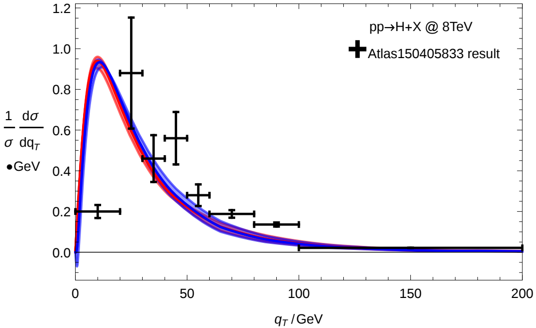




- Slightly larger impact of $F^{(3,0)}$.
- Uncertainty small compared to e.g. pdf-unc..
- Largest impact at small $q_T$.

**Further effects** not discussed here:

- EW-corrections, mass/off-shell effects, non-pert. effects, . . .

# Matched vs data - H ATLAS




$\epsilon^5$ $N^3LL_p^e + N^2LO$

$R_{ms}(q_T)$

$R_{ms}(\mu_h) = 1$

- Sizeable matching correction.
- Implies some matching scheme dependence (mainly on uncertainty bands).
- Low statistics for data:
  - ATLAS, hep-ex/1504.05833 $H$ at 8TeV.
  - Cuts unfolded.

# Alternatives and extensions

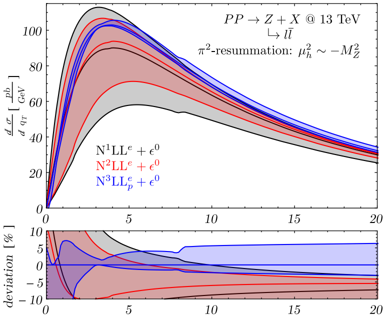
- Monte Carlo event generators. $\rightarrow$ Frank Krauss' talk.
- CSS: DYqT, DYRes, HRes [Catani,Cieri,de Florian,Ferrera,Grazzini], ResBos [Balazs, Yuan] ...
- Our method can be applied to **all** other processes with color neutral final states: $V$, $H$, $VV'$, $HH$, $Z'$, ...
- Currently available in CuTe: $\gamma^*$, $Z$, $W$, $H$.
- For others to include F.O. part.
  - $VV'$ at fixed order see e.g. Mainz/Zürich/Karlsruhe-groups [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, v.Manteuffel, Pozzorini, Rathlev, Tancredi, Torre, Weihs] [Caola, Henn, Melnikov, Smirnov, Smirnov].
- NNLO+N$^2$LL':
  - $\gamma\gamma$: [Cieri, Coradeschi, de Florian]
  - $ZZ$, $W^+W^-$ [Grazzini, Kallweit, Rathlev, Wiesemann]

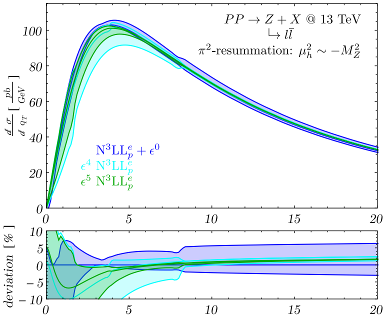
# Conclusions

- **Resummation** essential for small $q_T/M$.
- **Generic** framework to obtain precise $d\sigma/dq_T(/dy)$ for large class of processes at hadron colliders.
- Determine these to $\mathbf{N^2LO+N^3LL}_p$ precision for $\gamma^*$, $Z$, $W$, $H$ with **CuTe**.
- Obtained very accurate description of $q_T$ spectrum.
- Physical scale choice and improved power counting: Besides PDFs no non-perturbative parameters required.
- Code will become public.

# Appendix

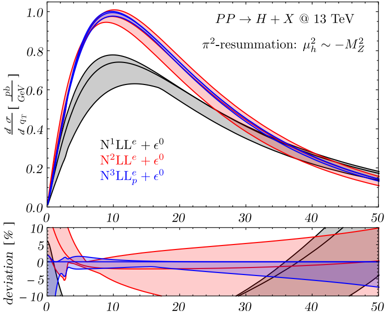
# Pure Resummed - different orders - Z

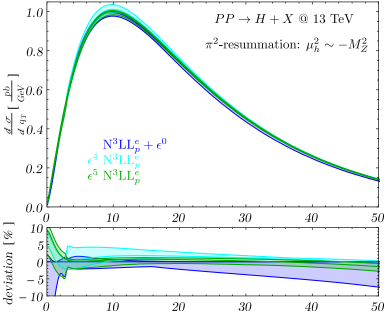



- $\mu(q_T) = q_T + q_* e^{-\frac{q_T}{q_*}}$
- Vary by factors 2, 1/2.
- $q_*^Z \sim 2GeV$.
- New channels $(q \to q', \bar{q})$ beyond $N^2LL + \epsilon^0$.

# Pure Resummed - different orders - H




- $\mu(q_T) = q_T + q_* e^{-\frac{q_T}{q_*}}$
- Vary by factors 2, 1/2.

- $q_*^H \sim 8 GeV$.
- Sizeable loop correction and new channels $(q \to g)$ at $N^2LL + \epsilon^0$.

# Non-perturbative effects

- TPDFs must vanish rapidly at $x_T > r_{\text{proton}}$. Ansatz:

$$B_{i/N}(z, x_T^2, \mu) = f_{\text{hadr}}(x_T \Lambda_{\text{NP}})\, B_{i/N}^{\text{pert}}(z, x_T^2, \mu) \ ,$$

- with

$$f_{\text{hadr}}^{\text{gauss}}(x_T \Lambda_{\text{NP}}) = \exp\left[-\Lambda_{\text{NP}}^2\, x_T^2\right] ,$$

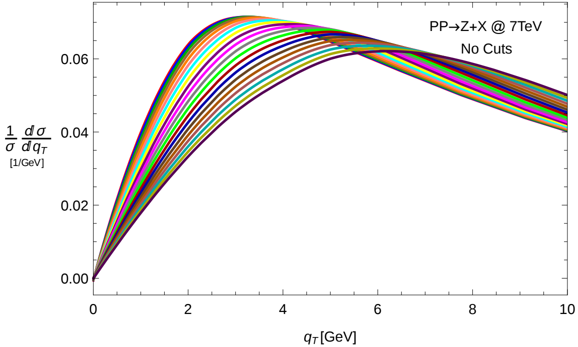
$$f_{\text{hadr}}^{\text{dipole}}(x_T \Lambda_{\text{NP}}) = \frac{1}{1 + \Lambda_{\text{NP}}^2\, x_T^2} ,$$

$$f_{\text{hadr}}^{\text{enhanced}}(x_T \Lambda_{\text{NP}}) = \exp\left[-\Lambda_{\text{NP}}^2\, x_T^2 \log(x_T^2 M^2 / b_0^2)\right] .$$

- Last see [Becher, Bell].
- $\Lambda_{\text{NP}} = 0\text{GeV}$: no correction.

# Non-perturbative effects

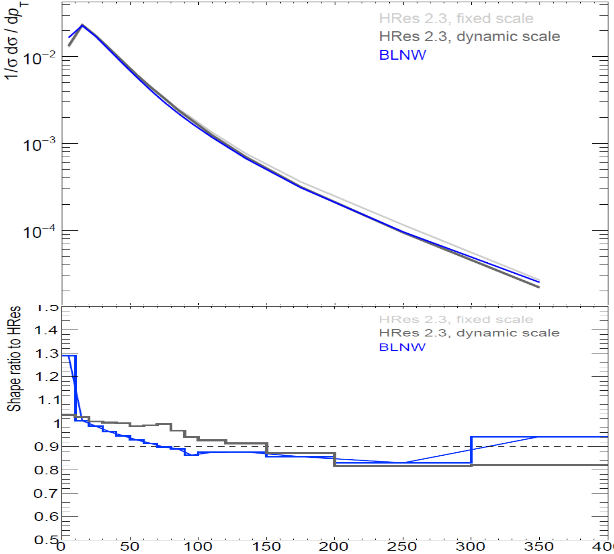
- Results for Gauss and Dipole basically equivalent.
- Gauss: $\Lambda_{\mathrm{NP}} = 0\,\mathrm{GeV} \longrightarrow 2\,\mathrm{Gev}$: **Shifts right** & damps.




- Enhanced for 'enhanced'. Form different, though.

# H: Our vs HRes (from HXWG, G. Petrucciani)

Normalized comparison of HRes, BLNW



# Relation to framework by Collins, Soper, Sterman

$$\widetilde{C}_{q\bar{q}\leftarrow ij}(z_1,z_2,x_T^2,q^2,\mu)$$

$$= \left|C_V(-q^2,\mu)\right|^2 \left(\frac{x_T^2 q^2}{b_0^2}\right)^{-F_{q\bar{q}}(x_T^2,\mu)} I_{q\leftarrow i}(z_1,x_T^2,\mu)\, I_{\bar{q}\leftarrow j}(z_2,x_T^2,\mu)\,,$$

compare this to [Collins, Soper, Sterman]:

$$= \exp\left\{ -\int_{\mu_b}^{q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\log\frac{q^2}{\bar{\mu}^2} A(\alpha_s(\bar{\mu})) + B(\alpha_s(\bar{\mu}))\right]\right\}$$
$$\times\, C_{qi}(z_1,\alpha_s(\mu_b))\, C_{\bar{q}j}(z_2,\alpha_s(\mu_b))\,,$$

$x_T$ dependence via $\mu_b = b_0 {x_T}^{-1}$.

**Relations:**

- $C_{qi}(z,\alpha_s(\mu_b)) = \left|C_V(-\mu_b^2,\mu_b)\right| I_{q/i}(z,x_T^2,\mu_b)\,,$
- $A$ & $B$ related to $F$ and anomalous dimensions.

# Dictionary CSS vs BN

Using $b_0 = 2e^{-\gamma_e}$, $\mu_b = b_0 x_T^{-1}$ and $\bar{x}_T = b_0 \bar{\mu}^{-1}$:

$$C_{qi}(z, \alpha_{s(\mu_b)}) = \left| C_V(-\mu_b^2, \mu_b) \right| I_{q/i}(z, \bar{x}_T^2, \mu_b)\,,$$

$$A(\alpha_{s(\bar{\mu})}) = \Gamma^q(\alpha_s) - \bar{\mu}^2 \frac{dF_{q\bar{q}}(\bar{x}_T, \bar{\mu})}{d\bar{\mu}^2}\,,$$

$$B(\alpha_{s(\bar{\mu})}) = 2\gamma_q(\alpha_s) + F_{q\bar{q}}(\bar{x}_T, \bar{\mu}) - \bar{\mu}^2 \frac{d \log |C_V(-\bar{\mu}^2, \bar{\mu})|^2}{d\bar{\mu}^2}\,.$$

$\Rightarrow$ Apply results in preferred resummation framework.

Can reconstruct $\mathcal{H}^{DY}_{q\bar{q}\leftarrow i\bar{j}}$, $\mathcal{H}^{H}_{gg\leftarrow i\bar{j}}$, ... in [Catani, Cieri, de Florian, Ferrera, Grazzini].

# Resummation

- RGEs $\Rightarrow$ Dependence on $\mu$ $\qquad i = i(c)\,,$

$$\frac{d}{d\log\mu}\log C_c(-M^2,\mu) = \Gamma^i(a_s)\log\frac{-M^2-i0^+}{\mu^2} + 2\gamma^i(a_s)\,,$$
$$\frac{d}{d\log\mu}F_i(L_\perp,a_s) = 2\,\Gamma^i(a_s)\,,$$
$$\frac{d}{d\log\mu}h_i(L_\perp,a_s) = \Gamma^i(a_s)L_\perp - 2\gamma^i(a_s)\,,$$
$$\frac{d}{d\log\mu}\bar{I}_{i/j}(z,L_\perp,\mu) = -2\sum_k \bar{I}_{i/k}(z,L_\perp,a_s)\otimes P_{k/j}(z,a_s)\,,$$
$$\frac{d}{d\log\mu}\phi_{i/j}(z,\mu) = 2\sum_k P_{ik}(z,\mu)\otimes\phi_{k/j}(z,\mu)\,.$$

$\Rightarrow$ Resum logarithms. E.g.:

$$|C_c(-M^2,\mu)|^2 = |C_c(-M^2,\mu_h)|^2\exp\left\{2\mathrm{Re}[\mathrm{E}_{\mathrm{C_c}}(-\mathrm{M}^2,\mu,\mu_\mathrm{h})]\right\}\,,$$
$$E_{C_c}(-M^2,\mu,\mu_h) = \int_{\mu_h}^{\mu}\frac{d\mu'}{\mu'}\left(\Gamma^i\log\frac{-M^2-i0^+}{\mu'^2} + 2\gamma^i\right)\,.$$

- Log indep. parts and anom. dims. from pert. calc..

# Towards N$^3$LL, required elements

Numbers refer to power $n$ in expansion $X = \sum_n a_s^n X^{(n)}$ .

| expression | needed to | known to | |
|---|---|---|---|
| $\Gamma^i$ | 4 | 3 | for RGEs |
| $\gamma_i$ | 3 | 3 | |
| $P_{i/j}(z)$ | 3 | 3 | |
| $\beta$ | 4 | 4 | |
| $C_c(M^2)$ | 2 | 2 | at appr. $\mu$ |
| $F_i(L_\perp)$ | 3 | 2 | |
| $h_i(L_\perp)$ | 2 | 2 | |
| $\bar{I}_{i/j}(z, L_\perp)$ | 2 | 2 | |
| $\bar{I}'_{g/j}(z, L_\perp)$ | 1 (2)* | 1 | |

*: $I'$ starts at $\alpha_s^1$.

$\Rightarrow$ $n = 1$ sufficient if $C_{gg}$ does not mix $I'$ & $I$. (E.g. for Higgs.)

# Transverse PDFs

## TPDF (quark, $n$ collinear, gauge invariant)

$$\mathcal{B}_{q/N_1}(z, x_T^2) = \frac{1}{2\pi}\int \mathrm{d}t\ e^{-izt\bar{n}\cdot p} \sum_{X_n} \frac{\bar{\not{n}}_{\alpha\beta}}{2} \times \langle N_1(p)|\,\bar{\chi}_\alpha(t\bar{n} + x_\perp)\,|X_n\rangle\,\langle X_n|\,\chi_\beta(0)\,|N_1(p)\rangle\,.$$

- $\chi_n = (\bar{\xi}_n W_n)$
- TPDF: Generalization of usual PDF $\phi_{i/N}(z)$.
- $k$ = momentum of $X_n$:
  $k_-$ and (after F.T.) $k_\perp$ fixed:
- $\int dt e^{-iztp_-} \bar{\chi}(t\bar{n} + x_\perp) \Rightarrow \delta(k_- - (1-z)p_-)\bar{\chi}$ ,
- $\bar{\chi}(t\bar{n} + x_\perp) \Rightarrow e^{-ik_\perp x_\perp} \stackrel{\text{F.T.}}{\to} \delta^{(2)}(k_\perp + q_\perp)\bar{\chi}$ .

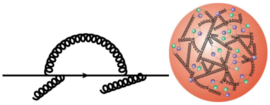
# Matching kernel

If $x_T^{-2} \gg \Lambda^2_{\mathrm{QCD}}$, refactorize these scales:

## Matching kernel $\mathcal{I}_{i/k}$

$$\mathcal{B}_{i/N}(z, x_T^2) = \sum_k \int_z^1 \frac{d\rho}{\rho} \mathcal{I}_{i/k}(\rho, x_T^2) \phi_{k/N}(z/\rho) + \mathcal{O}(\Lambda^2 x_T^2)$$

$$= \sum_k \mathcal{I}_{i/k}(z, x_T^2) \otimes \phi_{k/N}(z) \, .$$



- $\mathcal{I}_{i/k}$ perturbative: Extract from perturbative $\mathcal{B}_{i/j}$ and $\phi_{k/j}$.
- Determine to NNLO: [Gehrmann,TL,Yang].

# Analytic regulator

- Applied results for $I$ [Gehrmann, TL, Yang] use analytic regulator $\alpha$ [Becher, Bell]: $\times\left(\frac{\nu}{n \cdot l_i}\right)^{\alpha}$ for each external parton.
- Same LC vector $n$ for all functions. Breaks symmetry $(n \leftrightarrow \bar{n})$.
- Simple soft function $\mathcal{S} = 1$.
- $\alpha$ poles cancel in product:

## Refactorization

$$\left[\mathcal{S}(x_T^2)\mathcal{B}_{i/j}(z_1, x_T^2)\bar{\mathcal{B}}_{\bar{i}/k}(z_2, x_T^2)\right]_{q^2} \stackrel{\alpha=0}{=} \left(\frac{x_T^2 q^2}{b_0^2}\right)^{-F_{i\bar{i}}^b(x_T^2)} B_{i/j}^b(z_1, x_T^2) B_{\bar{i}/k}^b(z_2, x_T^2)\,,$$

$\nwarrow$ $b_0 = 2e^{-\gamma_e}$

- On refactorized RHS no $\alpha$ and $\nu$ dependence left.
- Hard scale $q^2$ generated.
- 'Collinear anomaly' [Becher, Neubert]. From RRG [Chiu, Jain, Neill, Rothstein]. $F_i = \gamma_\nu^{B_i}$.