Scanning the Heterotic String Landscape

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Based on:

- M. Fischer, M. Ratz, J. Torrado and P. V.: 1209.3906, JHEP 1301 (2013) 084
- M. Fischer, S. Ramos-Sánchez and P. V.: 1304.7742, JHEP 1307 (2013) 080
- H. P. Nilles and P. V.: 1403.1597, Mod.Phys.Lett. A30 (2015) 10, 1530008
- S. Groot Nibbelink, O. Loukas, F. Ruehle, P.V.: 1506.00879, PRD 92 (2015) 4, 046002

- Connect string theory to particle physics
- Framework: $E_8 \times E_8$ heterotic string
- Compactify from 10d to 4d using:
 - orbifolds
 - Calabi-Yaus
- Questions in this talk:
 - How many 6d orbifold geometries are there?
 - What are their common properties for model-building?
 - What is the situation for Calabi-Yau compactifications?

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Lattice $\Lambda \Rightarrow$ torus T^2 P rotational symmetry of Λ Orbifold T^2/P



Lattice $\Lambda \Rightarrow$ torus T^2 P rotational symmetry of Λ Orbifold T^2/P with 3 fixed points •



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Lattice $\Lambda \Rightarrow \text{torus } T^2$ *P* rotational symmetry of Λ Orbifold T^2/P with 3 fixed points • Heterotic string on orbifolds



Lattice $\Lambda \Rightarrow \text{torus } T^2$

P rotational symmetry of Λ

Orbifold T^2/P with 3 fixed points •

Heterotic string on orbifolds

untwisted string \bigcirc from E₈ × E₈ twisted string \bigcirc at fixed point



in 6D

- 1) Lattice Λ
- 2) P rotational symmetry of Λ

 $P \Leftrightarrow \mathcal{N} = 1 \text{ SUSY}$

- 3) in lcude roto-translations $x\mapsto \theta x+t$
- \Rightarrow Crystallography

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- 3) in lcude roto-translations $x\mapsto \theta x+t$
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Example in 2d:

- 1) 17 different tilings
- 2) $P = \mathbb{Z}_N$ with N = 2, 3, 4, 6E.g. no \mathbb{Z}_5 in 2d

Results in 6d:

- 1) 60 point groups P with $\mathcal{N} \geq 1$ SUSY
- 2) 186 lattices Λ

include roto-translations:

3) 520 inequivalent orbifold geometries in 6d

162 with P abelian

358 with P non-abelian

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Classification of Orbifolds Results in 6d: 1) 60 point groups P with $N \ge 1$ SUSY 2) 186 lattices Λ include roto-translations:

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 $h^{2,1}$ vs. $h^{1,1}$ for abelian orbifold geometries



M. Fischer, M. Ratz, J. Torrado and P. V. 2012

Number of generations for abelian orbifold geometries



M. Fischer, M. Ratz, J. Torrado and P. V. 2012

Number of generations for abelian orbifold geometries

- $h^{1,1} h^{2,1}$ always divisible by six
- Only exception: $(h^{1,1}, h^{2,1}) = (20, 0)$
- No geometry with three generations
 discrete Wilson lines needed for three generations
- computer program "orbifolder" to create and analyse orbifold models with abelian P H.P. Nilles, S. Ramos-Sanchez, P. V. and A. Wingerter 2012

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Non-abelian Orbifolds: untwisted Moduli $(h_{\mathbf{U}}^{(1,1)}, h_{\mathbf{U}}^{(2,1)})$

untwisted moduli	
$(h_{\sf U}^{(1,1)},h_{\sf U}^{(2,1)})$	non-abelian point groups
(2,2)	S ₃ , D ₄ , D ₆
(2,1)	$QD_{16}, (\mathbb{Z}_4 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2, \mathbb{Z}_4 \times S_3, (\mathbb{Z}_6 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2,$
	$\operatorname{GL}(2,3), \operatorname{SL}(2,3) \rtimes \mathbb{Z}_2$
(2,0)	$\mathbb{Z}_8 \rtimes \mathbb{Z}_2$, $\mathbb{Z}_3 \times S_3$, $\mathbb{Z}_3 \rtimes \mathbb{Z}_8$, SL(2, 3)–I, $\mathbb{Z}_3 \times D_4$,
	$\mathbb{Z}_3 imes Q_8$, $(\mathbb{Z}_4 imes \mathbb{Z}_4) times \mathbb{Z}_2$, $\mathbb{Z}_3 imes (\mathbb{Z}_3 times \mathbb{Z}_4)$, $\mathbb{Z}_6 imes S_3$,
	$\mathbb{Z}_3 \times \mathrm{SL}(2,3), \mathbb{Z}_3 \times ((\mathbb{Z}_6 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2), \mathrm{SL}(2,3) \rtimes \mathbb{Z}_4$
(1,1)	A4, S4
(1,0)	$T_7, \ \Delta(27), \ \mathbb{Z}_3 \times A_4, \ \Delta(48), \ \Delta(54), \ \mathbb{Z}_3 \times S_4, \ \Delta(96),$
	$\Sigma(36\phi)$, Δ(108), PSL(3,2), Σ(72 ϕ), Δ(216)

$h^{2,1}$ vs. $h^{1,1}$ for non-abelian orbifold geometries



M. Fischer, S. Ramos-Sánchez and P. V. 2013

Number of generations for all orbifold geometries

• 65 orbifold geometries with $h^{1,1} = h^{2,1}$

 \Rightarrow always non-chiral using standard heterotic CFT

Magnetized orbifolds to create chirality in blow-up

S. Groot Nibbelink and P. V. 2012

- 1) all \mathbb{Z}_N and some $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifolds
- 2) use "orbifolder" for MSSM models we find ≈ 12000 MSSM-like models
- 3) lessons from this OrbifoldLandscape?
- location of matter
- location of Higgs
- flavor symmetries

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			max. #	# models with				# MSSM	
orb	oifold	# MSSM	of indep.	0	1	2	3	\geq 4	without
			WLs	ine	dep. va	anishii	ng W	/Ls	$\mathrm{U}(1)_{anom}$
\mathbb{Z}_3	(1,1)	0	3	0	0	0	0	0	0
\mathbb{Z}_4	(1,1)	0	4	0	0	0	0	0	0
	(2,1)	128	3	128	0	0	0	0	0
	(3,1)	25	2	25	0	0	0	0	0
\mathbb{Z}_{6} -I	(1,1)	31	1	31	0	0	0	0	0
	(2,1)	31	1	31	0	0	0	0	0
ℤ ₆ -Ⅱ	(1,1)	348	3	13	335	0	0	0	1
	(2,1)	338	3	10	328	0	0	0	2
	(3,1)	350	3	18	332	0	0	0	2
	(4,1)	334	2	39	295	0	0	0	3
\mathbb{Z}_7	(1,1)	0		0	0	0	0 (0	0
ℤ ₈ -Ι	(1,1)	263	2	221	42	0	0	0	7
	(2,1)	164	2	123	41	0	0	0	5
	(3,1)	387	1	387	0	0	0	0	27
ℤ ₈ -Ⅱ	(1,1)	638	3	212	404	22	0	0	7
	(2,1)	260	2	92	168	0	0	0	3
ℤ ₁₂ -Ι	(1,1)	365	1	365	0	0	0	0	8
	(2,1)	385	1	385	0	0	0	0	9

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ℤ ₆ -Ⅱ	(1,1)	348	3	13	335	0	0	0	1
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	(3,1)	387	1	387	0	0	0	0	27
ℤ ₈ -Ⅱ	(1,1)	638	3	212	404	22	0	0	7
	(2,1)	260	2	92	168	0	0	0	3
\mathbb{Z}_{12} -I	(1,1)	365	1	365	0	0	0	0	8
	(2,1)	385	1	385	0	0	0	0	9

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			max. #	# models with					# MSSM
orbifo	ld	# MSSM	of indep.	0	1	2	3	\geq 4	without
			WLs		indep. v	anishing	WLs		$\mathrm{U}(1)_{anom}$
\mathbb{Z}_{12} -II	(1,1)	211	2	135	76	0	0	0	3
$\mathbb{Z}_2 imes \mathbb{Z}_2$	(1,1)	101	6	0	59	42	0	0	0
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(1,1)	3632	4	67	2336	1199	30	0	10
$\mathbb{Z}_2 \times \mathbb{Z}_6$ -I	(1,1)	445	2	332	113	0	0	0	5
$\mathbb{Z}_2 \times \mathbb{Z}_6$ -II	(1,1)	0	0	0	0	0	0	0	0
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1,1)	445	3	1	369	75	0	0	9
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(1,1)	465		441	24	0	0	0	0
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1,1)	1466	3	11	529	921	5	0	1
$\mathbb{Z}_6 \times \mathbb{Z}_6$	(1,1)	1128	0	1128	0	0	0	0	0
total		11940	CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX	XXXX	AND				102

Local GUTs

Fixed point with local GUT (e.g. SO(10) or SU(5)) and localized matter representation:



H.P. Nilles and P. V. 2014

Locations with split multiplets for Higgs

- Higgs in split GUT representation
- Origin? Location in higher dim. where GUT group is broken, i.e. 10d bulk





Complete Intersection Calabi-Yau: CICY 7862

Candelas, Dale, Lutken, Schimmrigk - 1987

- CY X described as hypersurface in product of projective spaces
- ► Hodge numbers $h_{11} = 4$ and $h_{21} = 68$ ⇒ four divisors: D_i , $i = 1, ..., h_{11} = 4$
- Non-vanishing triple intersection numbers and second Chern classes:

$$\kappa_{ijk} = \int_X D_i D_j D_k = 2$$
 and $c_{2i} = \int_{D_i} c_2 = 24$

for $i \neq j \neq k \neq i$ from 1 to $h_{11} = 4$

- CICY 7862 X with $S(U(1)^5) = U(1)^4$ gauge background.
- Specified by five vectors k_(a) = (k¹_(a),...,k⁴_(a)) ∈ Z⁴, a = 1,...,5:

$$\mathcal{V} = \bigoplus_{a=1}^{5} \mathcal{O}_X\left(k_{(a)}^1, \dots, k_{(a)}^4\right) \quad \text{with} \quad \sum_{a=1}^{5} k_{(a)} = 0$$

Alternative description:

$$\frac{\mathcal{F}}{2\pi} = D_i H_i$$
, $H_i = V_i^I H_I$ where

- D_i with i = 1, ..., h₁₁ = 4: two-forms, Poincaré-dual to the divisors
- H_I with I = 1, ..., 16: Cartan generators of $E_8 \times E_8$
- V_i with $i = 1, ..., h_{11} = 4$: 16-component line bundle vectors

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- ► Write line bundle vector V_i = (V'_i, V''_i) for observable and hidden E₈
- Given the five vectors k_(a) the corresponding line bundle vectors in the observable E₈ are

$$V_i' = (a_i^5, b_i, c_i, d_i)$$

(Exponent 5 indicates 5-times repetition)▶ Using

for $i = 1, ..., h_{11}$

Choose Model number 2, identifier {7862, 4, 5}

Corresponding line bundle vectors:

$$V_{1} = \left(-\frac{1}{2}^{5}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}\right) (0^{8})$$

$$V_{2} = \left(\frac{1}{2}^{5}, -\frac{3}{2}, -\frac{1}{2}, -\frac{5}{2}\right) (0^{8})$$

$$V_{3} = (0^{5}, -1, -2, -1) (0^{8})$$

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Anderson, Gray, Lukas, Palti - 2011

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Integrated BIs of the Kalb-Ramond field in the presence of NS5 branes:

$$\begin{array}{rcl} N_i &=& N'_i + N''_i \\ N'_i &=& c_{2i} + \kappa_{ijk} \; V'_j \cdot V'_k \\ N''_i &=& c_{2i} + \kappa_{ijk} \; V''_j \cdot V''_k \end{array}$$

for all divisors D_i , $i = 1, ..., h_{11} = 4$.

- Bls \Rightarrow anomaly cancellation in the 4D EFT
- Unbroken SUSY requires N_i ≥ 0
- We will choose V_i'' such that $N_i = 0$, i.e. $N_i'' = -N_i'$
- Chiral part of the 4D matter spectrum: multiplicity operator:

$$\mathcal{N} = \frac{1}{6} \kappa_{ijk} H_i H_j H_k + \frac{1}{12} c_{2i} H_i$$

evaluated on each root p of $E_8 \times E_8$ using $H_i(p) = V_i \cdot p$ (and checked with cohomCalg) Blumenhagen, Jurke, Rahn, Roschy - 2010

• Bls with $N_i = 0$ for CICY 7862:

$$24 + \sum_{i \neq j=1}^{4} V_i \cdot V_j = 0$$

Allows for infinite number of solutions, e.g.

$$V_{1} = \left(-\frac{1}{2}^{5}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}\right) \left(1^{4}, 0, -2, 0, 0\right)$$

$$V_{2} = \left(\frac{1}{2}^{5}, -\frac{3}{2}, -\frac{1}{2}, -\frac{5}{2}\right) \left(0^{4}, -1, 1, 0, 2k\right)$$

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for all $k \in \mathbb{Z}$

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for all $k \in \mathbb{Z}$

- Gauge group $SU(5) \times SU(4) \times U(1)^9$
- Chiral spectrum (omitting U(1) charges):

Observ	able E ₈	Hidden E ₈		
Mult.	Rep.	Mult.	Rep.	
12	(5, 1)	12k + 8	(1,4)	
12	(10, 1)	12k + 8	$(1, \overline{4})$	
		4k	(1,6)	
60	(1,1)	80k + 8	(1, 1)	

- Pure, mixed (non-)Abelian gauge and gravitational anomalies cancel for all k via generalized Green-Schwarz mechanism
- One-loop corrected DUY equations satisfied using VEVs of charged fields

Blumenhagen, Honecker, Weigand - 2005

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Conclusion

- ► Complete classification of orbifold geometries with N ≥ 1 SUSY (Abelian and non-Abelian)
- \Rightarrow 520 orbifold geometries
- Useful tool for abelian P: "orbifolder"
- The OrbifoldLandscape: \approx 12000 MSSM-like models
- Lessons:
 - Location of matter: local GUTs
 - Location of Higgs: 10d bulk
 - Discrete flavor symmetries
- Calabi-Yau case:
 - MSSM like-models
 - Hidden E₈: infinite set of models? At the boundary of the Kähler cone?