

Electroweak corrections to $pp \rightarrow W^+W^- \rightarrow 4 \text{ leptons}$ at the LHC

Lukas Salfelder



in collaboration with

B. Biedermann, M. Billoni, A. Denner, S. Dittmaier, L. Hofer and B. Jäger

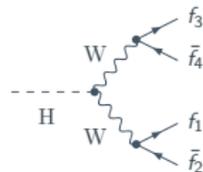
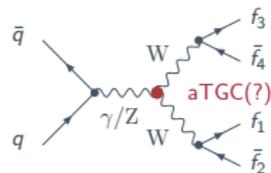
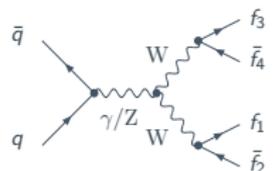
DESY Theory Workshop
Sept. 29 - Oct. 02, 2015

- 1 Introduction
- 2 Full EW corrections to $pp \rightarrow W^+W^- \rightarrow 4 \text{ leptons}$
- 3 Preliminary results
- 4 Summary

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Motivation: Why W^+W^- is interesting...

- Probe non-abelian structure of the Standard Model:
→ simplest process with trilinear gauge couplings
- Limits on anomalous gauge couplings
- Irreducible background to Higgs decay $H \rightarrow WW^*$



Motivation: Why W^+W^- is interesting...

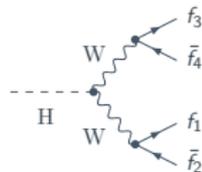
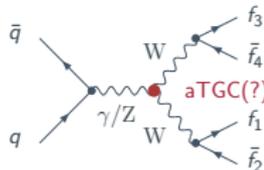
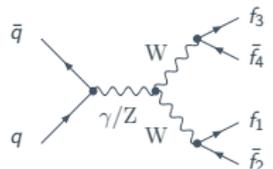
- Probe non-abelian structure of the Standard Model:
→ simplest process with trilinear gauge couplings

- Limits on anomalous gauge couplings

→ high energy region: $M_{4\ell} \gg 2M_W$

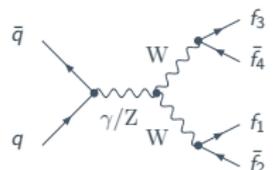
- Irreducible background to Higgs decay $H \rightarrow WW^*$

→ low energy region: $M_{4\ell} \leq 2M_W$



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- Probe non-abelian structure of the Standard Model:
→ simplest process with trilinear gauge couplings



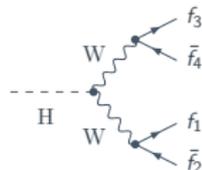
- Limits on anomalous gauge couplings

→ high energy region: $M_{4l} \gg 2M_W$



- Irreducible background to Higgs decay $H \rightarrow WW^*$

→ low energy region: $M_{4l} \leq 2M_W$



- at leading order $\mathcal{O}(\alpha^4)$ → little "QCD-pollution"

Status of W^+W^- predictions at the LHC

QCD corrections

- NLO corrections

[Ohnemus et al. '91; Mele et al. '91; Frixione et al. '92 '93 - Baur et al. '96; Dixon et al. '98]

- NLO corrections + parton shower matching

[Frixione et al. '03; Nason et al. '06 - Hamilton '10; Höche et al. '10; Melia et al. '11; Frederix et al. '11]

- NNLO corrections

[T. Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi '14]

EW corrections

- NLO corrections to on-shell W^+W^- production

[Bierweiler, Kasprzik, Kühn, Uccirati '12 '13]

- NLO corrections to on-shell W^+W^- including γq contributions

[Baglio, Ninh, Weber '13]

- NLO corrections to $pp \rightarrow W^+W^- \rightarrow 4l$ in DPA

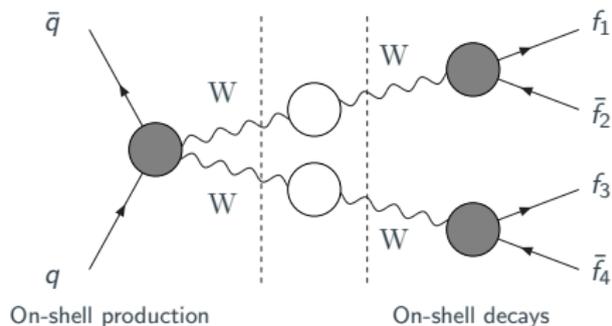
[Billoni, Dittmaier, Jäger, Speckner '13]

Measurements

- Measurements of W^+W^- production cross sections at 7 and 8 TeV

[ATLAS, CMS '11 '13]

NLO EW in double-pole approximation



- Double-pole approximation (DPA):
first applied in RacoonWW: $e^+e^- \rightarrow 4\ell$

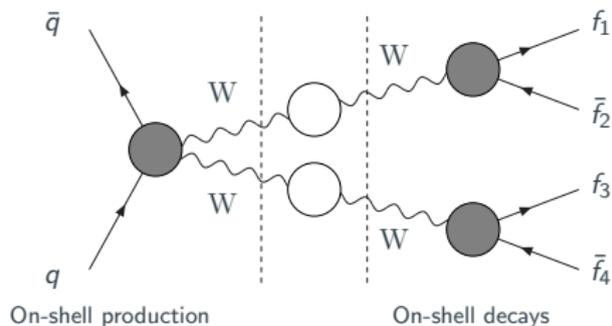
[Denner, Dittmaier, Roth, Wackerath '99-'02]

- Full EW corrections to $e^+e^- \rightarrow 4\ell$

[Denner, Dittmaier, Roth, Wieders '06]

$$\Rightarrow \Delta_{\text{DPA}} \sim \frac{\alpha}{2\pi} \frac{\Gamma_W}{M_W} \sim 0.5\% \quad [170 \text{ GeV} \leq \sqrt{s} \lesssim 500 \text{ GeV}]$$

NLO EW in double-pole approximation



⇒ virtual corrections only to doubly resonant diagrams!

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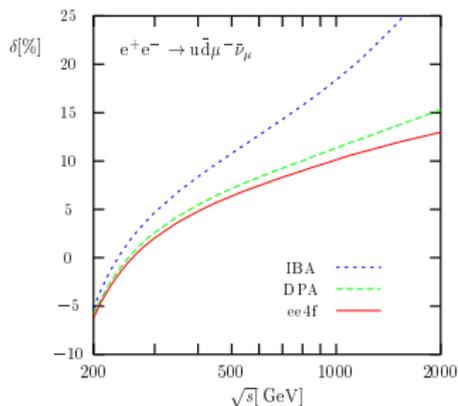
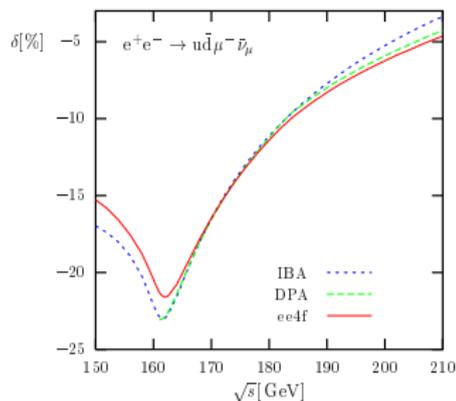
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NLO EW in double-pole approximation



[Denner, Dittmaier, Roth, Wieders '06]

$\sqrt{s} \leq 2M_W$:

- DPA not applicable!
- Improved Born approximation (IBA) used

$\sqrt{s} \gg 2M_W$:

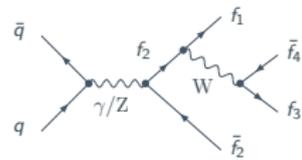
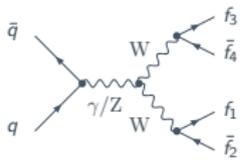
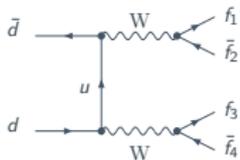
- Difference on integrated cross-section should stay $\lesssim 1\%$
- In distributions larger corrections in the TeV-range expected, especially for $p_T(\ell^\pm)$ or $p_T(\ell^+\ell^-)$

\Rightarrow Full EW corrections needed

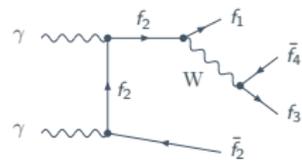
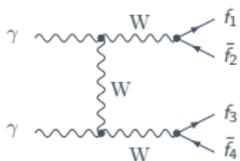
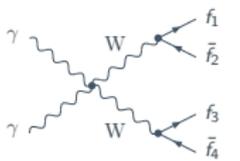
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Contributions to the cross-section

$$\sigma_{H_1 H_2}^{LO} = \left(\sum_{q, \bar{q}} f_q^{H_1} f_{\bar{q}}^{H_2} \otimes \left[\int_4 d\hat{\sigma}_{q\bar{q}}^{LO} \right] + (q \leftrightarrow \bar{q}) \right) \\ + \left(f_b^{H_1} f_{\bar{b}}^{H_2} \otimes \int_4 d\hat{\sigma}_{b\bar{b}}^{LO} + (b \leftrightarrow \bar{b}) \right) \\ + \left(f_\gamma^{H_1} f_\gamma^{H_2} \otimes \int_4 d\hat{\sigma}_{\gamma\gamma}^{LO} \right)$$



...



...

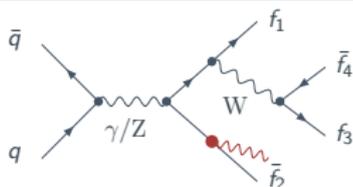
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$$\begin{aligned}\sigma_{H_1 H_2}^{NLO} &= \left(\sum_{q, \bar{q}} f_q^{H_1} f_{\bar{q}}^{H_2} \otimes \left[\int_4 d\hat{\sigma}_{q\bar{q}}^{LO} + \int_5 d\hat{\sigma}_{q\bar{q}}^{real} + \int_4 d\hat{\sigma}_{q\bar{q}}^{virt} + \int_0^1 dx \int_4 d\hat{\sigma}_{q\bar{q}}^{fact} \right] + (q \leftrightarrow \bar{q}) \right) \\ &+ \left(f_b^{H_1} f_{\bar{b}}^{H_2} \otimes \int_4 d\hat{\sigma}_{b\bar{b}}^{LO} + (b \leftrightarrow \bar{b}) \right) \\ &+ \left(f_\gamma^{H_1} f_\gamma^{H_2} \otimes \int_4 d\hat{\sigma}_{\gamma\gamma}^{LO} \right)\end{aligned}$$

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 \end{aligned}$$

Real corrections: $q\bar{q} \rightarrow \nu_\mu \mu^+ e^- \bar{\nu}_e + \gamma$



\Rightarrow soft and/or collinear singularities!

Dipole subtraction formalism for final-state photons

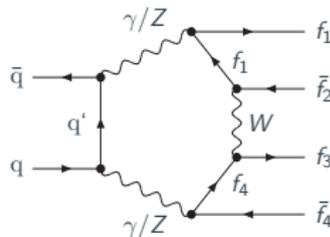
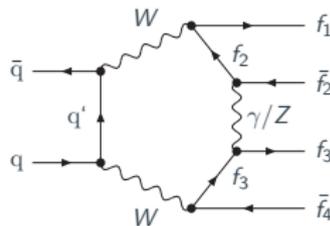
\Rightarrow suitable for MC-Integration!

[Dittmaier, Kabelschacht, Kasprzik '99 '08]

Virtual contribution: $q\bar{q} \rightarrow \nu_\mu \mu^+ e^- \bar{\nu}_e$

- amplitude generation: inhouse Mathematica routines
 - up to 6-point one-loop tensor integrals
 - after renormalization: UV finite
- reduction of one-loop tensor integrals

[Denner, Dittmaier '05]



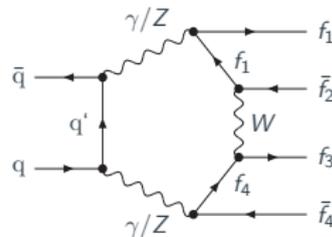
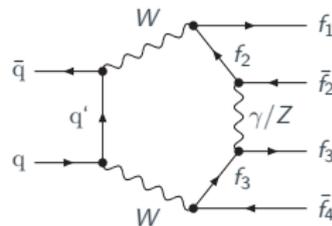
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[Denner, Dittmaier '05]

- $\int_4 d\hat{\sigma}_{q\bar{q}}^{\text{endp}}$ from subtraction function cancels remaining IR singularities per construction:

$$\int_5 d\hat{\sigma}_{q\bar{q}}^{\text{subtr}} = \underbrace{\int_0^1 dx \int_4 d\hat{\sigma}_{q\bar{q}}^{\text{conv}}}_{\rightarrow \hat{\sigma}^{\text{fact}}} + \underbrace{\int_4 d\hat{\sigma}_{q\bar{q}}^{\text{endp}}}_{\rightarrow \hat{\sigma}^{\text{virt}}}$$



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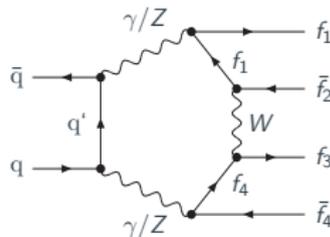
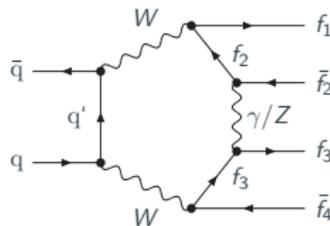
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$$\Rightarrow \int_4 d\hat{\sigma}_{q\bar{q}}^{\text{virt,fin}} = \int_4 d\hat{\sigma}_{q\bar{q}}^{\text{virt}} + \int_4 d\hat{\sigma}_{q\bar{q}}^{\text{endp}}$$



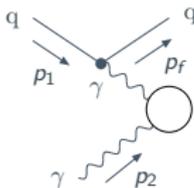
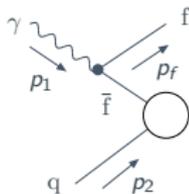
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 \end{aligned}$$

Real corrections (II): Incoming photons



Further details on the calculation...

- Multi-channel integrator based on Coffey $\gamma\gamma \rightarrow 4\ell$

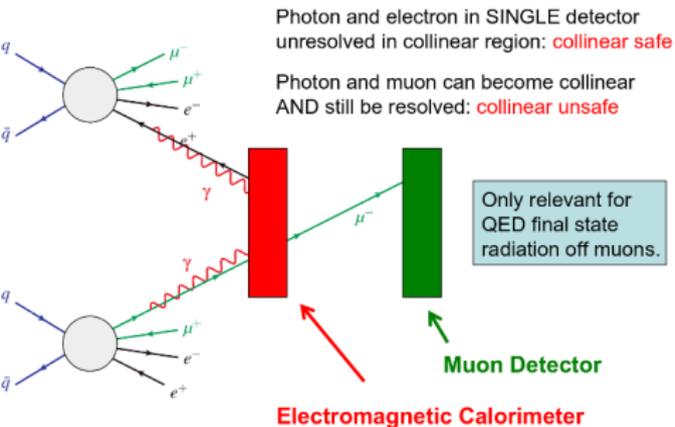
[Bredenstein, Dittmaier, Roth '05]

- Tree-level contributions:

- Weyl-van-der-Waerden spinor formalism
- complex-mass scheme:
 - $\rightarrow \mu_V^2 = M_V^2 - iM_V\Gamma_V$ for $V = W, Z$
 - \rightarrow all EW couplings $\propto c_W = \mu_W/\mu_Z$

[Denner, Dittmaier, Roth, Wackerth '99; Denner, Dittmaier, Roth, Wieders '05]

- generalized to non-collinear-safe observables



muon mass enters as regulator only in the subtraction part

⇒ no complete cancellation against virtual corrections

no recombination for final-state muons

⇒ affects phase-space cuts and binning of certain observables

⇒ impact on cross-section and differential distributions!

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- generalized to non-collinear-safe observables

- final-state leptons recombined if $\Delta_{\ell\gamma} < 0.1$ \rightarrow collinear safe
- final-state muons may remain un-recombined \rightarrow collinear unsafe

...extensively checked via second independent implementation
by Benedikt Biedermann!

[Recola: Actis et al.; Collier: Denner, Dittmaier, Hofer '14]

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Phenomenological setup

LHC Setup:

- NNPDF2.3QED,NLO
- $\mu_F = M_W$
- $\alpha_{G_\mu} = \frac{\sqrt{2} G_\mu M_W^2 s_W^2}{\pi} \approx \frac{1}{132.36}$
- jet def.: $p_{T,i} > 25 \text{ GeV}$,
 $|y_i| < 5$

Minimal cuts:

- $p_{T,\ell} > 20 \text{ GeV}$
- $|y_\ell| < 2.5$
- jet veto: $p_{T,\text{jet}} < 100 \text{ GeV}$,
 $R_{\text{jet}\ell} > 0.4$
 \Rightarrow avoid large QCD corrections

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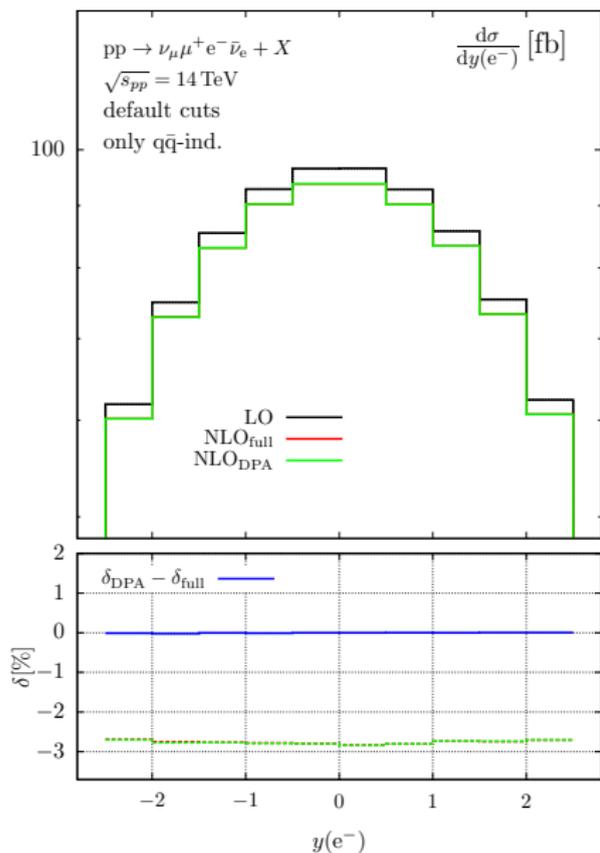
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additional ATLAS cuts (WW search):

[arXiv:1210.2979[hep-ex]]

- $p_{T,\ell}^{\text{leading}} > 25 \text{ GeV}$
- $E_T^{\text{miss}} = |\vec{p}_T^{\text{miss}}| > 25 \text{ GeV}$
- $R_{e\mu} > 0.1$, $M_{e\mu} > 10 \text{ GeV}$
- jet veto: $p_{T,\text{jet}} < 25 \text{ GeV}$
 \Rightarrow No Jets!

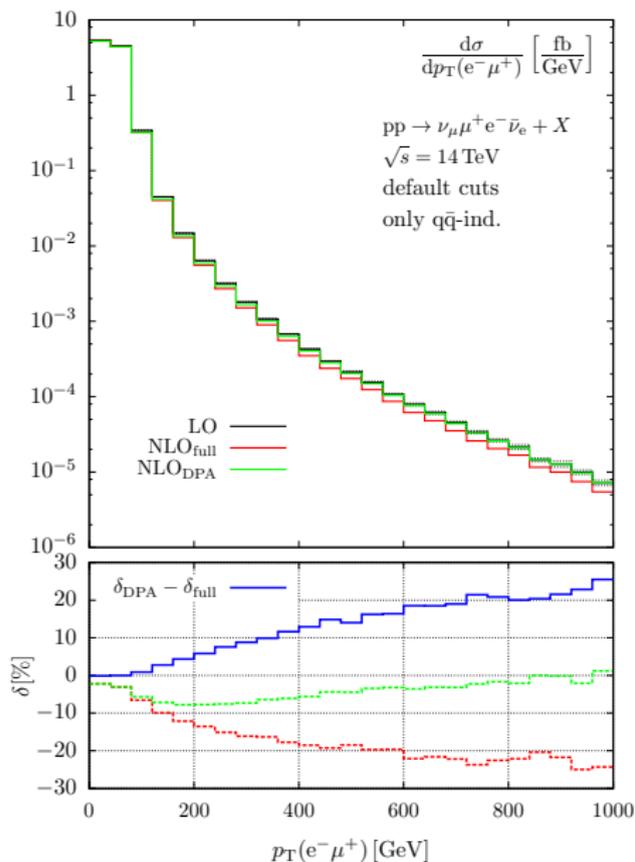
Preliminary results(I): Full corrections vs. DPA



- Rapidity: not sensitive to difference between DPA and full corrections

$$\delta_X(\mathcal{O}) = \frac{\frac{d\sigma_X^{\text{NLO}}}{d\mathcal{O}} - \frac{d\sigma^{\text{LO}}}{d\mathcal{O}}}{\frac{d\sigma^{\text{LO}}}{d\mathcal{O}}}$$

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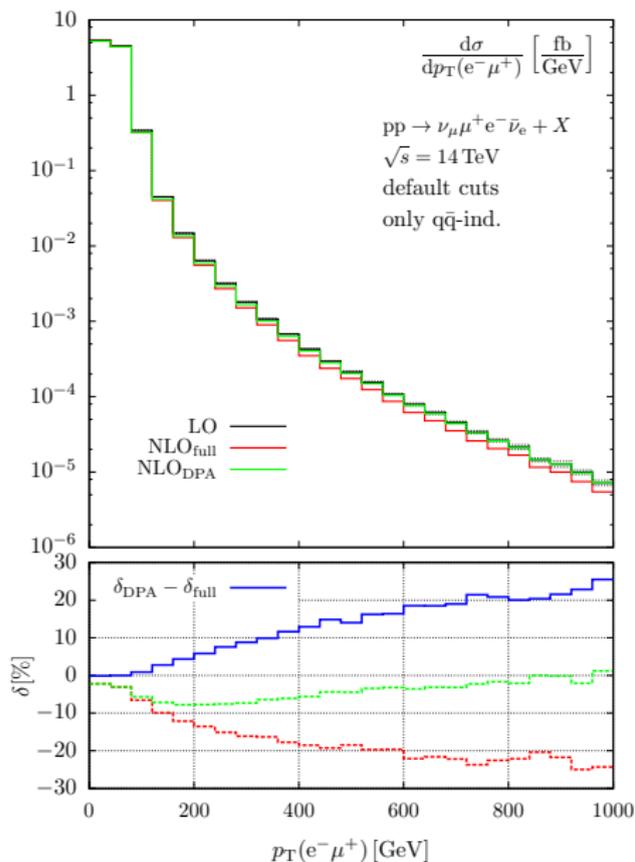
- $p_T(\ell^+\ell^-)$:

→ large difference in TeV-region

→ high $p_T(1^+1^-) \leftrightarrow$ non-doubly resonant contributions

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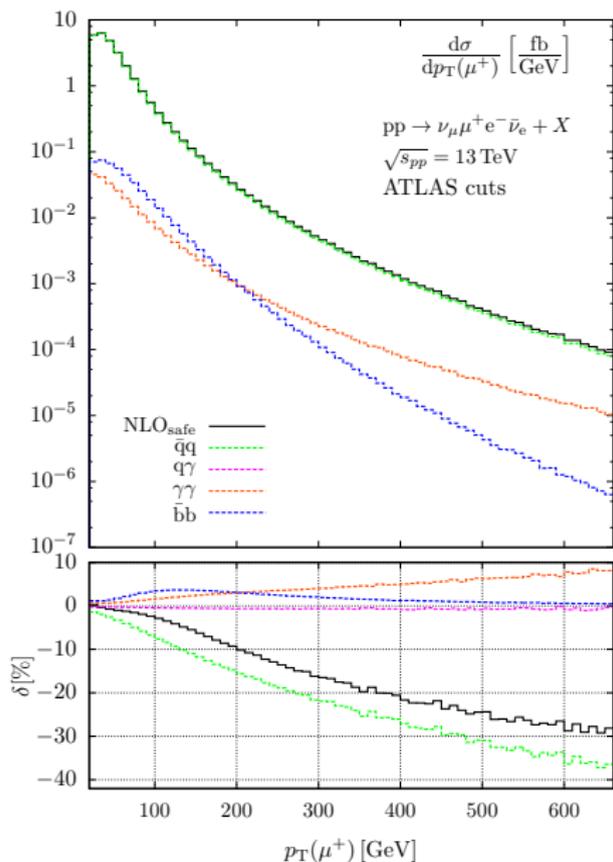
Preliminary results(I): Full corrections vs. DPA



- Rapidity: not sensitive to difference between DPA and full corrections
- $p_T(l^+l^-)$:
 - large difference in TeV-region
 - high $p_T(l^+l^-) \leftrightarrow$ non-doubly resonant contributions
 - large effect of virtual corrections to non-doubly resonant contributions

Only contained in full calculation!

Preliminary results(II): collinear safe vs. collinear unsafe

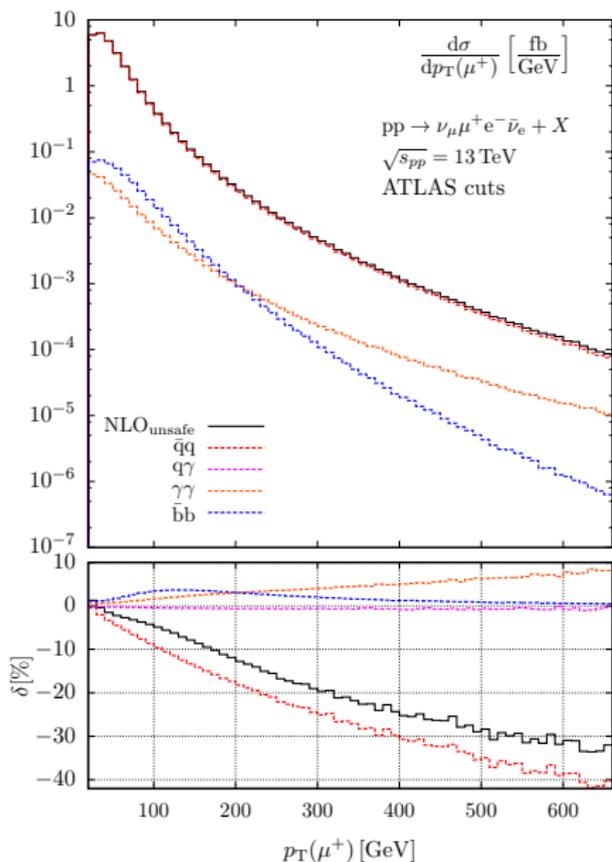


ATLAS-cuts @13 TeV

$\sigma_{\bar{q}q}^{\text{LO}}$	267.28(5) [fb]	
$\delta_{\bar{q}q}$	safe - 8.489(4) [fb]	($\approx -3.1\%$)
	unsafe - 9.913(5) [fb]	($\approx -3.7\%$)
$\delta_{q\gamma}$	- 0.744(2) [fb]	($\approx -0.3\%$)
$\delta_{\gamma\gamma}$	2.377(1) [fb]	($\approx +0.9\%$)
$\delta_{\bar{b}b}$	4.471(1) [fb]	($\approx +1.7\%$)

- $\delta_{q\gamma}$: very small contribution
[→ dedicated cuts!]
- $\delta_{\gamma\gamma}$: relevance increasing with energy
- $\delta_{\bar{b}b}$: $< 2\%$ (at $p_T \simeq 300 \text{ GeV}$)

Preliminary results(II): collinear safe vs. collinear unsafe

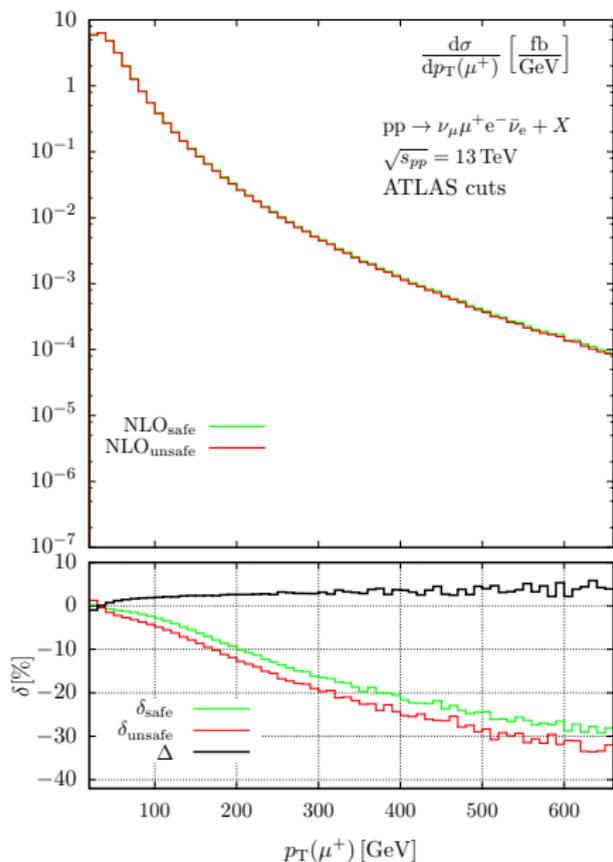


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- $\delta_{\gamma\gamma}$: relevance increasing with energy
- $\delta_{b\bar{b}}$: $< 2\%$ (at $p_T \simeq 300 \text{ GeV}$)
- $\delta_{q\bar{q}}$: $\approx 35\% - 40\%$ (at $p_T \simeq 600 \text{ GeV}$)

Preliminary results(II): collinear safe vs. collinear unsafe

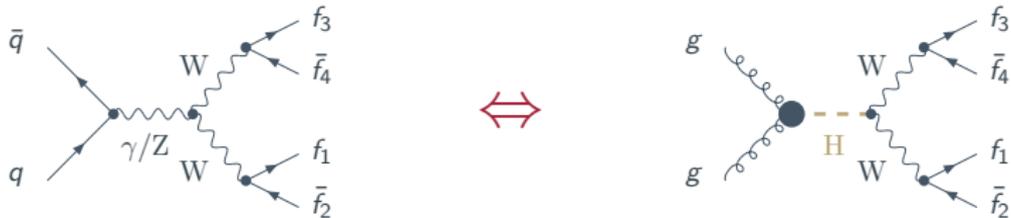


ATLAS-cuts @13 TeV

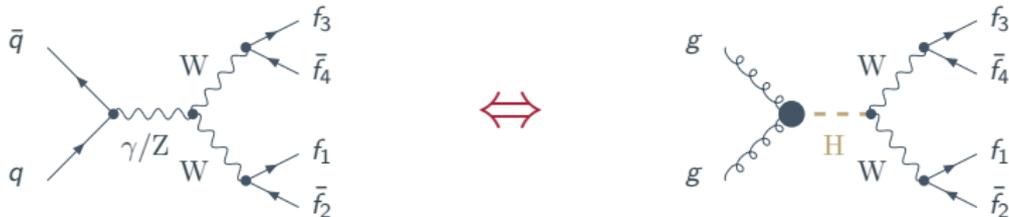
$\sigma_{\bar{q}q}^{\text{LO}}$		267.28(5) [fb]	
$\delta_{\bar{q}q}$	safe	- 8.489(4) [fb]	($\approx -3.1\%$)
	unsafe	- 9.913(5) [fb]	($\approx -3.7\%$)
$\delta_{q\gamma}$		- 0.744(2) [fb]	($\approx -0.3\%$)
$\delta_{\gamma\gamma}$		2.377(1) [fb]	($\approx +0.9\%$)
$\delta_{\bar{b}b}$		4.471(1) [fb]	($\approx +1.7\%$)

- $\delta_{q\gamma}$: very small contribution
 [→dedicated cuts!]
- $\delta_{\gamma\gamma}$: relevance increasing with energy
- $\delta_{\bar{b}b}$: $< 2\%$ (at $p_T \simeq 300 \text{ GeV}$)
- $\delta_{\bar{q}q}$: $\approx 35\% - 40\%$ (at $p_T \simeq 600 \text{ GeV}$)

Preliminary results(III): Higgs-background setup



Preliminary results(III): Higgs-background setup



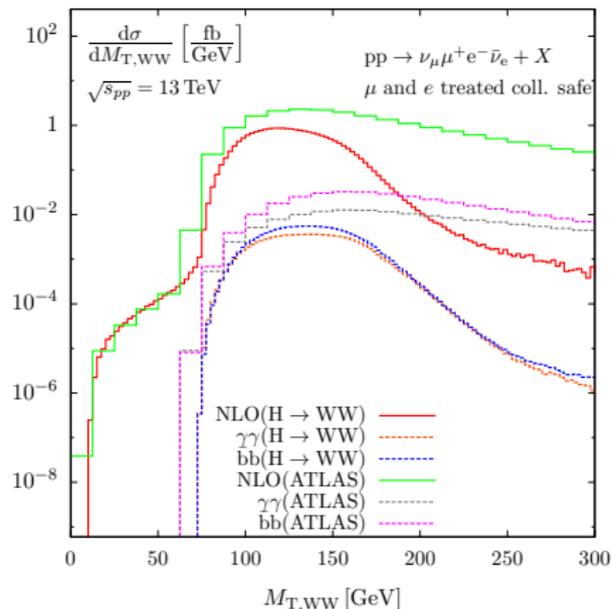
additional cuts:

[arXiv:1412.2641[hep-ex]]

- $10 \text{ GeV} < M_{e\mu} < 55 \text{ GeV}$
- $\Delta\phi_{e\mu} < 1.8$
- $E_{\text{T}}^{\text{miss}} = |\vec{p}_{\text{T}}^{\text{miss}}| > 25 \text{ GeV}$

Higgs-background-cuts @13 TeV

$\sigma_{\bar{q}q}^{\text{LO}}$		49.54(3) [fb]	
$\delta_{\bar{q}q}$	safe	- 0.978(4) [fb]	($\approx -2.0\%$)
$\delta_{q\gamma}$		- 0.112(2) [fb]	($\approx -0.2\%$)
$\delta_{\gamma\gamma}$		0.257(1) [fb]	($\approx +0.5\%$)
$\delta_{\bar{b}b}$		0.358(1) [fb]	($\approx +0.7\%$)



- 1 Introduction
- 2 Full EW corrections to $pp \rightarrow W^+W^- \rightarrow 4 \text{ leptons}$
- 3 Preliminary results
- 4 Summary

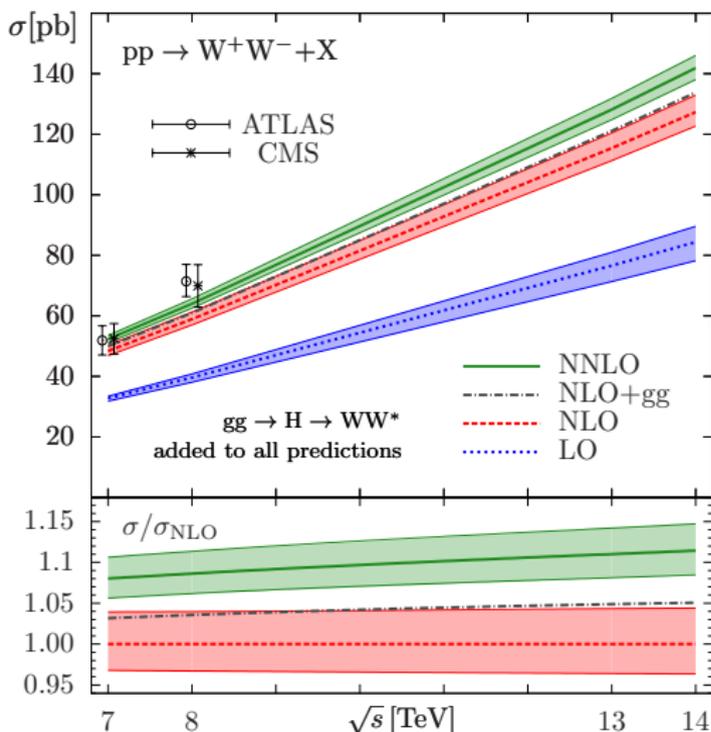
- W^+W^- production is very important for a detailed understanding of the EW gauge sector
- below $2M_W$ and in certain phase-space regions: DPA not sufficient
- full EW corrections can reach up to $\mathcal{O}(-40\%)$ for certain observables
- collinear unsafe generalization enhances corrections ($\sim \mathcal{O}(1\%)$)
- Higgs-background setup still dominated by on-shell W 's

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Thank you for your attention 😊

Backup...

QCD predictions and measurements



[T. Gehrmann et al. '14]

In NNLO-QCD:

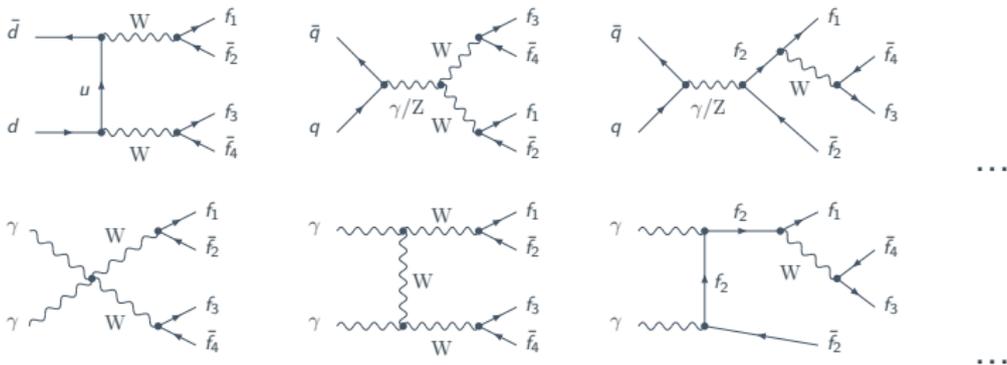
- gg-induced channel fully included
- realistic estimate of theoretical uncertainties
- tension almost disappeared

Measurements:

- for extrapolation from fiducial to inclusive cross section care needed...

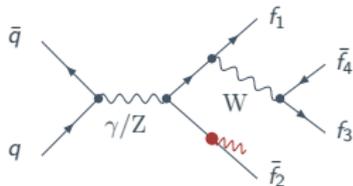
[Monni, Zanderighi '14]

Leading order contributions



- All external particles massless, CKM mixing negligible
 - \Rightarrow Need partonic matrix elements for: $\bar{u}u/\bar{d}d/\bar{b}b/\gamma\gamma \rightarrow \nu_\mu\mu^+e^-\bar{\nu}_e$
- Only small contributions from $b\bar{b}$ ($< 2\%$) and $\gamma\gamma$ ($\lesssim 1\%$) on integrated cross-section
 - \Rightarrow No higher order corrections considered for these channels
 - $\Rightarrow \gamma\gamma$ -induced: important on differential level

Real corrections: $q\bar{q} \rightarrow \nu_\mu \mu^+ e^- \bar{\nu}_e + \gamma$



\Rightarrow soft and/or collinear singularities!

Dipole subtraction formalism for final-state photons:

[Dittmaier, Kabelschacht, Kasprzik '99 '08]

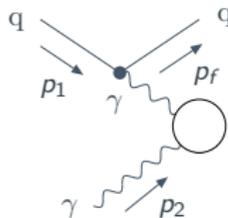
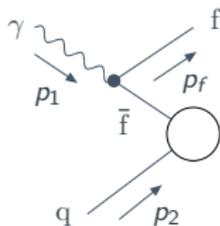
$$\int_5 d\hat{\sigma}_{q\bar{q}}^{\text{real,fin}} = \int_5 d\hat{\sigma}_{q\bar{q}}^{\text{real}} - \int_5 d\hat{\sigma}_{q\bar{q}}^{\text{subtr}}$$

\Rightarrow suitable for MC-Integration!

$$\int_5 d\hat{\sigma}_{q\bar{q}}^{\text{subtr}} = \underbrace{\int_0^1 dx \int_4 d\hat{\sigma}_{q\bar{q}}^{\text{conv}}}_{\rightarrow \hat{\sigma}^{\text{fact}}} + \underbrace{\int_4 d\hat{\sigma}_{q\bar{q}}^{\text{endp}}}_{\rightarrow \hat{\sigma}^{\text{virt}}}$$

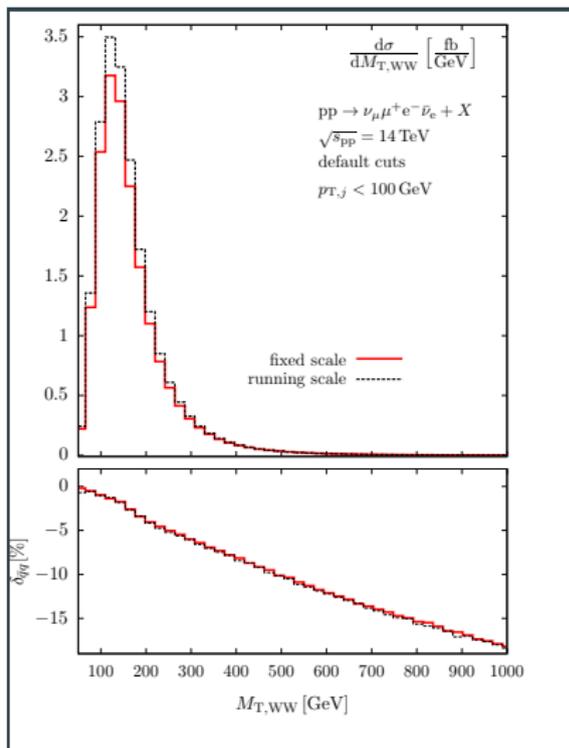
\Rightarrow both separately IR singular

- $\int_0^1 dx \int_4 (d\hat{\sigma}_{q\bar{q}}^{\text{fact}} + d\hat{\sigma}_{q\bar{q}}^{\text{conv}})$ per construction IR finite
- $\int_4 d\hat{\sigma}_{q\bar{q}}^{\text{endp}}$: all real-emission IR singularities with LO kinematics



- $\gamma \rightarrow f\bar{f}^*$ splitting:
 - $f = l$: no singularities if $p_T(l^\pm)$ -cuts applied
 - $f = q$: dipole-subtraction approach
- $q \rightarrow q\gamma^*$ splitting:
 - spin-correlated matrix elements for $\gamma\gamma \rightarrow 4l$ OR
 - effective collinear factor:
 - restores regulator mass dependence of amplitudes in collinear limit
 - no impact outside collinear regions

Factorization ansatz



[Billoni, Dittmaier, Jäger, Speckner '13]

- Dependence on μ_F only via PDFs
- $\pm 8\%$ for $\mu_F = \xi M_W$,
 $0.5 < \xi < 2$
- shape of distributions affected,
but relative corrections $\delta_{\bar{q}q}$ not!

$$d\sigma = d\sigma^{\text{QCD}_{q\bar{q}}} \times (1 + \delta_{q\bar{q}}^{\text{EW}}) \\ + d\sigma_{gg} + d\sigma_{\gamma\gamma} + d\sigma_{\gamma q}.$$

- only consider photons with $|y_\gamma| < 5$.
- determine separation R_{ij} in rapidity-azimuthal plane

$$R_{ij} = \sqrt{(y_i - y_j)^2 + (\Delta\phi_{ij})^2},$$

with y_k rapidity and $\Delta\phi_{ij}$ the azimuthal angle between i and j .

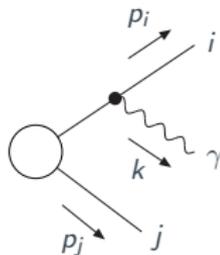
- recombine photons with the closest charged lepton ℓ whenever

$$R_{\gamma\ell} < 0.1.$$

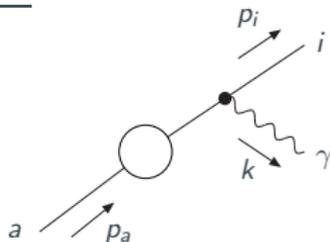
- We form dressed leptons by adding the momenta of the photon and the respective lepton. The momenta of the other particles in the event remain unaffected.

more on dipole subtraction...(i)

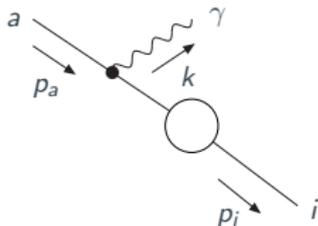
$\underline{g_{ij}^{(sub)}}$: Final-Final



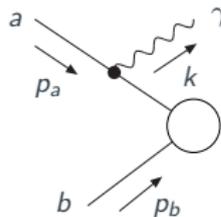
$\underline{g_{ia}^{(sub)}}$: Final-Initial



$\underline{g_{ai}^{(sub)}}$: Initial-Final



$\underline{g_{ab}^{(sub)}}$: Initial-Initial



$$\int_5 d\hat{\sigma}_{\bar{q}q}^{\text{real,fin}} = F(x_1 x_2) \int_5 d\Phi_5 \left[\sum |\mathcal{M}_{\text{real}}^{\bar{q}q \rightarrow \nu_\mu \mu^+ e^- \bar{\nu}_e \gamma}|^2 \Theta(\Phi_5) - \sum_{\substack{i,j=1 \\ i \neq j}}^6 \sum |\mathcal{M}_{\text{sub},ij}|^2 \Theta(\tilde{\Phi}_{4,ij}) \right]$$

more on dipole subtraction...(ii)

$$\int_5 d\hat{\sigma}_{q\bar{q}}^{\text{subtr}} = \underbrace{\int_0^1 dx \int_4 d\hat{\sigma}_{q\bar{q}}^{\text{conv}}}_{\rightarrow \hat{\sigma}^{\text{fact}}} + \underbrace{\int_4 d\hat{\sigma}_{q\bar{q}}^{\text{endp}}}_{\rightarrow \hat{\sigma}^{\text{virt}}}, \quad \int_5 d\Phi_5 = \int_0^1 dx \int_4 d\tilde{\Phi}_{4,ij}(x) \int_1 d\Phi_{\gamma,ij}(x)$$

$$\begin{aligned} \int_5 d\hat{\sigma}_{q\bar{q}}^{\text{subtr}} &= -\frac{\alpha}{2\pi} \sum_{\substack{i,j=1 \\ i \neq j}}^6 (-1)^{i+j} Q_i Q_j \\ &\times \int_0^1 dx \left[\int_4 d\tilde{\Phi}_{4,ij}(x) \mathcal{G}_{ij}(\tilde{s}_{ij}, x) F(x_1 x_2) \overline{\sum} |\mathcal{M}_{\text{Born}}(\tilde{\Phi}_{4,ij}(x))|^2 \Theta(\tilde{\Phi}_{4,ij}(x)) \right. \\ &\quad \left. - \int_4 d\tilde{\Phi}_{4,ij}(1) \mathcal{G}_{ij}(\tilde{s}_{ij}, x) F(x_1 x_2) \overline{\sum} |\mathcal{M}_{\text{Born}}(\tilde{\Phi}_{4,ij}(1))|^2 \Theta(\tilde{\Phi}_{4,ij}(1)) \right] \\ &- \frac{\alpha}{2\pi} \sum_{\substack{i,j=1 \\ i \neq j}}^6 (-1)^{i+j} Q_i Q_j F(x_1 x_2) \int_4 d\Phi_4 G_{ij}(s_{ij}) \overline{\sum} |\mathcal{M}_{\text{Born}}(\Phi_4)|^2 \Theta(\Phi_4) \end{aligned}$$

$$\mathcal{G}_{ij}(\tilde{s}_{ij}, x) = 8\pi^2 x \int_1 d\Phi_{\gamma,ij}(x) \mathbf{g}_{ij}(q_i, q_j, k). \quad G_{ij}(s_{ij}) = \int_0^1 dx \mathcal{G}_{ij}(s_{ij}, x) = \mathcal{L}(s_{ij}, m_i^2) + C_{ij},$$

$$\mathcal{L}(s, m^2) = \ln\left(\frac{m^2}{s}\right) \ln\left(\frac{\lambda^2}{s}\right) + \ln\left(\frac{\lambda^2}{s}\right) - \frac{1}{2} \ln^2\left(\frac{m^2}{s}\right) + \frac{1}{2} \ln\left(\frac{m^2}{s}\right)$$

Improved Born approximation

[Dittmaier, Böhm, Denner '92; Denner, Dittmaier, Roth, Wackerath '01]

$$d\hat{\sigma}_{\bar{q}q}^{\text{IBA}} = F(x_1 x_2) d\Phi_4 \sum \left| \mathcal{M}_{\text{IBA}}^{\bar{q}q \rightarrow \nu_\mu \mu^+ e^- \bar{\nu}_e} \right|^2 [1 + \delta_{\text{Coul}}(\hat{s}, k_+^2, k_-^2)] g(\bar{\beta}).$$

- $\frac{e^2}{s_W^2} \rightarrow 4\sqrt{2}G_\mu M_W^2, \quad e^2 \rightarrow 4\pi\alpha(\hat{s})$

- for photon exchange:

$$\alpha(Q^2) = \frac{\alpha(M_Z^2)}{1 - \frac{\alpha(M_Z^2)}{3\pi} \ln\left(\frac{Q^2}{M_Z^2}\right) \sum_{f \neq t} N_f^c Q_f^2}$$

- universal correction factor δ_{Coul} to account for the contribution of the Coulomb singularity from photon exchange near W -pair production threshold. Has only an impact in the threshold region.

