Precise Predictions for Higgs Masses in the Next-to-Minimal Supersymmetric Standard Model (NMSSM) with `NMSSM-FeynHiggs`

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Introduction: NMSSM

- NMSSM features an additional singlet and singlino compared to the MSSM
  - $\Rightarrow$ 3 $\mathcal{CP}$-even, 2 $\mathcal{CP}$-odd Higgs fields, 5 neutralino fields
- dynamic solution of the μ-problem by singlet vacuum expectation-value v_s: $\mu_{\text{eff}} \propto v_s$
- increased upper bound for tree-level mass of lightest Higgs field compared to MSSM

$$m_{h_1}^2 \lesssim M_Z^2 \left(\cos^2 2\beta + \frac{\lambda^2}{g^2} \sin^2 2\beta \right), \quad g^2 = \frac{e^2}{2 s_W c_W}$$

Introduction: Higgs Masses in the NMSSM

- Several codes are available so far. ☺

  `SPheno`, `FlexibleSUSY`, `NMSSMTools`, `SoftSUSY`, `NMSSMCalc`
- This means several (different) results are available so far! ☹

Q: What can `NMSSM-FeynHiggs` add as an additional code?

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Motivation

Q: What can `NMSSM-FeynHiggs` add as an additional code?

A1: Prediction for Higgs masses in NMSSM with a similar precision as in the MSSM in a (slightly) different calculation.

A2: Prediction for Higgs masses in NMSSM in a framework that is consistent with and (in a future step) contains the same functionality as `FeynHiggs`.

NMSSM-FeynHiggs: Higgs Mass Calculation

- masses obtained with diagrammatic methods from complex pole of full propagator

$$\Delta^{-1}\left(k^2\right) = \mathrm{i}\left[k^2\mathbb{1} - \mathcal{M}_{\phi\phi} + \hat{\Sigma}_{\phi\phi}\left(k^2\right)\right]$$

- renormalised self-energies $\hat{\Sigma}_{\phi\phi}$ approximated by

$$\hat{\Sigma}_{\phi\phi}\left(k^2\right) \approx \hat{\Sigma}^{(1L)}_{\phi\phi}\left(k^2\right)\Big|^{\text{NMSSM}} + \hat{\Sigma}^{(2L)}_{\phi\phi}\left(k^2\right)\Big|^{\text{MSSM}}_{k^2=0}$$

⇒ includes complete 1-loop NMSSM contributions

⇒ includes 2-loop MSSM contributions $\mathcal{O}\left(\alpha_s\alpha_t, \alpha_s\alpha_b, \alpha_t^2, \alpha_t\alpha_b\right)$

⇒ includes LL- and NLL-resummation

NMSSM: Leading Contributions

- Higgs-(s)top and Higgs-Higgs coupling constants

	fields	ϕ_i-ϕ_j-t	ϕ_i-ϕ_j-$\tilde{t}$(-$\tilde{t}$)	ϕ_i-ϕ_j-ϕ_k(-ϕ_l)
MSSM	ϕ_1, ϕ_2	Y_t	Y_t	g_1, g_2
NMSSM	ϕ_1, ϕ_2, ϕ_s	Y_t, (0)	Y_t, λ	g_1, g_2, λ, κ

- Yukawa-coupling of the top-quark:

$$Y_t = \frac{m_t}{v_2} \approx 1 \;\Rightarrow\; \alpha_t = \frac{{Y_t}^2}{4\pi}$$

- "Yukawa-couplings" of the Higgs-sector

$$\lambda^2 + \kappa^2 \lesssim 0.5 \;\Rightarrow\; \lambda,\; \kappa < 0.7$$

$\Longrightarrow$ for perturbative predictions up to high scales: $Y_t > \lambda$

NMSSM: Leading Contributions @ 1-Loop

- reminder: inverse propagator

$$\Delta^{-1}(k^2) = \mathrm{i}\left[k^2\mathbb{1} - \mathcal{M}_{\phi\phi} + \hat{\Sigma}_{\phi\phi}(k^2)\right]$$

- leading 1-loop self-energies are of "$\mathcal{O}(\alpha_t)$"$\hat{=}\, \mathcal{O}\left({Y_t}^2, \lambda Y_t, \lambda^2\right)$

$$\hat{\Sigma}^{(1L)}_{\phi\phi}(k^2) = \left(\begin{array}{cc|c} \mathcal{O}({Y_t}^2) & \mathcal{O}({Y_t}^2) & \mathcal{O}(\lambda Y_t) \\ \mathcal{O}({Y_t}^2) & \mathcal{O}({Y_t}^2) & \mathcal{O}(\lambda Y_t) \\ \hline \mathcal{O}(\lambda Y_t) & \mathcal{O}(\lambda Y_t) & \mathcal{O}(\lambda^2) \end{array}\right)$$

$\Rightarrow$ MSSM-like contributions of $\mathcal{O}\left({Y_t}^2\right)$ form 2×2 sub matrix of 3×3 self-energy matrix

NMSSM: Leading Contributions @ 2-Loop

- reminder: inverse propagator

$$\Delta^{-1}(k^2) = \mathrm{i}\left[k^2\mathbb{1} - \mathcal{M}_{\phi\phi} + \hat{\Sigma}_{\phi\phi}(k^2)\right]$$

- leading 2-loop self-energies are of "$\mathcal{O}(\alpha_t\alpha_s)$"$\hat{=}\,\mathcal{O}\left({Y_t}^2\alpha_s, \lambda Y_t\alpha_s, \lambda^2\alpha_s\right)$

$$\hat{\Sigma}^{(2L)}_{\phi\phi}(k^2) \approx \left(\begin{array}{cc|c} \mathcal{O}({Y_t}^2\alpha_s) & \mathcal{O}({Y_t}^2\alpha_s) & 0 \\ \mathcal{O}({Y_t}^2\alpha_s) & \mathcal{O}({Y_t}^2\alpha_s) & 0 \\ \hline 0 & 0 & 0 \end{array}\right)$$

$\Rightarrow$ MSSM-like contributions of $\mathcal{O}\left({Y_t}^2\alpha_s\right)$ form 2×2 sub matrix of 3×3 self-energy matrix

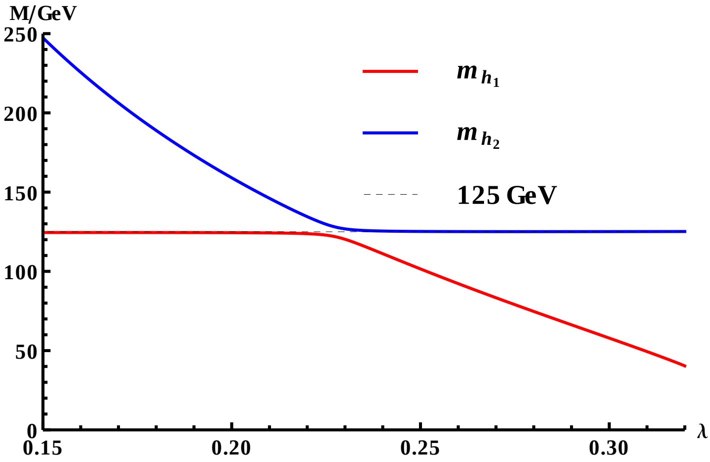
Sample Scenario

- genuine NMSSM-scenario with a second lightest CP-even state that can be interpreted as the Higgs-boson signal at 125 GeV and a lighter singlet-like state

$$M_{H^\pm} = 1000\ \mathrm{GeV}, \mu_{\text{eff}} = 125\ \mathrm{GeV},$$
$$A_\kappa = -300\ \mathrm{GeV},\ A_t = -2000\ \mathrm{GeV},$$
$$\tan\beta = 8,\ \kappa = 0.2$$

$$m_{\tilde{t}_1} \approx 1400\ \mathrm{GeV},\ m_{\tilde{t}_2} \approx 1600\ \mathrm{GeV}$$
$$m_{\tilde{b}_i} \approx 1500\ \mathrm{GeV},\ m_{\tilde{g}} \approx 1500\ \mathrm{GeV}$$

Lighter Masses @ 2-Loop Order



- mass decreasing with increasing λ belongs to singlet-like, constant mass to doublet-like state

Intermediate Summary: NMSSM-FeynHiggs

Discussed so far:

- The approximation of 2-loop NMSSM contributions by 2-loop MSSM contributions presented itself to be well motivated.

Discussed next: Comparison with other OS code (NMSSMCalc) . . .

- . . . as one (additional) validation of approximation.
- . . . for estimating theoretical uncertainties (work in progress).

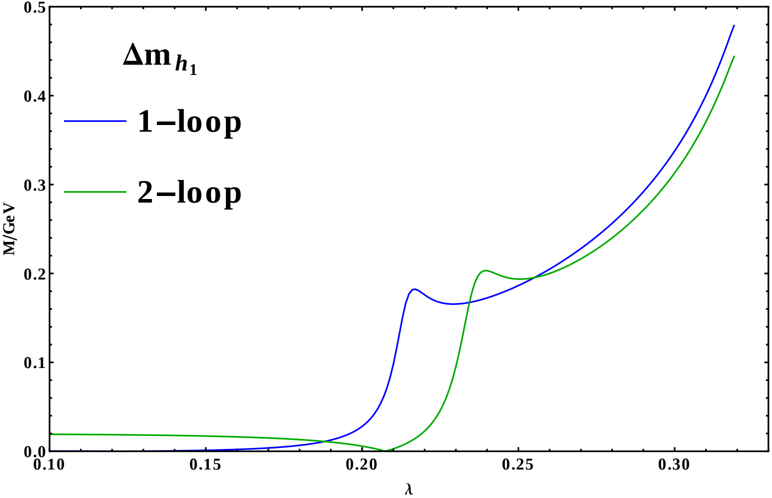
NB: Comparison of codes is a common effort between development teams!

Comparing Codes: Validation II

- Shown results are obtained with modified `NMSSMCalc` version that takes OS stop-parameters as input.
- `NMSSMCalc` and `NMSSM-FeynHiggs` use different renormalisation of the electroweak coupling constant.

	NMSSMCalc		NMSSM-FeynHiggs
1-loop	$\alpha_{\text{em}}(M_Z)$ renormalised	$\leftrightarrow$	$\alpha_{\text{em}}(M_Z)$ reparametrised
2-loop	NMSSM $\mathcal{O}(\alpha_s\alpha_t)$	$\leftrightarrow$	MSSM $\mathcal{O}(\alpha_s\alpha_t)$
			$+\ \mathcal{O}\left(\alpha_s\alpha_b, \alpha_t^2, \alpha_t\alpha_b\right)$
			+ LL- and NLL-resummation

Comparing Codes: Sample Scenario, $\Delta m_{h_1} = \left| m_{h_1}^{(\texttt{NMSSM-FH})} - m_{h_1}^{(\texttt{NMSSMCalc})} \right|$



$\Rightarrow$ Genuine NMSSM-Corrections at $\mathcal{O}(\alpha_s \alpha_t)$ account for a difference below 100 MeV.

Comparing Codes: Estimating theoretical Uncertainties

What has been done so far

- comparison of public $\overline{\text{DR}}$ codes [Staub et. al. '15]
  - $\Rightarrow$ SPheno, FlexibleSUSY, NMSSMTools, SoftSUSY, NMSSMCalc
  - $\Rightarrow$ thorough explanation of differences between codes

What needs to be done

- comparison of on-shell codes (NMSSMCalc, NMSSM−FeynHiggs)
  - $\Rightarrow$ work in progress, see this talk
- comparison of $\overline{\text{DR}}$ and on-shell codes
- estimation of theoretical uncertainty

Comparing Codes: *Preliminary Results*

- Scenario with SM-like Higgs around 125 GeV and slightly lighter singlet-like state (TP5 in [Staub et. al. '15]).

M_{SUSY}	$\tan\beta$	λ	μ_{eff}	M_1	M_2	M_3	A_t	$m_{\tilde{Q}_3}$
1500	3	0.67	200	135	200	1400	0	1500

- parameters run to and calculation performed at scale $m_t^{(\mathrm{OS})}$, `NMSSM-FeynHiggs` uses default value $\alpha_{\mathrm{em}}^{G_F}$

code	scheme	"singlet"	SM-Higgs
`NMSSM-FH` (2L)	OS-$t/\tilde{t}$	119.2 GeV	**121.6** GeV
`NMSSMCalc` (2L)	OS-$t/\tilde{t}$	118.8 GeV	**122.3** GeV
`NMSSMCalc` (2L)	$\overline{\mathrm{DR}}$-$t/\tilde{t}$	118.6 GeV	**125.3** GeV

⇒ First results indicate at numerical difference of 3 GeV of SM-Higgs mass for OS and $\overline{\mathrm{DR}}$-renormalised stops.

Conclusions

I `NMSSM-FeynHiggs`

- work in progress, release soon
- consistent with `FeynHiggs`, offers a good approximation for 2-loop contributions
- will contribute to the estimation of theoretical uncertainties in the NMSSM

II Status of estimating theoretical uncertainties for Higgs mass predictions in the NMSSM

- A common effort involving all (soon to be) available spectrum generators is on the way!

Backup

Motivation

- overview over different codes and their 2-loop contributions

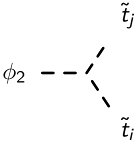
type	code	2-loop contributions
$\overline{\text{DR}}$	`SPheno`	complete w/o EW couplings
$\overline{\text{DR}}$/OS	`NMSSMCalc`	NMSSM $\mathcal{O}(\alpha_t\alpha_s)$
OS	`NMSSM-FH`	MSSM $\mathcal{O}\Big(\alpha_s\alpha_t, \alpha_s\alpha_b, \alpha_t^2, \alpha_t\alpha_b\Big)$, incl. LL- and NLL-resummation

Q: Why is using the MSSM corrections a valid option?

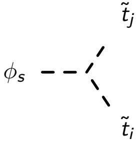
A: They are numerically leading for the fermion/sfermion contributions!

NMSSM: Leading Contributions

- In MSSM corrections of order $\mathcal{O}(\alpha_t \ldots)$ are dominant at 1- and 2-loop orders, $\alpha_t = Y_t{}^2/4\pi$
- In the NMSSM singlet-sfermion couplings are governed by λ instead of Y_t, e.g.



$$\approx i\sqrt{2}Y_t\,(m_t - A_t \sin 2\theta_{\tilde{t}}) = \mathcal{O}(Y_t)$$



$$\approx i\sqrt{2}\lambda m_t \sin 2\theta_{\tilde{t}} \qquad = \mathcal{O}(\lambda).$$

$\Longrightarrow$ Speaking of order $\mathcal{O}(\alpha_t \ldots)$ is misleading in the NMSSM!

Comparison of $\mathcal{O}(Y_t^2)$ with $\mathcal{O}(\lambda Y_t, \lambda^2)$

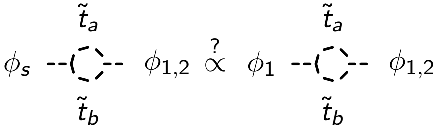
Q: Is neglecting the genuine NMSSM-contributions (3^{rd} column and row in self energy) a good approximation for all masses?

A: Compare MSSM-like and genuine NMSSM corrections at 1-loop order and apply to 2-loop order results.

Comparison of $\mathcal{O}\left(Y_t^2\right)$ with $\mathcal{O}\left(\lambda Y_t, \lambda^2\right)$

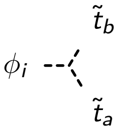
Q: Is there a relation between MSSM-like diagrams and genuine NMSSM-diagrams?

e.g.



A: Yes, since diagrams only differ in values for the Higgs-stop-stop vertices.

e.g. $i\Gamma_{\phi_i \tilde{t}_a \tilde{t}_b} =$



$\Rightarrow$ genuine NMSSM diagrams can be quantified by ratio between the diagrams

Comparison of $\mathcal{O}\left(Y_t^2\right)$ with $\mathcal{O}\left(\lambda Y_t, \lambda^2\right)$

- diagrams with trilinear couplings can be compared by ratio of the couplings

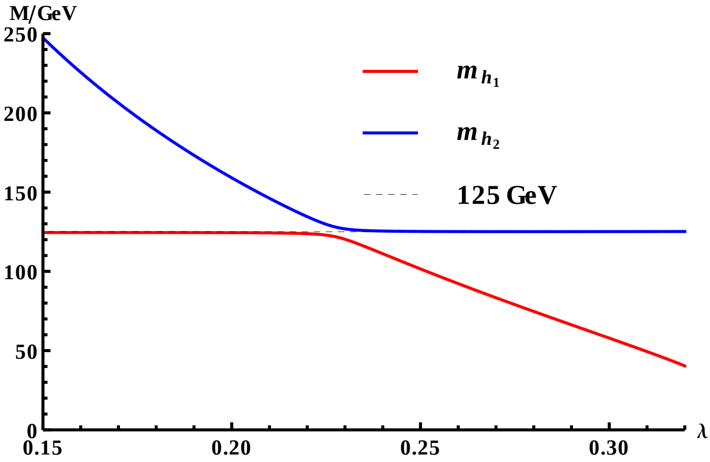
$$\frac{\Gamma_{\phi_s \tilde{t}_a \tilde{t}_b}}{\Gamma_{\phi_1 \tilde{t}_a \tilde{t}_b}} = \lambda \frac{v}{\mu_{\text{eff}}} \cos\beta$$

$$\frac{\Gamma_{\phi_s \tilde{t}_a \tilde{t}_b}}{\Gamma_{\phi_2 \tilde{t}_a \tilde{t}_b}} = \begin{cases} \lambda \frac{v}{A_t} \cos\beta & \text{if } a \neq b \\ \lambda \frac{v \sin 2\theta_{\tilde{t}}}{2m_t \pm A_t \sin 2\theta_{\tilde{t}}} \cos\beta & \text{if } a = b \end{cases}$$

- diagrams with quartic couplings can be compared by ratio of the couplings

$$\frac{\Gamma_{\phi_s \phi_1 \tilde{t}_a \tilde{t}_b}}{\Gamma_{\phi_2 \phi_2 \tilde{t}_a \tilde{t}_b}} = \frac{\lambda}{Y_t} \left| \frac{1}{2} \sin 2\theta_{\tilde{t}} \right|$$

Lighter Masses @ 2-Loop Order



- mass decreasing with increasing λ belongs to singlet-like, constant mass to doublet-like state

Suppression Factors

- diagrams with trilinear couplings can be compared by ratio of the couplings

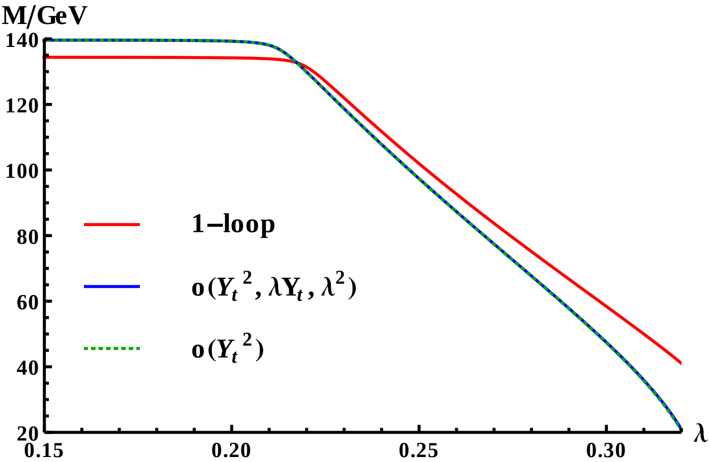
$$\frac{\Gamma_{\phi_s \tilde{t}_a \tilde{t}_b}}{\Gamma_{\phi_1 \tilde{t}_a \tilde{t}_b}} \lesssim 5.5\%$$

$$\frac{\Gamma_{\phi_s \tilde{t}_a \tilde{t}_b}}{\Gamma_{\phi_2 \tilde{t}_a \tilde{t}_b}} \lesssim \begin{cases} 4‰ & \text{if } a \neq b \\ 3‰ & \text{if } a = b \end{cases}$$

- diagrams with quartic couplings can be compared by ratio of the couplings

$$\frac{\Gamma_{\phi_s \phi_1 \tilde{t}_a \tilde{t}_b}}{\Gamma_{\phi_2 \phi_2 \tilde{t}_a \tilde{t}_b}} \lesssim 16.5\%$$

Lightest Mass @ 1-Loop Order

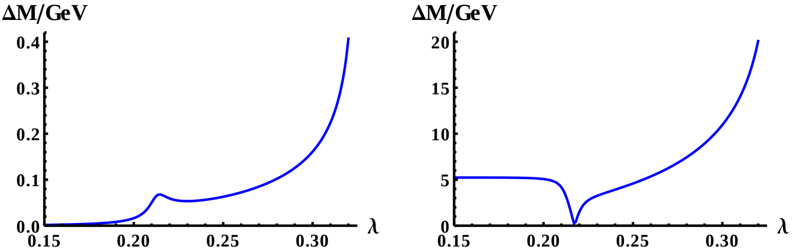


$\Longrightarrow$ prediction with or without corrections of order $\mathcal{O}\left(Y_t\lambda, \lambda^2\right)$ are not distinguishable in the plot

$\Longrightarrow$ influence of corrections $\propto Y_t\lambda,\ \lambda^2$ from stops is tiny

Lightest Mass @ 1-Loop Order

Absolute difference between different mass predictions

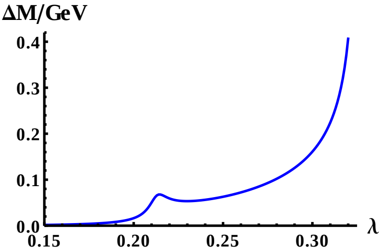


$$\Delta M = \left| m_{h_1}^{(Y_t)} - m_{h_1}^{(Y_t,\lambda)} \right| \qquad \Delta M = \left| m_{h_1}^{(Y_t,\lambda)} - m_{h_1}^{(1\mathrm{L})} \right|$$

$\Longrightarrow$ influence of corrections beyond top/scalar top-sector is by far larger than those of the order $\mathcal{O}\left(Y_t\lambda, \lambda^2\right)$

Lightest Mass @ 1-Loop Order

Absolute difference between different mass predictions



$$\Delta M = \left| m_{h_1}^{(Y_t)} - m_{h_1}^{(Y_t,\lambda)} \right| \qquad \Delta M = \left| m_{h_1}^{(Y_t,\lambda+\mathrm{H}+\mathrm{G})} - m_{h_1}^{(1\mathrm{L})} \right|$$

include Higgs- & gauge-sector

$\Longrightarrow$ influence of corrections beyond top/scalar top-sector is by far larger than those of the order $\mathcal{O}\left(Y_t\lambda, \lambda^2\right)$

Comparing Codes: Estimating theoretical Uncertainties

- Comparison takes place for several physical scenarios with different properties (e.g. MSSM-like, large stop-mass splitting, genuine NMSSM).
- Scenarios are defined generally for use in $\overline{\text{DR}}$- and OS-calculations!

  Scenarios defined at several scales by $\overline{\text{DR}}$-parameters according to standards of the SUSY Les-Houches Accord (SLHA).

$\Rightarrow$ Some $\overline{\text{DR}}$-parameters of the scenarios need to be converted into OS-parameters internally or externally to avoid higher-order effects!

Comparing Codes: *Preliminary Results*

- Scenario with SM-like Higgs around 125 GeV and slightly lighter singlet-like state (TP5 in [Staub et. al. '15]).

M_{SUSY}	$\tan\beta$	λ	μ_{eff}	M_1	M_2	M_3	A_t	$m_{\tilde{Q}_3}$
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- parameters run to and calculation performed at scale $m_t^{(\text{OS})}$, `NMSSM-FeynHiggs` uses default value $\alpha_{\text{em}}^{G_F}$

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`NMSSMCalc` (2L)	$\overline{\text{DR}}$-$t/\tilde{t}$	118.6 GeV	**125.3** GeV
`NMSSM-FH` (1L)	OS-$t/\tilde{t}$	120.4 GeV	**136.6** GeV
`NMSSMCalc` (1L)	OS-$t/\tilde{t}$	120.7 GeV	**137.7** GeV

NMSSM: Superpotential

$$\mathcal{W} = \; Y_t \widehat{Q}\widehat{H}_2\widehat{t} - Y_b \widehat{Q}\widehat{H}_1\widehat{b} - Y_\tau \widehat{L}\widehat{H}_1\widehat{\tau} + \lambda \widehat{S}\widehat{H}_2\widehat{H}_1 + \frac{\kappa}{3}\widehat{S}^3$$

$$H_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}\left(\phi_1 - \mathrm{i}\chi_1\right) \\ -\phi_1^- \end{pmatrix} \qquad H_2 = \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}\left(\phi_2 + \mathrm{i}\chi_2\right) \end{pmatrix}$$

$$S = v_s + \frac{1}{\sqrt{2}}\left(\phi_s + \mathrm{i}\chi_s\right)$$

$\Rightarrow$ conventions follow `FeynHiggs` for MSSM-like part

NMSSM: Superpotential

$$\mathcal{W} = Y_t \widehat{Q}\widehat{H}_2\widehat{t} - Y_b \widehat{Q}\widehat{H}_1\widehat{b} - Y_\tau \widehat{L}\widehat{H}_1\widehat{\tau} + \lambda \widehat{S}\widehat{H}_2\widehat{H}_1 + \frac{\kappa}{3}\widehat{S}^3$$

Yukawa-coupling of the top-quark:

$$Y_t = \frac{m_t}{v_2} \approx 1 \;\Rightarrow\; \alpha_t = \frac{Y_t^2}{4\pi}$$

"Yukawa-couplings" of the Higgs sector

$$\lambda^2 + \kappa^2 \lesssim 0.5 \;\Rightarrow\; \lambda,\ \kappa < 0.7$$

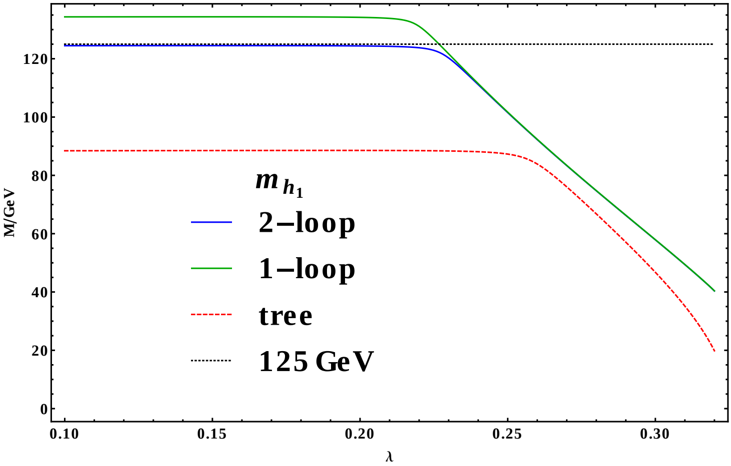
$\Longrightarrow$ for perturbative predictions up to high scales: $Y_t > \lambda$

Sample Scenario

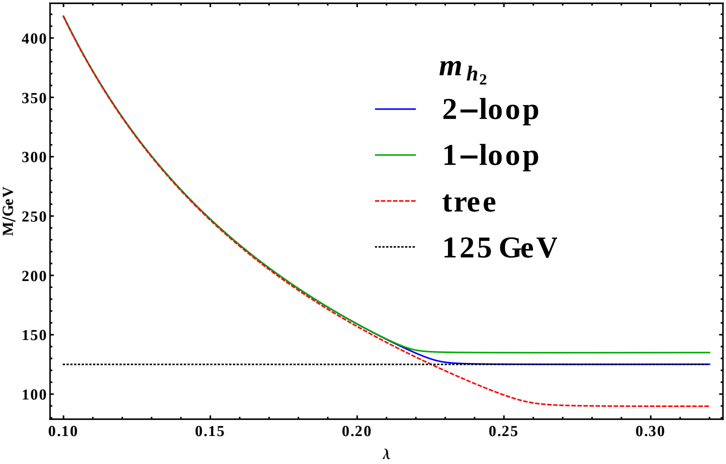
$$M_{H^\pm} = 1000 \text{ GeV}, \mu_{\mathrm{eff}} = 125 \text{ GeV},$$
$$A_\kappa = -300 \text{ GeV}, \; A_t = -2000 \text{ GeV},$$
$$\tan\beta = 8, \; \kappa = 0.2$$

$$m_{\tilde{t}_1} \approx 1400 \text{ GeV}, \; m_{\tilde{t}_2} \approx 1600 \text{ GeV}$$
$$m_{\tilde{b}_i} \approx 1500 \text{ GeV}, \; m_{\tilde{g}} \approx 1500 \text{ GeV}$$
$$M_{\chi_i^\pm} \approx \{110, \; 330\} \text{ GeV}$$
$$M_{\chi_i^0} \approx \{80, \; 140, \; 160, \; \mathit{190}, \; 330\} \text{ GeV}$$

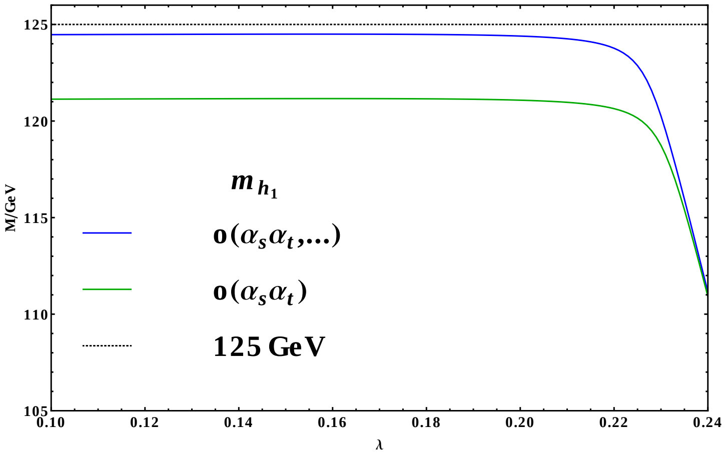
2-Loop Results: Lightest State h_1



2-Loop Results: Lightest State h_2

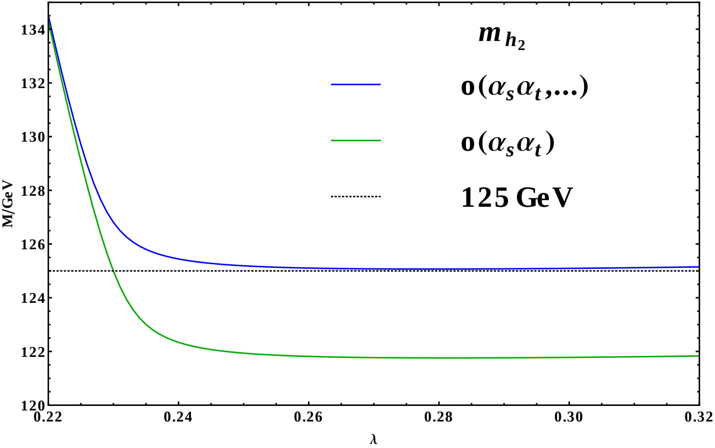


Mass of lightest State h_1



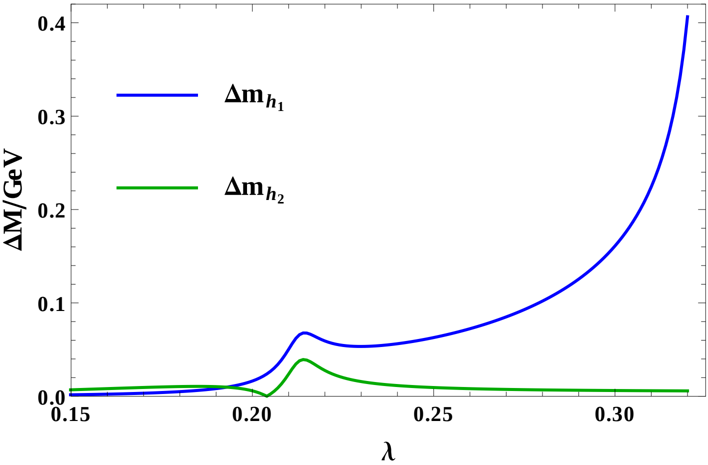
- corrections at 2-loop order $\mathcal{O}\left(\alpha_s\alpha_t, \alpha_s\alpha_b, \alpha_t^2, \alpha_t\alpha_b\right)$ including resummed logarithms from the MSSM have sizeable impact on doublet fields while overall reliability for very light singlet-mass decreases with λ

Mass of next-to lightest State h_2



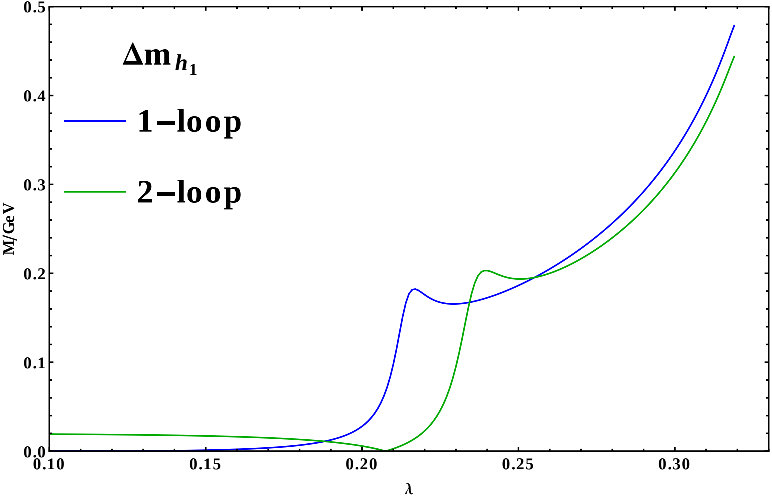
- corrections at 2-loop order $\mathcal{O}\left(\alpha_s\alpha_t, \alpha_s\alpha_b, \alpha_t^2, \alpha_t\alpha_b\right)$ including resummed logarithms from the MSSM have sizeable impact on doublet fields while overall reliability for very light singlet-mass decreases with λ

Backup: Comparison with `NMSSMCalc` I



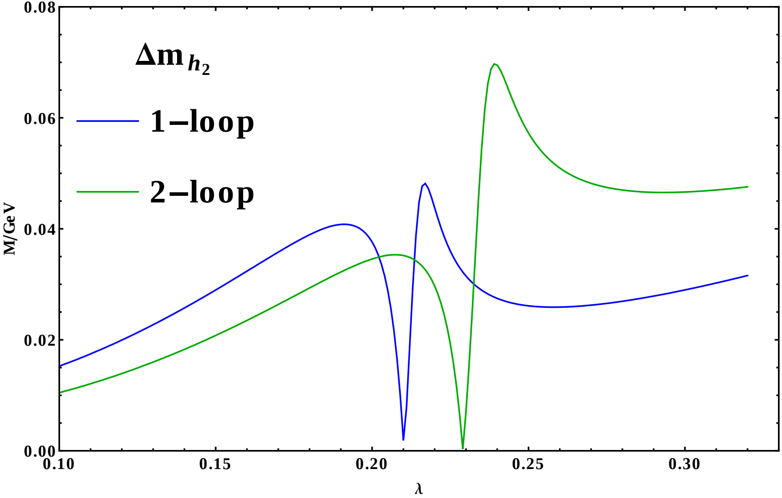
Backup: Comparison with NMSSMCalc II

- difference 1-loop: δZ_e, difference 2-loop: $\mathcal{O}\Big(Y_t\lambda, \lambda^2\Big)$
- $\Delta m_{h_1} = \Big| m_{h_1}^{(\mathrm{FH})} - m_{h_1}^{(\mathrm{NC})} \Big|$



Backup: Comparison with NMSSMCalc III

- difference 1-loop: δZ_e, difference 2-loop: $\mathcal{O}\Big(Y_t\lambda, \lambda^2\Big)$
- $\Delta m_{h_2} = \Big| m_{h_2}^{(\mathrm{FH})} - m_{h_2}^{(\mathrm{NC})} \Big|$



TP5

M_{SUSY}	$\tan\beta$	λ	κ	A_λ	A_κ	μ_{eff}	M_1	M_2	M_3	A_t	A_b	$m_{\tilde{Q}_3}$	$m_{\tilde{t}_R}$
1500	3	0.67	0.2	570	-25	200	135	200	1400	0	0	1500	1500

Backup: Reparametrisation I

- generic bare coupling g_0 expressed in renormalisation schemes I and II

$$g_0 = g^I \left(1 + \delta Z_g^I\right) = g^{II} \left(1 + \delta Z_g^{II}\right)$$

- relation between the renormalised couplings g in both schemes

$$g^I = g^{II} \left(1 + \delta Z_g{}^I - \delta Z_g{}^{II}\right) \equiv g^{II} \left(1 + \Delta Z_g\right)$$

- result applied for α

$$\Delta Z_\alpha = \left(2\delta Z_e^I - 2\delta Z_e^{II}\right) \ .$$

Backup: Reparametrisation II

$$\Delta Z_\alpha = \left(2\delta Z_e^I - 2\delta Z_e^{II}\right)$$

- charge renormalisation constant in our calculation (II)

$$\delta Z_e^{II} = \frac{1}{2}\left(\frac{\delta s_W^2}{s_W^2} + \frac{\delta M_W^2}{M_W^2} - \frac{\delta v^2}{v^2}\right) = \frac{1}{2}\left(\frac{\delta s_W^2}{s_W^2} + \frac{\delta M_W^2}{M_W^2}\right)$$

- charge renormalisation constant for reparametrisation to $\alpha(M_Z)$

$$\delta Z_e^I = \delta Z_e^{\text{Thomson}} - \frac{1}{2}\left(\Delta\alpha_{\text{had}}^{(5)} + \Delta\alpha_{\text{lep}}\right)$$

- charge renormalisation constant for reparametrisation to α_{G_F}

$$\delta Z_e^I = \delta Z_e^{\text{Thomson}} - \frac{1}{2}\Delta r$$

Renormalisation Scheme: Charge Renormalisation I

- $\overline{\text{DR}}$-renormalisation of the vacuum expectation-value

$$v = \frac{\sqrt{2} s_W M_W}{e}, \quad v \to v + \delta v$$

- implicit renormalisation of the electric charge

$$\left[\frac{\delta v^2}{v^2}\right]^{\overline{\text{DR}}} = \frac{\delta s_W^2}{s_W^2} + \frac{\delta M_W^2}{M_W^2} - 2\delta Z_e$$

$$\Rightarrow [\delta Z_e]^{\text{fin.}} = \frac{1}{2}\left[\frac{\delta s_W^2}{s_W^2} + \frac{\delta M_W^2}{M_W^2}\right]$$

- reparametrisation for electric charge necessary to use $\alpha(G_F)$

Renormalisation Scheme: Charge Renormalisation II

- renormalised self-energy at 1-loop order from fermions/sfermions

$$\hat{\Sigma}_{\phi_i\phi_s}\left(k^2\right) = \Sigma_{\phi_i\phi_s}\left(k^2\right) - \delta V_{\phi_i\phi_s}$$

$\Rightarrow$ $\Sigma_{\phi_i\phi_s}$ *independent* of κ

- counterterm

$$\delta V_{\phi_i\phi_s} \supset -\kappa\lambda v_s v_j \left(\frac{\delta\lambda}{\lambda} + \frac{\delta\kappa}{\kappa} + \frac{\delta v}{v} + \mathcal{O}(\delta\tan\beta)\right) \stackrel{!}{=} 0$$

$\Rightarrow$ has to be finite, since $\Sigma_{\phi_i\phi_s}$ is independent of κ

$\Rightarrow$ κ-dependent finite contribution if term in brackets gives rise to finite contributions

Backup: Constraints on Parameters

- constraint from demanding stable minimum of potential [Ellwanger, Hugonie, Teixeira, '09]

$$A_\kappa^2 \gtrsim 9m_S^2$$
$$-2\sqrt{2}\kappa v_S \lesssim A_\kappa \lesssim 0$$

- constraint from demand for no Landau-pole of Yukawa-couplings below the GUT-scale [Miller, Nevzorov, Zerwas '03]

$$\lambda^2 + \kappa^2 \lesssim 0.5$$
$$\lambda^2,\ \kappa^2 \lesssim 0.7$$

Backup: Couplings

- Higgs-sfermion-couplings

$$\Gamma_{\phi_i \tilde{f}_L \tilde{f}_R} \propto \lambda v_s Y_f = \mu_{\text{eff}} Y_f$$
$$\Gamma_{\phi_s \tilde{f}_L \tilde{f}_R} \propto \lambda v_i Y_f \propto m_f \lambda$$

- Higgs-Higgs-couplings

$$\Gamma_{\phi_1 \phi_2 \phi_s \phi_s} \propto \lambda \kappa, \quad \Gamma_{\phi_1 \phi_2 \phi_1 \phi_2} \propto \lambda^2$$

Backup: Codes & References

FeynArts 3.9	hep-ph/0012260
FormCalc 7.4	hep-ph/9807565
LoopTools 2.12	hep-ph/9807565
FeynHiggs 2.10.3	feynhiggs.de
NMSSMTools 4.5.1	hep-ph/0406215
NMSSMCALC 1.03	arXiv:1312.4788
HiggsBounds 4.2.0	higgsbounds.hepforge.org

[Ellwanger, Hugonie, Teixeira, '09]	arXiv:0910.1785
[Heinemeyer, Rzehak, Weiglein, et. al. '10]	arXiv:1007.0956
[Heinemeyer, Weiglein, Zeune, et. al. '12]	arXiv:1207.1096
[Miller, Nevzorov, Zerwas '03]	hep-ph/0304049