

# $h \rightarrow \mu\tau$ as an indication for $S_4$ flavor symmetry

Erik Schumacher, TU Dortmund

Based on Phys. Rev. D 91, 116011  
In collaboration with M. Campos,  
A. E. Cárcamo Hernández and H. Päs

DESY Hamburg, 30.09.15

# Motivation

## Could lepton flavor be violated?

- $2.4\sigma$  anomaly in  $h \rightarrow \mu\tau$

CMS:  $\text{Br}(h \rightarrow \mu\tau) = (0.84 \pm 0.39)\%$

[CMS Collaboration, Phys.Lett. B749 (2015)]

ATLAS:  $\text{Br}(h \rightarrow \mu\tau) = (0.77 \pm 0.62)\%$

[ATLAS Collaboration, ArXiv:1508.03372 (2015)]

- A typical signature of Multi-Higgs models

## Objective

- Build a Multi-Higgs model (here: 3HDM) that predicts  $h \rightarrow \mu\tau$  avoiding FCNC constraints

# Main tool: Lepton Flavor Triality

- Global symmetry group (e.g.,  $S_4$ ) is broken down to a residual  $Z_3$  subgroup in the lepton sector

[Cao, Damanik, Ma, Wegman, Phys.Rev. D83, 093012 (2011)]

$$U_l = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \quad \omega = e^{2i\pi/3}$$

$$(e, \mu, \tau) \sim (1, \omega^2, \omega) \quad (\phi_a, \phi_b, \phi_c) \sim (1, \omega, \omega^2)$$

- Preserved  $Z_3$  quantum numbers forbid certain interactions, among them the  $l \rightarrow l' \gamma$  decays  $\rightarrow$  FCNCs suppressed!
- Additional scalar doublets can decay into LFV final states

# Why $S_4$ ?

## Requirements:

Correct mixing, Triality,  $Z_3$  symmetry, 3 Higgs doublets, ...

- $S_4$  naturally leads to TBM mixing, perturbations account for  $\theta_{13} \approx 9^\circ$
- The global minimum  $v(1, 1, 1)$  of the  $S_4$  potential preserves a  $Z_3$  subgroup

$$U_{\text{PMNS}} \approx \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$S_4$  is (together with  $A_4$  and  $\Delta(27)$ ) the smallest group that fulfills our requirements

# The Model (charged leptons)

## Ingredients:

- $L = (L_e, L_\mu, L_\tau) : \mathbf{3}, \quad \tau_R : \mathbf{1}, \quad (e_R, \mu_R) : \mathbf{2}$
- Three  $SU(2)$  doublets to induce LFV  $\phi = (\phi_1, \phi_2, \phi_3) : \mathbf{3}$
- VEV alignment  $\frac{v}{\sqrt{3}}(1, 1, 1)$  of the  $\phi_i$  breaks  $S_4$  down to  $Z_3$

## Result:

$$M_l = \frac{v_{SM}}{\sqrt{6}} \begin{pmatrix} 1 & 1 & 1 \\ \omega^2 & \omega & 1 \\ \omega & \omega^2 & 1 \end{pmatrix} \text{diag}(y_e, y_\mu, y_\tau)$$

$\Rightarrow$  Charged lepton and scalar mass bases  
coincide with  $Z_3$  basis

$$(e, \mu, \tau) \sim (\omega, \omega^2, 1), \quad (\phi_a, \phi_b, \phi_c) \sim (\omega^2, \omega, 1)$$

# Yukawa Couplings

In the mass bases (of scalars and leptons) the charged lepton Yukawa couplings are:

$$\phi_{(a)b} : Y_{(a)b}^0 = (i) \frac{1}{v\sqrt{2}} \begin{pmatrix} 0 & m_\mu \omega^2 & (-)m_\tau \omega \\ (-)m_e \omega & 0 & m_\tau \omega^2 \\ m_e \omega^2 & (-)m_\mu \omega & 0 \end{pmatrix}$$
$$\phi_c : Y_c^0 = \frac{1}{v} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

$\Rightarrow \phi_c^0$  couplings are SM Higgs-like  $\Rightarrow \phi_c^0 \equiv h_{\text{SM}}$

$\Rightarrow \phi_{a,b}^0$  can decay into LFV final states, **but  $\phi_c^0 \equiv h_{\text{SM}}$  cannot!**

$\Rightarrow$  Since  $\phi_{a,b}$  and  $\phi_c$  are distinct  $Z_3$  states, they do not mix

*How do we explain  $h \rightarrow \mu\tau$ ?*  
 $\Rightarrow$  **Triality needs to be broken!**

# Breaking $Z_3$

*What about quarks and neutrinos?*

- $Z_3$  actually broken by extra (not  $Z_3$ -symmetric) terms required to generate quark and neutrino mixings
- Global minimum not exactly  $(1, 1, 1)$  but  $(1 + 2\epsilon, 1 - \epsilon, 1 - \epsilon)$
- In scalar mass basis  $\langle(\phi_a, \phi_b, \phi_c)\rangle = v(0, \sin \theta, \cos \theta)$   
[Heeck et al, Nucl. Phys. B896 (2015) 281-310]
  - $\Rightarrow$   $\theta$  accounts for deviation from triality
  - $\Rightarrow$  Deviation leads to new mixing of  $\phi_b$  and  $\phi_c$
  - $\Rightarrow$  New mass eigenstates  $h$  (SM Higgs) and  $H$
  - $\Rightarrow$  Small fraction of  $h$  decays into  $\mu\tau$  depending on  $\theta$

# Flavor violation

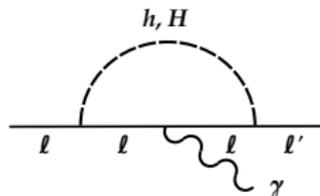
**Problem:** With flavor violation also come **FCNCs** ...

- Dominant LFV Yukawa Higgs couplings ( $y_{ll'}, hll'$ )

$$y_{e\tau}, y_{\mu\tau} \propto \frac{m_\tau}{v_{\text{SM}}} \sin(\theta), \quad y_{e\mu} \propto \frac{m_\mu}{v_{\text{SM}}} \cdot f(\theta)$$

- $y_{\mu\tau}$  decisive for  $\text{Br}(h \rightarrow \mu\tau)$   
and  $y_{e\mu}$  tightly constrained by  
 $\text{Br}(\mu \rightarrow e\gamma) < 5.7 \times 10^{-13}$

[MEG, Phys.Rev.Lett. 110 (2013)]

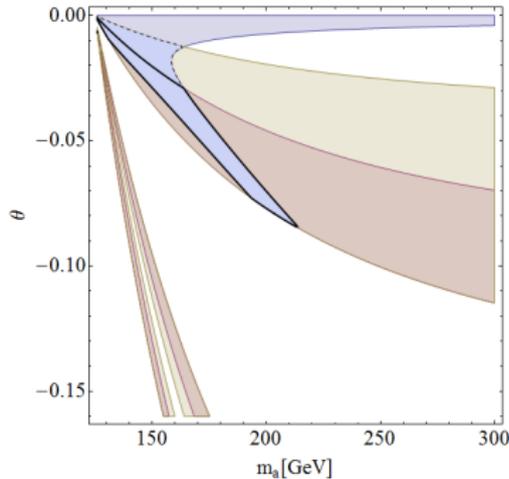
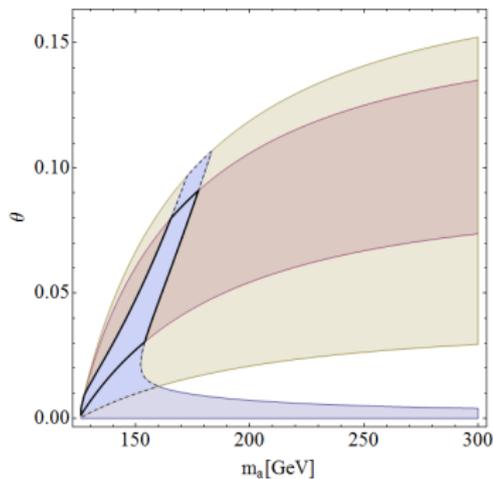


**Objective:**

Find regions in the  $\theta$  parameter space that explain  $\text{Br}(h \rightarrow \mu\tau)$  and at the same time comply with  $\text{Br}(\mu \rightarrow e\gamma)$  data!

# Experimental constraints

- 1  $\text{Br}(h \rightarrow \mu\tau)_{\text{CMS}} \Leftrightarrow 0.0019 < \sqrt{|y_{\mu\tau}|^2 + |y_{\tau\mu}|^2} < 0.0032$  (@ $1\sigma$ )  
[I. Dorsner et al., 1502.07784]
- 2  $\text{Br}(\mu \rightarrow e\gamma) = \frac{\tau_\mu \alpha_{\text{EM}} m_\mu^5}{64\pi^4} (|c_L|^2 + |c_R|^2) < 5.7 \times 10^{-13}$  (MEG)



# Predictions

@ $1\sigma$  and  $m_\eta = 200$  GeV

$$\begin{array}{ccccc} 0.002 & < & \theta & < & 0.090, \\ 126 \text{ GeV} & < & m_H & < & 204 \text{ GeV} \end{array}$$

- With these parameters we predict

$$\begin{array}{ll} \text{Br}(\tau \rightarrow \mu\gamma) & \in (0.3 - 1.3) \times 10^{-8}, & < 4.4 \times 10^{-8} \\ \text{Br}(\tau \rightarrow e\gamma) & \in (0.3 - 1.3) \times 10^{-8}, & < 1.2 \times 10^{-7} \\ \text{Br}(\tau \rightarrow 3\mu) & \in (1.9 - 8.1) \times 10^{-10}, & < 2.1 \times 10^{-8} \\ \text{Br}(\tau \rightarrow 3e) & \in (0.9 - 4.1) \times 10^{-9}, & < 2.7 \times 10^{-8} \\ \text{Br}(h \rightarrow e\tau) & \in (0.4 - 1.2) \times 10^{-2} \\ \text{Br}(h \rightarrow e\mu) & \in (1.5 - 4.2) \times 10^{-5} \end{array}$$

$\Rightarrow$  Large LFV also expected in  $h \rightarrow e\tau$  channel

# Conclusion

- We have investigated a flavor model based on the discrete  $S_4$  symmetry that can give rise to LFV Higgs decays
- Lepton Flavor Triality is employed to suppress the constrained radiative decays  $l \rightarrow l' \gamma$
- A small breaking of triality can explain the  $2.4\sigma$  anomaly in  $h \rightarrow \mu\tau$  reported by CMS if the extra neutral scalars are light
- Large branching fractions are also expected for  $h \rightarrow e\mu$  and  $h \rightarrow e\tau$  in this framework

**Thank you!**

**Backup**

# $S_4$ Symmetry

- Irreducible presentations of  $S_4$

$$1, 1', 2, 3, 3'$$

- The  $S_4$  product rules

$$\begin{aligned} 3' \otimes 3' &= 1 \oplus 2 \oplus 3 \oplus 3', & 3 \otimes 3' &= 1' \oplus 2 \oplus 3 \oplus 3', \\ 2 \otimes 2 &= 1 \oplus 1' \oplus 2, & 2 \otimes 3 &= 3 \oplus 3', & 2 \otimes 3' &= 3' \oplus 3, \\ 3 \otimes 1' &= 3', & 3' \otimes 1' &= 3, & 2 \otimes 1' &= 2. \end{aligned}$$

## Scalar Sector (Triality limit)

$$\phi_j = \left[ \phi_j^+, \quad \frac{1}{\sqrt{2}} \left( \frac{v}{\sqrt{3}} + \phi_{jR}^0 + i\phi_{jI}^0 \right) \right]^T$$

- 3 CP even neutral scalars  $\phi_{jR}^0$  ( $j = 1, 2, 3$ ), 3 CP odd neutral scalars  $\phi_{jI}^0$ , 3 complex charged scalars  $\phi_j^+$
- 3 degrees of freedom eaten by the SM gauge bosons  
 $\Rightarrow$  In total 9 massive (low-energy) scalars  $\phi_{a,b,c}$   
with  $\langle \phi_a \rangle = \langle \phi_b \rangle = 0$  and  $\langle \phi_c \rangle = v_{SM}$

## Scalar Sector (Triality limit)

$$V(\phi) = -\mu_1^2 \sum_{i=1}^3 \phi_i^\dagger \phi_i + \alpha \left( \sum_{i=1}^3 \phi_i^\dagger \phi_i \right)^2 \\ + \sum_{i,j=1, i \neq j}^3 (\beta (\phi_i^\dagger \phi_i) (\phi_j^\dagger \phi_j) + \gamma |\phi_i^\dagger \phi_j|^2 + \delta (\phi_i^\dagger \phi_j)^2)$$

$$\begin{aligned} m_{\phi_{(a,b)R}}^2 &= -\frac{v^2}{3} \kappa, & m_{\phi_{cR}}^2 &= \frac{v^2}{3} (3\alpha + 2\kappa), \\ m_{\phi_{a,b}^\pm}^2 &= -v^2 (\kappa - \beta), & m_{\phi_{cl}^0}^2 &= 0, \\ m_{\phi_{(a,b)l}^0}^2 &= -v^2 \delta, & m_{\phi_c^\pm}^2 &= 0. \end{aligned}$$

with  $\kappa := \beta + \gamma + \delta$ .

# Scalar Sector

- Mass eigenstates  $\phi_{a,b,c}$

$$\begin{pmatrix} \phi_a \\ \phi_b \\ \phi_c \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sqrt{2}}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \cdot \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

- Comparing with the  $Z_3$  eigenstates  $\phi_{x,y,z} \sim 1, \omega, \omega^2$

$$\begin{pmatrix} \phi_x \\ \phi_y \\ \phi_z \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

we find that  $Z_3$  remains unbroken

$$\phi_c \equiv \phi_x, \quad \phi_b \equiv \frac{1}{\sqrt{2}}(\phi_y + \phi_z), \quad \phi_a \equiv \frac{1}{\sqrt{2}}(\phi_y - \phi_z)$$

$$\text{with } \langle \phi_a \rangle = \langle \phi_b \rangle = 0 \quad \text{and} \quad \langle \phi_c \rangle = v_{\text{SM}}.$$

# Charged leptons

- The particle assignments in the notation  $(S_4, SU(2), Z_{12})$  are

$$L = (L_e, L_\mu, L_\tau) : (3', 2, 1), \quad \tau_R : (1, 1, -i), \quad (e_R, \mu_R) : (2, 1, e^{\frac{7i\pi}{6}}),$$
$$\phi = (\phi_1, \phi_2, \phi_3) : (3', 2, 1), \quad \eta_1 : (1, 1, e^{\frac{i\pi}{6}}), \quad \eta_2 : (1', 1, e^{\frac{i\pi}{6}}).$$

- Three Higgs  $SU(2)$  doublets  $\phi_i$  induce LFV
- $Z_{12}$  and  $\eta_{1,2}$  necessary for a realistic lepton mass hierarchy

# Charged leptons

## Charged lepton Lagrangian

$$\begin{aligned}\mathcal{L} \supset y_1 [L\phi]_1 \tau_R \frac{\eta_1^3 + \varepsilon_0 \eta_1 \eta_2^2}{\Lambda^3} \\ + y_2 [L\phi]_2 \begin{pmatrix} e_R \\ \mu_R \end{pmatrix} \frac{\eta_1^5 + \varepsilon_1 \eta_1^3 \eta_2^2 + \varepsilon_2 \eta_1 \eta_2^4}{\Lambda^5} \\ + y_3 [L\phi]_2 \begin{pmatrix} e_R \\ \mu_R \end{pmatrix} \frac{\eta_2^5 + \varepsilon_3 \eta_2^3 \eta_1^2 + \varepsilon_4 \eta_2 \eta_1^4}{\Lambda^5}\end{aligned}$$

leads to the following mass matrix for charged leptons:

$$\begin{aligned}M_l &= \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 (\tilde{y}_2 - \tilde{y}_3) \lambda^5 & v_1 (\tilde{y}_2 + \tilde{y}_3) \lambda^5 & v_1 \tilde{y}_1 \lambda^3 \\ v_2 \omega^2 (\tilde{y}_2 - \tilde{y}_3) \lambda^5 & v_2 \omega (\tilde{y}_2 + \tilde{y}_3) \lambda^5 & v_2 \tilde{y}_1 \lambda^3 \\ v_3 \omega (\tilde{y}_2 - \tilde{y}_3) \lambda^5 & v_3 \omega^2 (\tilde{y}_2 + \tilde{y}_3) \lambda^5 & v_3 \tilde{y}_1 \lambda^3 \end{pmatrix} \\ &= v \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 1 & 1 \\ \omega^2 & \omega & 1 \\ \omega & \omega^2 & 1 \end{pmatrix} \text{diag}((\tilde{y}_2 - \tilde{y}_3) \lambda^5, (\tilde{y}_2 + \tilde{y}_3) \lambda^5, \tilde{y}_1 \lambda^3)\end{aligned}$$

with  $v_1 = v_2 = v_3 = \frac{v}{\sqrt{3}}$ ,  $\omega = e^{2i\pi/3}$

$$\tilde{y}_1 = (1 + \varepsilon_0) y_1, \quad \tilde{y}_2 = (1 + \varepsilon_1 + \varepsilon_2) y_2, \quad \tilde{y}_3 = (1 + \varepsilon_3 + \varepsilon_4) y_3$$

# Neutrino sector

The  $S_4 \otimes Z_2 \otimes Z'_2 \otimes Z_{12}$  assignments are

$$N_{1R} : (1, -1, 1, 1), \quad N_{2R} : (1, -1, -1, 1), \\ \chi : (3', -1, 1, 1), \quad \xi : (3, -1, 1, 1), \quad \sigma : (3', -1, -1, 1), \quad \zeta : (3, -1, -1, 1).$$

$$\mathcal{L} \quad \supset y_1^{(\nu)} [L\phi]_{3'} N_{1R} \frac{\chi}{\lambda} + y_2^{(\nu)} [L\phi]_3 N_{1R} \frac{\xi}{\lambda} + y_3^{(\nu)} [L\phi]_{3'} N_{2R} \frac{\sigma}{\lambda} \\ + y_4^{(\nu)} [L\phi]_3 N_{2R} \frac{\zeta}{\lambda} + \frac{y_5^{(\nu)}}{\lambda} L[\phi\chi]_{3'} N_{1R} + \frac{y_6^{(\nu)}}{\lambda} L[\phi\xi]_{3'} N_{1R} \\ + \frac{y_7^{(\nu)}}{\lambda} L[\phi\sigma]_{3'} N_{2R} + \frac{y_8^{(\nu)}}{\lambda} L[\phi\zeta]_{3'} N_{2R} + M_1 \bar{N}_{1R} N_{1R}^c + M_2 \bar{N}_{2R} N_{2R}^c.$$

# Neutrino sector

Mass matrices

$$M_\nu^D = \frac{v}{\sqrt{3}} = \begin{pmatrix} 0 & ae^{i\tau} \\ b & 0 \\ 0 & ae^{-i\tau} \end{pmatrix}, \quad M_R = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$$

$$\begin{aligned} M_L^{(\nu)} &= M_\nu^D M_R^{-1} (M_\nu^D)^T = \begin{pmatrix} a^2 e^{2i\tau} & 0 & a^2 \\ 0 & b^2 \frac{M_2}{M_1} & 0 \\ a^2 & 0 & a^2 e^{-2i\tau} \end{pmatrix} \frac{v^2}{3M_2} \\ &= \begin{pmatrix} Ae^{2i\tau} & 0 & A \\ 0 & B & 0 \\ A & 0 & Ae^{-2i\tau} \end{pmatrix} \end{aligned}$$

Mixing matrix:

$$V_\nu = \begin{pmatrix} \cos \psi & 0 & \sin \psi e^{-i\phi} \\ 0 & 1 & 0 \\ -\sin \psi e^{i\phi} & 0 & \cos \psi \end{pmatrix} \quad \text{with} \quad \psi = \pm \frac{\pi}{4}$$

# Neutrino sector

$$J = \text{Im} (U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^*) = \frac{1}{6\sqrt{3}} \cos 2\psi, \quad \sin \delta = \frac{8J}{\cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13}}.$$

$$\text{NH} : m_{\nu_1} = 0, \quad m_{\nu_2} = B = \sqrt{\Delta m_{21}^2} \approx 9 \text{ meV},$$

$$m_{\nu_3} = 2|A| = \sqrt{\Delta m_{31}^2} \approx 50 \text{ meV}$$

$$\text{IH} : m_{\nu_2} = B = \sqrt{\Delta m_{21}^2 + \Delta m_{13}^2} \approx 50 \text{ meV},$$

$$m_{\nu_1} = 2|A| = \sqrt{\Delta m_{13}^2} \approx 49 \text{ meV}, \quad m_{\nu_3} = 0.$$

By varying  $\phi$  we obtain the following best-fit result:

$$\text{NH} : \quad \phi = -0.453\pi, \quad \sin^2 \theta_{12} \approx 0.34, \quad \sin^2 \theta_{23} \approx 0.61, \quad \sin^2 \theta_{13} \approx 0.0232$$

$$\text{IH} : \quad \phi = 0.546\pi, \quad \sin^2 \theta_{12} \approx 0.34, \quad \sin^2 \theta_{23} \approx 0.61, \quad \sin^2 \theta_{13} \approx 0.024.$$

# Quark sector

The  $S_4 \otimes Z_2'' \otimes Z_6 \otimes Z_{12}$  assignments are:

$$\begin{aligned}t_R & : (1, 1, 1, 1), & c_R & : (1, 1, 1, 1), & u_R & : (1, 1, 1, -1), \\b_R & : (1, 1, -1, 1), & s_R & : (1, 1, -1, 1), & d_R & : (1, 1, -1, -1), \\Q & : (3', -1, 1, 1), & \rho & : (3', -1, 1, 1), & \varphi & : (3, -1, 1, 1), \\ \Omega_1 & : (1, 1, 1, i), & \Omega_2 & : (1, -1, 1, \omega^{\frac{1}{2}}), & \Omega_3 & : (1, 1, \omega^{\frac{1}{2}}, 1)\end{aligned}$$

with the VEV patterns

$$\begin{aligned}\langle \rho \rangle & = v_\rho (i, 0, 0), & \langle \varphi \rangle & = v_\varphi (1, 0, 0), \\ \langle \Omega_1 \rangle & = v_{\Omega_1}, & \langle \Omega_2 \rangle & = v_{\Omega_2} e^{i\theta_\Omega}, & \langle \Omega_3 \rangle & = v_{\Omega_3}.\end{aligned}$$

lead to the quark mass matrices

$$M_q = \begin{pmatrix} C_q e^{i\theta_{1q}} & 0 & 0 \\ D_q e^{-i\theta_{2q}} & E_q e^{-i\theta_{3q}} & F_q e^{-i\theta_{4q}} \\ D_q e^{i\theta_{2q}} & E_q e^{i\theta_{3q}} & F_q e^{i\theta_{4q}} \end{pmatrix}, \quad q = U, D.$$

which can be diagonalized in closed form

## Breaking $Z_3$

- To generate the large leptonic mixing angles the  $S_4$  symmetry has to be broken in a different direction in the neutrino sector
- Accommodating quarks and neutrinos requires extra scalars ( $\mathbf{3} \in S_4$  and  $\mathbf{1} \in SU(2)$ ) with  $Z_3$  breaking VEV alignments
- Thanks to flavor symmetries the scalars communicate with each other only through portal-like interactions

**Example:**  $\rho : \mathbf{3}$  with  $\langle \rho \rangle = v_\rho(1, 0, 0)$

$$V_{\text{int}} \supset \sum_{i=1,2,3,3'} \lambda_{\rho_i} (\phi^\dagger \phi)_i (\rho^\dagger \rho)_i$$
$$(\phi^\dagger \phi)_2 (\rho^\dagger \rho)_2 = \sum_{j,k=1, j \neq k}^3 2|\phi_j|^2 |\rho_j|^2 - |\phi_j|^2 |\rho_k|^2$$
$$\sim 2v_\rho^2 \lambda_\rho |\phi_1|^2 - v_\rho^2 \lambda_\rho |\phi_2|^2 - v_\rho^2 \lambda_\rho |\phi_3|^2$$

## Breaking $Z_3$

- We can interpret this breaking as a perturbation of the  $\phi$  VEVs [Heeck et al, Nucl. Phys. B896 (2015) 281-310]

$$\langle \phi \rangle = \langle (\phi_1, \phi_2, \phi_3) \rangle = v(1 + 2\epsilon, 1 - \epsilon, 1 - \epsilon)$$

$$\begin{pmatrix} \langle \phi_a \rangle \\ \langle \phi_b \rangle \\ \langle \phi_c \rangle \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sqrt{2}}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \frac{v}{\sqrt{3}}(1 + 2\epsilon) \\ \frac{v}{\sqrt{3}}(1 - \epsilon) \\ \frac{v}{\sqrt{3}}(1 - \epsilon) \end{pmatrix} = v \begin{pmatrix} 0 \\ \sqrt{2}\epsilon \\ 1 \end{pmatrix}$$

$\Rightarrow$  Choose parametrization

$$\langle (\phi_a, \phi_b, \phi_c) \rangle = v(0, \sin \theta, \cos \theta)$$

- $\theta$  accounts for the deviation from LFT (Triality limit  $\theta \rightarrow 0$ )

## Breaking $Z_3$

- $Z_3$  breaking induces mixing of the scalars  $\phi_b$  and  $\phi_c$   
( $\cos \theta := c_\theta$ ,  $\sin \theta := s_\theta$ )

$$\begin{pmatrix} H^\pm \\ \pi^\pm \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} \phi_b^\pm \\ \phi_c^\pm \end{pmatrix}$$

$$\begin{pmatrix} \eta_I^0 \\ \pi_I^0 \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} \phi_{b,I}^0 \\ \phi_{c,I}^0 \end{pmatrix}$$

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} \phi_{b,R}^0 \\ \phi_{c,R}^0 \end{pmatrix}$$

- Result: FCNCs, but also LFV from Higgs decays that can be controlled with the  $\theta$  parameter
- Triality limit  $\theta \rightarrow 0$  (also  $\vartheta \rightarrow 0$ ) yields original masses

$$m_h^2 \rightarrow m_{\phi_{c,R}^0}^2, \quad m_H^2 \rightarrow m_{\phi_{b,R}^0}^2 = m_{\phi_{a,R}^0}^2$$

$\Rightarrow$  Consider  $h$  the SM like Higgs with  $m_h \approx 125$  GeV

# Corrections to charged lepton mixing

Because of the perturbed alignment  $v(0, s_\theta, c_\theta)$  the charged leptons mass matrix is no longer diagonal in the  $Z_3$  basis of the charged leptons

$$M_l = \frac{v}{\sqrt{2}} \underbrace{\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ \omega^2 & \omega & 1 \\ \omega & \omega^2 & 1 \end{pmatrix}}_{U_\omega} \begin{pmatrix} c_\theta(\tilde{y}_2 - \tilde{y}_3)\lambda^5 & \frac{s_\theta(\tilde{y}_2 + \tilde{y}_3)\lambda^5}{\sqrt{2}} & \frac{s_\theta\tilde{y}_1\lambda^3}{\sqrt{2}} \\ \frac{s_\theta(\tilde{y}_2 - \tilde{y}_3)\lambda^5}{\sqrt{2}} & c_\theta(\tilde{y}_2 + \tilde{y}_3)\lambda^5 & \frac{s_\theta\tilde{y}_1\lambda^3}{\sqrt{2}} \\ \frac{s_\theta(\tilde{y}_2 - \tilde{y}_3)\lambda^5}{\sqrt{2}} & \frac{s_\theta(\tilde{y}_2 + \tilde{y}_3)\lambda^5}{\sqrt{2}} & c_\theta\tilde{y}_1\lambda^3 \end{pmatrix}.$$

The correction to  $V_L$  is approximately

$$V_L = U_\omega W_L \quad \text{with} \quad W_L \simeq \begin{pmatrix} 1 & \frac{\theta}{\sqrt{2}} - \frac{3\theta^2}{4} & \frac{\theta}{\sqrt{2}} \\ -\frac{\theta}{\sqrt{2}} + \frac{3\theta^2}{4} & 1 & \frac{\theta}{\sqrt{2}} \\ -\frac{\theta}{\sqrt{2}} & -\frac{\theta}{\sqrt{2}} & 1 \end{pmatrix}.$$

The deviations are the largest in  $\theta_{12}$  but are within  $3\sigma$  for  $\theta < 0.08$ . The breaking also affects the Jarlskog invariant

$$J \approx \frac{\cos(2\psi)}{6\sqrt{3}} + \frac{1}{12}\theta \left( \sqrt{6} \sin(2\psi) \cos(\phi) - \sqrt{2} \sin(2\psi) \sin(\phi) \right) + \mathcal{O}(\theta^2)$$

For  $\theta \approx 0.07$  we can obtain CP violation of up to  $\delta_{CP} \approx 0.3$

## Z3 Breaking details

In leading order ( $m_e \ll m_\mu \ll m_\tau$ ) the dominant flavor violating couplings are

$$y_{e\tau} \simeq -\frac{m_\tau}{v} (c_{\alpha_L} - s_{\alpha_L}) s_{\vartheta+\theta},$$

$$y_{\mu\tau} \simeq -\frac{m_\tau}{v} (c_{\alpha_L} + s_{\alpha_L}) s_{\vartheta+\theta},$$

$$\begin{aligned} y_{e\mu} \simeq & \frac{m_\mu}{4v} (2c_\vartheta (c_{\alpha_L} (c_\theta - 2s_\theta^2 - 1) - s_{\alpha_L} (c_\theta - 2s_\theta^2 + 1))) \\ & - s_\vartheta (c_{\alpha_L} (c_\theta (4s_\theta + \sqrt{2}) - 2s_\theta + \sqrt{2})) \\ & + s_{\alpha_L} (-c_\theta (4s_\theta + \sqrt{2}) + 2s_\theta + \sqrt{2}). \end{aligned}$$

$$\tan 2\vartheta = -\frac{4s_\theta(\sqrt{2}\kappa s_\theta - 2c_\theta(3\alpha + \kappa))}{6(-1 + 2s_\theta^2)\alpha + (-6 - 2\sqrt{2}c_\theta s_\theta + 11s_\theta^2)\kappa}$$

$$\tan 2\alpha_L = \frac{s_\theta (4\sqrt{2}c_{3\theta} + 12\sqrt{2}c_\theta - 7s_{3\theta} - 3s_\theta)}{10c_{3\theta} + 6c_\theta + 8\sqrt{2}s_\theta^3}.$$

# Mass spectrum after $Z_3$ breaking

$$\begin{aligned}m_{\phi_{a,l}^0}^2 &= m_{\eta_l^0}^2 = -v^2\delta, & m_{\pi_l^0}^2 &= 0, \\m_{\phi_a^\pm}^2 &= m_{H^\pm}^2 = -v^2(\kappa - \beta), & m_{\pi^\pm}^2 &= 0, \\m_{\phi_{a,R}^0}^2 &= \frac{1}{6}v^2(-2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2)\kappa, \\m_h^2 &= \frac{1}{2}(v^2\alpha - m_{\phi_{a,R}^0}^2) + \Delta(\theta), & m_H^2 &= \frac{1}{2}(v^2\alpha - m_{\phi_{a,R}^0}^2) - \Delta(\theta)\end{aligned}$$

## Observations:

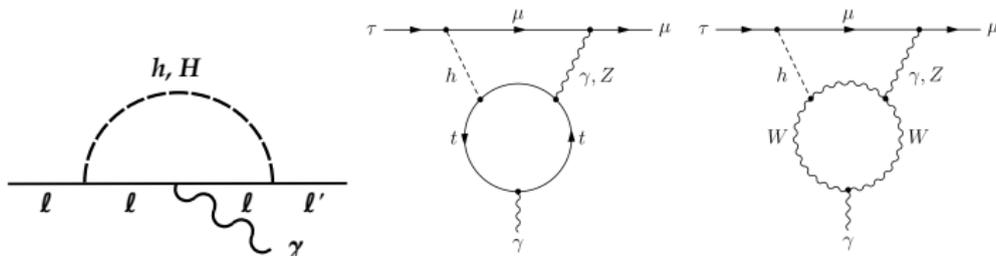
- Only the CP even scalar masses depend on  $\theta$
- Nonzero  $\theta$  causes a mass splitting between  $h$  and  $H$
- Triality limit  $\theta \rightarrow 0$  yields original masses

$$m_h^2 \rightarrow m_{\phi_{c,R}^0}^2, \quad m_H^2 \rightarrow m_{\phi_{b,R}^0}^2 = m_{\phi_{a,R}^0}^2$$

$\Rightarrow$  Consider  $h$  the SM like Higgs with  $m_h \approx 125$  GeV

# Experimental constraints

Strongest constraints from radiative decays  $l \rightarrow l' \gamma$

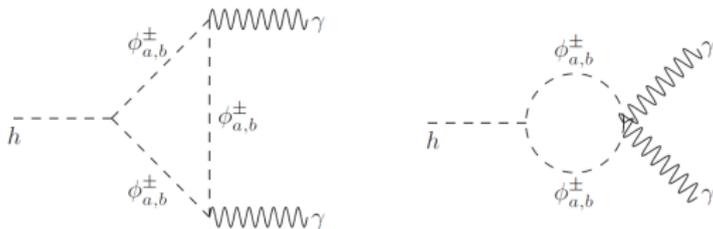


[R. Harnik, J. Kopp, J. Zupan., 10.1007/JHEP03(2013)026]

**Objective:**

Find regions in the  $\theta$  parameter space that explain  $\text{Br}(h \rightarrow \mu\tau)$  and at the same time comply with  $\text{Br}(\mu \rightarrow e\gamma)$  data!

$h \rightarrow \gamma\gamma$



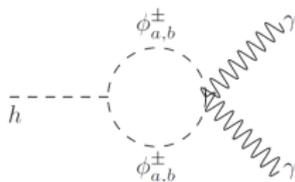
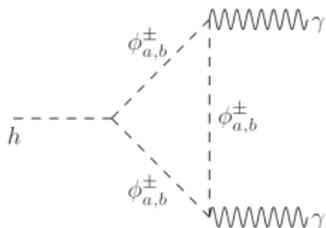
$$\Gamma(h \rightarrow \gamma\gamma) = \frac{\alpha_{em}^2 m_h^3}{256\pi^3 v^2} \left| \sum_f a_{hff} N_c Q_f^2 F_{1/2}(\beta_f) + F_1(\beta_W) + \sum_{s=a,b} \frac{\lambda_{h\phi_s^\pm \phi_s^\mp} v}{2m_{\phi_s^\pm}^2} F_0(\beta_{\phi_s^\pm}) \right|^2$$

$$\text{with } \lambda_{h\phi_s^\pm \phi_s^\mp} = \frac{2}{v} (m_h^2 - m_{\phi_s^\pm}^2)$$

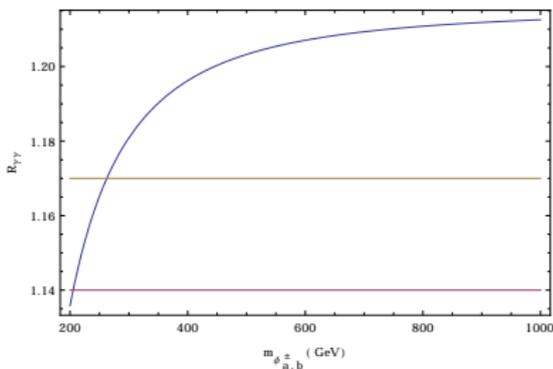
$$R_{\gamma\gamma} = \frac{\sigma(pp \rightarrow h) \Gamma(h \rightarrow \gamma\gamma)}{\sigma(pp \rightarrow h)_{SM} \Gamma(h \rightarrow \gamma\gamma)_{SM}} \simeq a_{htt}^2 \frac{\Gamma(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow \gamma\gamma)_{SM}}$$

$$R_{\gamma\gamma}^{\text{CMS}} = 1.14_{-0.23}^{+0.26} \quad \text{and} \quad R_{\gamma\gamma}^{\text{ATLAS}} = 1.17 \pm 0.27,$$

$h \rightarrow \gamma\gamma$



$$R_{\gamma\gamma} = \frac{\sigma(pp \rightarrow h) \Gamma(h \rightarrow \gamma\gamma)}{\sigma(pp \rightarrow h)_{SM} \Gamma(h \rightarrow \gamma\gamma)_{SM}} \simeq a_{htt}^2 \frac{\Gamma(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow \gamma\gamma)_{SM}},$$



$$R_{\gamma\gamma}^{CMS} = 1.14^{+0.26}_{-0.23}$$

$$R_{\gamma\gamma}^{ATLAS} = 1.17 \pm 0.27$$

$h \rightarrow \gamma\gamma$  measurement  
favors charged Higgses  
between 200 – 205 GeV