

Quantum structure of the minimal calculable unified model

*work in progress**

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Can we improve the theoretical predictions for the
proton lifetime?

Introduction

Problems of SM :

neutrino oscillations, DM, charge quantization, hierarchy problem, parity violation, ...



BSM

GUTs :

experimental signature → **proton decay**

large *uncertainties* in τ_p estimates !!!

Motivation

Experiment :

Future proton decay searches (DUNE, Hyper-K, ...)



$\mathcal{O}(10)$ increase in $\tau_p(p^+ \rightarrow \pi^0 e^+, K^+ \bar{\nu})$ sensitivity

Theory :

matching experiment: first ever *NLO* calculation



requires reducing the large uncertainties in predicted τ_p

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Outline

- 1 Theoretical uncertainties
- 2 The minimal model
- 3 Conclusions

Theoretical uncertainties

Main sources of uncertainties in τ_p estimates in GUTs:

- **hadronic matrix elements** :

$$\langle \pi^{+,0}, K^{+,0}, \eta | \dots | p^+ \rangle$$

$$lattice \quad \longrightarrow \quad \Delta \sim \mathcal{O}(20 - 40\%)$$

Theoretical uncertainties

Main sources of uncertainties in τ_p estimates in GUTs:

- unification scale determination :

M_{GUT} = effective mediator mass $\longrightarrow \tau_p \propto M_{GUT}^4$
 (in non-SUSY predominantly *heavy gauge bosons*:
 $(3, 2, -5/6)_{45}$ or $(3, 2, 1/6)_{45}$)

@ NLO:

- * running of λ_i **@ 2-loop** ($\Delta\alpha_i^{\text{exp}}(m_Z)$)
- * *threshold effects* **@ 1-loop** (scalar spectrum @ tree level/1-loop)
- * *Planck scale effects* : gauge kinetic form operators

$$\mathcal{L} \ni \frac{\kappa}{\Lambda} F^{\mu\nu} \langle \Phi \rangle F_{\mu\nu}$$

\longrightarrow uncontrolled & inhomogeneous *shifts in matching*

$$\Delta\alpha_i^{-1}(M_{GUT}) \sim \mathcal{O}(1)$$

\longrightarrow orders of magnitude uncertainty in M_{GUT}

\longrightarrow tame them **or** no sense in NLO

Theoretical uncertainties

Main sources of uncertainties in τ_p estimates in GUTs:

- **flavour structure of the BLV currents** :

- * partial widths $\Gamma(p^+ \rightarrow \text{final state})$ in general depend on *rotation matrices* \rightarrow some channels are more *robust to perturbations of the mass matrices*

$\Gamma(p^+ \rightarrow K^+\bar{\nu}, \pi^+\bar{\nu})$ independent of rotation matrices for symmetric quark-sector mass matrices \rightarrow governed only by mediator masses

- * *Planck scale effects* : scalar & fermionic kinetic form, higher-order Yukawa operators \rightarrow *shifts in $d = 4$ matching*

$$\mathcal{L} \ni \frac{\rho}{\Lambda} (D_\mu \phi)^\dagger \langle \Phi \rangle (D^\mu \phi)$$

$$\mathcal{L} \ni \frac{1}{\Lambda} i \bar{\psi}_{R_i} \not{D}_{R_i} Y' \psi_{R_i} \langle \Phi \rangle$$

$$\mathcal{L} \ni \frac{1}{\Lambda} \psi_{R_1} Y'' \psi_{R_2} \phi \langle \Phi \rangle$$

- * Yukawa sector fits inaccurate to $\langle \Phi \rangle / \Lambda \sim 1\%$ due to shifted matching \rightarrow potentially large changes in the *mixing angles* and *BLV currents* !

The minimal renormalizable non-SUSY SO(10) model

SO(10) GUT spontaneously broken by **45** :

leading Planck-scale effects in gauge matching *absent* (group theory argument)

$$\mathcal{L} \ni \frac{\kappa}{\Lambda} F^{\mu\nu} \langle 45 \rangle F_{\mu\nu} = 0$$

Higgs sector :

- SO(10) broken by $\omega_Y \equiv (15, 1, 1)_{PS}$, $\omega_R \equiv (1, 3, 1)_{PS} \subset \langle 45 \rangle$

$$SO(10) \xrightarrow{\omega_Y} 3_C \times 2_L \times 2_R \times 1_{B-L} \xrightarrow{\omega_R} 3_C \times 2_L \times 1_R \times 1_{B-L}$$

OR

$$SO(10) \xrightarrow{\omega_R} 4_C \times 2_L \times 1_R \xrightarrow{\omega_Y} 3_C \times 2_L \times 1_R \times 1_{B-L}$$

- rank reduced by $(\bar{10}, 3, 1)_{PS} \subset \langle 126 \rangle$

$$3_C \times 2_L \times 1_R \times 1_{B-L} \xrightarrow{\langle 126 \rangle} 3_C \times 2_L \times 1_Y$$

Scalar potential :

$$V = V_{45} + V_{126} + V_{mix}$$

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Is the *minimal renormalizable non-SUSY SO(10)*
really ruled out?

Spectrum computation

@ tree level :

either **tachyonic** spectrum or **flipped SU(5)** vacuum

Yasue '81

Anastaze et al. '83

Babu, Ma '85

$$\left. \begin{aligned} m_{(8,1,0)_{45}}^2 &\propto (\omega_Y - \omega_R)(\omega_R + 2\omega_Y) \\ m_{(1,3,0)_{45}}^2 &\propto (\omega_R - \omega_Y)(\omega_Y + 2\omega_R) \end{aligned} \right\} \rightarrow -2 < \frac{\omega_Y}{\omega_R} < -\frac{1}{2}$$

@ 1-loop :

large corrections \rightarrow quantum salvation *Bertolini, Di Luzio, Malinsky '10*

$$\Delta m_{(8,1,0)_{45}}^2 = \frac{1}{(4\pi^2)} \left[\tau^2 + \beta^2 (\omega_R^2 - \omega_R \omega_Y + 3\omega_Y^2) + g^4 (13\omega_R^2 + \omega_R \omega_Y + 22\omega_Y^2) \right] + \text{logs}$$

$$\Delta m_{(1,3,0)_{45}}^2 = \frac{1}{(4\pi^2)} \left[\tau^2 + \beta^2 (2\omega_R^2 - \omega_R \omega_Y + 2\omega_Y^2) + g^4 (16\omega_R^2 + \omega_R \omega_Y + 19\omega_Y^2) \right] + \text{logs}$$

RGE running with realistic see-saw scale \rightarrow some "light" scalars in the desert: $(8, 2, 1/2)_{126}$ or $(6, 3, 1/3)_{126}$

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Summary & outlook

τ_p calculation accuracy :

not enough to *rule out* any particular GUT model



i) the *mediator mass* at best @ *LO*

or

ii) *flavour structure* of BLV currents not robust

||

too sensitive to generic $\mathcal{O}(1)$ *Planck-scale effects*

Summary & outlook

*** minimal realistic GUT @ NLO :

renormalizable non-SUSY SO(10) model with a $126 \oplus 45$

▷▷▷ the only *perturbative* unified theory, with at least some classes of Planck scale operators (*gauge kinetic form operators*) under control



can compute *radiative corrections* to masses and τ_p

▷▷▷ genuinely *quantum* theory = no available *tree level* description (*tachyons*)



1-loop

▷▷▷ *robustness* in the ν *channel* = prerequisite for *proton decay upper limit*

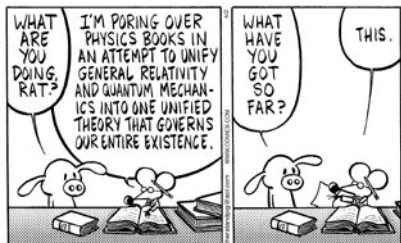


DUNE sensitive to “*flavour robust*” final states

Summary & outlook

TO DO :

- compute the whole **spectrum** (V_{eff} approach)
- show it's **realistic** & check **vacuum state** longevity
- provide the first ever **NLO computation** of corresponding τ_p



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