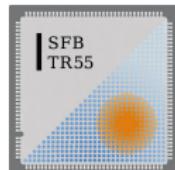


# Lattice QCD for the LHC

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# Outline

- Introduction
- Proton structure
- $\Lambda_b$  decay
- Two topics of High Precision QCD
- Summary

# Lattice QCD for non-LHC

- Fundamental quantum field theory questions: orbifold equivalence, SUSY QCD, Large  $N$  QCD, phase diagrams etc.
- Nonperturbative dynamics beyond the standard model: mostly technicolour. Recently axions: topological susceptibility  $\chi_t = f_a^2 m_a^2$  at high temperatures of interest. [E Berkowitz et al, 1505.07455, R Kitano, N. Yamada, 1506.00370, S Borsanyi et al, 1508.06917]
- Low energy standard model tests: matrix elements relevant for (B)SM  $\beta$ -decay,  $(g - 2)_\mu$ , dark matter couplings,  $K$  physics ( $K \rightarrow \pi\pi, \epsilon'/\epsilon$  [T Blum et al, 1502.00263; I Ishizuka et al, 1505.05289; C Lehner et al 1508.01801]),  $D$  physics.
- Fundamental parameters: connect experiment to  $m_u, m_d, m_s, m_c, V_{cs}, V_{cd}$ , running of  $\alpha_{em}$ , weak charge.
- Hadron spectroscopy, decay constants and distribution amplitudes: BES III, JLAB, Belle 2, PANDA@FAIR etc.
- Generalized parton distributions (GPDs), transverse momentum distributions (TMDs) etc.: relevant for COMPASS 2, JLAB, BNL, MAMI etc.

# Lattice QCD for LHC

Important for LHC physics but not for BSM searches

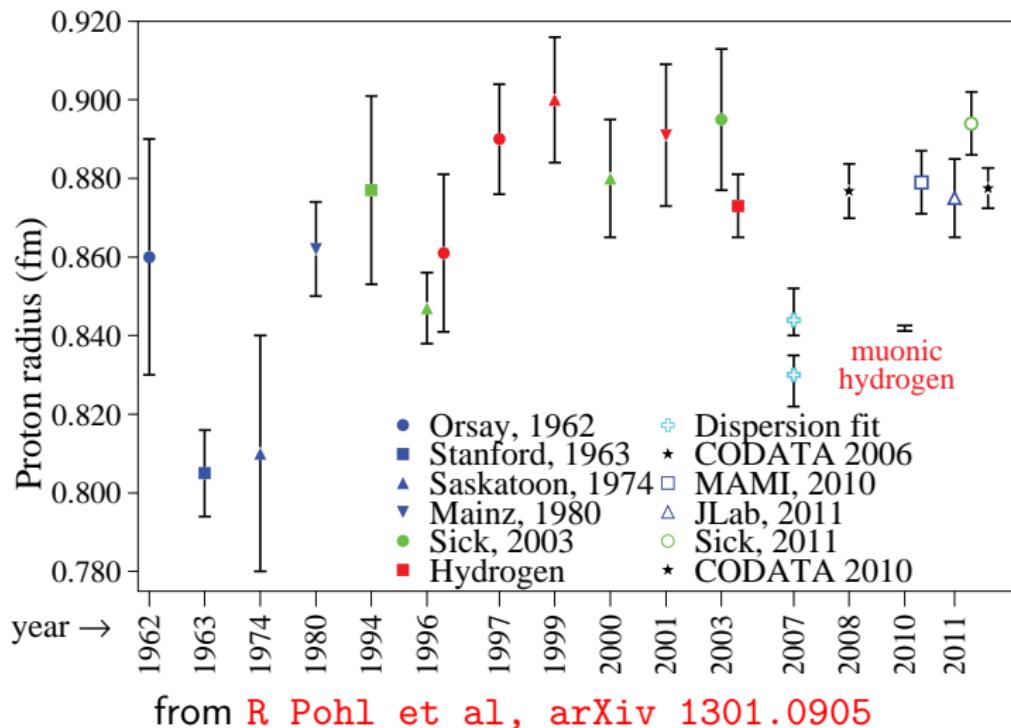
- ALICE (also ATLAS, CMS). QCD at high temperatures: transition temperature  $T_c$  to the QGP, hadron modifications at high  $T$ , Debye screening lengths, fluctuations of conserved charges, freezeout curve, equation of state, conductivity, role of magnetic fields etc.
- Spectroscopy of mesons and baryons with open and closed charm and bottom (LHC by-products) and some of their properties.

Important for SM tests at LHC

- Fundamental parameters:  $\alpha_s$ ,  $m_b$ .
- $b$ -decays (LHC-b):  $f_B$ ,  $f_{B_s}$ , electroweak decay form factors etc. to connect experiment to  $V_{cb}$ ,  $V_{ub}$  etc.
- Hadron/proton structure: PDFs, DPDs (also distribution amplitudes for particle production).

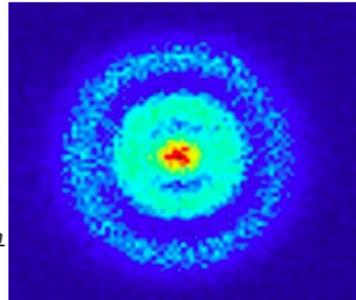
# Proton radius puzzle

We may not understand the probe used at LHC to discover new physics!



mass of the hydrogen atom:

$$\underbrace{938.29 \text{ MeV}}_{\text{proton}} + \underbrace{0.51 \text{ MeV}}_{\text{electron}} - \underbrace{0.0000136 \text{ MeV}}_{\text{binding energy: } \frac{m_e \alpha_{em}^2}{2}}$$



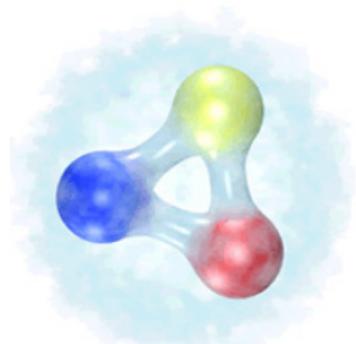
$$\text{RMS radius} \approx 0.92 \cdot 10^{-10} \text{ m} = 0.092 \text{ nm}$$

AS Stodolna et al,  
PRL (13) 213001

mass of the proton  $m_p$ :

$$\underbrace{2 \times 2.2 \text{ MeV}}_{\text{up quarks}} + \underbrace{4.7 \text{ MeV}}_{\text{down quark}} \underbrace{+}_{!!!} \underbrace{929.2 \text{ MeV}}_{???$$

$$\text{RMS charge radius} \approx 0.84 \cdot 10^{-15} \text{ m} = 0.84 \text{ fm}$$



artist's impression

Hydrogen:  $|E|\langle r_H^2 \rangle^{1/2} = \sqrt{3} \underbrace{(m_e \alpha_{em})^{-1}}_{a_B \approx 0.053 \text{ fm}} |E| = \frac{\sqrt{3}}{2} \alpha_{em} \approx 0.006$

Proton:  $m_p \langle r_p^2 \rangle^{1/2} \approx 4 = \mathcal{O}(1)$

Solving a strongly coupled, non-linear, relativistic four-dimensional quantum system is not so easy

⇒ numerical simulation.

Lattice spacing  $a < 1/m_p$ , i.e.  $a \Lambda_{\text{QCD}} \ll 1$ , for polynomial cut-off effects.

⇒  $a^{-1} \gtrsim 2 \text{ GeV}$

Lattice extent  $L \gg r_p$ . Actually, finite size effects  $\propto e^{-M_\pi L}$ .

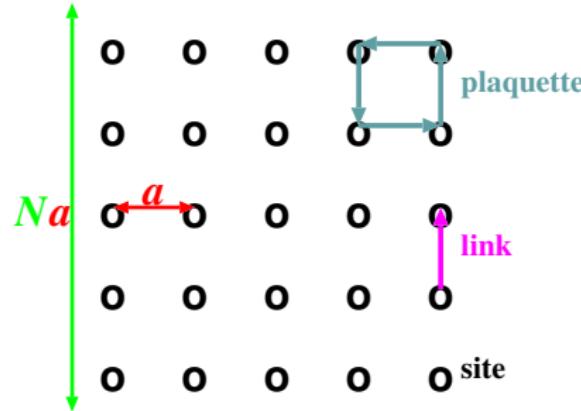
⇒  $L \gtrsim 4/M_\pi$

⇒  $L \gtrsim 5.8 \text{ fm}$  for physical pion mass.

This means  $N = L/a > 5.8 \text{ fm} \cdot 2 \text{ GeV} \approx 60$ .

We are lucky:  $m_p/M_\pi \ll 60 \ll M_W/\Lambda_{\text{QCD}} \ll M_{\text{Planck}}/M_W$ .

# Lattice QCD



typical values:

$$a^{-1} = 2-5 \text{ GeV}, Na = 2-7 \text{ fm}$$

continuum limit:  $a \rightarrow 0$ ,  $Na$  fixed

infinite volume:  $Na \rightarrow \infty$

$$\langle O \rangle = \frac{1}{Z} \int [dU] [d\psi] [d\bar{\psi}] O[U] e^{-S[U, \psi, \bar{\psi}]}$$

“Measurement”: average over a representative ensemble of gluon configurations  $\{U_i\}$  with probability  $P(U_i) \propto \int [d\psi] [d\bar{\psi}] e^{-S[U, \psi, \bar{\psi}]}$

$$\langle O \rangle = \frac{1}{n} \sum_{i=1}^n O(U_i) + \Delta O$$

$$\Delta O \propto \frac{1}{\sqrt{n}} \xrightarrow{n \rightarrow \infty} 0$$

**Input:** discretized  $\mathcal{L}_{QCD} = \frac{1}{16\pi\alpha_L(a)} FF + \bar{q}_f(\not{D} + m_f(a))q_f$

$$m_N^{\text{latt}} = m_N^{\text{phys}} \longrightarrow a$$

$$M_\pi^{\text{latt}} / m_N^{\text{latt}} = M_\pi^{\text{phys}} / m_N^{\text{phys}} \longrightarrow m_u(a) \approx m_d(a)$$

...

**Output:** hadron masses, matrix elements, decay constants, etc...

**Required:**

- ①  $L = Na \rightarrow \infty$ : FSE suppressed with  $\exp(-LM_\pi) \Rightarrow LM_\pi \gtrsim 4$ .
- ②  $m_q^{\text{latt}} \rightarrow m_q^{\text{phys}}$ : chiral perturbation theory ( $\chi$ PT) helps for  $m_{ud}$  but  $m_{ud}^{\text{latt}}$  must be sufficiently small to start with ( $M_\pi \lesssim 200$  MeV?).
- ③  $a \rightarrow 0$ : functional form known:  $\mathcal{O}(a^2), \mathcal{O}(\alpha_s a) \Rightarrow \approx 4$  lattice spacings.

Only in very few calculations all the above is done as yet, e.g., light hadron spectrum, meson decay constants,  $\alpha_s, m_{u,d,s,c}$ .

# Computational challenges

Cost of simulation is proportional to

- number of points:  $(L/a)^4$
- condition number of linear system:  $1/M_\pi^2$
- $L^{1/2}/M_\pi$  in (Omelyan) time integration within hybrid Monte Carlo
- $1/a^{>2}$  critical slowing down (autocorrelations)

Adjusting  $L \propto 1/M_\pi$  this means:

$$\text{cost} \propto \frac{1}{a^{>6} M_\pi^{7.5}}$$

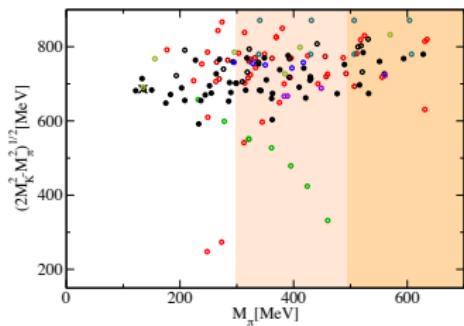
NB: for baryonic observables at small  $M_\pi$  additional noise/signal problems.

State of the art:  $64^3 \times 128$  sites, corresponding to  $\approx (4 \times 10^9)^2$  (sparse) complex matrices.

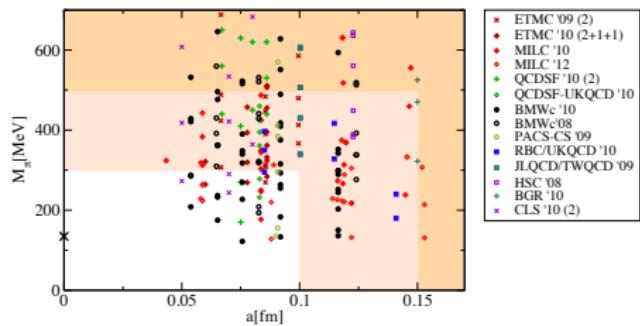
Tremendous progress in Hybrid Monte Carlo, solver, noise reduction.

# Landscape of recent lattice simulations

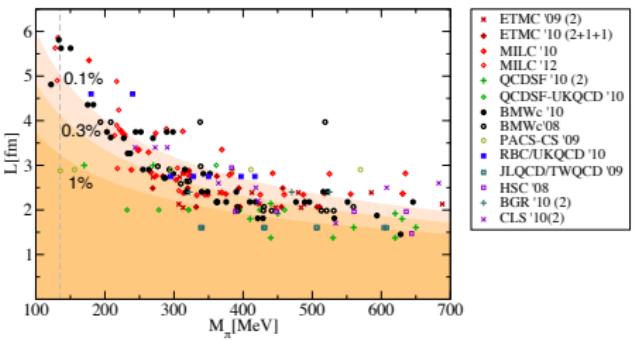
$$M_{ss} \propto m_s \text{ vs. } M_\pi$$



$$M_\pi \text{ vs. } a$$



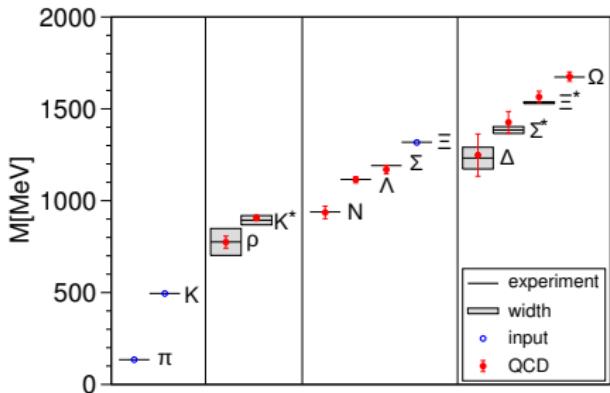
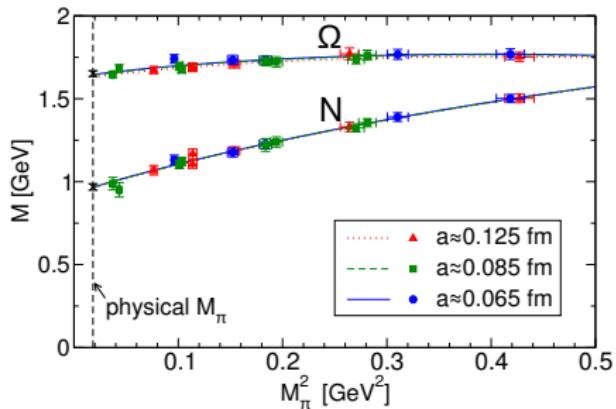
$$L \text{ vs. } M_\pi$$



Figures taken from

[C Hoelbling, arXiv:1410.3403]

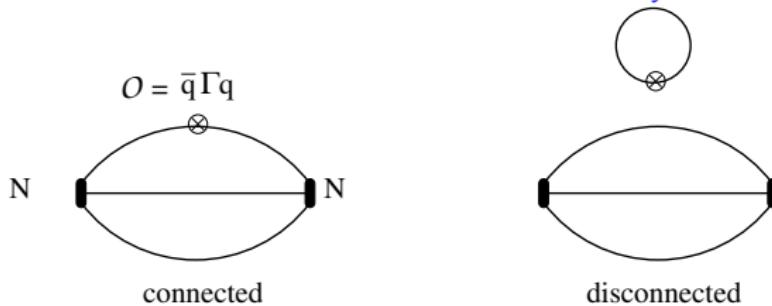
# Light hadron spectrum



[BMW-c: S Dürr et al, arXiv:0906.3599]

# Topic I: Proton structure. Three point functions

Evaluate  $\langle N | \bar{q} \Gamma q | N \rangle$  (Lines: quark “propagators”  $M_{xy}^{-1}$ ,  $M = \not{D} + m_q$ )



$q \in \{u, d\}$ : both quark-line connected and disconnected terms.

$q = s$ : only the disconnected term.

“Connected” requires only 12 rows (spin  $\times$  colour) of  $M^{-1}$ .

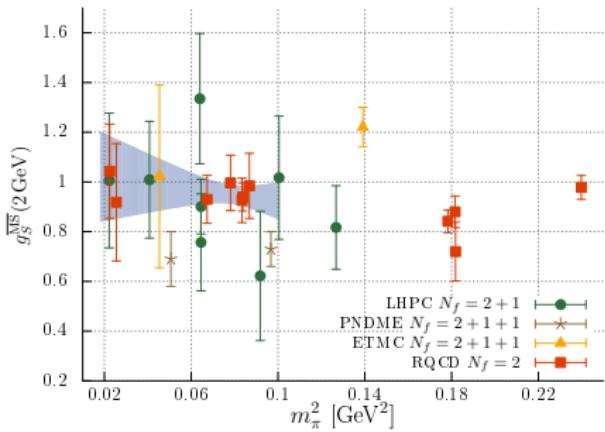
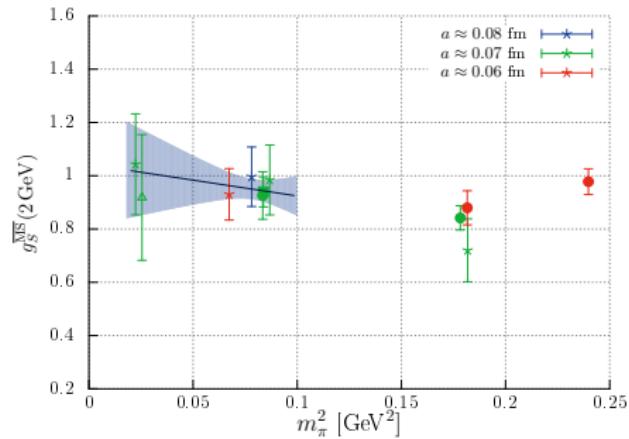
“Disconnected”  $12N^3$  rows (timeslice): stochastic “all-to-all” methods.

“Disconnected” cancels ( $m_u = m_d$ , QED) from isovector combinations:

“proton minus neutron”, i.e.  $\langle p | (\bar{u} \Gamma u - \bar{d} \Gamma d) | p \rangle = \langle p | \bar{u} \Gamma d | n \rangle$ .

# Isovector scalar charge $g_S = G_S(0)$

$$\langle p | \bar{u}d | n \rangle = G_S(q^2) \bar{u}_p(\mathbf{p}_f) u_n(\mathbf{p}_i)$$



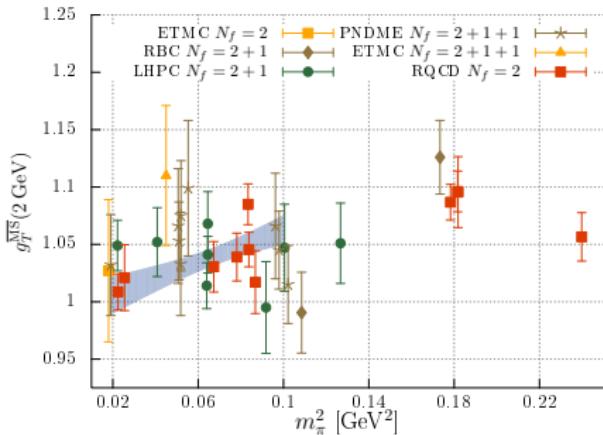
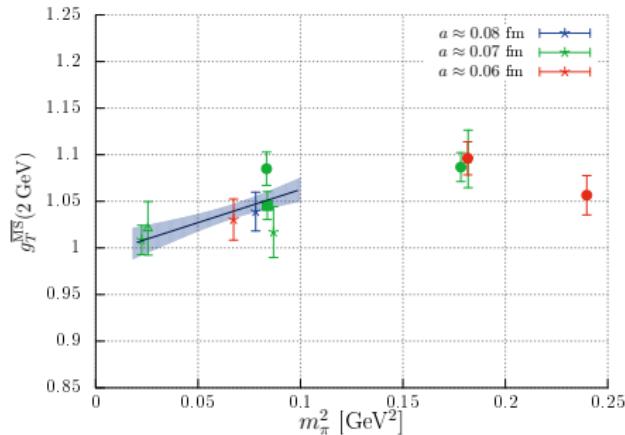
LHPC: 1206.4527, PNDME: 1306.5435, ETMC: 1411.3494,

RQCD: 1412.7336

See also: ETMC: 1507.04936

# Isovector tensor charge $g_T = G_T(0)$

$$\langle p | \bar{u} \sigma_{\mu\nu} d | n \rangle = G_T(q^2) \bar{u}_p(\mathbf{p}_f) \sigma_{\mu\nu} u_n(\mathbf{p}_i)$$



ETMC: 1507.04936, RBC: 1003.3387, LHPC: 1206.4527,  
 PNDME: 1506.06411, RQCD: 1412.7336

General remark: we vary  $a^2$  only by a factor 1.8  $\Rightarrow$  we cannot exclude lattice spacing effects of up to  $0.071^2 / (0.081^2 - 0.060^2) \cdot \Delta g \approx 1.7 \cdot \Delta g$ .

# Decomposition of the proton mass

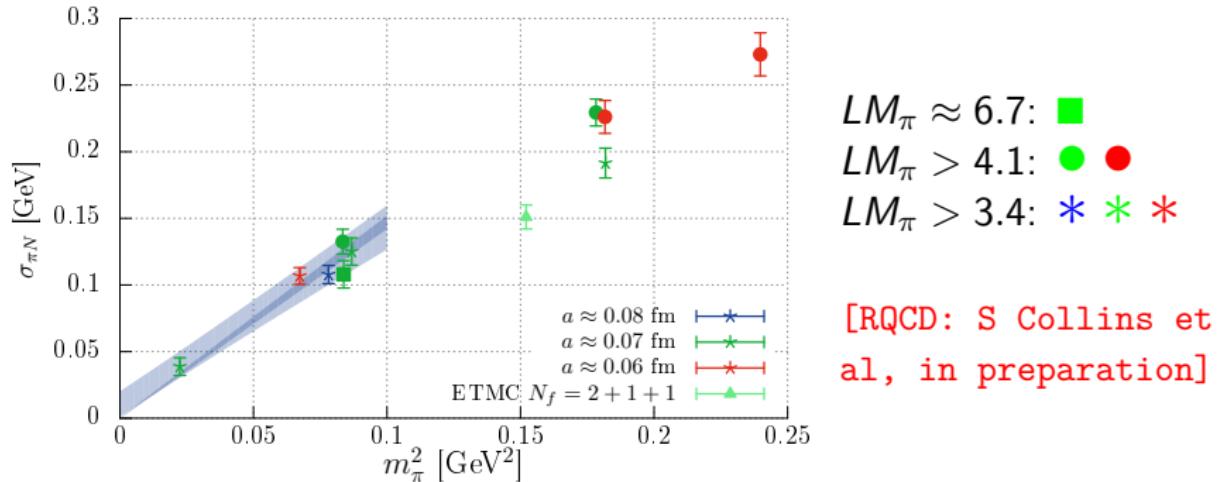
$$m_N = \underbrace{\sum_{q \in \{u,d,s,\dots\}} m_q \langle N | \bar{q} \mathbb{1} q | N \rangle}_{\text{quarks}} + \underbrace{\left\langle N \left| \frac{1}{8\pi\alpha_L} (\mathbf{E}^2 - \mathbf{B}^2) + \sum_q \bar{q} \mathbf{D} \cdot \gamma q \right| N \right\rangle}_{\text{gluon interactions (Eucl. spacetime)}} \\ + \underbrace{\frac{1}{4} \left( m_N - \sum_q m_q \langle N | \bar{q} \mathbb{1} q | N \rangle \right)}_{\text{trace anomaly}}$$

VEV  $\langle 0 | \bar{q} q | 0 \rangle$  is understood to be subtracted from  $\langle N | \bar{q} q | N \rangle$ .

Pion-nucleon  $\sigma$ -term:  $\sigma_{\pi N} = m_u \langle N | \bar{u} u | N \rangle + m_d \langle N | \bar{d} d | N \rangle = \sigma_u + \sigma_d$ .

Requires computation of disconnected quark loops, unless obtained via derivative  $\sigma_q = \partial m_N / \partial \ln m_q$ .

Scalar particles (Higgs, neutralino etc.) couple  $\propto$  quark matrix elements.



The non-vanishing light quark masses are directly responsible for only  $\approx 35$  MeV of the nucleon mass but for 68 MeV of the pion mass!

This may not be too surprising since  $m_N \not\rightarrow 0$  as  $m_{ud} \rightarrow 0$ : Most nucleon mass is from the glue.

The scalar matrix elements  $m_q \langle N | \bar{q}q | N \rangle$  determine the coupling of the nucleon to scalar particles at zero recoil:

$$\frac{f_N}{m_N} \approx \sum_{q \in \{u,d,s\}} f_{T_q} \frac{\alpha_q}{m_q} + \frac{2}{27} f_{T_G} \frac{\alpha_c}{m_c} + \frac{2}{25} f_{T_G} \frac{\alpha_b}{m_b} + \frac{2}{23} f_{T_G} \frac{\alpha_t}{m_t}.$$

Cross section  $\propto |f_N|^2$ . Higgs example:  $\alpha_q \propto m_q/m_W$ .

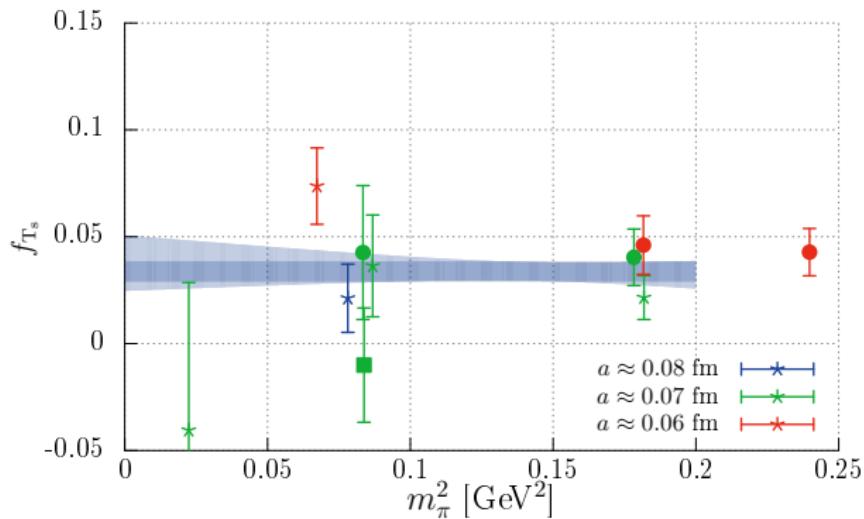
$$f_{T_q} \equiv \frac{m_q \langle N | \bar{q}q | N \rangle}{m_N}$$

are the contributions of the light quark masses to the proton mass and

$$f_{T_G} \approx 1 - \sum_{q \in \{u,d,s\}} f_{T_q}.$$

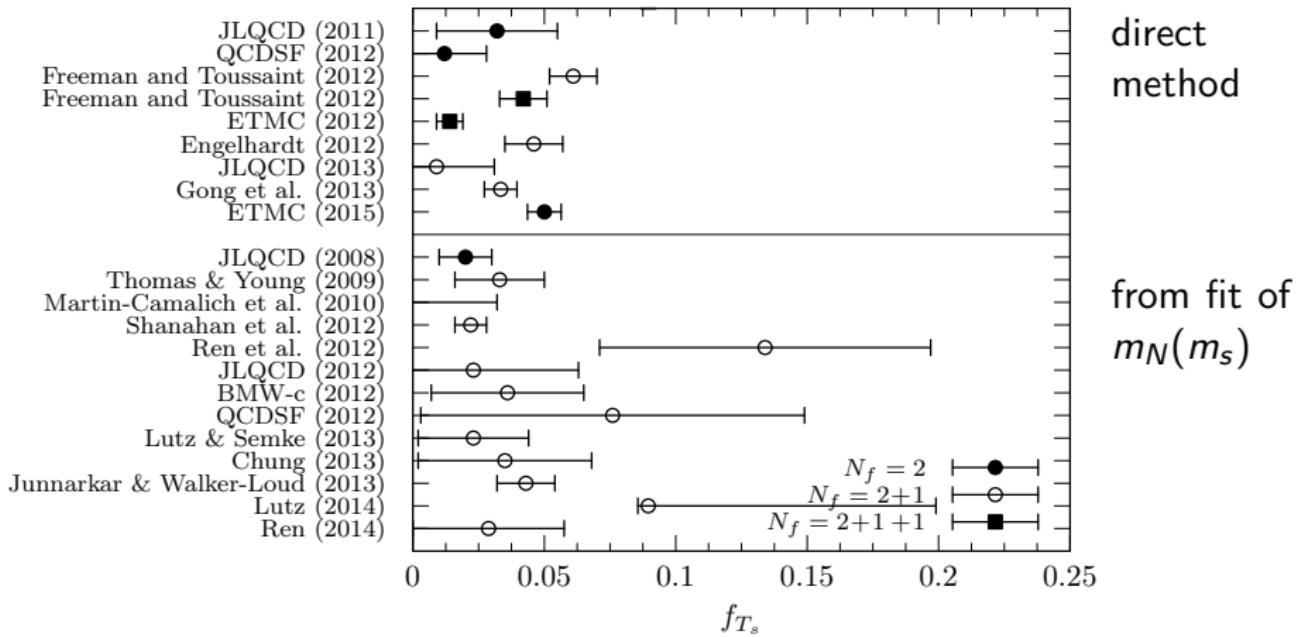
Little about  $f_{T_q}$  is known experimentally.

# Scalar strangeness content



[QCDSF: GB et al, arXiv:1111.1600,  
RQCD: S Collins et al, in preparation]

# $f_{T_s}$ : Other lattice results



# Spin of the nucleon

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \sum_{q,\bar{q}} L_q + J_g :$$

Ji decomposition into the contributions of the (longitudinal) quark spins

$$\Delta \Sigma = \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} + \dots ,$$

the (longitudinal) quark and antiquark orbital angular momenta

$L_q = J_q - \frac{1}{2} \Delta q$  and the (longitudinal) gluon total angular momentum  $J_g$ .

Naïve non-relativistic SU(6) quark model:  $\Delta \Sigma = 1$ ,  $L_q = J_g = \Delta s = 0$ .

Relativistic quark models:  $\Delta \Sigma \sim 0.6$ ,  $L_{\text{quarks}} \sim 0.2$ .

I will say nothing about the Jaffe and Manohar decomposition:

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \mathcal{L}_{\text{quarks}} + \Delta G + \mathcal{L}_g \quad \left( J_g \neq \Delta G + \mathcal{L}_g, J_q \neq \frac{1}{2} \Delta q + \mathcal{L}_q \right).$$

## Individual quark spin contributions ( $q \in \{u, d, s\}$ )

$$(\Delta q + \Delta \bar{q}) s_\mu = \frac{1}{m_N} \langle N, s | \bar{q} \gamma_\mu \gamma_5 q | N, s \rangle = F_A^q(0) = \tilde{A}_{10}^q(0)$$

Axial charges:

$$a_3 = -s_\mu \frac{1}{m_N} \langle N, s | \bar{\psi} \gamma_\mu \gamma_5 \lambda_3 \psi | N, s \rangle = \Delta u - \Delta d = g_A$$

$$\begin{aligned} a_8 &= -s_\mu \frac{\sqrt{3}}{m_N} \langle N, s | \bar{\psi} \gamma_\mu \gamma_5 \lambda_8 \psi | N, s \rangle \\ &= \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} - 2\Delta s - 2\Delta \bar{s} \end{aligned}$$

$$\begin{aligned} a_0(Q^2) &= -s_\mu \frac{1}{m_N} \langle N, s | \bar{\psi} \gamma_\mu \gamma_5 \mathbb{1} \psi | N, s \rangle \\ &= \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} = \Delta \Sigma(Q^2). \end{aligned}$$

$\psi = (u, d, s)^t$ ,  $\lambda_j$  are Gell-Mann flavour matrices.

$a_3 = g_A$  known from neutron  $\beta$  decay, assuming isospin symmetry.

$a_8$  usually estimated from hyperon  $\beta$  decay, assuming  $SU(3)_F$  symmetry.

# Extraction of the $\Delta q$ 's from experiment

DIS gives spin structure functions of proton and neutron  $g_1^{p,n}(x, Q^2)$ .

First moment related to  $a_i$ 's via OPE (leading twist):

$$\Gamma_1^{p,n}(Q^2) = \int_0^1 dx g_1^{p,n}(x, Q^2) = \frac{1}{36} [(a_8 \pm 3a_3) C_{NS} + 4a_0 C_S]$$

Use **models** to extrapolate  $g_1$  from experimental  $x_{\min}$  to  $x = 0$ !

$$C_{S/NS} = C_{S/NS}(\alpha_s(Q^2)).$$

Combinations of  $a_i$  give  $\Delta q$ 's, e.g.,  $(\Delta s + \Delta \bar{s})(Q^2) = \frac{1}{3}[a_0(Q^2) - a_8]$

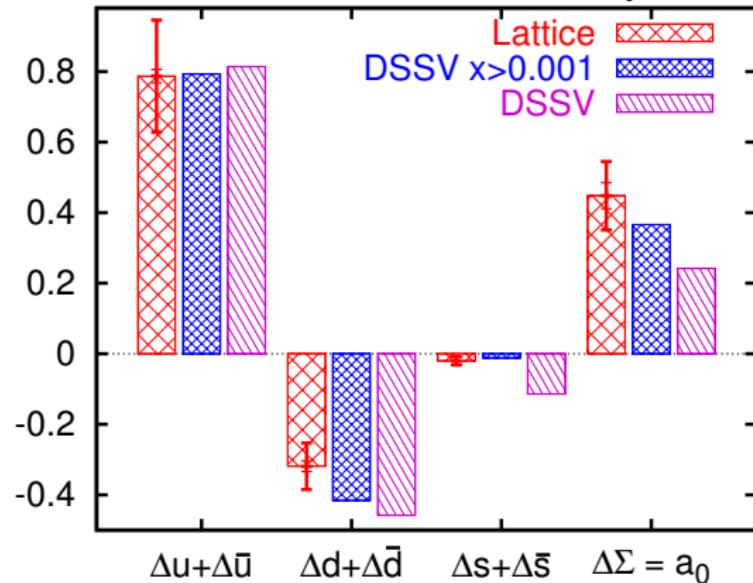
SIDIS allows for direct measurements of the  $\Delta q(x)$  but requires fragmentation functions.

[COMPASS, arXiv 1001.4654]

	Naive Extrapol.	combined with DSSV
$(\Delta s + \Delta \bar{s})(5 \text{ GeV}^2)$	$-0.02 \pm 0.02 \pm 0.02$	$-0.10 \pm 0.02 \pm 0.02$

DSSV: [de Florian et al, arXiv:0904.3821]

No continuum limit,  $M_\pi \approx 290$  MeV  $\Rightarrow$  add 20 % systematic error.

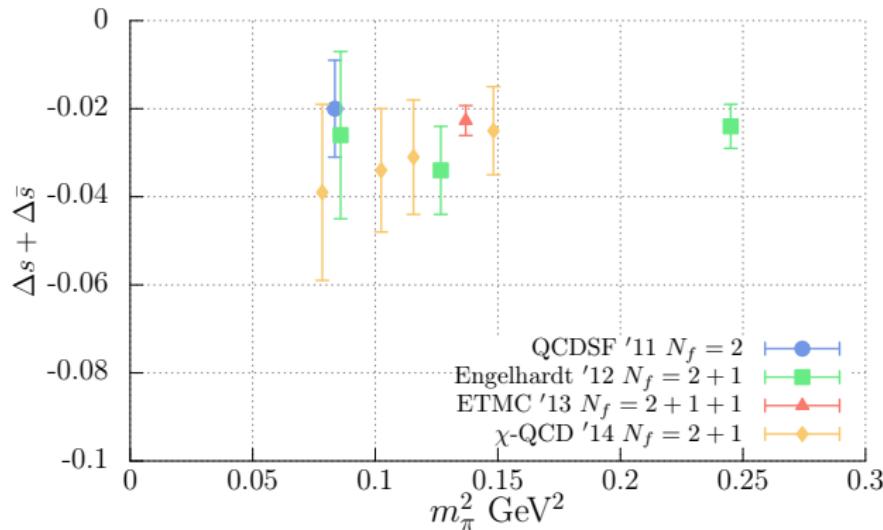


[QCDSF: GB et al, 1112.3354]

Result in the  $\overline{\text{MS}}$  scheme at  $\mu^2 = 7.4$  GeV $^2$ :

$$\begin{aligned}\Delta \Sigma &= \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} &= & 0.45(4)(9) \\ \Delta s + \Delta \bar{s} &= & -0.020(10)(4)\end{aligned}$$

# Comparison of recent lattice calculations



Consistency between different determinations: small  $\Delta s + \Delta \bar{s}$ .

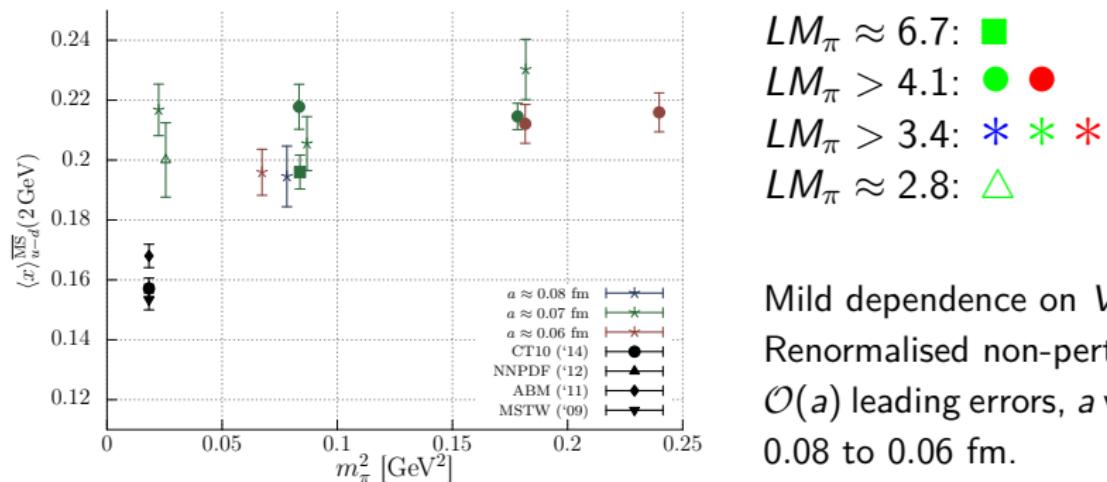
ETMC result shows statistical accuracy that is possible. Systematics!

[QCDSF: GB et al, 1112.3354; M Engelhardt, 1210.0025; ETMC: A Abdel-Rehim et al, 1310.6339;  $\chi$ QCD: Y Yang et al, unpublished.]

$\exists$  similar results by [UKQCD/CSSM: A Chambers et al, 1508.06856; T Bhattacharya et al, 1503.05975]

# Isovector quark momentum fraction: $\langle x \rangle_{u-d}^{\overline{\text{MS}}}(2 \text{ GeV})$

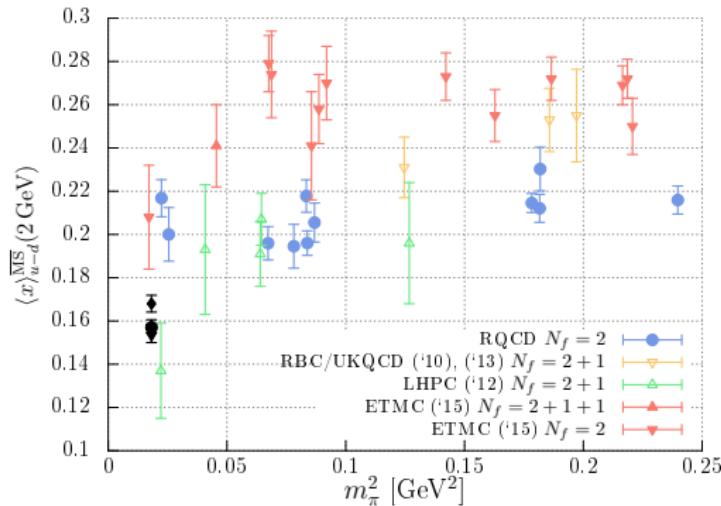
[RQCD: GB et al, arXiv:1408.6850]  $N_f = 2$



Improvement on earlier calculations which suffered from excited state contamination, obtaining  $\langle x \rangle_{u-d}^{\overline{\text{MS}}}(2 \text{ GeV}) \sim 0.25$ .

Near physical point but more work needs to be done — lattice spacing dependence? Lattice artefacts linear in  $a$  could exceed statistical errors by a factor of three.

# $\langle x \rangle_{u-d}^{\overline{MS}}(2 \text{ GeV})$ : summary of recent lattice results



RQCD: GB et al, 1408.6850;  
RBC/UKQCD: Y Aoki et al,  
arXiv:1003.3387;  
LHPC: J Green et al,  
arXiv:1209.1687;  
ETMC: A Abdel-Rehim et al,  
arXiv:1507.05068.

PDFs from

S Alekhin et al, 1310.3059; CT10: J Gao et al, 1302.6246;  
NNPDF: R Ball et al 1207.1303; A Martin et al 0905.3531.

# Isosinglet $\langle x \rangle_q$

Huge uncertainties. Example  $\langle x \rangle_s$ :

NNPDF2.3 [R Ball et al, arXiv:1207.1303]:

$$\frac{\langle x \rangle_s}{\langle x \rangle_{u-d}} = 0.111(5)$$

CT10 [H-L Lai et al, arXiv:1007.2241]:

$$\frac{\langle x \rangle_s}{\langle x \rangle_{u-d}} \approx 0.17$$

MSTW2008 [A Martin et al, arXiv:0901.0002]:

$$\frac{\langle x \rangle_s}{\langle x \rangle_{u-d}} \approx 0.13$$

(I was too lazy to upload NNPDF3.0/CT14 etc. into LHAPDF)

First lattice results on  $\langle x \rangle_s / \langle x \rangle_{u,d}$  [ $\chi$ QCD: M Sun et al, 1502.05482; ETMC: C Alexandrou et al, 1410.8761]: Mixing of  $\langle x \rangle_s$  with  $\langle x \rangle_g$  and  $\langle x \rangle_{u,d}$  ignored.

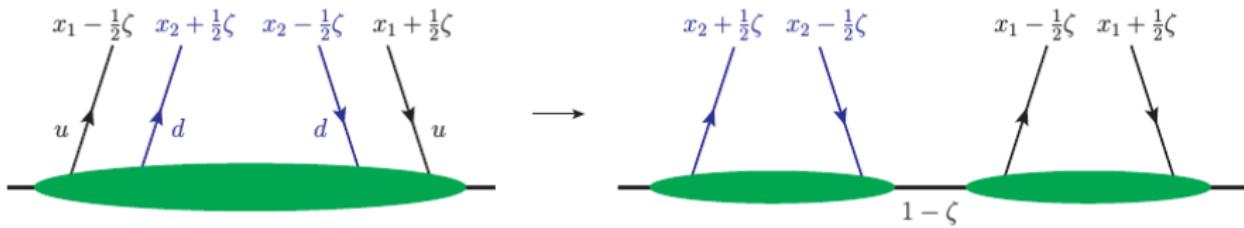
# Double hard scattering: DPDs

Needed to describe background (underlying event, cf. **Frank Krauss'** talk)

Example  $W^+ W^+$  production.  $qq \rightarrow qqW^+W^+$ . Double parton scattering is important! [G Gaunt et al, arXiv:1003.3953]

Also (considering gluons)  $c\bar{c}c\bar{c}$  over  $c\bar{c}$  production  $\sim 100$  times higher than theory predictions [LHCb, arXiv:1205.0975]!

Typically convolution of DPD into two generalized form factors is assumed.



$$A_{VV}^{ud}(y^2, py = 0) = \int_{-1}^1 d\zeta \int_{-1}^1 dx_1 dx_2 F_{ud}(x_1, x_2, \zeta, \mathbf{y}^2)$$

can be extracted from four point functions on the lattice. For the pion:

$$\langle \pi^- | [\bar{u} \gamma_{mu} d](\mathbf{y}) [\bar{u} \gamma_\nu d(\mathbf{0})] | \pi^+ \rangle$$

# Quark line diagrams to be considered

$$C_1^{ij}(y) = \text{---} \otimes \text{---}$$

Two horizontal quark lines connect two green ovals. The top line has an arrow pointing left and is labeled  $\mathcal{O}_i(0)$ . The bottom line has an arrow pointing right and is labeled  $\mathcal{O}_j(y)$ .

$$C_2^{ij}(y) = \text{---} \otimes \text{---} \otimes \text{---}$$

Three horizontal quark lines connect two green ovals. The top line has an arrow pointing right and is labeled  $\mathcal{O}_i(0)$ . The middle line has an arrow pointing right and is labeled  $\mathcal{O}_j(y)$ . The bottom line has an arrow pointing left.

$$A^{ij}(y) = \text{---} \otimes \text{---}$$

Two quark lines connect two green ovals. The left line has an arrow pointing right and is labeled  $\mathcal{O}_i(0)$ . The right line has an arrow pointing right and is labeled  $\mathcal{O}_j(y)$ .

$$S_1^{ij}(y) = \text{---} \otimes \text{---}$$

Two horizontal quark lines connect two green ovals. The top line has an arrow pointing right and is labeled  $\mathcal{O}_i(0)$ . The bottom line has an arrow pointing left. A circular loop with an arrow points clockwise around the two lines, labeled  $\mathcal{O}_j(y)$ .

$$S_2^{ij}(y) = \text{---} \otimes \text{---}$$

Two horizontal quark lines connect two green ovals. The top line has an arrow pointing right and is labeled  $\mathcal{O}_i(0)$ . The bottom line has an arrow pointing left. A circular loop with an arrow points clockwise around the bottom line, labeled  $\mathcal{O}_j(y)$ .

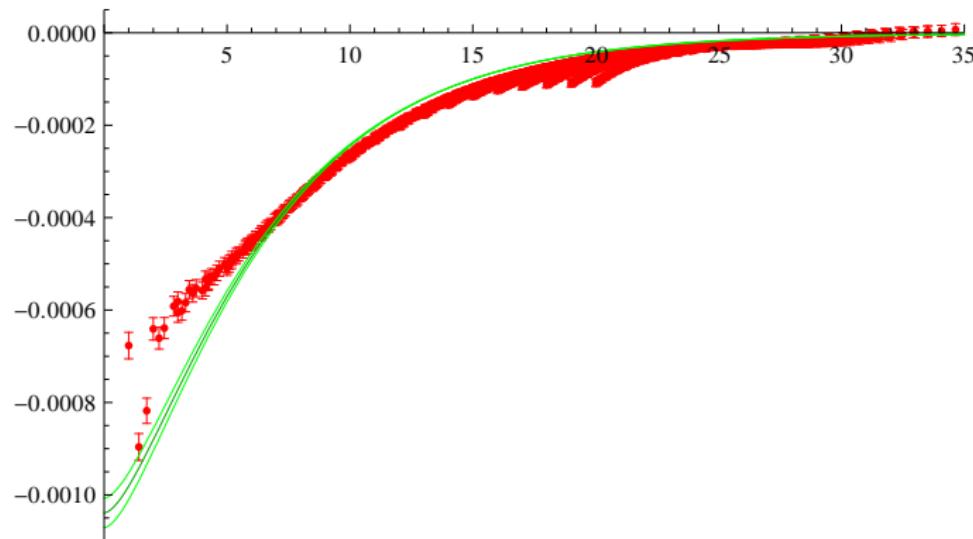
$$D^{ij}(y) = \text{---} \otimes \text{---}$$

Two horizontal quark lines connect two green ovals. The left line has an arrow pointing right and is labeled  $\mathcal{O}_i(0)$ . The right line has an arrow pointing right and is labeled  $\mathcal{O}_j(y)$ .

# Comparison of $A_{VV}$ with convolution of EM formfactors

G Bali, L Castagnini, M Diehl, J Gaunt, B Gläßle, A Schäfer,  
C Zimmermann,.... very preliminary

$M_\pi \approx 290$  MeV, lattice extent:  $L = 40a$ ,  $a \approx 0.071$  fm.



Seems to work for  $|y| \gtrsim 0.4$  fm

(Small  $|y|$ : lattice artefacts. Large  $|y|$ : finite size effects.)

## Topic II: Semileptonic $\Lambda_b$ decay form factors

[W Detmold, C Lehner, S Meinel, 1503.01421]

$$q^2 \in [14, 22], [7, 15] \text{ GeV}^2$$

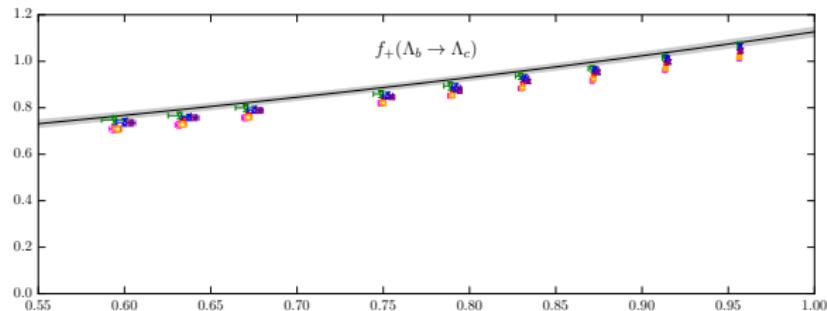
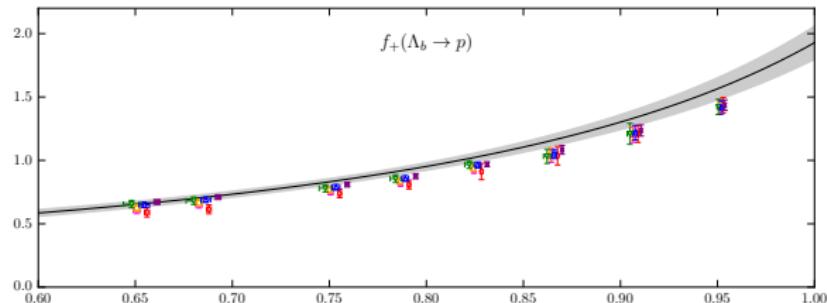
$$a \approx 0.112, 0.085 \text{ fm}$$

$$M_\pi \in [230, 350] \text{ MeV}$$

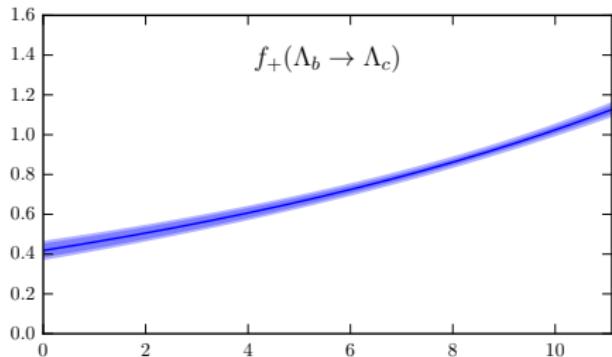
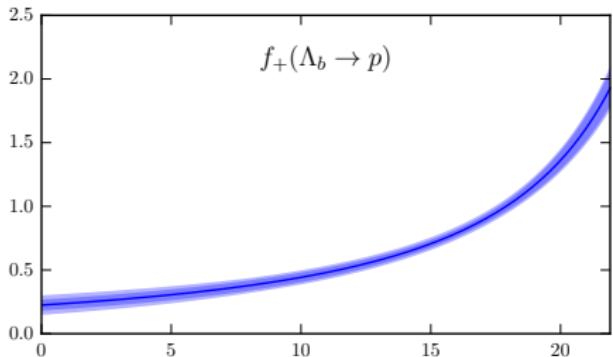
FNAL heavy quark action  
tuned to give correct  
meson masses.

Curves: extrapolations to  
 $a = 0, M_\pi = 135 \text{ MeV}$ .

Also  $f_0, f_\perp, g_+, g_0, g_\perp$  have been computed.



## Parametrization of the lattice data:

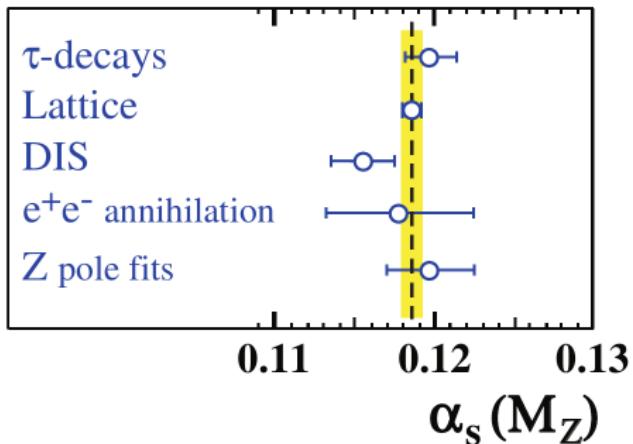
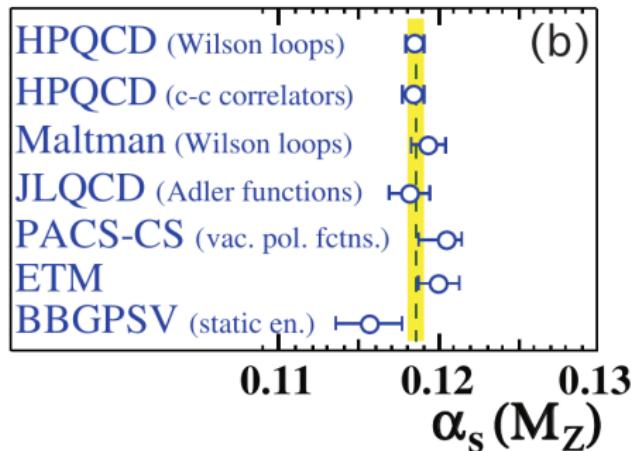


Note that lattice data covers  $q^2 \in [14, 22] \text{ GeV}^2$  and  $q^2 \in [7, 15] \text{ GeV}^2$

Authors claim, including all systematics and combining with data for smaller  $q^2$ , theory accuracy of 2.2% for  $|V_{cb}|$  and 4.3% for  $|V_{ub}|$  can be achieved.

Results used in [\[LHCb: R Aaij et al, 1504.01568\]](#).  
(also talk by [Marco Ciuchini](#)).

# Topic III: High precision QCD. Strong coupling parameter



[PDG 2014]: Lattice average:  $\alpha_s(M_Z) = 0.1185(5)$

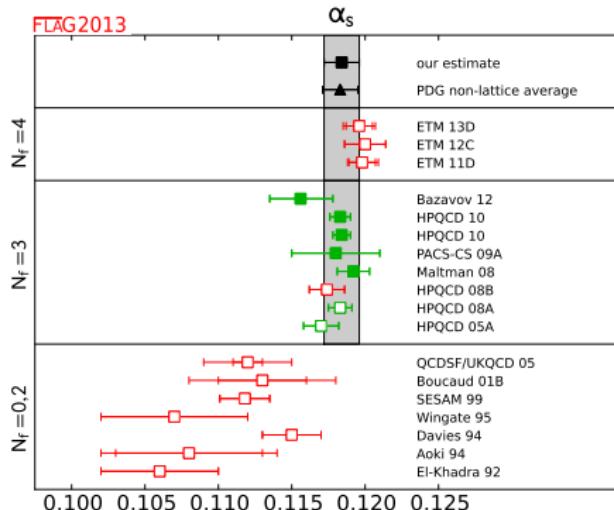
World average:  $\alpha_s(M_Z) = 0.1185(6)$

(cf. talk by Giulia Zanderighi)

World average without Lattice:  $\alpha_s(M_Z) = 0.1183(12)$

# Strong coupling parameter II

FLAG 2 report: [FLAG: S Aoki et al, 1310.8555]



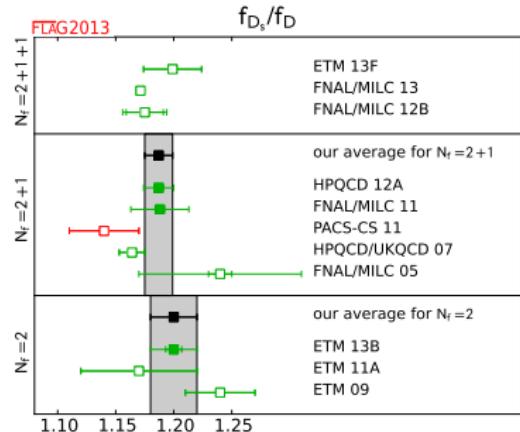
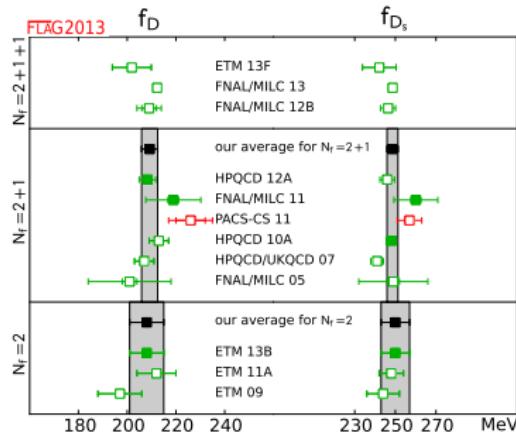
Lattice average:  $\alpha_s(M_Z) = 0.1184(12)$

PDG world average without lattice:  $\alpha_s(M_Z) = 0.1183(12)$

Lattice QCD will improve on this. Present limitation is scale  $\mu$  of  $\alpha(\mu)$  relative to the order of the perturbative matching to the  $\overline{\text{MS}}$  scheme.

# The precision frontier: $D$ and $D_s$ decay constants

FLAG 2 report: [FLAG: S Aoki et al, 1310.8555]



$$\text{FLAG 2: } f_{D_s} = 248.6(2.7), \quad f_{D_s}/f_D = 1.187(12)$$

Preliminary FLAG 3 analysis [FLAG: A Vladikas, 1509.01155]:

$$N_f = 2 + 1 : f_{D_s} = 249.8(2.3), \quad N_f = 2 + 1 + 1 : f_{D_s} = 248.8(1.3)$$

Latest CKMfitter <http://ckmfitter.in2p3.fr/> (did not check UTfit):

$f_{D_s}$

Reference	Article	$N_f$	Mean	Stat	Syst
ETMC09	[61]	2	244	3	9
HPQCD10	[71]	2+1	248.0	1.4	4.5
FNAL-MILC11	[70]	2+1	260.1	8.9	16.2
FNAL-MILC14	[69]	2+1+1	249.0	0.3	$^{+1.7}_{-2.1}$
ETMC14	[67]	2+1+1	247.2	3.9	2.2
Our average			248.2	0.3	1.9

$f_{D_s}/f_D$

Reference	Article	$N_f$	Mean	Stat	Syst
ETMC09	[61]	2	1.24	0.03	0.01
FNAL-MILC11	[70]	2+1	1.188	0.014	0.054
HPQCD12	[72]	2+1	1.187	0.004	0.023
FNAL-MILC14	[69]	2+1+1	1.1712	0.0010	$^{+0.0037}_{-0.0040}$
ETMC14	[67]	2+1+1	1.192	0.019	0.017
Our average			1.175	0.001	0.004

Potentially dangerous: [69] dominates.

Difference  $f_{D^+}$  vs  $f_{D^0}$ ? Are  $f_{D^+}$ ,  $f_{D_s}$  defined at all?

Electromagnetic corrections, e.g., [N Carrasco et al, 1502.00257]!

# Summary

- Many quantities that are hard to constrain by experiment, e.g.,  $\sigma_{\pi N}$ ,  $f_{T_s}$ ,  $g_S$ ,  $g_T$ , are accessible in Lattice QCD.
- Lattice calculations are important to determine the spin content of the nucleon:  $\Delta q$ ,  $\Delta \Sigma$ ,  $J_q$ ,  $\langle x \rangle_{\Delta q}$ , ...  
**People should take  $\Delta s + \Delta \bar{s} \approx -0.02$  result more seriously!**
- In the past disconnected quark line diagrams were often omitted and differences quoted:  $g_A$ ,  $\langle x \rangle_{u-d}$ , ..., but no  $\Delta s$ ,  $\Delta \Sigma$ ,  $J_q$ ,  $\langle x \rangle_q$  etc.  
**Improved methods now allow for the calculation of these contributions.**
- $\langle x \rangle_{u-d}$  comes out almost 20% bigger than expected.  
**Lattice spacing effects? Renormalization?**
- First studies of double hard parton scattering/quark correlations.
- High Mellin moments almost impossible to compute  $\Rightarrow$  recent interest also in “quasi” parton distribution functions.
- Lattice QCD is major input in flavour physics.  
**Reliable error estimates are very important!**
- Electromagnetic corrections are now of major interest.