Semi-Automated Two-Loop Calculations for FeynHiggs

Sebastian Paßehr[†]
in collaboration with Thomas Hahn[‡]
arXiv:1508.00562 [hep-ph]

[†]DESY, Hamburg, [‡]Max Planck Institute for Physics, Munich

HELMHOLTZ | ASSOCIATION

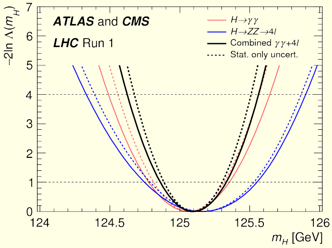
DESY Theory Workshop:
Physics at the LHC and beyond,

DESY Hamburg, Germany,
1st of October 2015

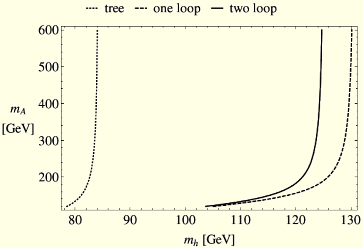
DESY

Contents

1. Motivation
2. Two-loop Higgs-mass corrections in the MSSM
   - Prerequisites
   - Step 0: the model
   - Step 1: the diagrams
   - Step 2: the amplitude
   - Step 3: tensor reduction
   - Step 4: simplification
   - Step 5: renormalization
   - Step 6: combination
   - Step 7: the result
3. Summary



[ATLAS, CMS, arxiv:1503.07589 [hep-ex]]



Motivation

Why do we want higher-order corrections?

two examples:

- Higgs mass m_h:
  measurement $m_h = (125.09 \pm 0.21 \pm 0.11)\text{GeV}$ [ATLAS, CMS, arxiv:1503.07589 [hep-ex]],
  theory uncertainty in MSSM: $\mathcal{O}(\text{GeV})$,

  Does the model fit with all data?

- anomalous magnetic moment of the muon a_μ:
  extremely precise measurement and calculation in the SM;
  deviation of $\Delta a_\mu = (28.7 \pm 8.0) \times 10^{-10}$ [Davier, Hoecker, Malaescu, Zhang, arxiv:1010.4180 [hep-ph]],

  Is there new physics?

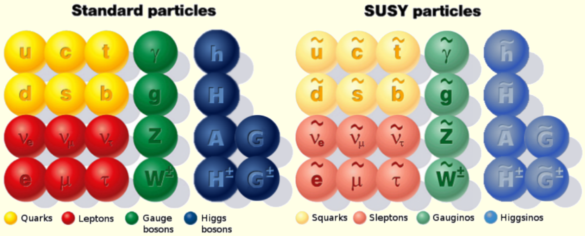
MSSM: Minimal Supersymmetric Standard Model

supersymmetry requires (at least) two $SU(2)$ Higgs doublets for

- holomorphy of superpotential,
- anomaly cancellation of Higgsinos.

five physical Higgs bosons:

- neutral CP-even h and H,
- neutral CP-odd A,
- charged $H^{\pm}$.



Two-loop Higgs-mass corrections in the MSSM

Higgs masses at k-loop order given by poles of propagator matrix

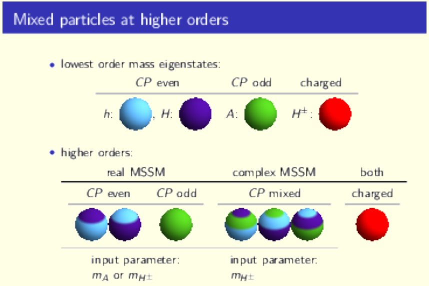
$$\boldsymbol{\Delta}^{(k)}_{hHA}\left(p^2\right) = i\,\left[p^2\mathbf{1} - \mathbf{M}^{(k)}_{hHA}\left(p^2\right)\right]^{-1},$$

with higher-order corrected mass matrix

$$\mathbf{M}^{(k)}_{hHA}\left(p^2\right)\Big|_{k\,\geq\,1} = \mathbf{M}^{(0)}_{hHA} - \sum_{j\,=\,1}^{k} \hat{\boldsymbol{\Sigma}}^{(j)}_{hHA}\left(p^2\right),$$

and renormalized self-energies

$$\hat{\boldsymbol{\Sigma}}^{(j)}_{hHA}\left(p^2\right) = \begin{pmatrix} \hat{\Sigma}^{(j)}_{h}(p^2) & \hat{\Sigma}^{(j)}_{hH}(p^2) & \hat{\Sigma}^{(j)}_{hA}(p^2) \\ \hat{\Sigma}^{(j)}_{hH}(p^2) & \hat{\Sigma}^{(j)}_{H}(p^2) & \hat{\Sigma}^{(j)}_{HA}(p^2) \\ \hat{\Sigma}^{(j)}_{hA}(p^2) & \hat{\Sigma}^{(j)}_{HA}(p^2) & \hat{\Sigma}^{(j)}_{A}(p^2) \end{pmatrix}$$



status of FeynHiggs for complex MSSM:

$$\mathbf{M}_{hHA}^{(2)} = \mathbf{M}_{hHA}^{(0)} - \hat{\mathbf{\Sigma}}_{hHA}^{(1)}\left(p^2\right) - \hat{\mathbf{\Sigma}}_{hHA}^{(2),\,\alpha_t\alpha_s}(0) - \hat{\mathbf{\Sigma}}_{hHA}^{(2),\,\alpha_t^2}(0)\,.$$

physical details and numerical analyses of $\hat{\mathbf{\Sigma}}_{hHA}^{(2),\,\alpha_t^2}$ available at
Hollik, SP, arxiv:1401.8275, 1409.1687, 1502.02394 [hep-ph],

in the following:
implementation in FeynHiggs, cf. Hahn, SP, arxiv:1508.00562 [hep-ph].

Prerequisites

utilized programs and packages:

- `Mathematica` [Wolfram],
- `FeynArts` [Hahn, Schappacher, arxiv:0105349 [hep-ph]],
- `FormCalc` [Hahn, Illana, arxiv:0607049 [hep-ph]],
- `TwoCalc` [Weiglein, Scharf, Böhm, arxiv:9310358 [hep-ph]].

how to use them for loop calculations:
$\rightarrow$ Computer Algebra and Particle Physics - CAPP 2015, DESY Hamburg
talks by T. Hahn.

Step 0: the model

Leading Higgs-mass corrections enhanced by top-Yukawa coupling h_t:

- neglect gauge couplings,
- set bottom mass equal to zero,

apply to `FeynArts` model file before any calculation:

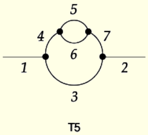
- less amount of data,
- much faster evaluation.

command line: `model/0-glmod`

main packages:	`packages/Gaugeless.m`, `packages/XtSimplify.m`
input:	`model/MSSMCT.mod.in`
output:	`model/MSSMCTgl.mod`

Step 1: the diagrams

- Feynman diagrams generated with the help of `FeynArts`,
- define process, e. g. `proc["h0h0"] = S[1] -> S[1]` for self-energy of h,
- besides usual selection options of `FeynArts`,
  e. g. `LastSelections -> F[3|4, {3, _}]` to require a top and/or bottom,
  also assignments of certain particles to lines, e. g.



`t[3] && htb[5|6]`

requires top/stop on line 3 and

Higgs or top/stop or bottom/sbottom on lines 5 and/or 6.

command line: `scripts/1-amps se cto`, (`se`: e. g. `h0h0`, `cto`: 0 or 1)

main packages:	`FeynArts`, `packages/FASettings.m`, `packages/Gaugeless.m`
input:	(none)
output:	`m/se/1-amps.cto` (amplitude),
	`m/se/1-amps.cto.ps.gz` (diagrams)

Step 2: the amplitude

apply several approximations to full amplitude:

- vanishing external momentum $\Rightarrow$ different (easier) integral reduction,
- sfermion mixing matrix elements $U_{ij} \equiv$ `USf[i,j]`, $i, j = 1, 2$:
  define products `UCSf[i,j] = USf[i,j] USf[i,j]`* (and others),
  easily exploit unitarity: `UCSf[2,2] = UCSf[1,1]`, `UCSf[1,2] = UCSf[2,1]`,
  expressions much shorter

command line: `scripts/2-prep se cto`

main packages:	`packages/USfSimplify.m`, `packages/U2Simplify.m`, `packages/SimplificationDefinitions.m`
input:	`m/se/1-amps.cto`
output:	`m/se/2-prep.cto`

Step 3: tensor reduction

for this project:

- `TwoCalc` for genuine two-loop diagrams, `cto` = 0,
- `FormCalc` for one-loop diagrams with counterterm insertion, `cto` = 1,
- feature of this script:
  name-space conflicts of these two `Mathematica` packages avoided,
- modular structure allows for use of other reduction algorithms

command line: `scripts/3-calc se cto`

main packages:	`TwoCalc`, `packages/UseSimplePackage.m`, `FormCalc`, `packages/FCSettings.m`
input:	`m/se/2-prep.cto`
output:	`m/se/3-calc.cto`

Step 4: simplification

apply simplifications to the expression for each type of diagram:

- simplify color structure,
- collect common factors,
- ...

types of diagrams defined by tag function `DiagMark[`m_i`]`,
m_i: list of all masses of particles inside the loop
(acknowledgment to Dominik Stöckinger)

command line: `scripts/4-simp se cto`

main packages:	`packages/SimplificationDefinitions.m`, `packages/FCSettings.m`
input:	`m/se/3-calc.cto`
output:	`m/se/4-simp.cto`

Step 5: renormalization

- one-loop renormalization constants defined in `model/MSSMCT.rc1`, originally part of `FeynArts` model file for complex MSSM,

  [Fritzsche, Hahn, Heinemeyer, von der Pahlen, Rzehak, Schappacher, arXiv:1309.1692 [hep-ph]],

  diagrams generated and evaluated with the help of `FeynArts` and `FormCalc`,
- special treatment of δM_V^2, $V = W, Z$:
  $\delta M_V^2 / M_V^2$ appearing in Yukawa couplings not zero in gauge-less limit,
- two-loop renormalization constants defined in `model/MSSMCT.rc2`, complete list without approximations recorded in Hollik, SP, arxiv:1409.1687,
- expansion in dimension parameter $\epsilon = (D-4)/2$

command line:	`scripts/5-rc se`
main packages:	`FeynArts`, `FormCalc`, `packages/RenConst.m`, `packages/ExpandDel.m`
input:	`m/se/4-simp.1`
output:	`m/se/5-rc`, `m/rc/*`

Step 6: combination

- combine
  - the genuine two-loop self-energies,
  - the one-loop self-energies with inserted renormalization constants,
  - the genuine two-loop counterterms with inserted renormalization constants,
- expansion in dimension parameter ϵ (library for integrals necessary),
- default: return only finite part (coefficient of ϵ^0),
- if environmental parameter `DEBUG` is set to -1 or -2:
  return divergent parts of order ϵ^{-1} or ϵ^{-2} respectively

command line: `scripts/6-comb se`

main packages: `packages/FinalSimp.m`, `packages/ExpandDel.m`
input: `m/se/4-simp.0`, `m/se/4-simp.1`, `m/rc/*`
output: `m/se/6-comb`

Step 7: the result

- translate the final result into optimized `Fortran` code using `FormCalc` functionality [Hahn, arXiv:1006.2231 [hep-ph]],
- arrange expressions of the seven self-energies and three tadpoles to suit different options of `FeynHiggs`,
- introduce abbreviations for common subexpressions,
- disambiguate parameters in gauge-less limit from normal ones used in `FeynHiggs`,
- output can be directly copied into `FeynHiggs`' source tree

command line: `scripts/7-code`

main packages:	`FormCalc`, `packages/FCSettings.m`
input:	`m/*/6-comb`, `static/*`
output:	`f/*`

semi-automatic two-loop calculation:

- based on Mathematica,
- modularized structure,
- highly customizable,
- user supplies model file and renormalization, defines process and types make (some more adjustments might be necessary),
- template included in FeynHiggs source code, available at → http://feynhiggs.de