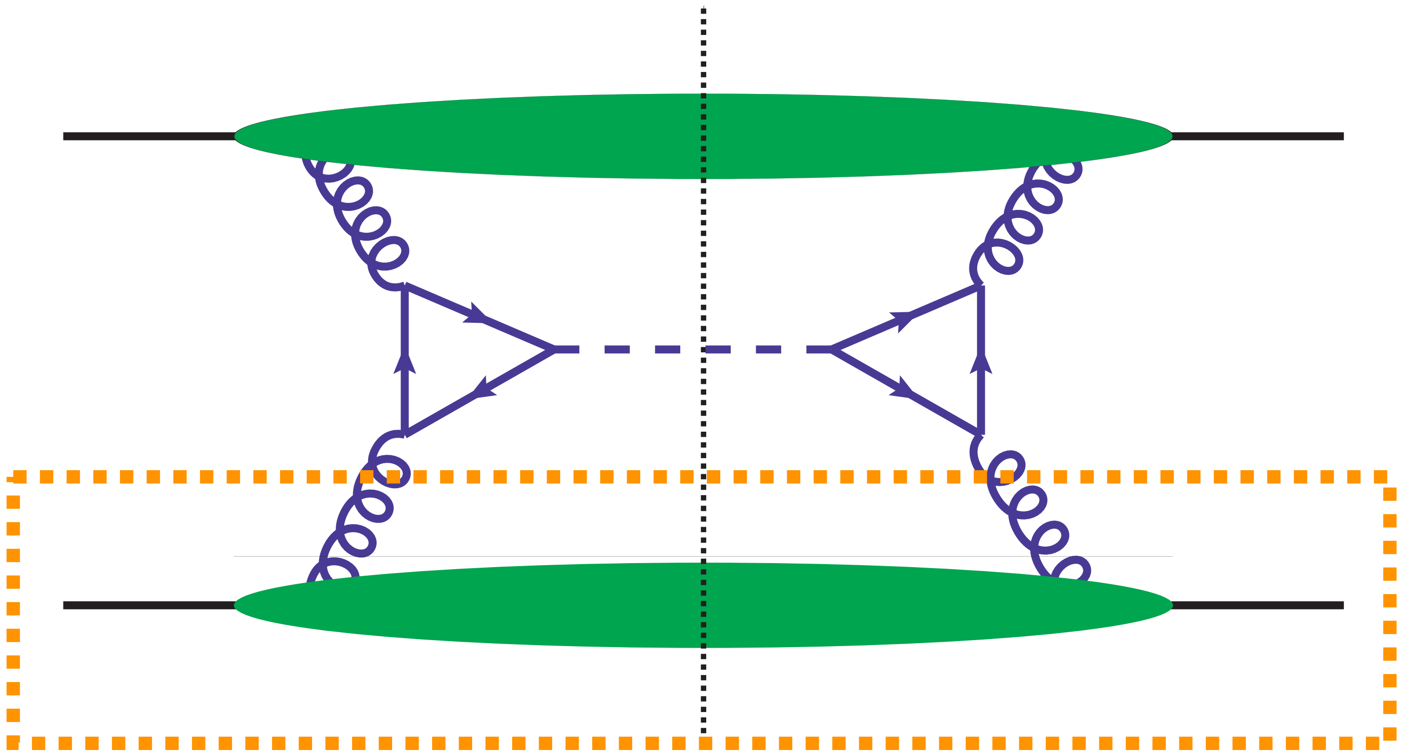
Transverse momentum dependent (un)polarized gluon distributions in Higgs production

Tomas Kasemets
Nikhef / VU

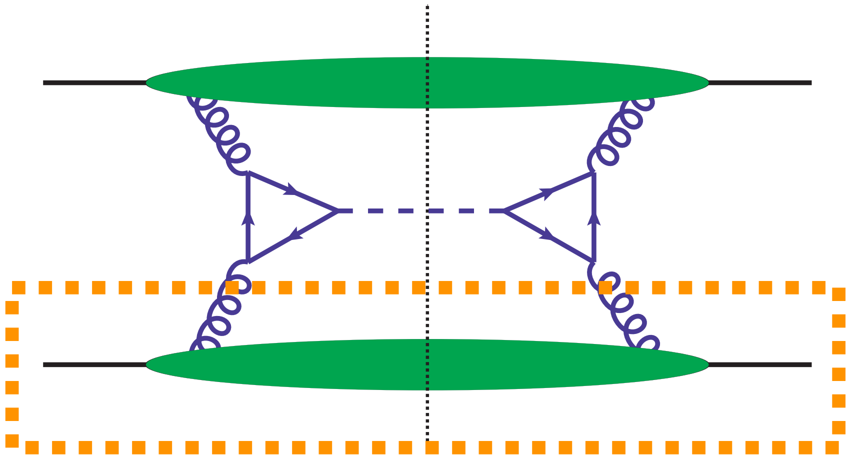


Echevarria, TK, Mulders and Pisano, arXiv:1502.05354;

DESY Theory Workshop
Hamburg, October 1, 2015

Outline

- Motivation and introduction to TMDs
- Factorization as multistep matching
  - Proper definition of TMDs
  - Evolution of gluon TMDs
  - Re-factorization in terms of PDFs
- Results for Higgs boson production at LHC
- Summary and outlook



- *Disclaimer:* Main interest of study - to properly define gluon TMDs and derive their scale evolution. Higgs boson production is a suitable prototype process.

Introduction

- Cross section calculations based on factorization

$$\text{cross section} = \text{parton distributions} \times \text{partonic cross section}$$

- Integrated over transverse momenta of Higgs boson

  $\Rightarrow$ PDFs describing the partons inside the protons

- Measured transverse momenta of Higgs boson

  - Sensitive to transverse momenta of the two partons

  $\Rightarrow$ TMDs describing the partons inside the protons

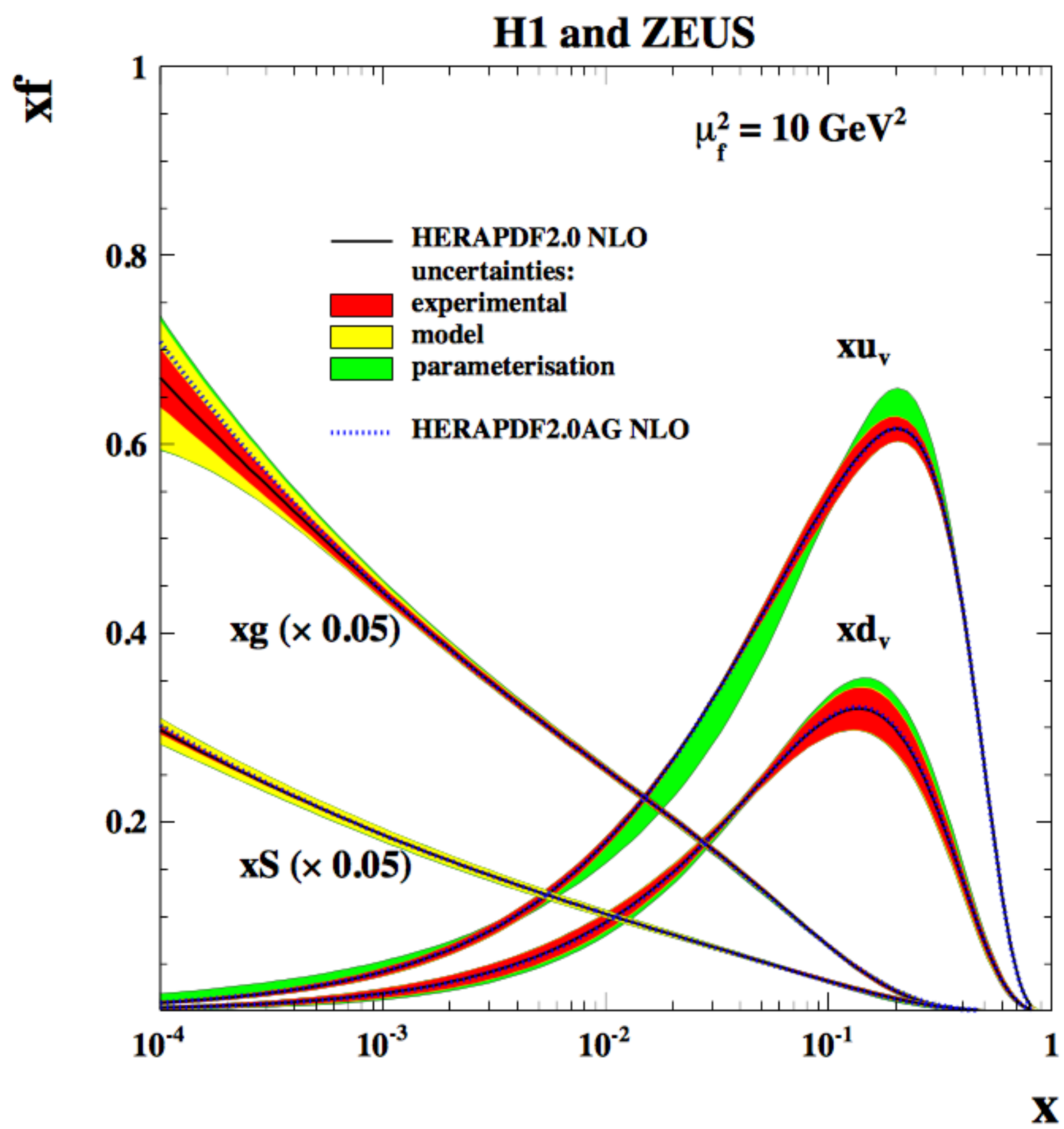
  - Depend on momentum fraction x and transverse momenta of parton

- Schematically, at leading order

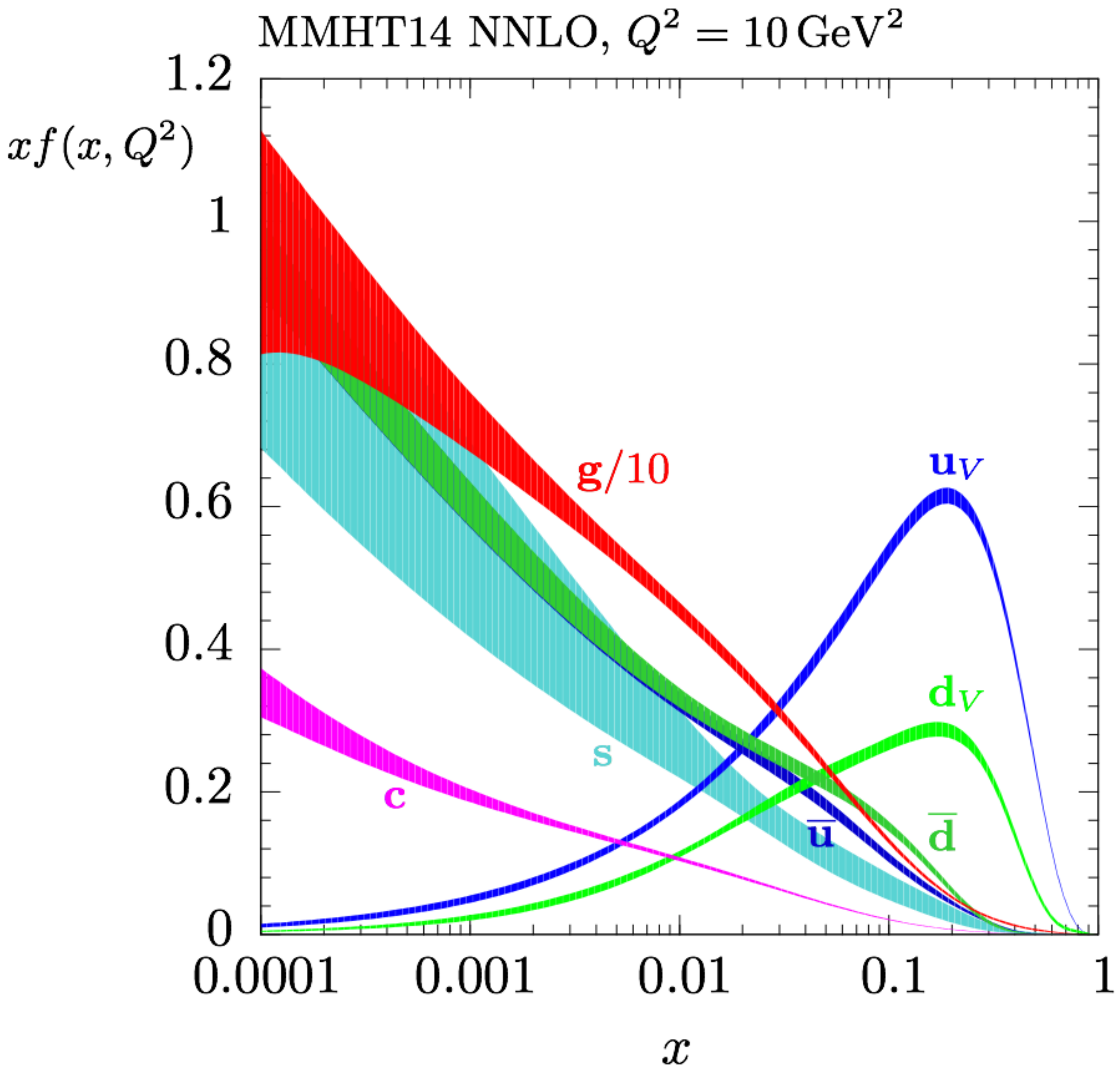
$$\frac{d\sigma}{dx d\bar{x} d^2\boldsymbol{q}_T} = \hat{\sigma} \int d^2\boldsymbol{k}_{aT} d^2\boldsymbol{k}_{bT} \delta^{(2)}(\boldsymbol{q}_T - \boldsymbol{k}_{aT} - \boldsymbol{k}_{bT}) f_{a/A}(x, \boldsymbol{k}_{aT}; \mu) f_{b/B}(\bar{x}, \boldsymbol{k}_{bT}; \mu)$$
$$+ \mathcal{O}(q_T/Q)$$

Motivation

- Map of the proton (momentum space): **PDF = 1D**



H1 and ZEUS Collaborations, 2015



Harland-Lang, Martin, Motylinski, Thorne, 2014

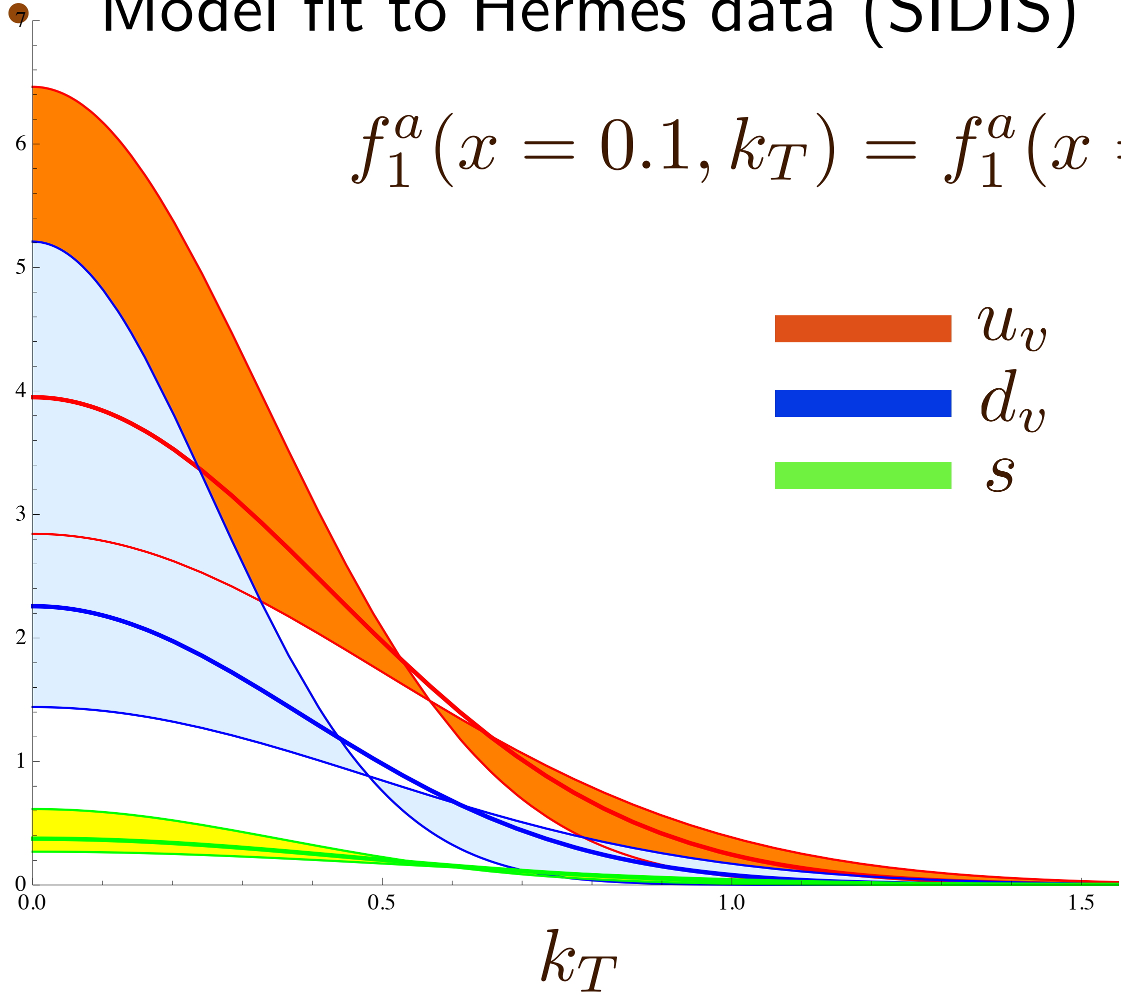
Motivation

- Map of the proton: **Township of Proton, Canada = 2D**

Motivation

- Map of the proton (momentum space): **TMD = 3D**
- How does the parton density depend on the transverse momenta?
- Quark TMDs have received much attention
  - Model fit to Hermes data (SIDIS)

$$f_1^a(x=0.1, k_T) = f_1^a(x=0.1)\frac{1}{\pi\left\langle k_T^2\right\rangle_a} e^{-k_T^2/\langle k_T\rangle_a}$$



$$\left\langle k_T^2\right\rangle_{u_v} = 0.36 \pm 0.14\text{GeV}^2$$
$$\left\langle k_T^2\right\rangle_{d_v} = 0.30 \pm 0.17\text{GeV}^2$$
$$\left\langle k_T^2\right\rangle_{s} = 0.41 \pm 0.16\text{GeV}^2$$

Signori, Bacchetta, Radici, Schnell, 2013

- Gluon TMDs much less explored

Polarization: PDFs vs TMDs

- PDFs: Unpolarized and longitudinally polarized
- TMDs: Many different distributions (correlations between spin of patron, transverse momenta and spin of proton)

$$G_{g/A}^{\mu\nu[U]}(x, \boldsymbol{k}_{n\perp}) = -\frac{g_\perp^{\mu\nu}}{2} f_1^g(x, k_{nT}) + \frac{1}{2}\left(g_\perp^{\mu\nu} - \frac{2k_{n\perp}^\mu k_{n\perp}^\nu}{k_{nT}^2}\right) h_1^{\perp g}(x, k_{nT})$$

$$G_{g/A}^{\mu\nu[L]}(x, \boldsymbol{k}_{n\perp}) = -i\frac{\epsilon_\perp^{\mu\nu}}{2}\lambda\, g_{1L}^g(x, k_{nT}) + \frac{\epsilon_\perp^{k_{n\perp}\{\mu}k_{n\perp}^{\nu\}}}{2k_{nT}^2}\lambda\, h_{1L}^{\perp g}(x, k_{nT})$$

$$G_{g/A}^{\mu\nu[T]}(x, \boldsymbol{k}_{n\perp}) = -g_\perp^{\mu\nu}\frac{\epsilon_\perp^{k_{n\perp}S_\perp}}{k_{nT}} f_{1T}^{\perp g}(x, k_{nT}) - i\epsilon_\perp^{\mu\nu}\frac{\boldsymbol{k}_{n\perp}\cdot\boldsymbol{S}_\perp}{k_{nT}} g_{1T}^g(x, k_{nT})$$
$$+\frac{\epsilon_\perp^{k_{n\perp}\{\mu}k_{n\perp}^{\nu\}}}{2k_{nT}^2}\frac{\boldsymbol{k}_{n\perp}\cdot\boldsymbol{S}_\perp}{k_{nT}} h_{1T}^{\perp g}(x, k_{nT}) + \frac{\epsilon_\perp^{k_{n\perp}\{\mu}S_\perp^{\nu\}} + \epsilon_\perp^{S_\perp\{\mu}k_{n\perp}^{\nu\}}}{4k_{nT}} h_{1T}^g(x, k_{nT})$$

Mulders, Rodrigues, 2000

- A gluon in an unpolarized proton is either unpolarized or linearly polarized

TMDs in Higgs production

$$h_A(P, S_A) + h_B(\bar{P}, S_B) \to H(q_T) + X$$

- Gluon channel via top-loop dominates $gg \to H$
- Factorization with effective theories, stepwise matching
- Higgs color singlet - makes it suitable process.
- Color singlet quarkonium production (c-cbar or b-bbar bound states) follows similar steps
- Previous works already addressed this process

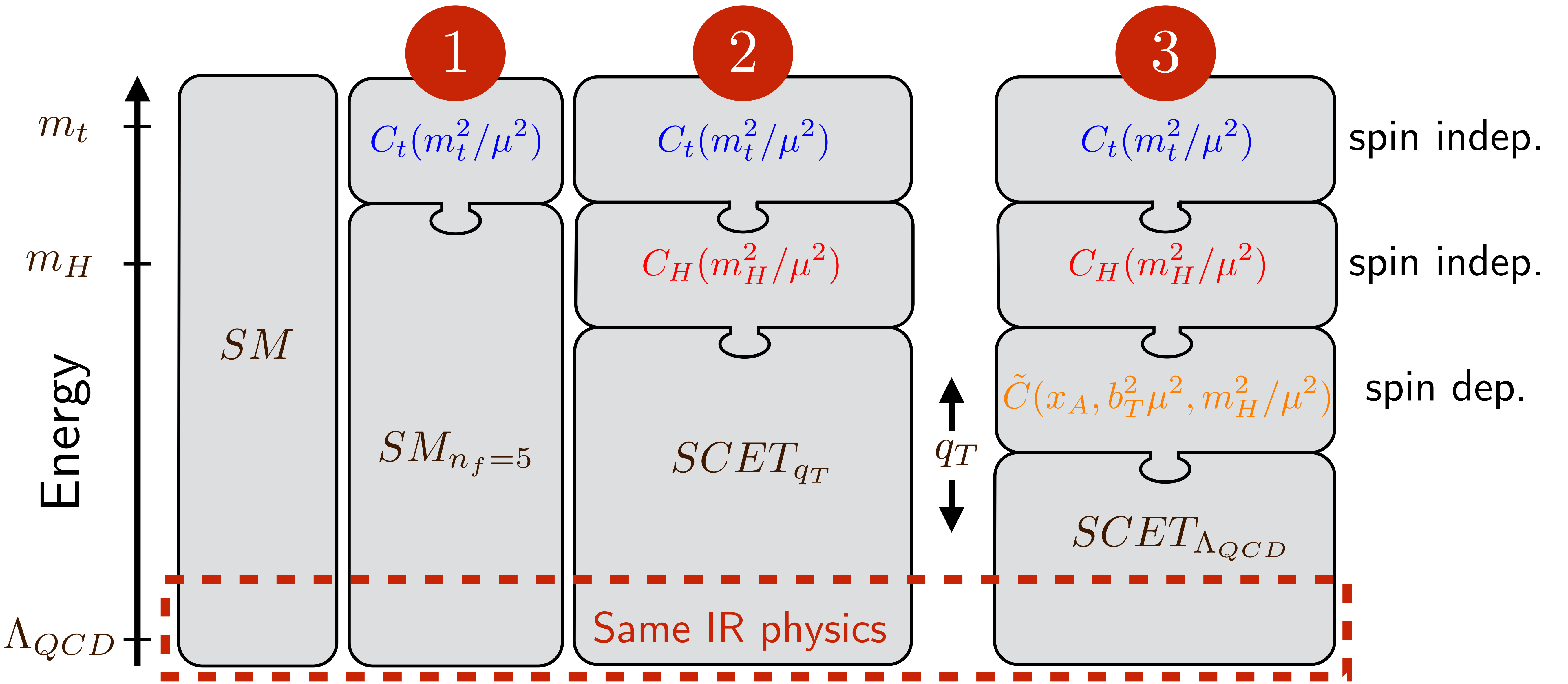
Neill, Rothstein, Vaidya 15; Becher, Neubert 12;
Chiu, Jain, Neill, Rothstein 12; Mantry, Petriello 11;
Sun, Xiao, Yuan 11; Ji, Ma, Yuan 05;
Catani, de Florian, Grazzini, Nason 03 etc.

We define TMDs (on the light cone) free from spurious rapidity divergencies

Factorization theorem

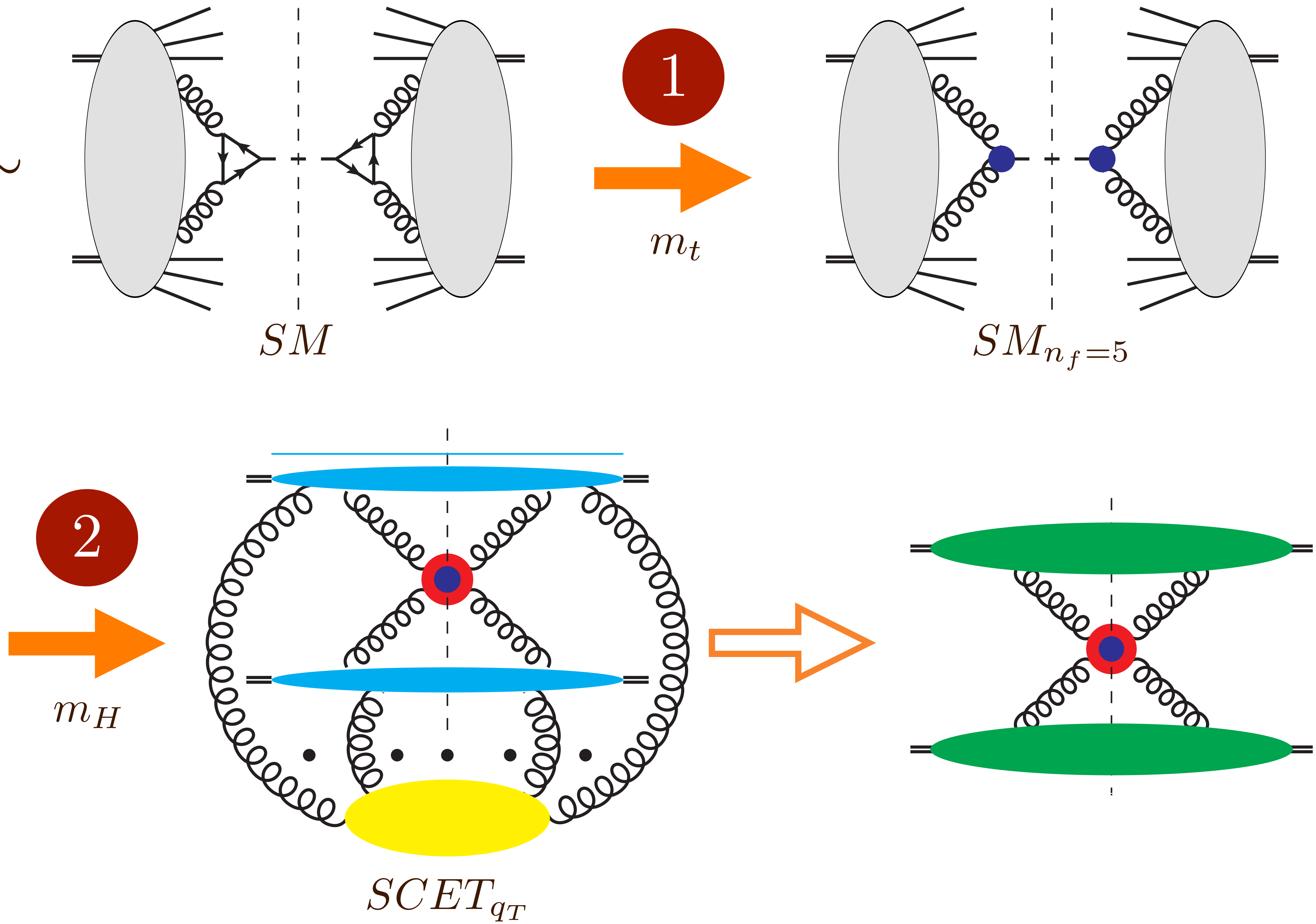
Factorization theorem = stepwise matching procedure

$$\mathrm{QCD}(n_f=6) \to \mathrm{QCD}(n_f=5) \to \mathrm{SCET}_{q_T} \to \mathrm{SCET}_{\Lambda_{QCD}}$$



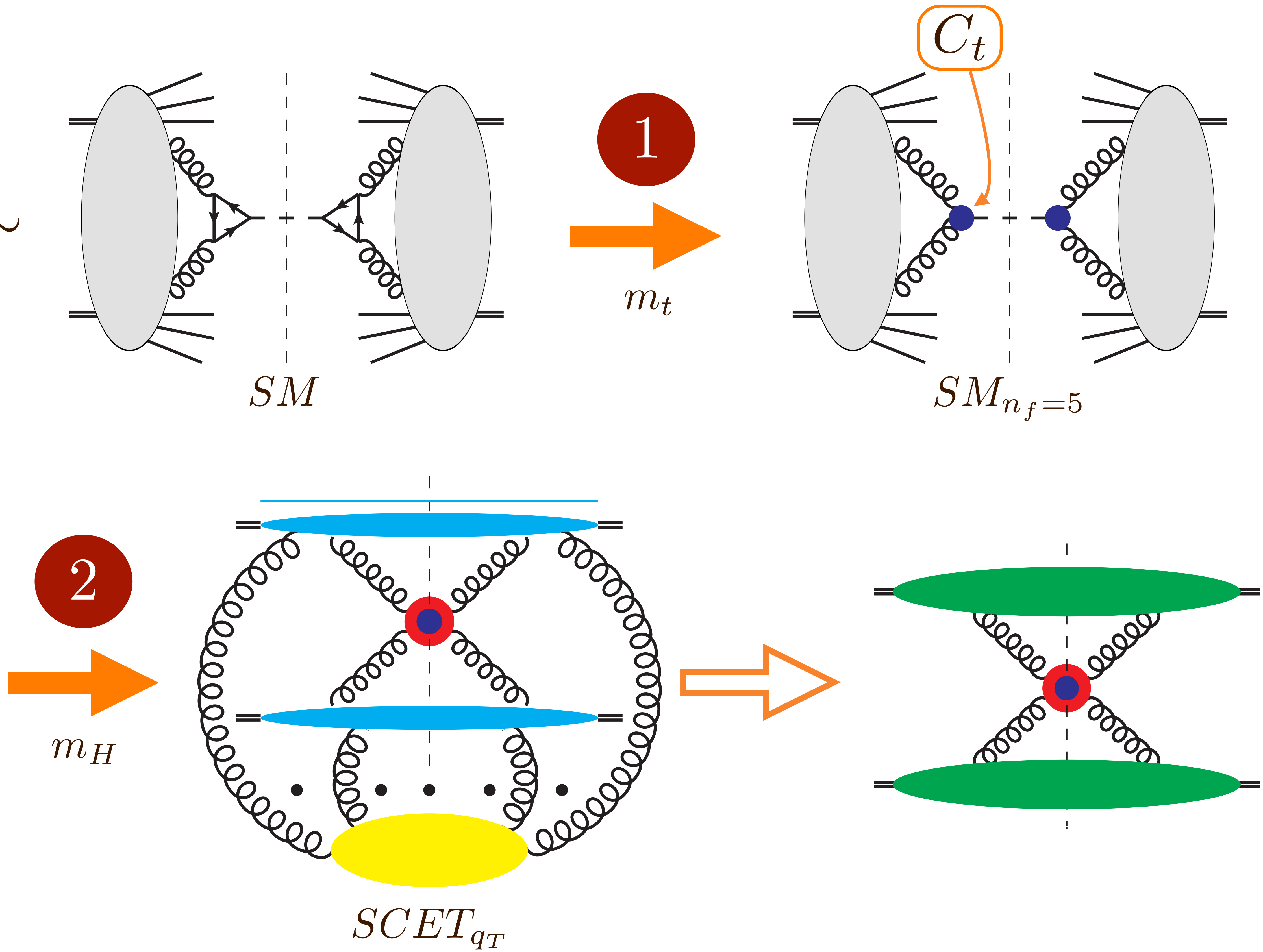
Factorization theorem

$d\sigma \sim$

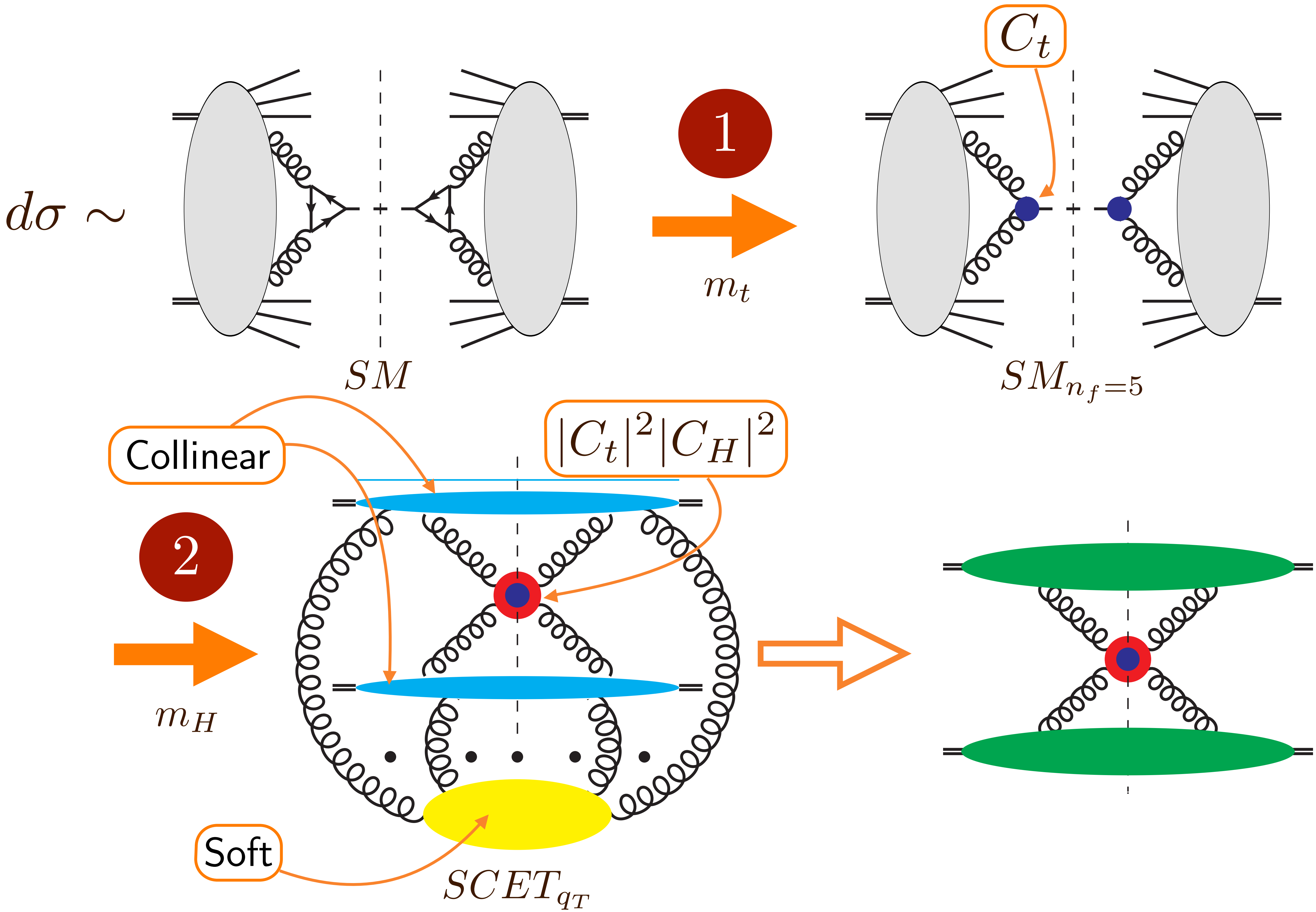


Factorization theorem

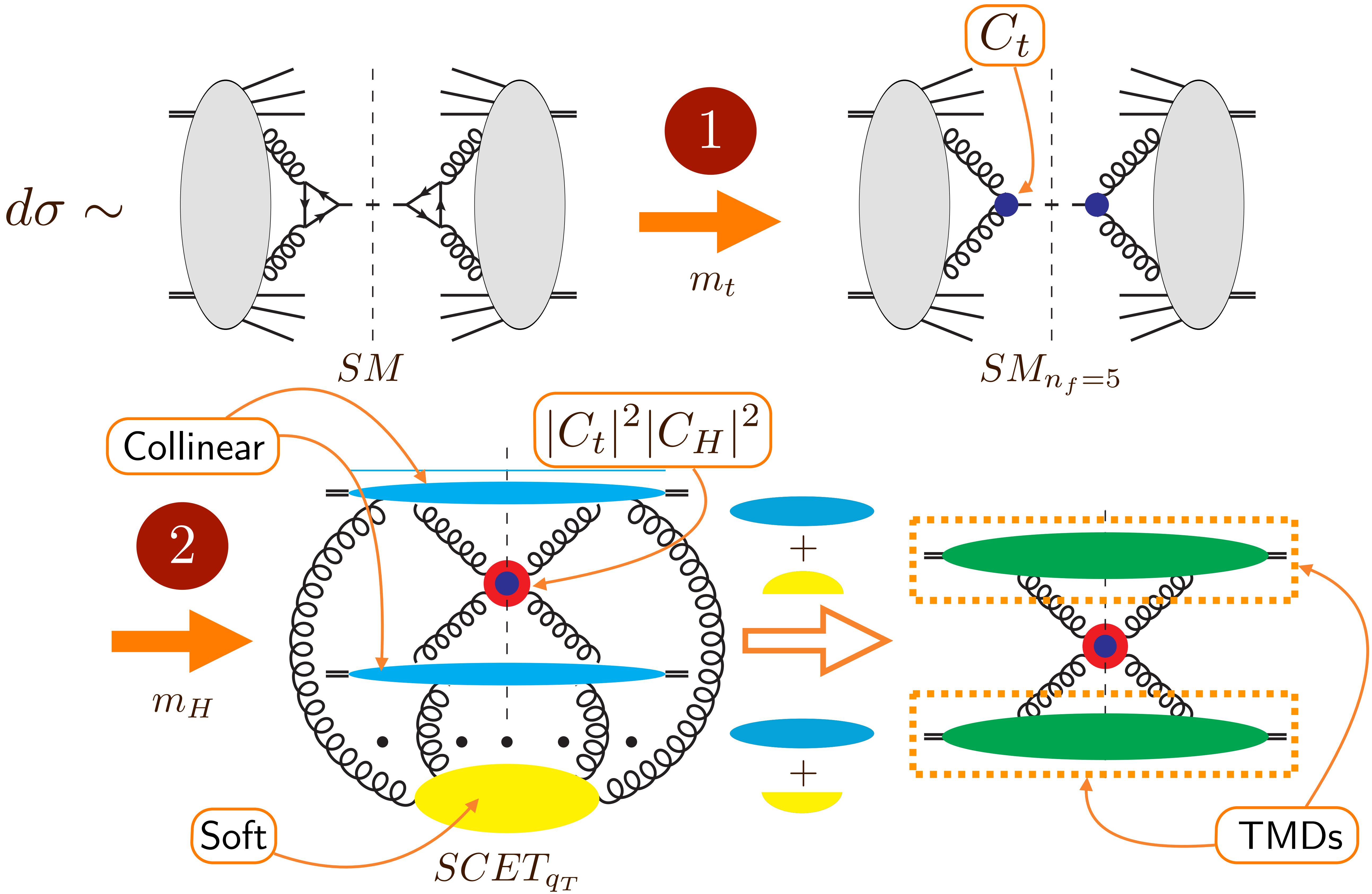
$d\sigma \sim$



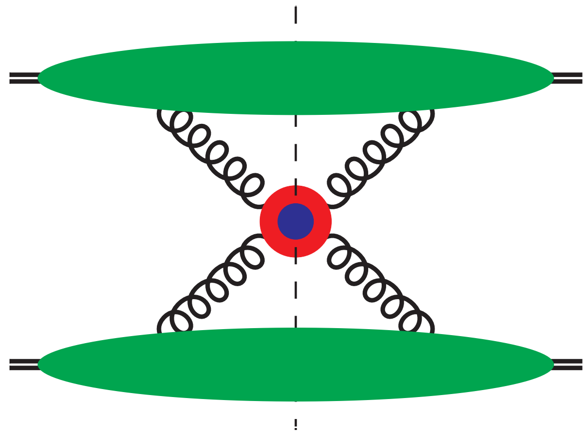
Factorization theorem



Factorization theorem



Step 2 $\mathrm{QCD}(n_f = 6) \to \mathrm{QCD}(n_f = 5) \to \mathrm{SCET}_{q_T} \to \mathrm{SCET}_{\Lambda_{QCD}}$



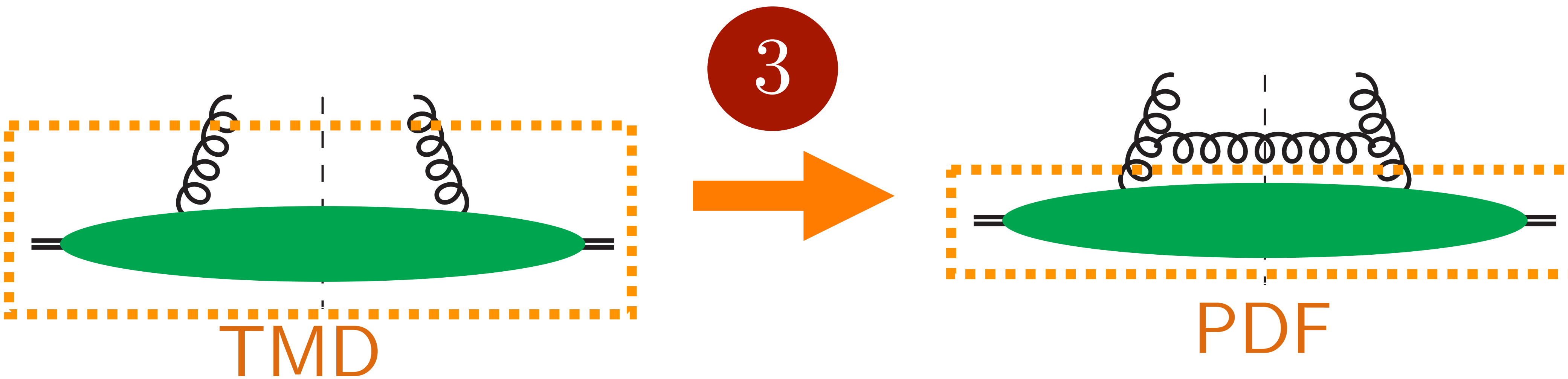
- Cross section in terms of properly defined TMDs

$$\frac{d\sigma}{dy\, d^2q_\perp} = 2\sigma_0(\mu)\, C_t^2(m_t^2,\mu) H(m_H,\mu)\, \frac{1}{(2\pi)^2} \int d^2b_\perp\, e^{i\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}$$
$$\times \frac{1}{2}\left[\tilde{f}_1^{g/A}(x_A, b_T; \zeta_A, \mu)\, \tilde{f}_1^{g/B}(x_B, b_T; \zeta_B, \mu) + \tilde{h}_1^{\perp\, g/A}(x_A, b_T; \zeta_A, \mu)\, \tilde{h}_1^{\perp\, g/B}(x_B, b_T; \zeta_B, \mu)\right]$$
$$+\,\mathcal{O}(q_T/m_H)$$

- The TMDs depend on two scales μ and ζ
- Combined, gives us the evolution of the gluon TMDs (universal evolution kernel = same for all polarizations)
  - Resums large logaritms
- Evolution contains non-perturbative piece, at large b

Step 3 $\mathrm{QCD}(n_f = 6) \to \mathrm{QCD}(n_f = 5) \to \mathrm{SCET}_{q_T} \to \mathrm{SCET}_{\Lambda_{QCD}}$

- When $\Lambda_{QCD} << k_T << Q$ we have a perturbative scale in TMDs
  - TMDs can be expressed in terms of usual PDFs



$$\tilde{f}_{g/A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \tilde{C}^f_{g/j}(\bar{x}, b_T; \zeta, \mu) \otimes f_{j/A}(x/\bar{x}; \mu) + \mathcal{O}(b_T \Lambda_{\mathrm{QCD}})$$

$$\tilde{h}_{g/A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \tilde{C}^h_{g/j}(\bar{x}, b_T; \zeta, \mu) \otimes f_{j/A}(x/\bar{x}; \mu) + \mathcal{O}(b_T \Lambda_{\mathrm{QCD}})$$

$$\tilde{g}^{g/A}_{1L}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \tilde{C}^g_{g\leftarrow j}(\bar{x}, b_T; \zeta, \mu) \otimes g_{j/A}(x/\bar{x}; \mu) + \mathcal{O}(b_T \Lambda_{\mathrm{QCD}})$$

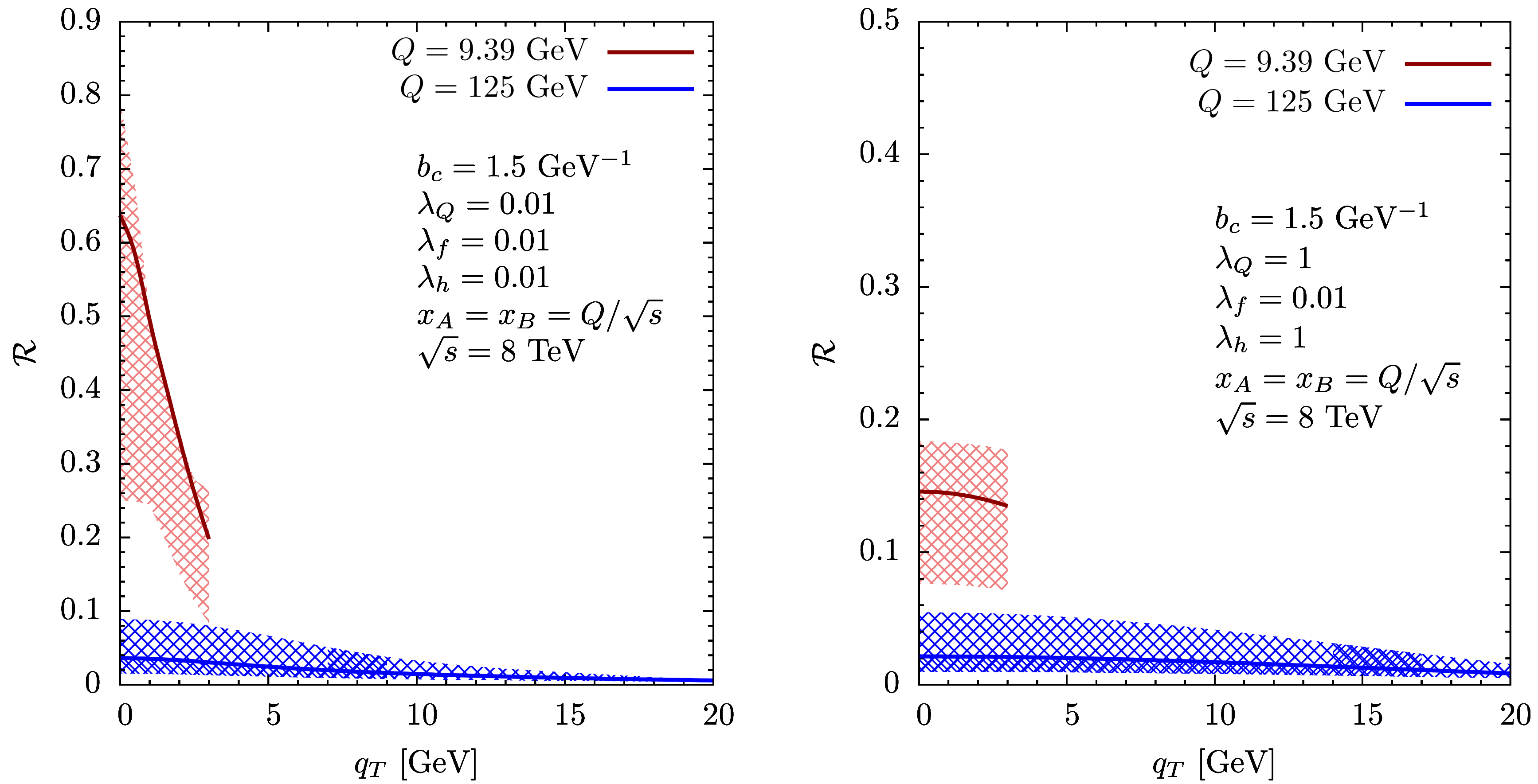
Linear polarization vs unpolarized

- Non-perturbative model:

$$\tilde{f}_{g/A} = \tilde{f}_{g/A}^{Pert}(x_A, \hat{b}_T; \zeta_A, \mu)\, e^{-b_T^2(\lambda_f + \lambda_Q \ln(Q^2/Q_0^2))} \qquad \hat{b}_T(b_T) = b_c\left(1 - e^{-(b_T/b_c)^2}\right)^{1/2}$$

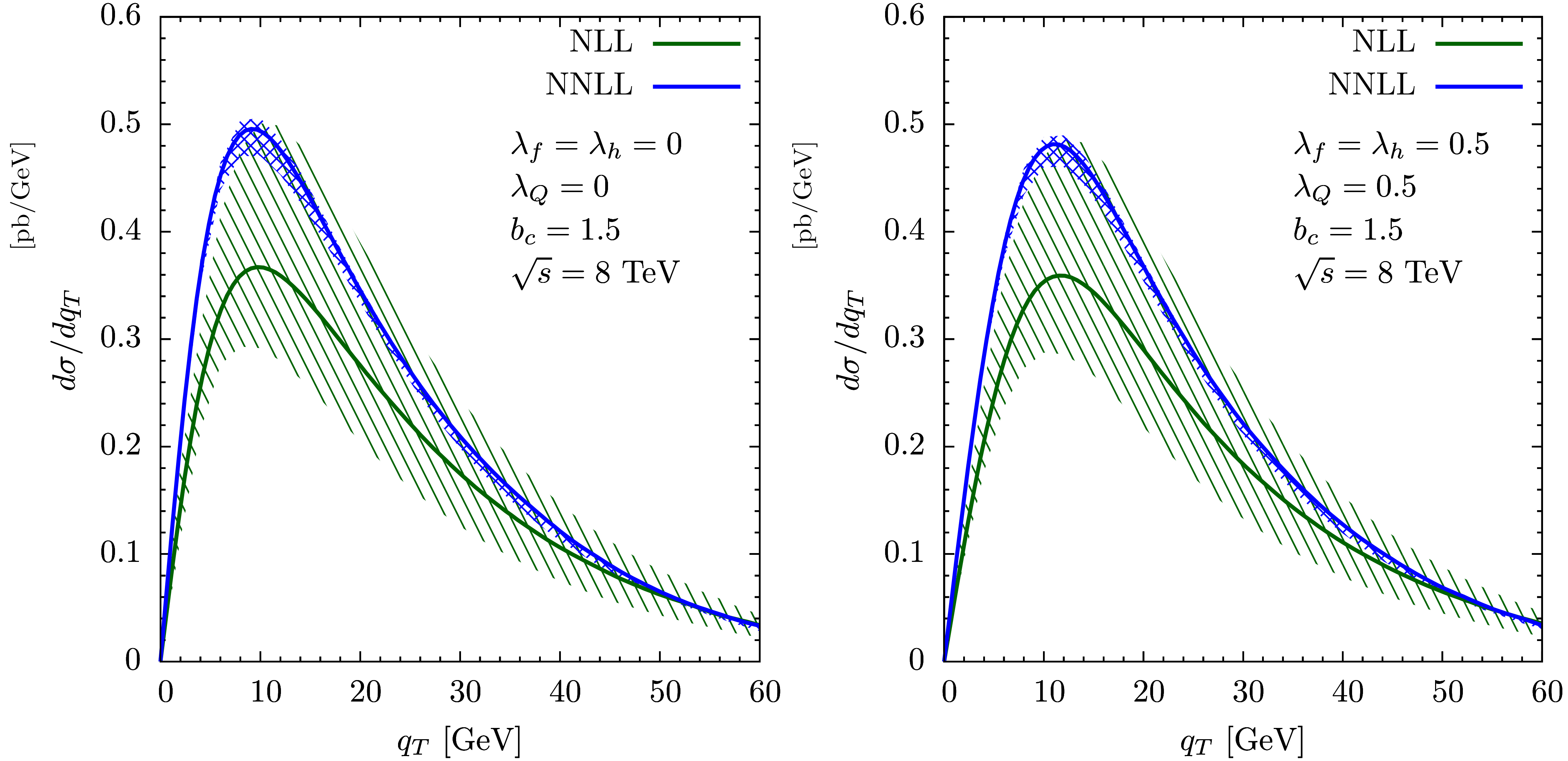
- Ratio:

$$R = \frac{d\sigma_{\text{lin. pol.}}}{d\sigma_{\text{unpol.}}}$$



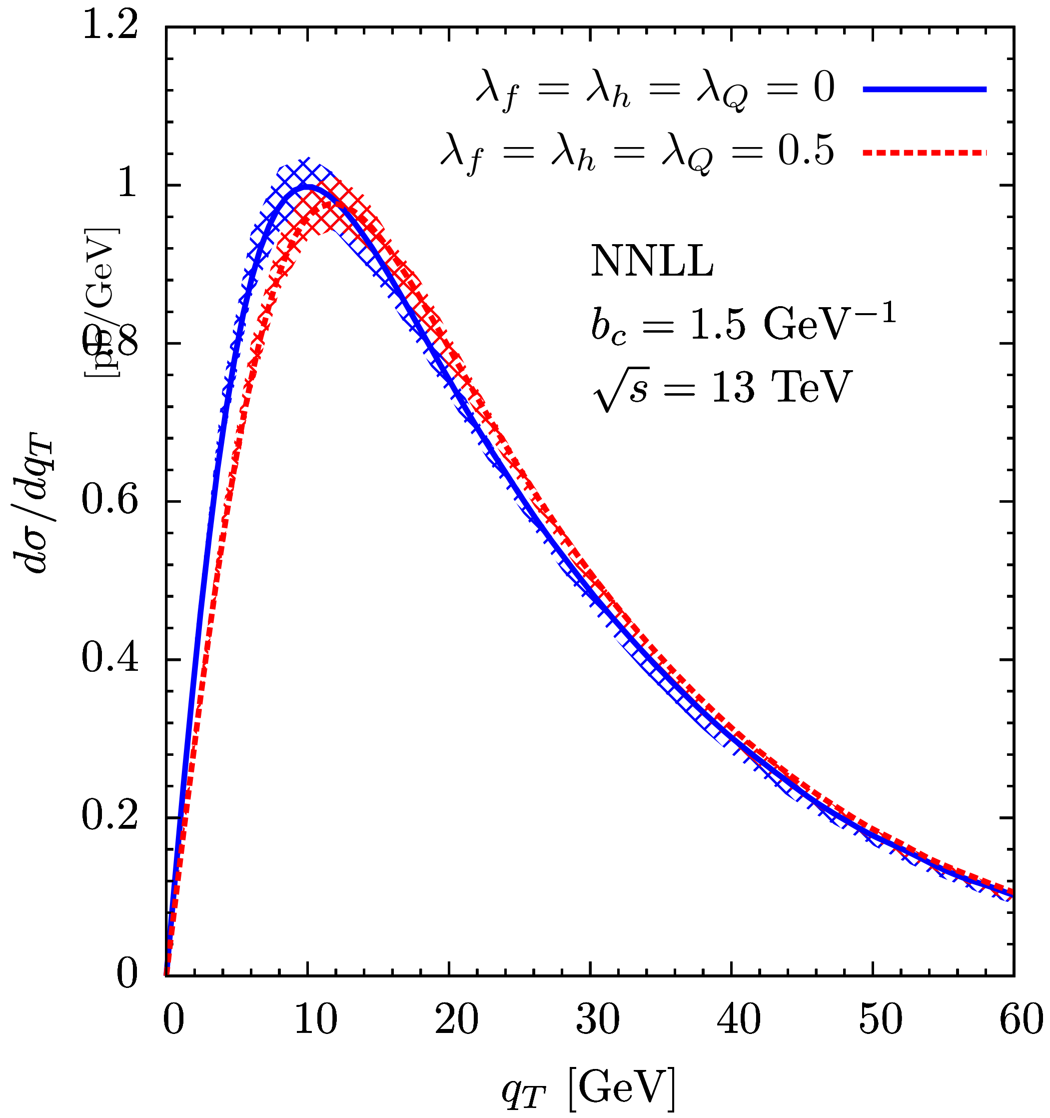
- Higgs boson x-section, linear polarization contributes a few precent
- Lower scales, linear polarization larger (and dependent on non-perturbative parameters) - could be measurable in quarkonium production

Higgs pT spectrum



- Uncertainty band from varying μ and ζ independently
- Small impact of non-perturbative parameters

Higgs pT spectrum



- Uncertainty band from varying μ and ζ independently
- qT spectrum has only small dependence on the non-perturbative input
- At the current level of precision, resummed result is good enough

Summary

- Defined gluon TMDs (on the light-cone), free from rapidity divergencies
- Derived their QCD evolution
  - The same for all gluon TMDs (all polarizations)
- Derived one-loop TMD factorization for Higgs production
- Examined the impact of linearly polarized gluons in Higgs production
  - Generically at the level of a few percent
- Results for the Higgs transverse momentum spectrum
  - Non-perturbative impact on spectrum relatively small,

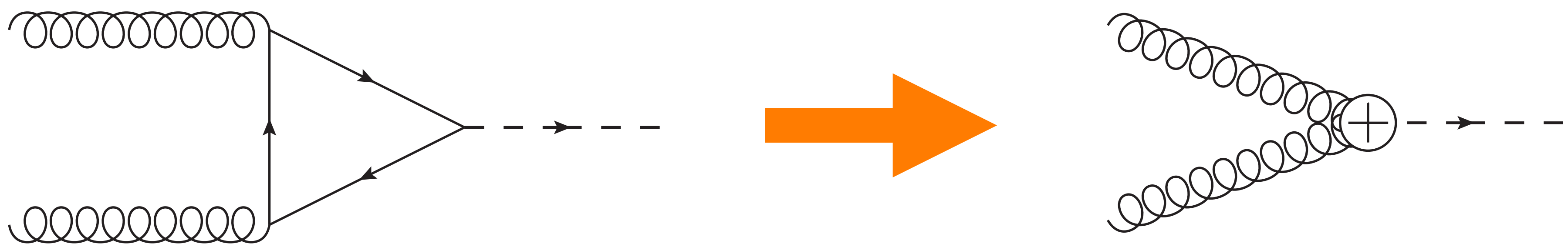
    ⇒ not the most promising process to measure gluon TMDs

    ⇒ justifies use of resummed collinear cross section
- Non-perturbative parameters much larger impact at lower scales,
  - for example in quarkonia production

Backup

Step 1 $\mathrm{QCD}(n_f = 6) \to \mathrm{QCD}(n_f = 5) \to \mathrm{SCET}_{q_T} \to \mathrm{SCET}_{\Lambda_{QCD}}$

- Integrating out the top



- Effective Lagrangian

Matching coefficient (known to 3-loops)

$$\mathcal{L}_{\mathrm{eff}} = C_t(m_t^2, \mu)\, \frac{H}{v}\, \frac{\alpha_s(\mu)}{12\pi}\, F^{\mu\nu,a}\, F^a_{\mu\nu}$$

Schroder, Steinhauser, 2006; Chetyrkin, Kuhn, Sturm, 2005

- Cross section:

$$d\sigma = \frac{1}{2s}\left(\frac{\alpha_s(\mu)}{12\pi v}\right)^2 C_t^2(m_t^2, \mu)\, \frac{d^3q}{(2\pi)^3 2E_q} \int d^4y\, e^{-iq\cdot y}$$
$$\times \sum_X \langle P, \bar{P}|\, F^a_{\mu\nu} F^{\mu\nu,a}(y)\, |X\rangle \langle X|\, F^b_{\alpha\beta} F^{\alpha\beta,b}(0)\, |P, \bar{P}\rangle$$

SCET crash course

- Effective theory of QCD
- Describes interactions between soft (low energy) and collinear (very energetic in one light-cone direction)
- Systematic expansion of Lagrangian in powers of small parameter λ (q_T/Q)
- Separate Lagrangians for (ultra-) soft quarks/gluons and (anti)collinear quarks/gluons

Bauer, Fleming, Pirjol, Stewart, 2001; BPS, 2002;
Beneke, Chapovsky, Diehl, Feldmann, 2002;

- Assumption SCET captures the IR physics of QCD (no Glaubers)

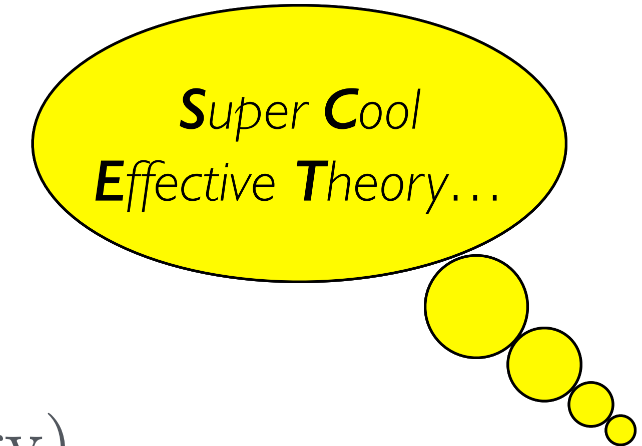
  Holds for Higgs production

  $\Rightarrow$ Matching is possible

- Useful to resum logarithms

$p_n^\mu = Q(1, \lambda^2, \lambda)$	n-collinear
$p_{\bar{n}}^\mu = Q(\lambda^2, 1, \lambda)$	$\bar{n}$ -collinear
$p_{us}^\mu = Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft (SCET-I)
$p_s^\mu = Q(\lambda, \lambda, \lambda)$	soft (SCET-II)

SCET crash course



- Effective theory of QCD
- Describes interactions between soft (low energy) and collinear (very energetic in one light-cone direction)
- Systematic expansion of Lagrangian in powers of small parameter λ (q_T/Q)
- Separate Lagrangians for (ultra-) soft quarks/gluons and (anti)collinear quarks/gluons

Bauer, Fleming, Pirjol, Stewart, 2001; BPS, 2002;
Beneke, Chapovsky, Diehl, Feldmann, 2002;

- Assumption SCET captures the IR physics of QCD (no Glaubers)

  Holds for Higgs production

  $\Rightarrow$ Matching is possible

- Useful to resum logarithms

$p_n^\mu = Q(1, \lambda^2, \lambda)$	n-collinear
$p_{\bar{n}}^\mu = Q(\lambda^2, 1, \lambda)$	$\bar{n}$ -collinear
$p_{us}^\mu = Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft (SCET-I)
$p_s^\mu = Q(\lambda, \lambda, \lambda)$	soft (SCET-II)

Step 2 $\mathrm{QCD}(n_f = 6) \to \mathrm{QCD}(n_f = 5) \to \mathrm{SCET}_{q_T} \to \mathrm{SCET}_{\Lambda_{QCD}}$

- Integrating out the Higgs mass

$$F^{\mu\nu,a} F^a_{\mu\nu} = -2q^2 C_H(-q^2, \mu^2)\, g^{\perp}_{\mu\nu} \mathcal{B}^{\mu,a}_{n\perp} \left(\mathcal{S}^\dagger_n \mathcal{S}_{\bar{n}}\right)^{ab} \mathcal{B}^{\nu,b}_{\bar{n}\perp}$$

Matching coefficient (known to 3-loops)

- With the fields

$$\mathcal{B}^\mu_{n\perp} = i\bar{n}_\alpha g^\mu_{\perp\beta} t^a (\mathcal{W}^\dagger_n)^{ab} F^{\alpha\beta,b}_n$$

Baikov et. al., 2009; Lee et. al, 2010; Gehrmann et. al, 2010

and collinear and soft Wilson lines

$$\mathcal{W}_n(x) = P \exp\left[ig \int_{-\infty}^{0} ds\, \bar{n} \cdot A^a_n(x + \bar{n}s) t^a\right],$$

$$\mathcal{S}_n(x) = P \exp\left[ig \int_{-\infty}^{0} ds\, n \cdot A^a_s(x + ns) t^a\right]$$

Adjoint representation
$$(t^a)^{bc} = -if^{abc}$$

(phase picked up by the gluons moving in the background field of the other hadron)

Step 2 $\text{QCD}(n_f=6) \to \text{QCD}(n_f=5) \to \text{SCET}_{q_T} \to \text{SCET}_{\Lambda_{QCD}}$

- Cross section

$$\frac{d\sigma}{dy\,d^2q_\perp} = 2\sigma_0(\mu)\,C_t^2(m_t^2,\mu)\,H(m_H,\mu)\,\frac{m_H^2}{\tau s}\,(2\pi)^2\int d^2b_\perp\,e^{-i\boldsymbol{b}_\perp\boldsymbol{q}_\perp}$$
$$\times\,J_n^{(0)\mu\nu}(x_A,\boldsymbol{b}_\perp;\mu)\,J^{(0)}_{\bar{n}\,\mu\nu}(x_B,\boldsymbol{b}_\perp;\mu)\,S(\boldsymbol{b}_\perp;m_H^2,\mu)+\mathcal{O}(q_T/m_H)$$

$H(m_H,\mu) = C_H^2$

- Collinear and soft matrix elements

$$J_n^{(0)\mu\nu}(x_A,\boldsymbol{b}_\perp;\mu) = \frac{x_A P^+}{2N_c}\int\frac{dy^-}{(2\pi)}\,e^{-i\left(\frac{1}{2}x_A y^- P^+\right)}$$
$$\times\sum_{X_n}\langle P|\,\mathcal{B}^{\mu,a}_{n\perp}(y^-,\boldsymbol{b}_\perp)\,|X_n\rangle\,\langle X_n|\,\mathcal{B}^{\nu,a}_{n\perp}(0)\,|P\rangle$$

$$S(\boldsymbol{b}_\perp;m_H^2,\mu) = \frac{1}{N_c^2-1}\sum_{X_s}\langle 0|\left(\mathcal{S}_n^\dagger\mathcal{S}_{\bar{n}}\right)^{ab}(\boldsymbol{b}_\perp)\,|X_s\rangle\,\langle X_s|\left(\mathcal{S}_{\bar{n}}^\dagger\mathcal{S}_n\right)^{ba}(0)\,|0\rangle$$

- Individually ill-defined. They contain rapidity divergencies!

Step 2 $\text{QCD}(n_f = 6) \to \text{QCD}(n_f = 5) \to \text{SCET}_{q_T} \to \text{SCET}_{\Lambda_{QCD}}$

- Soft function can be separated in two pieces (to all orders)

$$\tilde{S}(b_T; m_H^2, \mu) = \tilde{S}_-\left(b_T; \zeta_A, \mu; \Delta^-\right) \tilde{S}_+\left(b_T; \zeta_B, \mu; \Delta^+\right)$$

- Combination of soft and collinear matrix elements to form TMDs

$$\tilde{G}^{\mu\nu}_{g/A}(x_A, \boldsymbol{b}_\perp; \zeta_A, \mu^2) = \tilde{J}_n^{(0)\mu\nu}(x_A, \boldsymbol{b}_\perp; \mu^2; \Delta^-)\, \tilde{S}_-(b_T; \zeta_A, \mu^2; \Delta^-)\,,$$
$$\tilde{G}^{\mu\nu}_{g/B}(x_B, \boldsymbol{b}_\perp; \zeta_B, \mu^2) = \tilde{J}_{\bar{n}}^{(0)\mu\nu}(x_B, \boldsymbol{b}_\perp; \mu^2; \Delta^+)\, \tilde{S}_+(b_T; \zeta_B, \mu^2; \Delta^+)\,.$$

  - Rapidity divergencies cancelled in the combination

- Cross section in terms of properly defined TMDs

$$\frac{d\sigma}{dy\, d^2q_\perp} = 2\sigma_0(\mu)\, C_t^2(m_t^2, \mu) H(m_H, \mu)\, \frac{1}{(2\pi)^2} \int d^2b_\perp\, e^{i\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp}$$
$$\times\, \tilde{G}^{\mu\nu}_{g/A}(x_A, \boldsymbol{b}_\perp; \zeta_A, \mu)\, \tilde{G}_{g/B\,\mu\nu}(x_B, \boldsymbol{b}_\perp; \zeta_B, \mu) + \mathcal{O}(q_T/m_H)$$

Step 2 $\mathrm{QCD}(n_f = 6) \to \mathrm{QCD}(n_f = 5) \to \mathrm{SCET}_{q_T} \to \mathrm{SCET}_{\Lambda_{QCD}}$

- Comparing the result of the full QCD(nf=5) result with the SCET result for the two TMDs gives the matching coefficient at one loop

$$C_H(-q^2, \mu) = 1 + \frac{\alpha_s C_A}{4\pi}\left[-\ln^2 \frac{-q^2 + i0}{\mu^2} + \frac{\pi^2}{6}\right] \qquad H = C_H^2$$

- Gives the Higgs cross section in terms of well defined TMDs

$$\frac{d\sigma}{dy\, d^2q_\perp} = 2\sigma_0(\mu)\, C_t^2(m_t^2, \mu) H(m_H, \mu)\, \frac{1}{(2\pi)^2} \int d^2b_\perp\, e^{i\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp} \times \frac{1}{2}\left[\tilde{f}_1^{g/A}(x_A, b_T; \zeta_A, \mu)\, \tilde{f}_1^{g/B}(x_B, b_T; \zeta_B, \mu) + \tilde{h}_1^{\perp\, g/A}(x_A, b_T; \zeta_A, \mu)\, \tilde{h}_1^{\perp\, g/B}(x_B, b_T; \zeta_B, \mu)\right] + \mathcal{O}(q_T/m_H)$$

- The TMDs depend on two scales μ and ζ.
- Scale invariance of cross section gives evolution of the gluon TMDs in the two scales

Gluon TMD Evolution

- The evolution in μ

$$\frac{d}{d\ln\mu}\ln\tilde{G}^{[pol]}_{g/A}(x_n, \boldsymbol{b}_\perp, S_A; \zeta_A, \mu) \equiv \gamma_G\left(\alpha_s(\mu), \ln\frac{\zeta_A}{\mu^2}\right)$$

and in ζ

$$\frac{d}{d\ln\zeta_A}\ln\tilde{G}^{[pol]}_{g/A}(x_A, \boldsymbol{b}_\perp, S_A; \zeta_A, \mu) = -D_g(b_T; \mu)$$

- Combined, gives us the evolution of the gluon TMDs (universal evolution kernel = same for all polarizations)

$$\tilde{G}^{[pol]}_{g/A}(x_n, \boldsymbol{b}_\perp, S_A; \zeta_{A,f}, \mu_f) = \tilde{G}^{[pol]}_{g/A}(x_n, \boldsymbol{b}_\perp, S_A; \zeta_{A,i}, \mu_i)\, \tilde{R}^g\,(b_T; \zeta_{A,i}, \mu_i, \zeta_{A,f}, \mu_f)$$

where

$$\tilde{R}^g\,(b_T; \zeta_{A,i}, \mu_i, \zeta_{A,f}, \mu_f) = \exp\left\{\int_{\mu_i}^{\mu_f}\frac{d\bar{\mu}}{\bar{\mu}}\,\gamma_G\left(\alpha_s(\bar{\mu}), \ln\frac{\zeta_{A,f}}{\bar{\mu}^2}\right)\right\}\left(\frac{\zeta_{A,f}}{\zeta_{A,i}}\right)^{-D_g(b_T;\mu_i)}$$

- Evolution contains non-perturbative piece, in D at large b