On Soft Limits of Large-Scale Structure Correlation Functions

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Based on 1411.3225 and 1508.06306

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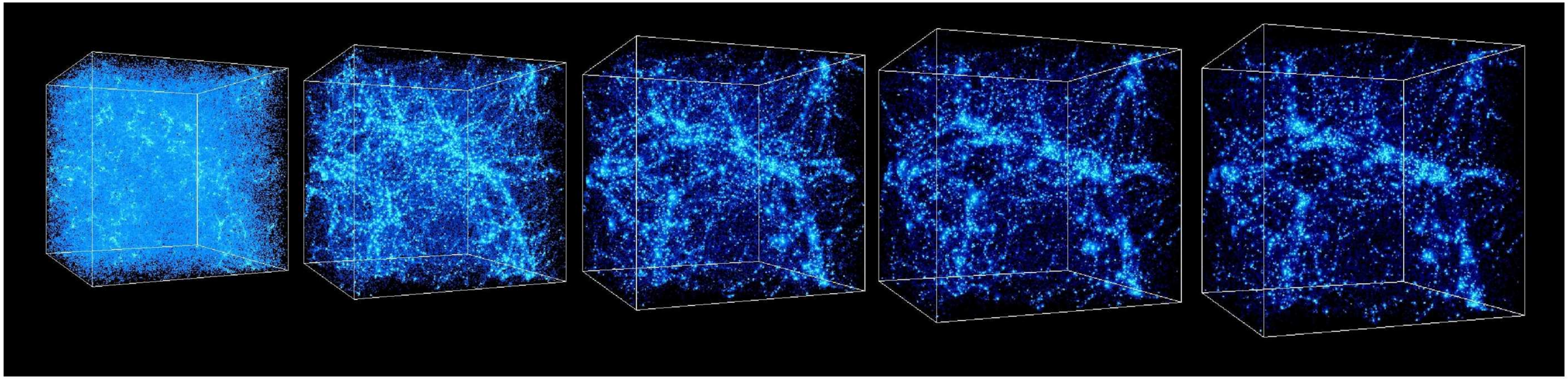
Introduction

Initial matter density perturbations → Present large-scale structure



[cosmicweb.uchicago.edu/filaments.html]

Fluid equations for LSS

- Solution: cosmological perturbation theory

→ Standard perturbation theory (SPT)

Standard perturbation theory

- Matter density contrast $\delta = (\rho - \bar{\rho})/\bar{\rho}$

$\rightarrow$ Power spectrum $P \sim \langle \delta\delta \rangle$

Standard perturbation theory

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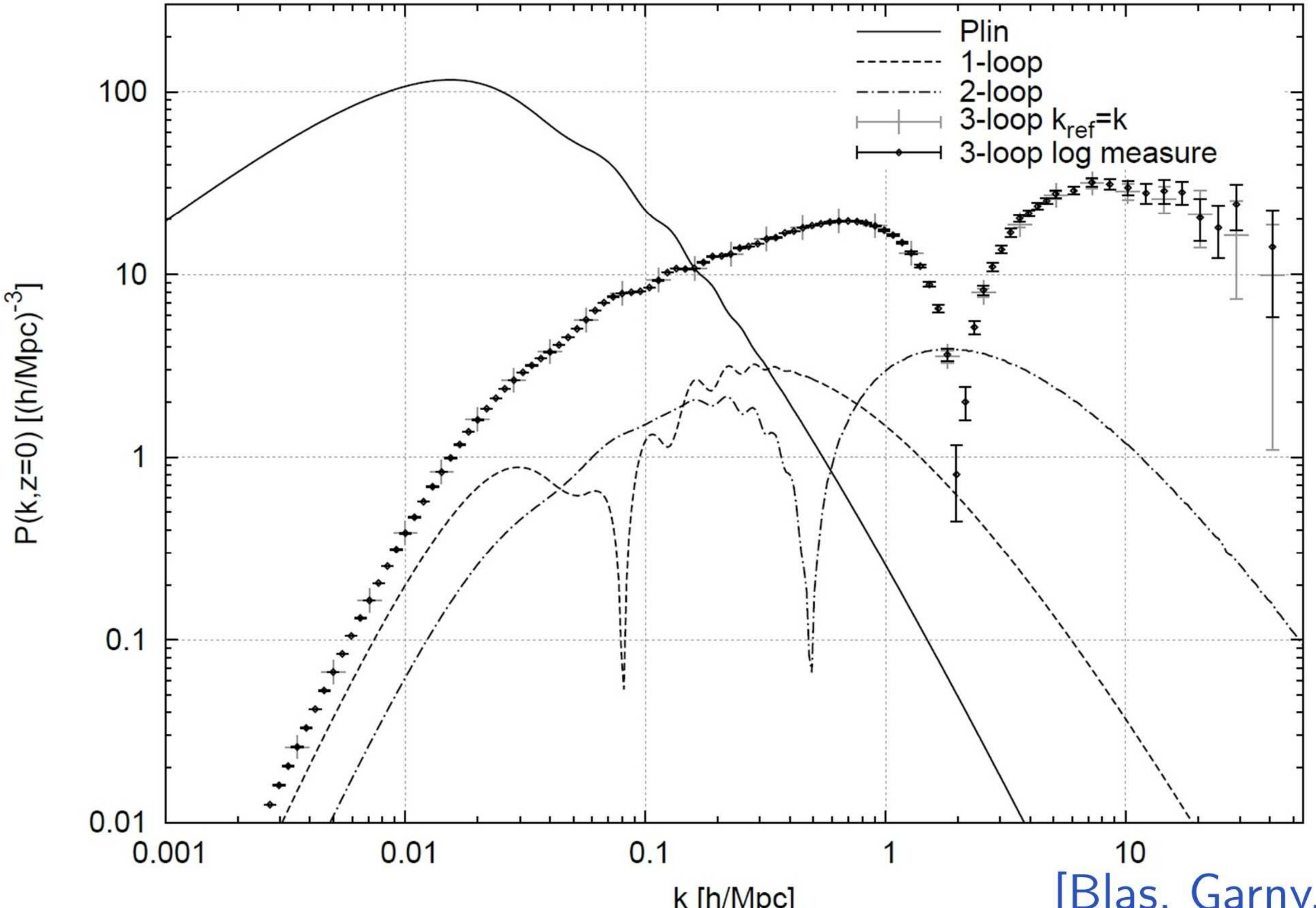

[Blas, Garny, Konstandin, '13]

Standard perturbation theory

- Matter density contrast $\delta = (\rho - \bar{\rho})/\bar{\rho}$

  $\rightarrow$ Power spectrum $P \sim \langle \delta\delta \rangle$

- No convergence of the loop expansion

  $\rightarrow$ SPT does not work appropriately

  $\rightarrow$ Non-perturbative methods?

Outline

1. Fluid equations

2. Non-perturbative soft limits

    - Bispectrum

    - Power spectrum

    $\longrightarrow$ Implications for perturbation theory

3. Conclusions

Fluid equations

Fluid equations

- Basic quantities of cosmological perturbation theory

$$\delta \qquad \text{and} \qquad \theta \equiv \boldsymbol{\nabla} \cdot \boldsymbol{v}$$

as doublet

$$\psi_a(\boldsymbol{k}, \eta) \equiv \begin{pmatrix} \delta \\ -\theta/\mathcal{H} \end{pmatrix}$$

$\longrightarrow$ Conformal expansion rate $\mathcal{H} = a\,H$

$\longrightarrow$ Time variable $\eta \equiv \ln a$

Fluid equations

- Fluid equations (continuity, Euler and Poisson equation)

  in a compact form:

$$\partial_\eta \psi_a = -\Omega_{ab}\, \psi_b + \gamma_{abc}\, \psi_b \psi_c$$

[Bernardeau et al., '02]
[Scoccimarro, '06]

→ Ω_{ab} depends on cosmological model

→ Vertex functions γ_{abc}

Fluid equations

- Fluid equations:

$$\partial_\eta \psi_a = -\Omega_{ab}\,\psi_b + \gamma_{abc}\,\psi_b\psi_c$$

- Iteration of correlation functions:

$$\begin{aligned}\partial_\eta \langle \psi_a\psi_b\rangle &= -\Omega_{ac}\langle\psi_c\psi_b\rangle - \Omega_{bc}\langle\psi_a\psi_c\rangle \\ &\quad + \gamma_{acd}\langle\psi_c\psi_d\psi_b\rangle + \gamma_{bcd}\langle\psi_a\psi_c\psi_d\rangle, \\ \partial_\eta\langle\psi_a\psi_b\psi_c\rangle &= \ldots\end{aligned}$$

With power spectrum, bispectrum:

$$\langle\psi_a\psi_b\rangle \sim P_{ab}\,, \quad \langle\psi_a\psi_b\psi_c\rangle \sim B_{abc}$$

Soft limit

of the

bispectrum

Physical picture

Goal:

Understand influence of a long-wavelength (soft) mode

on the local dynamics of the universe

[Baldauf et al., '11]

Ansatz:

- Isotropic long-wavelength (soft) perturbation:

$$\Phi_L \simeq \frac{1}{4} H^2 a^2 \delta_L \boldsymbol{x}^2 \ll 1$$

- Flat FRW cosmology in Newtonian gauge:

$$ds^2 = -\big[1 + 2\Phi_L\big] dt^2 + a^2 \big[1 - 2\Phi_L\big] d\boldsymbol{x}^2$$

Physical picture

- Coordinate transformation:

$$t = t_K + f(t_K, \boldsymbol{x}_K)\,, \quad \boldsymbol{x} = \boldsymbol{x}_K(1 + g(t_K, \boldsymbol{x}_K))$$

$\longrightarrow$ Locally curved universe:

$$ds^2 = -dt_K^2 + a_K^2 \frac{d\boldsymbol{x}_K^2}{\left(1+\frac{1}{4}K\boldsymbol{x}_K^2\right)^2}\,,$$

$$K \simeq \frac{5}{3}H^2 a^2 \delta_L\,, \quad a_K \simeq a\left(1 - \frac{1}{3}\delta_L\right)\,, \quad \delta_K = \delta\,(1 - \delta_L)$$

 + flat FRW $\longrightarrow$ locally curved universe

Soft limit of the bispectrum

- Correlation functions in Fourier space:

$$\langle\delta\delta\rangle \sim P\,, \quad \langle\delta\delta\delta\rangle \sim B$$

- Non-perturbative relation for the bispectrum in the soft limit:

$$B_{111\;q\to 0} = P^L(q)\left[\left(1 - \frac{1}{3}k\;\partial_k - \frac{1}{3}\partial_\eta\right)P(k) + \frac{5}{3}\;\frac{\partial}{\partial\kappa}P_K(k)\bigg|_{K=0}\right]$$

With $\eta \equiv \ln a\,,\; \kappa = K/(a^2H^2)$

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Hypothetical curved universe

Soft limit of the bispectrum

$$B_{111\ q\to 0} = P^L(q)\left[\left(1-\frac{1}{3}k\ \partial_k - \frac{1}{3}\partial_\eta\right)P(k) + \frac{5}{3}\ \frac{\partial}{\partial\kappa}P_K(k)\Big|_{K=0}\right]$$

- Proposal by Valageas, and by Kehagias, Perrier, Riotto (VKPR):

$$B_{111\ q\to 0} = \left(1-\frac{1}{3}k\,\partial_k + \frac{13}{21}\partial_\eta\right)P(k)\,P^L(q)$$

$$\rightarrow\ \frac{\partial}{\partial\kappa}P_K(k)\Big|_{K=0} = \frac{4}{7}\,\partial_\eta P(k)$$

[Valageas '13]

[Kehagias, Perrier, Riotto '13]

Soft limit
of the power
spectrum

Soft limit of the power spectrum

Physical picture:

Directional soft mode $\longrightarrow$ curved anisotropic universe

Derivation:

- Insert soft-limit bispectrum in fluid equations
- Use VKPR relation to approximate curvature dependence

  $\longrightarrow$ Non-perturbative equation for the power spectrum

Soft limit of the power spectrum

- Non-perturbative equation for the power spectrum:

$$
\begin{aligned}
\partial_\eta P_{ab}(q) &= -\Omega_{ac} P_{cb}(q) - \Omega_{bc} P_{ac}(q) \\
&-q^2 P^L(q) \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} \left(\frac{6}{7} \sigma_{22}^2(\eta) + \frac{13}{14} \partial_\eta \sigma_{22}^2(\eta) \right)
\end{aligned}
$$

With $\sigma_{22}^2(\eta) \equiv \frac{4\pi}{3} \int dk \; P_{22}(k,\eta)$

Soft limit of the power spectrum

- Non-perturbative equation for the power spectrum:

$$\begin{aligned}\partial_\eta P_{ab}(q) \quad &= \quad -\Omega_{ac} P_{cb}(q) - \Omega_{bc} P_{ac}(q) \\ -q^2 P^L(q) &\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} \left(\frac{6}{7} \sigma_{22}^2(\eta) + \frac{13}{14} \partial_\eta \sigma_{22}^2(\eta) \right)\end{aligned}$$

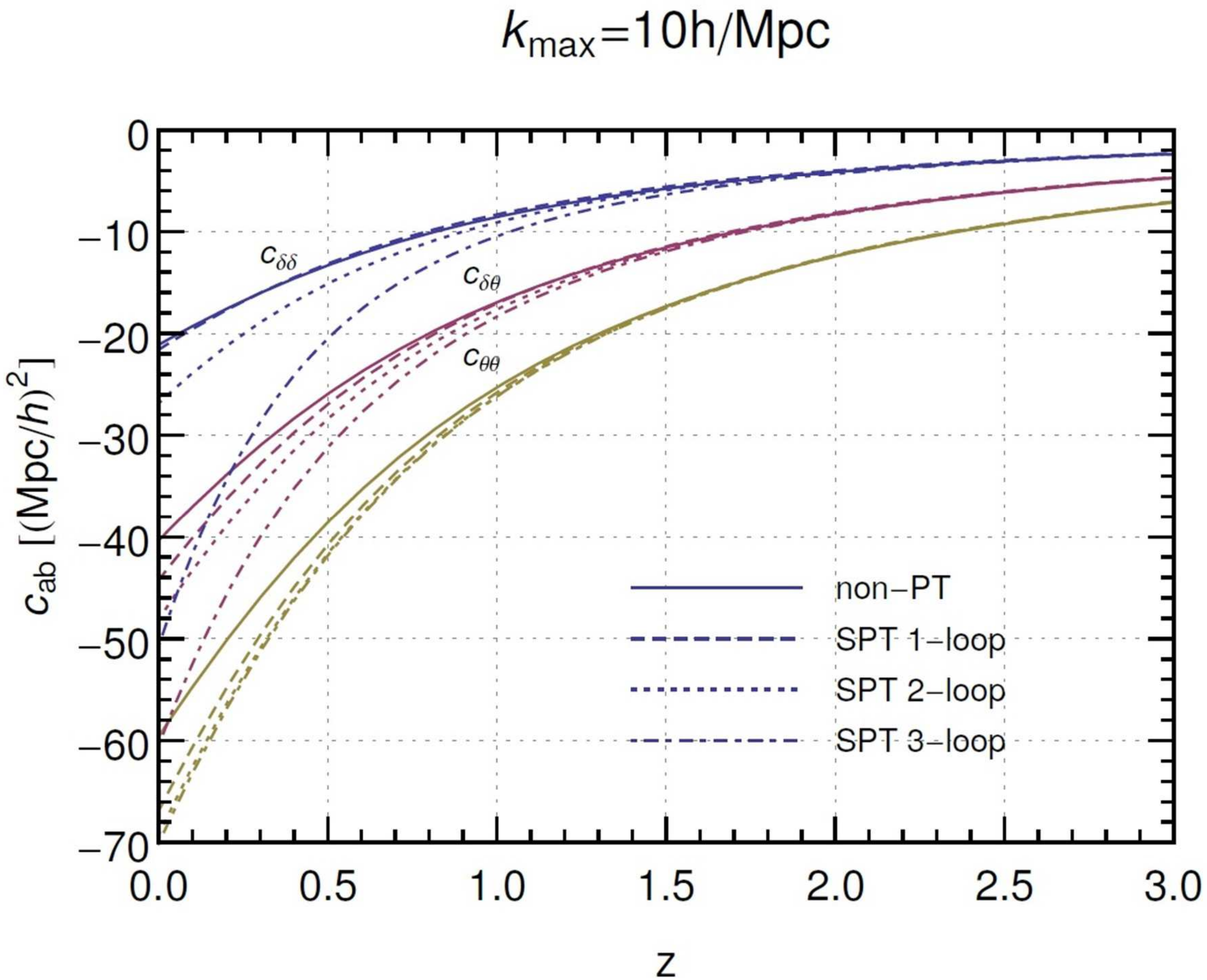
With $\sigma_{22}^2(\eta) \equiv \frac{4\pi}{3} \int dk \; P_{22}(k, \eta)$

- Parameterize power spectrum by

$$P_{ab}(q) = P^L(q) \Big(1 + q^2 \, c_{ab}(\eta) \Big)$$

$\rightarrow$ Differential equation for $c_{ab}(\eta)$

Comparison to SPT



- UV sensitivity in SPT: artifact

$\rightarrow$ perturbative techniques inapplicable beyond non-linear scale

Conclusions

Conclusions

Non-perturbative soft limits of LSS correlation functions:

- Bispectrum:

  $\rightarrow$ Simple consistency relation

  as probe for upcoming LSS surveys

- Power spectrum:

  $\rightarrow$ UV dependence in SPT: artifact

Future work: non-perturbative relations in the hard limit

Thank you for your attention!

Backup

Backup

❶ Fluid equations

- Continuity, Euler and Poisson equation

❷ Soft limit of the bispectrum

- Linear order
- 1-loop oder

❸ Soft limit of the power spectrum

- Comparison to N-body simulations
- Implications for perturbation theory

Fluid equations

Fluid equations

Continuity equation

= conservation of mass:

$$\partial_\tau \delta + \nabla \cdot \left[(1+\delta)\, \boldsymbol{v}\right] = 0$$

Euler equation

= conservation of momentum:

$$\partial_\tau v + \mathcal{H}\, \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\, \boldsymbol{v} = -\nabla \Phi$$

Poisson equation:

$$\triangle \Phi = \frac{3}{2} \Omega_m \mathcal{H}^2\, \delta$$

$\longrightarrow$ highly non-linear differential equations

Physical quantities:

- Density contrast $\delta = \dfrac{\rho - \bar{\rho}}{\bar{\rho}}$
- Conformal time τ: $d\tau = \dfrac{dt}{a}$
- Peculiar velocity $\boldsymbol{v}$
- Expansion rate $\mathcal{H} = a\, H$
- Gravitational potential Φ

Soft limit

of the

bispectrum

Time-flow approach

[Pietroni, '08]

- Correlation functions:

$$\langle \psi_a \psi_b \rangle \sim P_{ab},$$
$$\langle \psi_a \psi_b \psi_c \rangle \sim B_{abc},$$
$$\langle \psi_a \psi_b \psi_c \psi_d \rangle \sim P_{ab}\, P_{cd} + P_{ac}\, P_{bd} + P_{ad}\, P_{bc} + Q_{abcd}$$

- Closure approximation:

$$Q_{abcd} = 0$$

$\rightarrow$ Solution of fluid equations

Soft limit of the bispectrum

Derivation:

- Gaussian initial conditions: $B_{abc}(\eta = 0) = 0$
- Angular average: $(\ldots)^{\mathrm{av}} \equiv \int d\Omega_{kq} / (4\pi)$
- Soft limit: $q \to 0$
- Linear order: $P_{ab}(k) \simeq P^L(k) \quad \forall\, a, b$
- Flat universe (EdS, ΛCDM)

Soft limit of the bispectrum

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- Gaussian initial conditions: $B_{abc}(\eta = 0) = 0$
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$$\rightarrow \; B^L_{abc\; q\to 0} \simeq \left(\frac{1}{21} \begin{pmatrix} 47 & 39 \\ 39 & 31 \end{pmatrix}_{ac} - \frac{1}{3} k \, \partial_k \right) P^L(k) \, P^L(q)$$

Soft limit of the bispectrum

For a flat universe (EdS, ΛCDM):

- At linear order:

$$P_{ab}(k) \simeq P^L(k) \quad \forall\, a, b$$

$$B^L_{abc}\,{}^{\rm av}_{q\to 0} \simeq \left(\frac{1}{21} \begin{pmatrix} 47 & 39 \\ 39 & 31 \end{pmatrix}_{ac} - \frac{1}{3} k\, \partial_k \right) P^L(k)\, P^L(q)$$

$\rightarrow$ For $B^L_{111}\,{}^{\rm av}_{q\to 0} \sim \langle \delta\delta\delta \rangle^{\rm av}_{q\to 0}$:

Coincides with standard perturbation theory (SPT)

[Sherwin, Zaldarriaga, '12]

Soft limit of the bispectrum

For a flat universe (EdS, ΛCDM):

- At linear order:

$$P_{ab}(k) \simeq P^L(k) \quad \forall\, a, b$$

$$B^{L\ \text{av}}_{abc\ q\to 0} \simeq \left(\frac{1}{21} \begin{pmatrix} 47 & 39 \\ 39 & 31 \end{pmatrix}_{ac} - \frac{1}{3} k\, \partial_k \right) P^L(k)\, P^L(q)$$

$\rightarrow$ For $B^{L\ \text{av}}_{111\ q\to 0} \sim \langle \delta\delta\delta \rangle^{\text{av}}_{q\to 0}$:
Reproduces VKPR relation since $P^L(k) = e^{2\eta} P_0(k)$,

$$B_{111\ q\to 0} = \left(1 - \frac{1}{3} k\, \partial_k + \frac{13}{21} \partial_\eta \right) P(k)\, P^L(q)$$

Locally curved universe

- VKPR relation:

  → At linear order: exact

  → Beyond linear order: very good approximation

  ⟹ Tested with "separate universe" simulations

  [Li et al., '14]
  [Wagner et al., '14]
  [Chiang et al '14]

- Generalization for density and velocity fields:

$$\frac{\partial}{\partial \kappa} \psi_{ab}^{K} \Big|_{K=0} \simeq \frac{4}{7} \begin{pmatrix} \partial_\eta & 0 \\ 0 & \partial_\eta + 1 \end{pmatrix}_{ab} \psi_b$$

Soft limit of the bispectrum

For a flat universe (EdS, ΛCDM):

- At 1-loop order:

$$B_{111}^{\text{1-loop}} \underset{q\to 0}{\overset{\text{av}}{\simeq}} \Bigg[k^2\,(\alpha + \beta\, k\,\partial_k)\,P^L(k)\,\times \int dl\, l^2 \left(\frac{P^L(l)}{l^2}\right) + k^4\,\gamma\,\times \int dl\, l^2 \left(\frac{P^L(l)}{l^2}\right)^2\Bigg] P^L(q)$$

for $l \gg k \gg q$

$\rightarrow$ Coefficients α, β, γ:

Deviation of $\mathcal{O}(1)$ between time-flow approach and SPT

$\rightarrow$ Trispectrum not negligible

Soft limit of the bispectrum

For a flat universe (EdS, ΛCDM):

- At 1-loop order:

$$B_{111}^{\text{1-loop}} \underset{q\to 0}{\text{av}} \simeq \left[k^2 \left(\alpha + \beta\, k\, \partial_k\right) P^L(k) \;\times \int dl\, l^2 \left(\frac{P^L(l)}{l^2}\right) + k^4\, \gamma \;\times \int dl\, l^2 \left(\frac{P^L(l)}{l^2}\right)^2 \right] P^L(q)$$

for $l \gg k \gg q$

$\rightarrow$ Coefficients α, β, γ:

Deviation of $\mathcal{O}\left(10^{-2}\right)$ between SPT and VKPR

$\rightarrow$ VKPR: very good approximation

Soft limit of the bispectrum

Coefficients α, β, γ at 1-loop order:

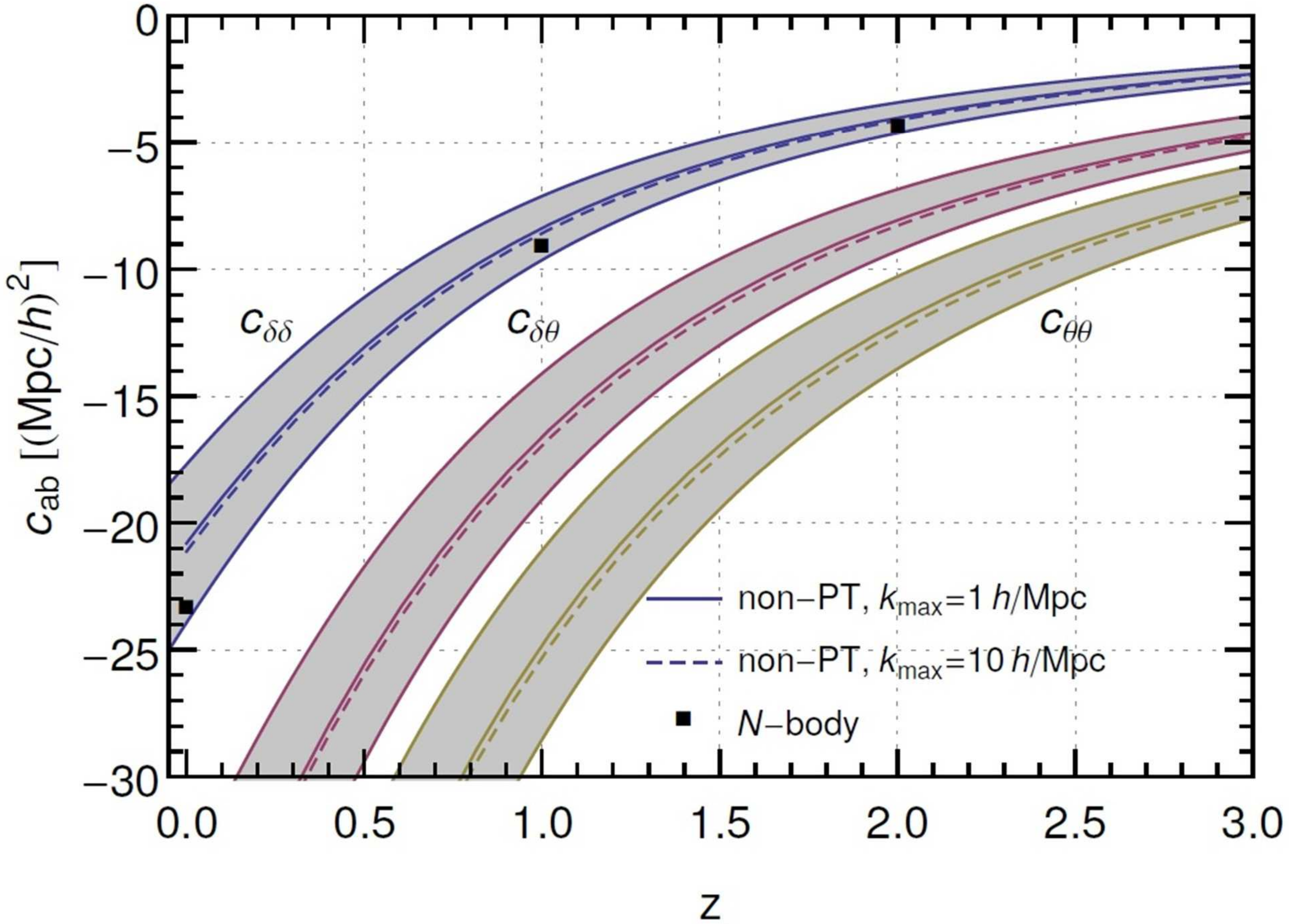
Approach	α	β	γ
TF	$-\frac{233}{1890} \simeq -0.123$	$\frac{103}{6930} \simeq 0.015$	$\frac{271}{19404} \simeq 0.014$
SPT	$-\frac{3719}{13230} \simeq -0.281$	$\frac{61}{1890} \simeq 0.032$	$\frac{515}{5292} \simeq 0.097$
VKPR	$-\frac{3599}{13230} \simeq -0.272$	$\frac{61}{1890} \simeq 0.032$	$\frac{135}{1372} \simeq 0.098$

Soft limit

of the power

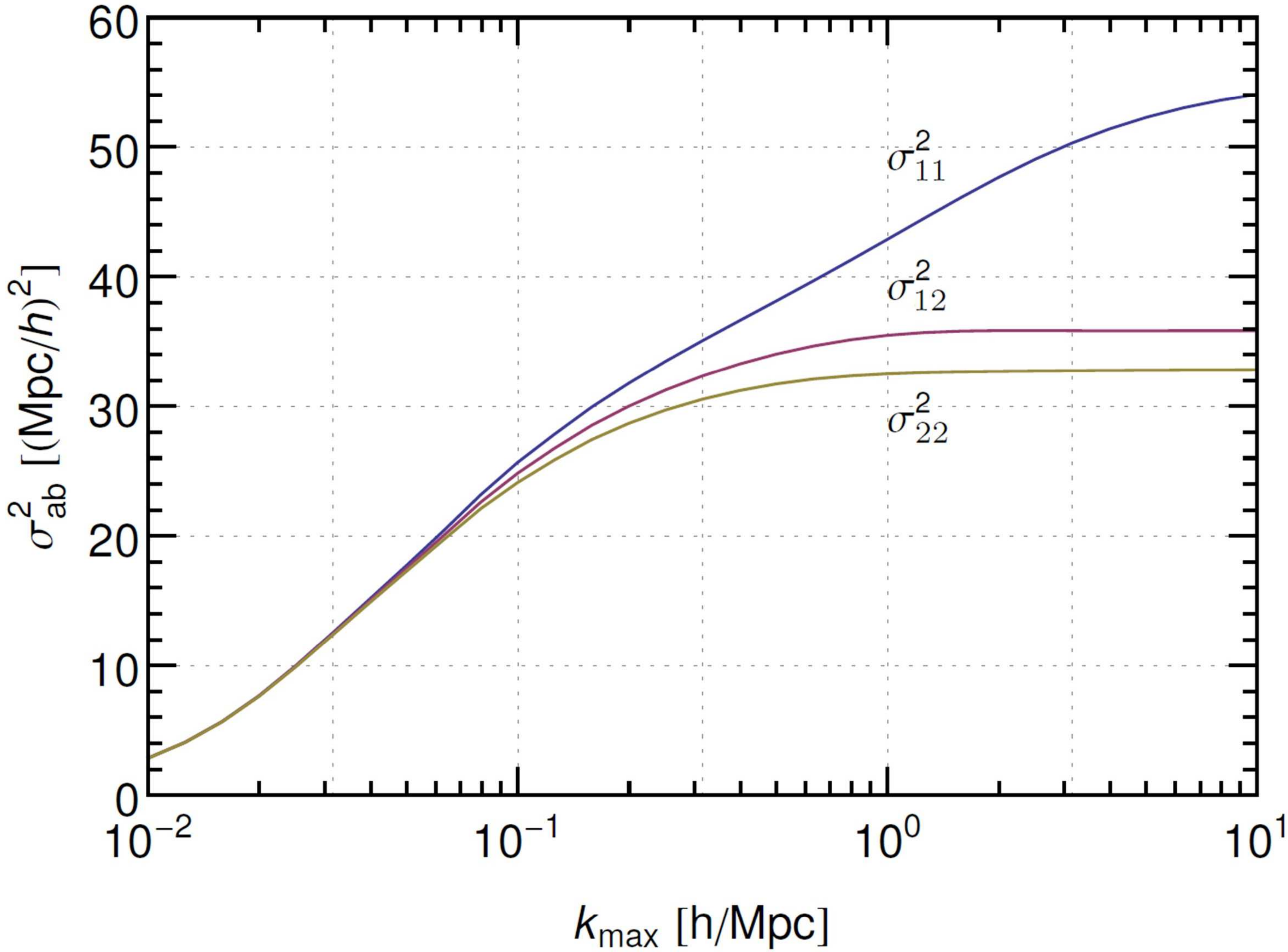
spectrum

Comparison to N-body simulations



- Very good agreement, especially at higher redshifts
- Error bands: neglection of terms $\sim \sigma_{12}^2(\eta)$, VKPR relations

Soft limit of the power spectrum



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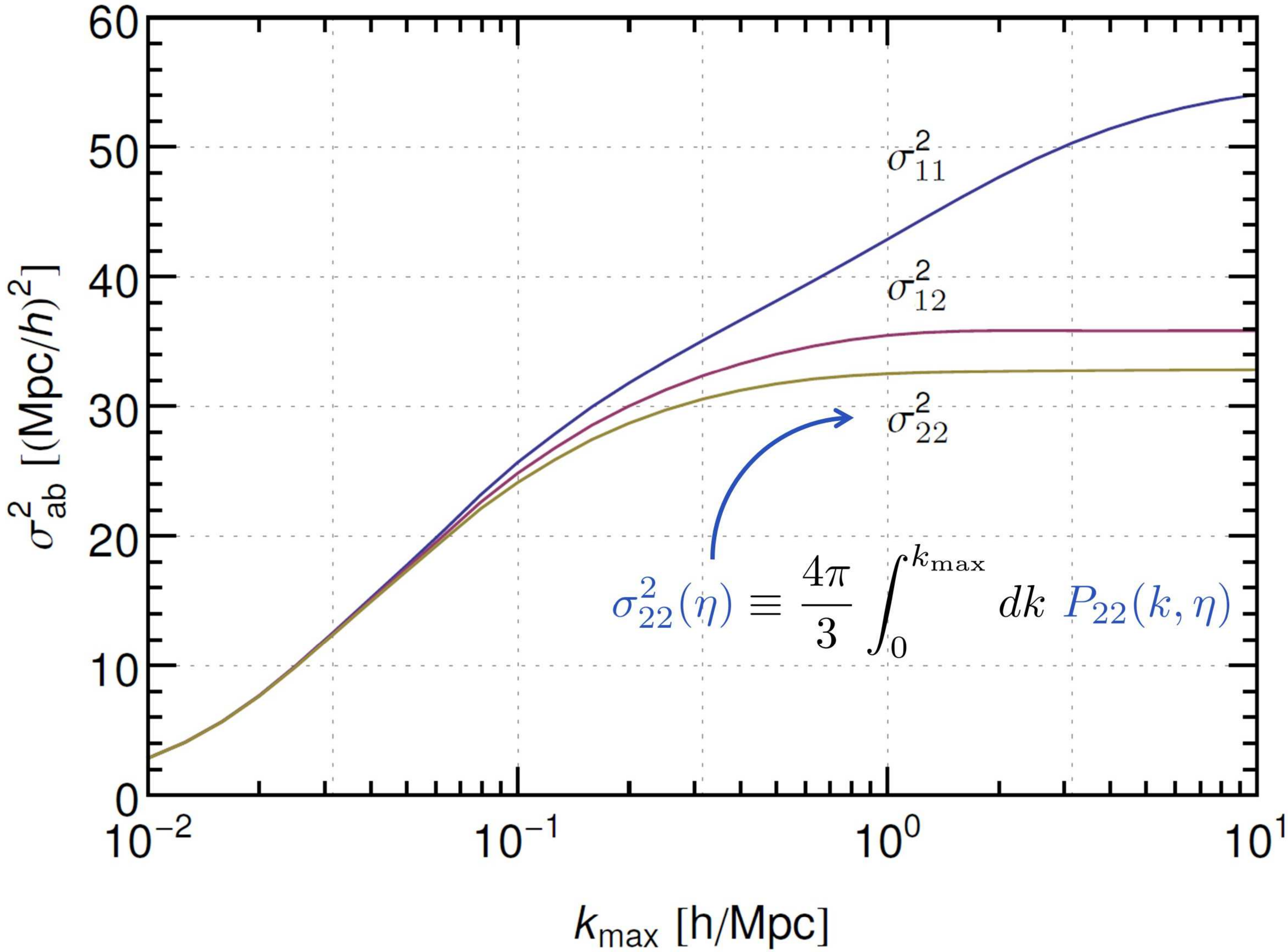

Implications for perturbation theory

σ_{22}^2 and hence $P(q)$ depend only weakly on UV modes

- UV sensitivity in standard perturbation theory (SPT): artifact

  $\rightarrow$ perturbative techniques inapplicable beyond non-linear scale

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  $\rightarrow$ perturbative techniques inapplicable beyond non-linear scale

- In effective field theory (EFT) of LSS:

  $\rightarrow$ counter-terms for UV dependence

  $\rightarrow$ leading-order renormalized coefficients:

  mainly modes up to non-linear scale contribute

Implications for perturbation theory

Future work:

- Non-perturbative power spectrum

  + "anisotropic universe" simulations

  → precisely infer leading-order EFT coefficients

- Non-perturbative relations in the hard limit