Resummation of double-differential cross sections

based on M. Procura, W. J. Waalewijn and L. Z., JHEP 1502 (2015) 117, [arXiv:1410.6483]

Lisa Zeune

DESY Theory Workshop 2015
Physics at the LHC and beyond

Hamburg, 30 September 2015

Motivation

Why shall we study multi-differential cross sections?

- LHC analyses often involve several measurements/cuts

  Example:

  $Z + 0$ jet: Jet veto using beam thrust and measurement of the transverse momentum

- If the measurements lead to widely separated energy scales
  $\rightarrow$ resummation required

- So far: resummed calculation mostly restricted to single variables

Motivation

Why shall we study multi-differential cross sections?

- Another important reason to study the resummation of multi/double - differential cross sections: Jet substructure

  ⟶ One goal: Discriminate QCD jets from heavy boosted particles (W, Z, H, t)

- Most powerful discrimination observables are ratios of infrared and collinear (IRC) safe observables

  Examples:

  Ratio of N-subjettiness, ratio of two angularities, ...

- These observables are not IRC safe (cannot be computed order-by-oder in α_S), but can calculated in a well-defined way by marginalising over the **resummed** double differential cross section. Larkoski, Thaler, '13

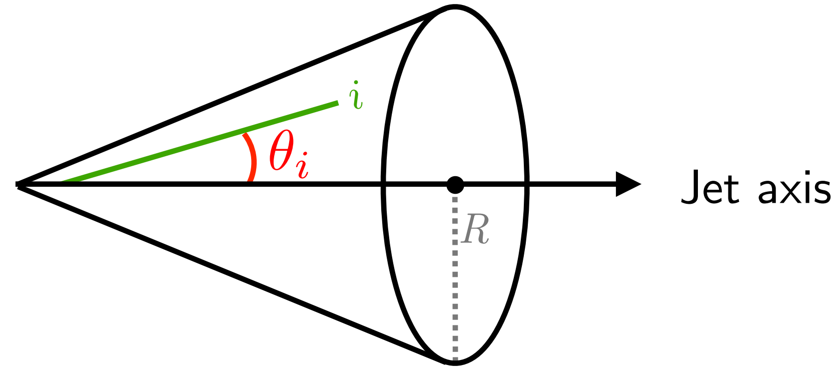
Measuring two angularities on one jet

- Definition of angularities:

Almeida, Lee, Perez, Sterman, Sung, Virzi, '09; Ellis,Vermilion, Walsh, Hornig, Lee, '10; Berger, Kucs, Sterman, '03;

$$e_\alpha = \frac{1}{Q}\sum_{i\in J} E_i \left(\frac{\theta_i}{R}\right)^\alpha$$

Jet energy



- Simultaneous measurement of two different angularities provides information about the jet structure: $r = e_\alpha/e_\beta$ (not IRC safe)

- Differential cross section is calculable, by resuming large logs in the double differential cross section to all orders:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}r} = \int \mathrm{d}e_\alpha \mathrm{d}e_\beta \frac{\mathrm{d}^2\sigma}{\mathrm{d}e_\alpha \mathrm{d}e_\beta}\delta\left(r - \frac{e_\alpha}{e_\beta}\right)$$

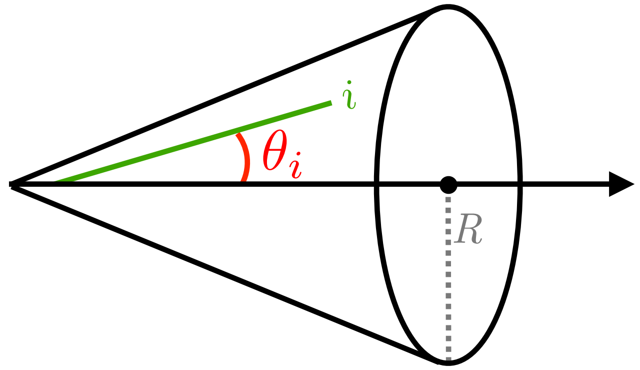
Measuring two angularities on one jet

- Definition of angularities:

Almeida, Lee, Perez, Sterman, Sung, Virzi, '09; Ellis, Vermilion, Walsh, Hornig, Lee, '10; Berger, Kucs, Sterman, '03;

$$e_\alpha = \frac{1}{Q} \sum_{i \in J} E_i \left(\frac{\theta_i}{R} \right)^\alpha$$

Jet energy

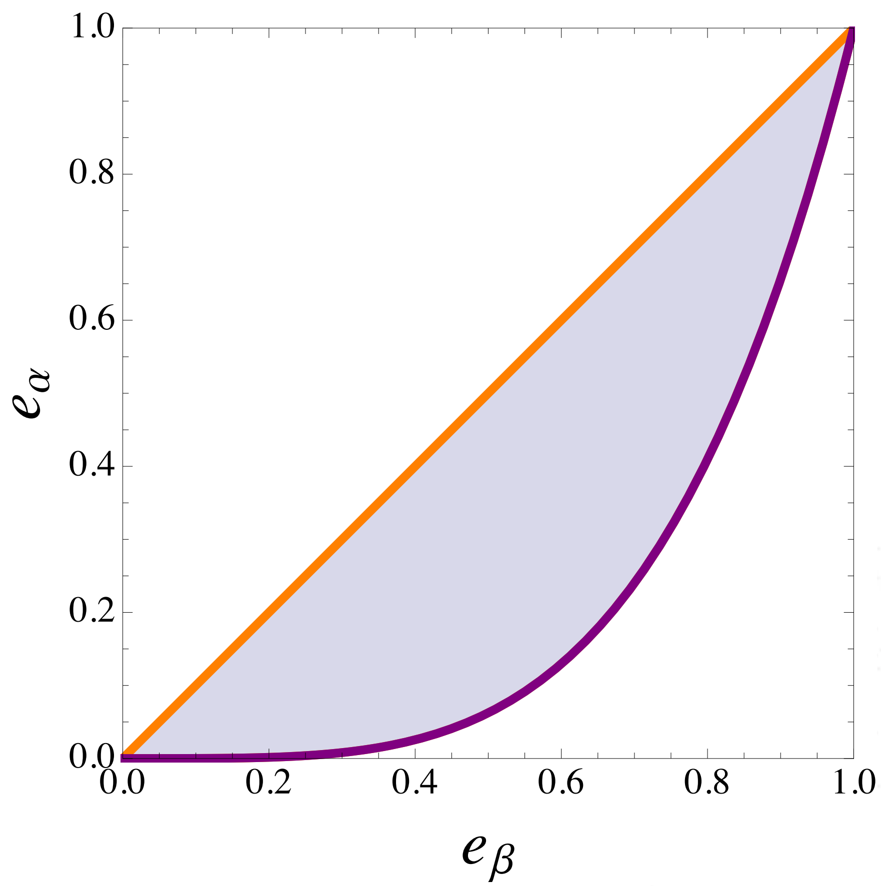


- Phase space for the measurement of two angularities e_α and e_β between

$$\alpha > \beta : e_\beta > e_\alpha$$

Boundary B1: $e_\alpha = e_\beta$
(from jet radius requirement)

Boundary B2: $e_\alpha^\beta = e_\beta^\alpha$
(from energy conservation)



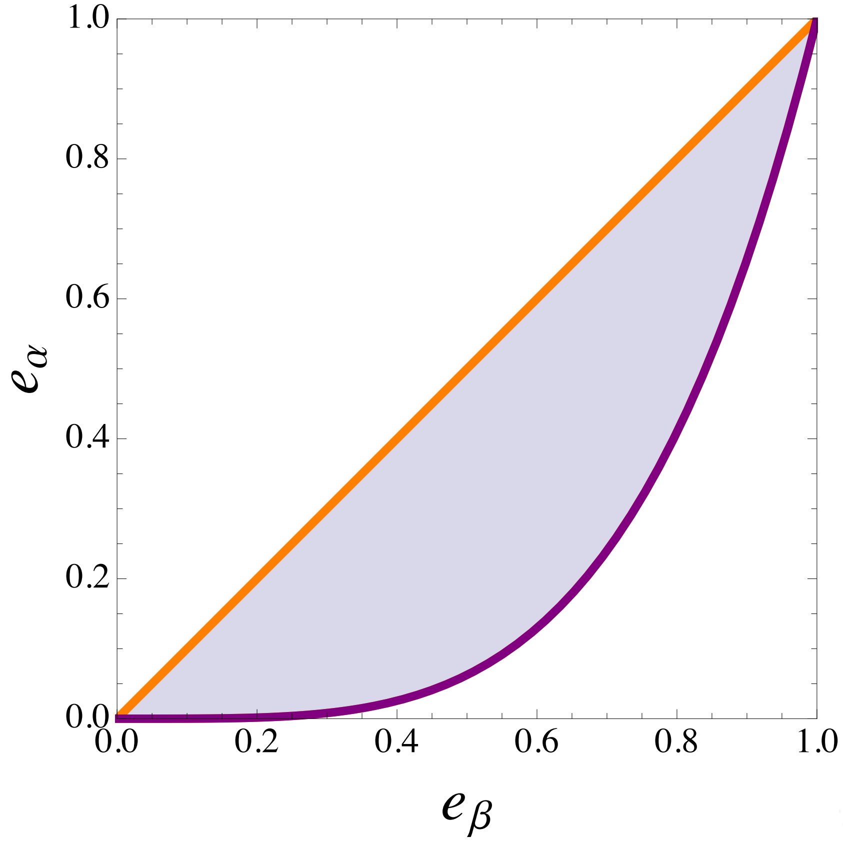
Boundary factorization theorems

- At the B1 identify relevant SCET modes:

Mode	Scaling $(-,+,\perp)$	Measurement
collinear	$Q(1,\lambda^{2/\beta},\lambda^{1/\beta})$	e_β
soft	$Q(\lambda,\lambda,\lambda)$	e_β, e_α

Factorization theorem: Larkoski, Moult, Neill, '14

$$\boxed{\frac{1}{\sigma_0}\frac{d^2\sigma}{de_\alpha\, de_\beta} = H \times J(e_\beta) \otimes S(e_\alpha, e_\beta)}$$



- Similarly at B2:

$$\boxed{\frac{1}{\sigma_0}\frac{d^2\sigma}{de_\alpha\, de_\beta} = H \times J(e_\alpha, e_\beta) \otimes S(e_\alpha)}$$

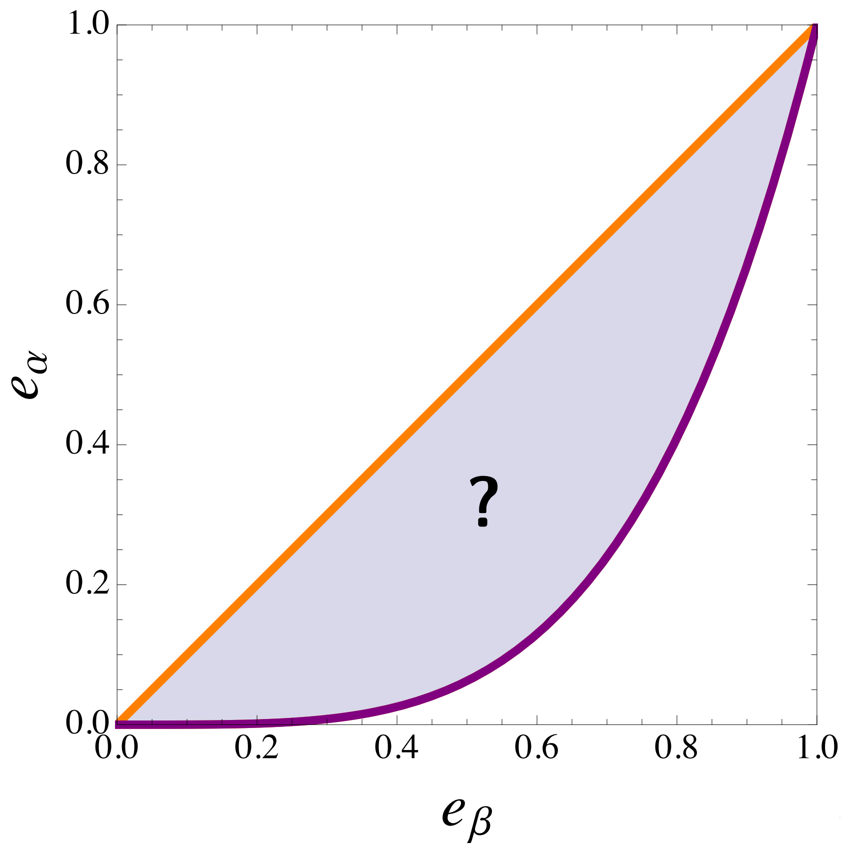
- In the bulk: Factorization of the cross section not possible using only soft and collinear modes Larkoski, Moult, Neill, '14

Factorization theorem in the bulk

- What to do in the bulk?

  ⟶ Larkoski, Moult, Neill: Interpolate

  ⟶ Our approach: Additional mode



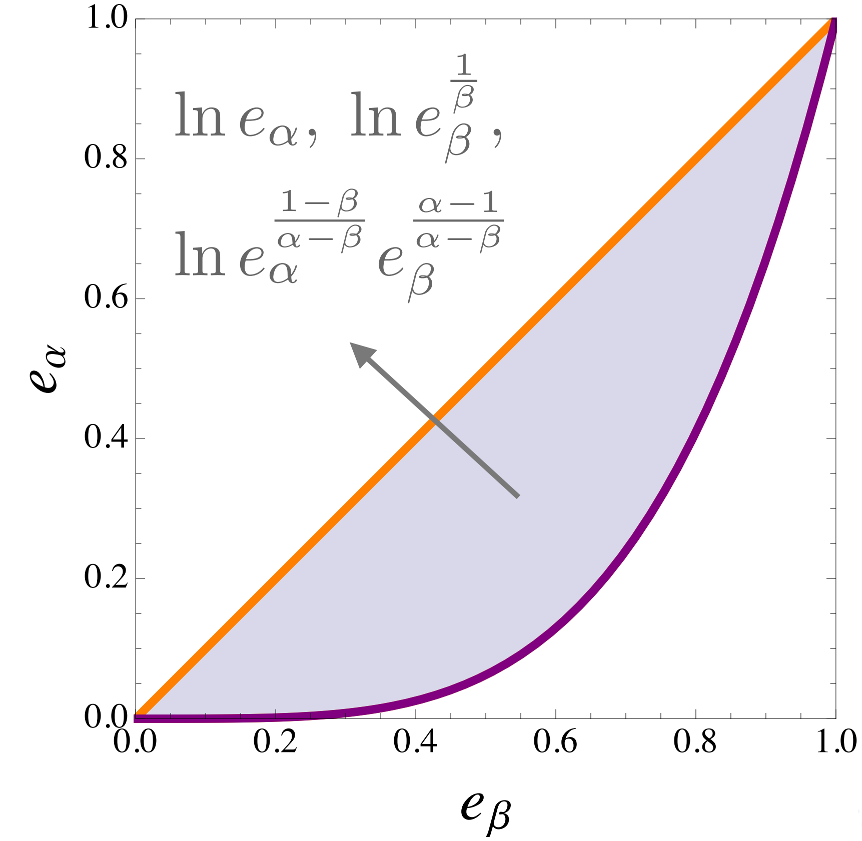
Factorization theorem in the bulk

- What to do in the bulk?

  ⟶ Larkoski, Moult, Neill: Interpolate

  ⟶ Our approach: Additional mode

- Extension of SCET (SCET+) containing additional collinear-soft mode resums all logarithms in the bulk



Mode	Scaling $(-,+,\perp)$	Measurement
n-collinear	$Q(1, \lambda^{2r/\beta}, \lambda^{r/\beta})$	e_β
n-collinear-soft	$Q\left(\lambda^{\frac{\alpha r-\beta}{\alpha-\beta}}, \lambda^{\frac{(\alpha-2)r-(\beta-2)}{\alpha-\beta}}, \lambda^{\frac{(\alpha-1)r-(\beta-1)}{\alpha-\beta}}\right)$	e_β, e_α
soft	$Q(\lambda, \lambda, \lambda)$	e_α

$\beta/\alpha < r < 1$

and $\lambda \sim e_\alpha \sim e_\beta^{1/r}$

Collinear-soft modes (with different scaling) are introduced also in other contexts to describe multi-scale problems **Bauer, Tackmann, Walsh, Zuberi, '12**; Larkoski, Moult, Neill, '15, Becher, Neubert, Rothen, Shao, '15; Chien, Hornig, Lee, '15

Factorization theorem in the bulk

- Factorization formula (valid to NLL)

$$\frac{\mathrm{d}^2\sigma_i}{\mathrm{d}e_\alpha\,\mathrm{d}e_\beta} = \hat{\sigma}_i^{(0)} H_i(Q^2) \int \mathrm{d}e_\beta^{\mathrm{c}} Q^\beta\, \mathrm{d}e_\alpha^{\mathrm{cs}} Q\, \mathrm{d}e_\beta^{\mathrm{cs}} Q^\beta\, \mathrm{d}e_\alpha^{\mathrm{s}} Q$$

i = q (quarks)
i = g (gluons)

$$J_i(e_\beta^{\mathrm{c}} Q^\beta)\, \mathscr{S}_i(e_\alpha^{\mathrm{cs}} Q, e_\beta^{\mathrm{cs}} Q^\beta)\, S_i(e_\alpha^{\mathrm{s}} Q)$$

$$\times\, \delta(e_\alpha - e_\alpha^{\mathrm{cs}} - e_\alpha^{\mathrm{c}})\delta(e_\beta - e_\beta^{\mathrm{c}} - e_\beta^{\mathrm{cs}})$$

- NLL resummation:
  Evolve all to the collinear-soft scale $\rightarrow$ double cumulative distribution

$$\Sigma(e_\alpha, e_\beta) = \int_0^{e_\alpha} \mathrm{d}e'_\alpha \int_0^{e_\beta} \mathrm{d}e'_\beta\, \frac{\partial^2\sigma}{\partial e'_\alpha \partial e'_\beta}$$

$$= \hat{\sigma}^{(0)} \frac{e^{K_H + K_J + K_S - \gamma_E\,\eta_J - \gamma_E\,\eta_S}}{\Gamma(1+\eta_J)\Gamma(1+\eta_S)} \Big(\frac{Q}{\mu_H}\Big)^{2\eta_H} \Big(\frac{e_\beta^{1/\beta} Q}{\mu_J}\Big)^{\beta\,\eta_J} \Big(\frac{e_\alpha Q}{\mu_S}\Big)^{\eta_S}$$

Hard scale Jet scale Soft scale

Measurement of $\mathrm{p_T}$ and thrust

- Consider Z + 0 jet production:
  Transverse momentum of Z measured
  and global jet veto imposed using beam thrust $\mathcal{T}$

$$\mathcal{T} = \sum_i p_{iT}\, e^{-|\eta_i|} = \sum_i \min\{p_i^+, p_i^-\}$$

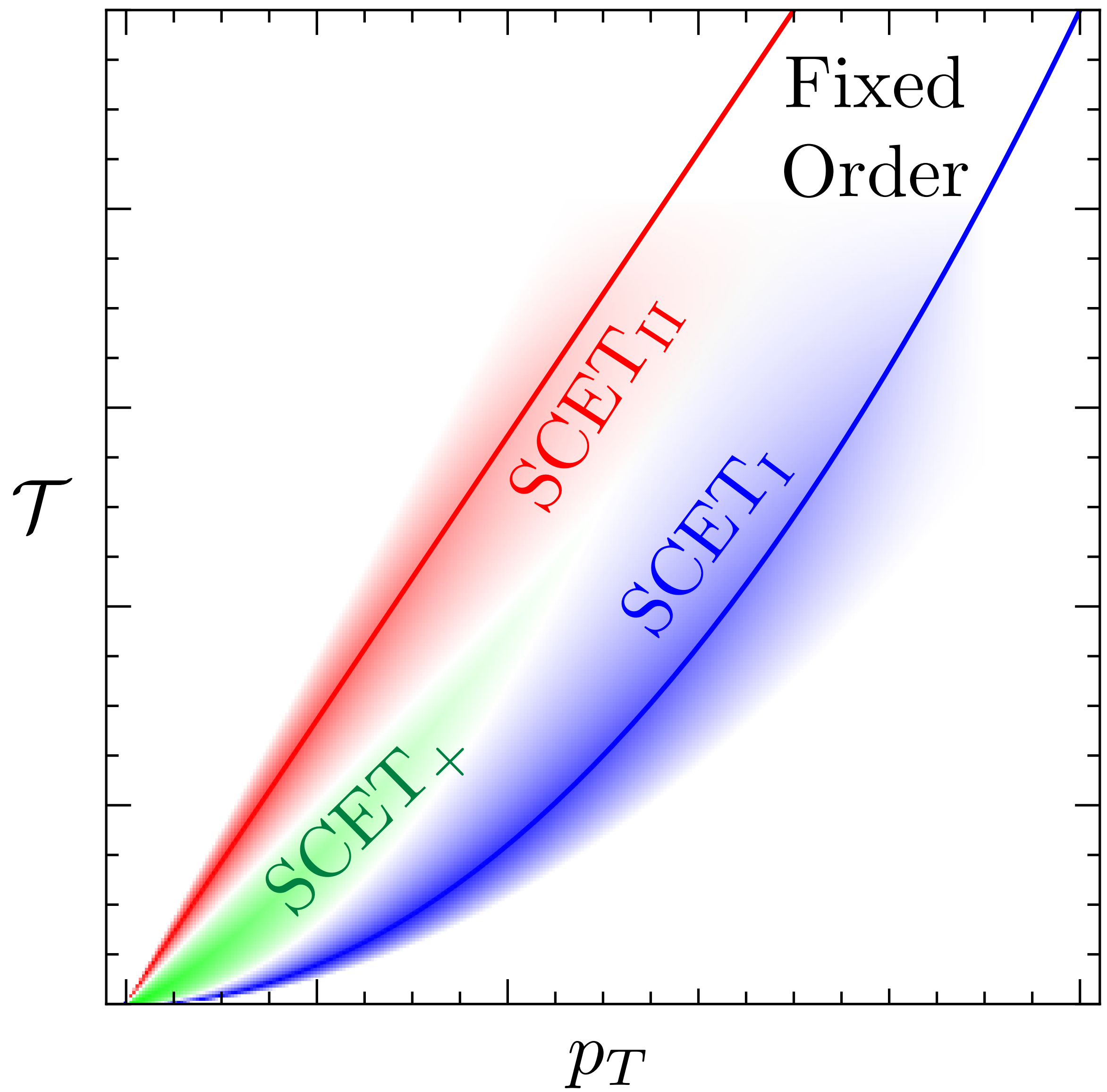
Stewart, Tackmann, Waalewijn, '09

Hierarchy between $\mathcal{T}$ and p_T determines the appropriate SCET version:

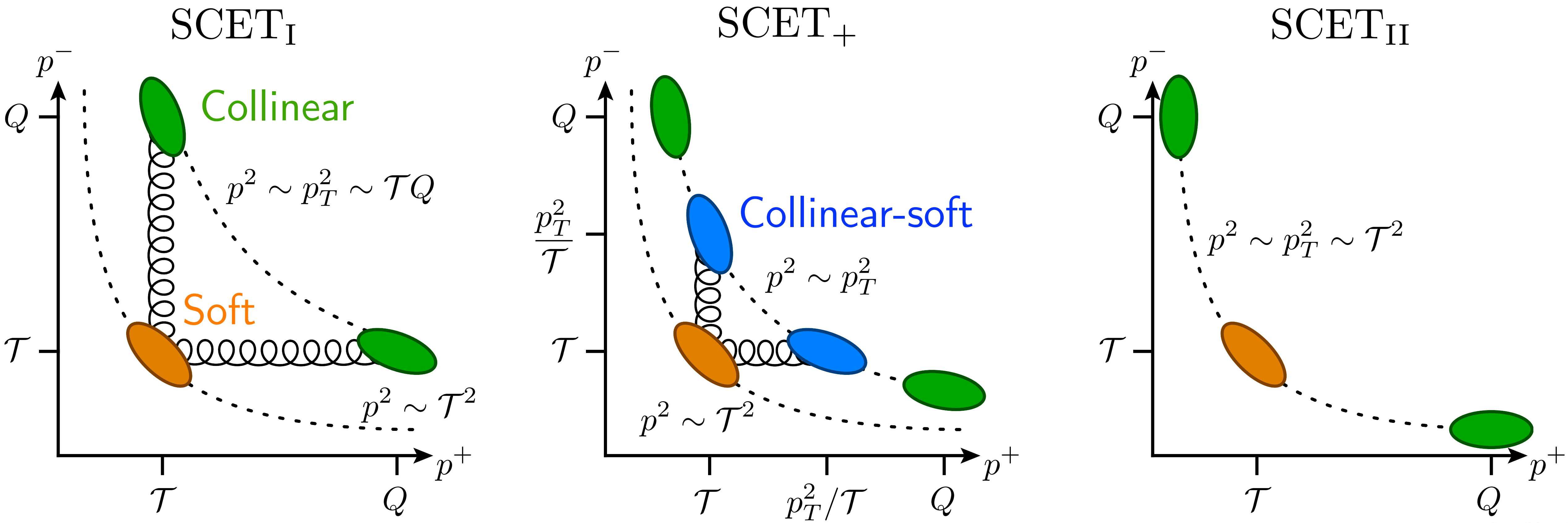
→ **SCET I:**
$p_T \sim Q^{1/2}\mathcal{T}^{1/2}$

→ **SCET+:**
$p_T \sim Q^{1-r}\mathcal{T}^{r}$
with $1/2 < r < 1$

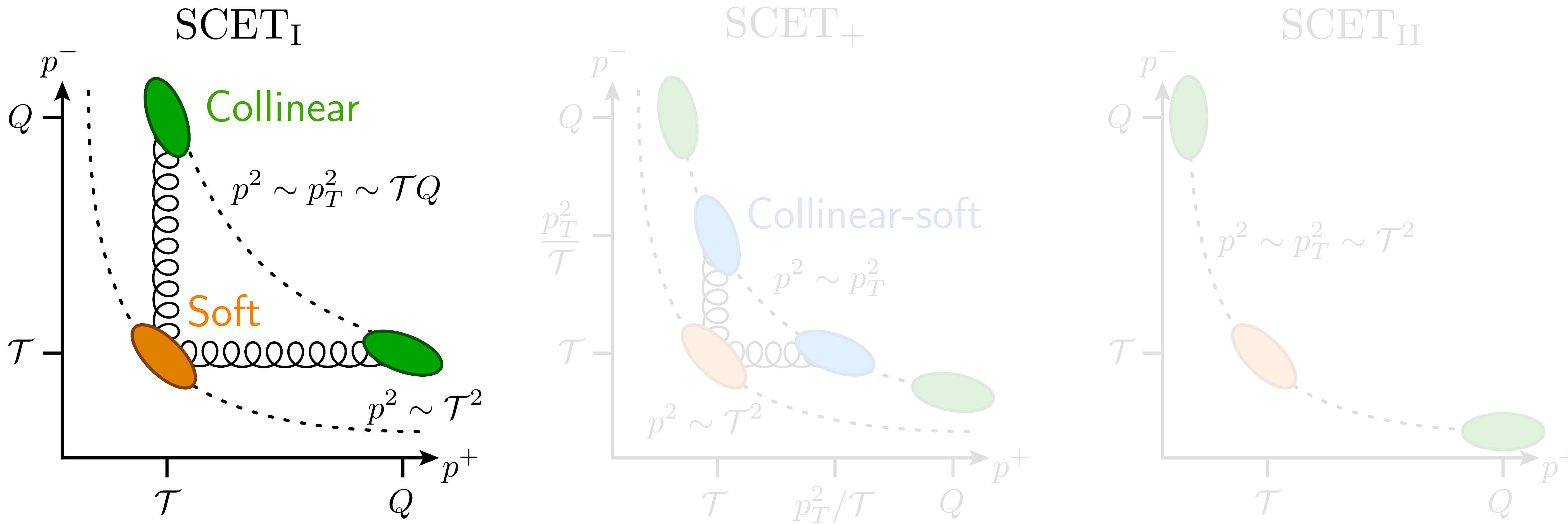
→ **SCET II:**
$p_T \sim \mathcal{T}$



Modes and factorization theorems

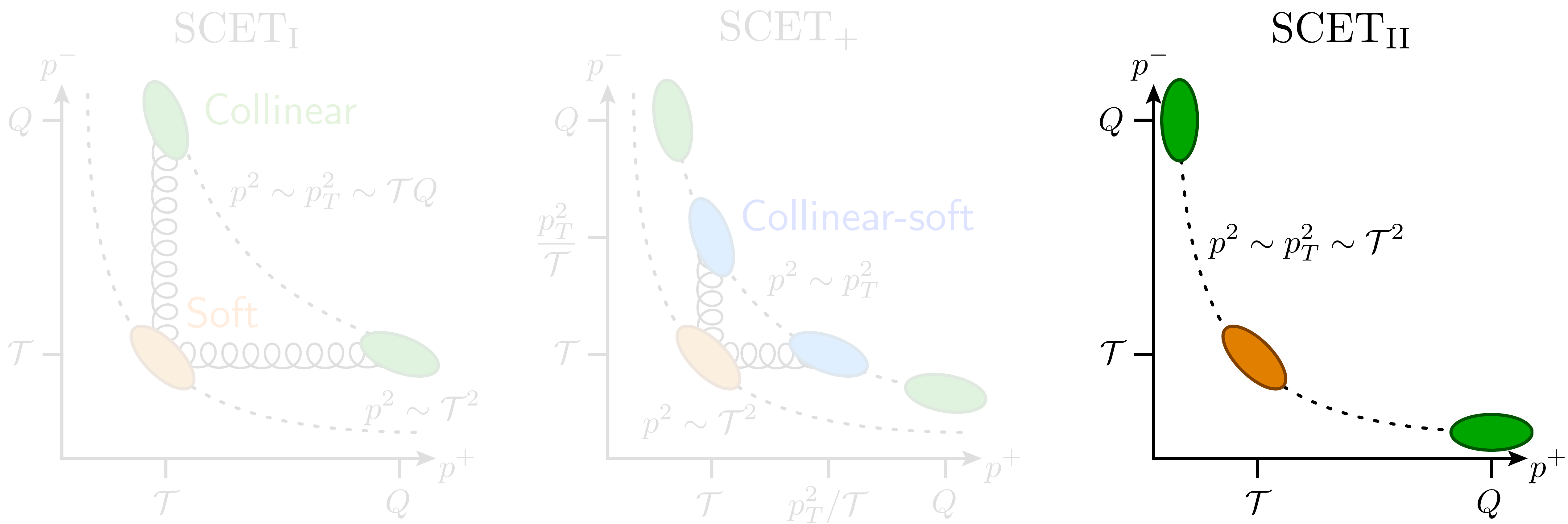


Modes and factorization theorems



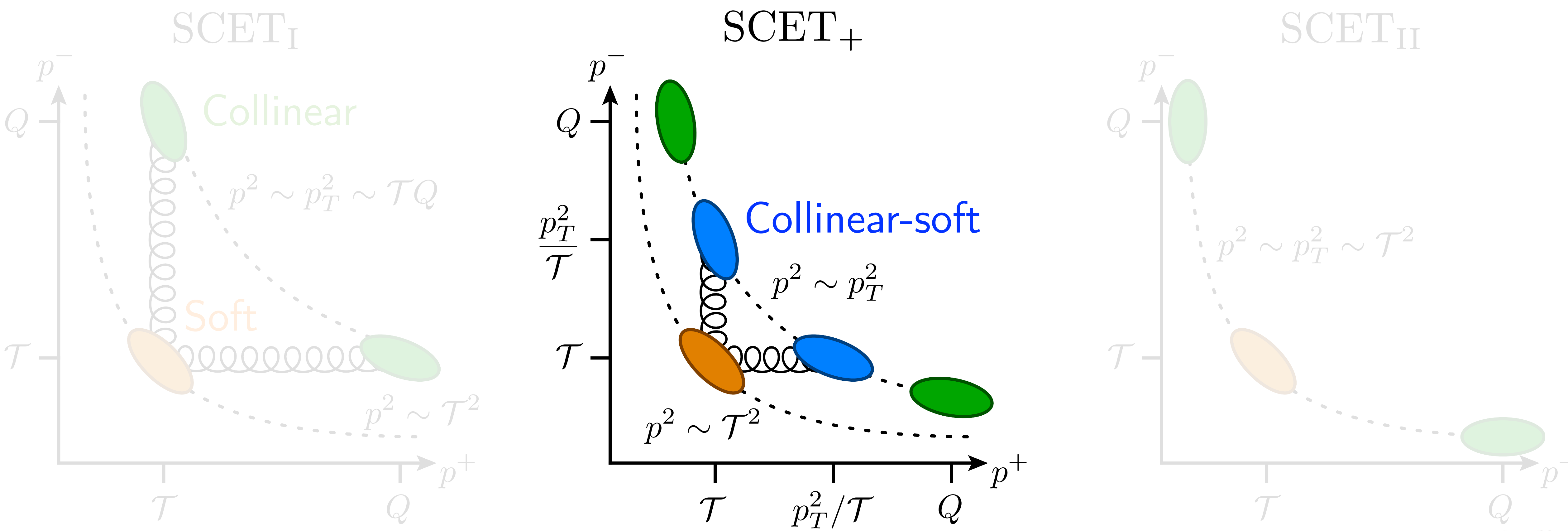
Mode	Scaling $(-,+,\perp)$	Measurement
n-collinear	$Q(1,\lambda^2,\lambda)$	$p_T \sim Q\lambda$
soft	$Q(\lambda^2,\lambda^2,\lambda^2)$	$\mathcal{T} \sim Q\lambda^2$

Modes and factorization theorems



Mode	Scaling $(-,+,\perp)$	Measurement
n-collinear	$Q(1,\lambda^2,\lambda)$	$p_T \sim Q\lambda$
soft	$Q(\lambda,\lambda,\lambda)$	$\mathcal{T} \sim Q\lambda$

Modes and factorization theorems

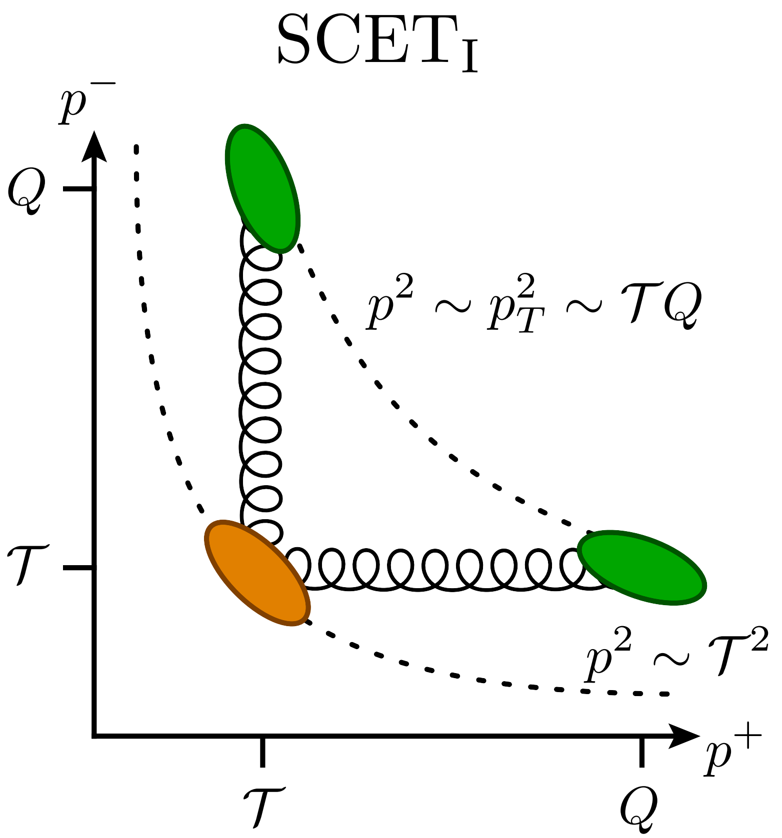


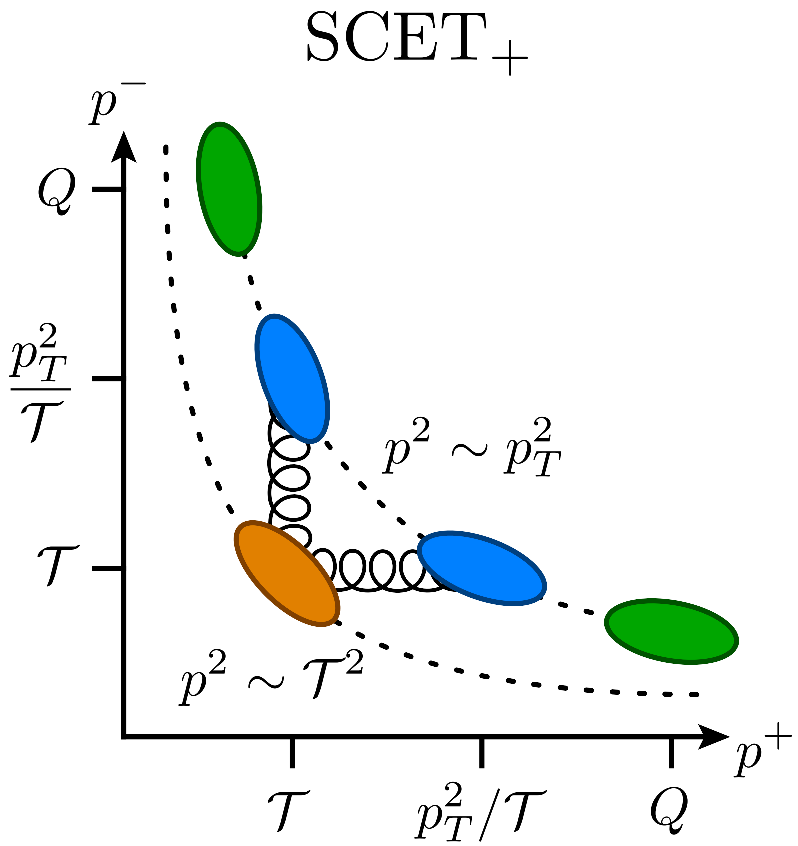
Mode	Scaling $(-,+,\perp)$	Measurement
n-collinear	$Q(1,\lambda^{2r},\lambda^r)$	p_T
n-collinear-soft	$Q(\lambda^{2r-1},\lambda,\lambda^r)$	
soft	$Q(\lambda,\lambda,\lambda)$	$\mathcal{T}$

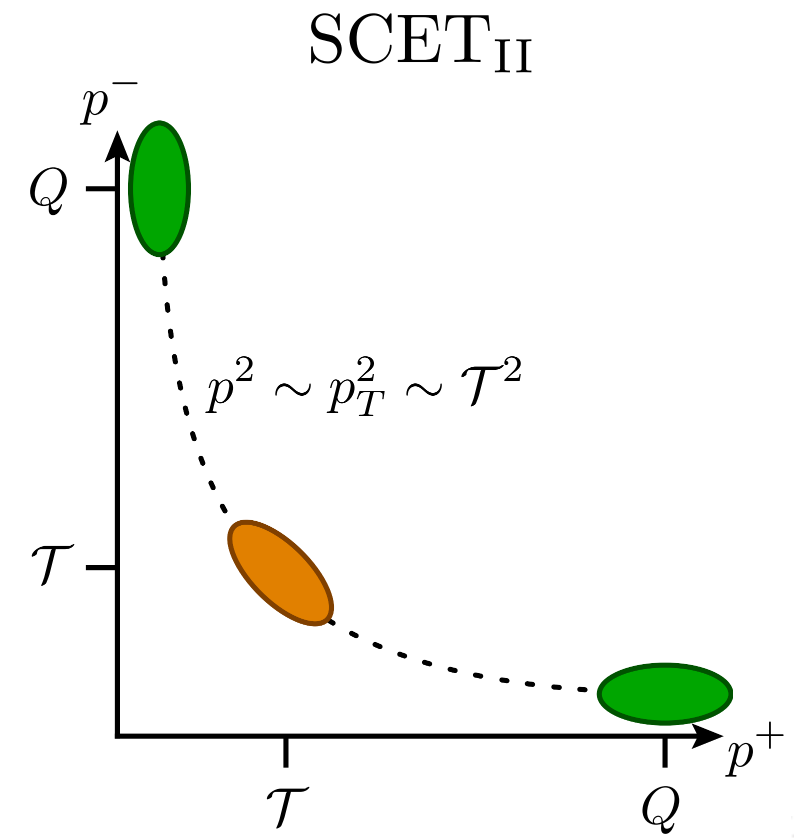
with $1/2 < r < 1$,

$\lambda \sim \mathcal{T}/Q \sim (p_T/Q)^{1/r}$

Modes and factorization theorems







$$d\sigma = H(Q^2) \times B_q(t_1, \vec{k}_{1\perp})\, B_{\bar{q}}(t_2, \vec{k}_{2\perp}) \times S(k^+)$$

Fully-unintegrated (FU) beam functions
Soft function

$$d\sigma = H(Q^2) \times B_q(\vec{k}_{1\perp})\, B_{\bar{q}}(\vec{k}_{2\perp}) \times \mathscr{S}(k_1^+, \vec{k}_{1\perp}^{\mathrm{cs}})\, \mathscr{S}(k_2^+, \vec{k}_{2\perp}^{\mathrm{cs}}) \times S(k^+)$$

TMD beam functions
Collinear-soft function
Soft function

$$d\sigma = H(Q^2) \times B_q(\vec{k}_{1\perp})\, B_{\bar{q}}(\vec{k}_{2\perp}) \times S(k^+, \vec{k}_{\perp})$$

TMD beam functions
FU soft function

NNLL resummation and consistency checks

- The SCET I, SCET+ and SCET II factorization theorems can be matched achieving a continuous cross section description

- All ingredients entering the factorisation calculated to the accuracy needed for NNLL resummation

  $\hookrightarrow$ New pieces: FU soft function and collinear soft function, both calculated at one-loop

  $\hookrightarrow$ No more details here $\rightarrow$ see paper

- Checks of our SCET+ framework

  $\hookrightarrow$ Cancellation of anomalous dimensions between the various ingredients

  $\hookrightarrow$ Full differential NLO cross section calculated and expanded in the SCET I, SCET+ and SCET II regions of phase space: Agreement with the predictions from factorization theorems

Conclusions

- Resummation of double-differential measurements achieved via a new effective theory framework SCET+ containing collinear-soft modes
- Two applications we studied:
  - Measurement of two angularities on a single jet
  - $pp \to Z + 0$ jets: jet veto is imposed through the beam thrust and transverse momentum of the Z measured

Thank you!

Back-up slides

RG equations

Hard function

non-cusp piece

$$\gamma^i_X(\alpha_s) = \sum_n \gamma^i_{X,n} \left(\frac{\alpha_s}{4\pi}\right)^{n+1}$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} H(Q^2,\mu) = \gamma_H(Q^2,\mu)\, H(Q^2,\mu)\,,$$

$$\gamma_H(Q^2,\mu) = \Gamma_{\text{cusp}}(\alpha_s) \ln\frac{Q^2}{\mu^2} + \gamma_H(\alpha_s)$$

cusp piece

Jet function

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} J(e_\beta Q^\beta,\mu) = \int_0^{e_\beta} \mathrm{d}e'_\beta\, Q^\beta\, \gamma_J(e_\beta Q^\beta - e'_\beta Q^\beta,\mu)\, J(e'_\beta Q^\beta,\mu)\,,$$

$$\gamma_J(e_\beta Q^\beta,\mu) = -\frac{2}{\beta-1}\,\Gamma_{\text{cusp}}(\alpha_s)\,\frac{1}{\mu^\beta}\mathcal{L}_0\left(\frac{e_\beta Q^\beta}{\mu^\beta}\right) + \gamma_J(\alpha_s)\,\delta(e_\beta Q^\beta)$$

Soft function

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} S(e_\alpha Q,\mu) = \int_0^{e_\alpha} \mathrm{d}e'_\alpha Q\, \gamma_S(e_\alpha Q - e'_\alpha Q,\mu)\, S(e'_\alpha Q,\mu)\,,$$

$$\gamma_S(e_\alpha Q,\mu) = \frac{2}{\alpha-1}\,\Gamma_{\text{cusp}}(\alpha_s)\,\frac{1}{\mu}\mathcal{L}_0\left(\frac{e_\alpha Q}{\mu}\right) + \gamma_S(\alpha_s)\,\delta(e_\alpha Q)$$

Collinear-soft function constrained by consistency

Comparison to Larkoski, Moult, Neill

- Their NLL conjecture:

$$\Sigma(e_\alpha, e_\beta)^{\text{conjecture}} = \frac{e^{-\gamma_E \tilde{R}(e_\alpha, e_\beta)}}{\Gamma(1+\tilde{R}(e_\alpha, e_\beta))} e^{-R(e_\alpha, e_\beta)-\gamma_i T(e_\alpha, e_\beta)}$$

- This mostly agrees with our result with

$$R(e_\alpha, e_\beta) + \gamma T(e_\alpha, e_\beta) \overset{\text{NLL}}{=} -K_H(\mu_H, \mu_{\mathscr{S}}) - K_J(\mu_J, \mu_{\mathscr{S}}) - K_S(\mu_S, \mu_{\mathscr{S}}) \,,$$
$$\tilde{R}(e_\alpha, e_\beta) \overset{\text{NLL}}{=} \eta_J(\mu_J, \mu_{\mathscr{S}}) + \eta_S(\mu_S, \mu_{\mathscr{S}})$$

- Difference in the denominator:

  (ignoring power-suppressed terms and terms beyond NLL)

Our result: $\Gamma(1+\eta_J)\Gamma(1+\eta_S)$

JHEP 1409 (2014) 046: $\Gamma(1+\eta_J+\eta_S)$

Difference at $\mathcal{O}(\alpha_s^2)$ in the bulk

Scale choices

- **Boundary conditions**

$$\Sigma(e_\alpha, e_\beta = e_\alpha^{\beta/\alpha}) = \Sigma(e_\alpha)$$

(e_β has been integrated over its entire range)

$$\Sigma(e_\alpha = e_\beta, e_\beta) = \Sigma(e_\beta)$$

(e_α has been integrated over its entire range)

derivative:

$$\frac{\partial}{\partial e_\alpha}\Sigma(e_\alpha, e_\beta)\Big|_{e_\beta = e_\alpha^{\beta/\alpha}} = \frac{d\sigma}{de_\alpha}$$

$$\frac{\partial}{\partial e_\alpha}\Sigma(e_\alpha, e_\beta)\Big|_{e_\beta = e_\alpha} = 0$$

and similarly for $\partial/\partial e_\beta$ with B1 $\leftrightarrow$ B2

- Boundary conditions in JHEP 1409 (2014) 046 fulfilled by adding power-suppressed terms

- **Profile scales**

Boundary conditions can be fulfilled by appropriate scale choice:

$$\mu_{\mathscr{S}}(e_\alpha, e_\beta)\Big|_{\text{B1}} = \mu_S(e_\alpha, e_\beta)\Big|_{\text{B1}}$$

$$\mu_{\mathscr{S}}(e_\alpha, e_\beta)\Big|_{\text{B2}} = \mu_J(e_\alpha, e_\beta)\Big|_{\text{B2}}$$

$$\frac{\partial}{\partial e_\alpha}\,\mu_J(e_\alpha, e_\beta)\Big|_{\text{B2}} = \frac{\mathrm{d}}{\mathrm{d}e_\alpha}\,\mu_J(e_\alpha, e_\alpha^{\beta/\alpha})$$

$$\frac{\partial}{\partial e_\alpha}\,\mu_{\mathscr{S}}(e_\alpha, e_\beta)\Big|_{\text{B2}} = \frac{\mathrm{d}}{\mathrm{d}e_\alpha}\,\mu_J(e_\alpha, e_\alpha^{\beta/\alpha})$$

$$\frac{\partial}{\partial e_\alpha}\,\mu_S(e_\alpha, e_\beta)\Big|_{\text{B2}} = \frac{\mathrm{d}}{\mathrm{d}e_\alpha}\,\mu_S(e_\alpha, e_\alpha^{\beta/\alpha})$$

$$\frac{\partial}{\partial e_\alpha}\,\mu_X(e_\alpha, e_\beta)\Big|_{\text{B1}} = 0 \quad , \; X = J, \mathscr{S}, S$$

and similarly for $\partial/\partial e_\beta$

Effective theory framework

I. Matching the QCD quark current onto SCET+

$$\bar{\Psi}\,\Gamma\,\Psi = C(Q^2,\mu)\,\bar{\xi}_{\bar{n}} W_{\bar{n}} S_{\bar{n}}^{\dagger} X_{\bar{n}}^{\dagger} V_{\bar{n}}\,\Gamma\, V_n^{\dagger} X_n S_n W_n^{\dagger} \xi_n$$

QCD quark fields | Dirac structure | matching coefficient | collinear antiquark moving in $\bar{n}$-direction | collinear quark moving in n-direction

Wilson lines

$W_n^{\dagger}$: n-collinear gluons emitted from $\bar{\Psi}$ ($\bar{n}$-collinear)

$V_n^{\dagger}$: n-collinear-soft gluons emitted from $\bar{\Psi}$ ($\bar{n}$-collinear)

S_n: soft gluons emitted from Ψ (n-collinear)

X_n: n-collinear-soft gluons emitted from Ψ (n-collinear)

The ordering of the Wilson lines is fixed by gauge invariance of SCET+

Effective theory framework

- n-collinear gauge transformation:

  Groups together $W_n^\dagger \xi_n$ ($W_n^\dagger \to W_n^\dagger U_n^\dagger$)

  $$\boxed{\xi_n \to U_n \xi_n\,, \qquad W_n \to U_n W_n\,,} \qquad S_n \to S_n\,, \qquad V_n \to V_n\,, \qquad X_n \to X_n$$

  Similarly $\bar{\xi}_{\bar{n}} W_{\bar{n}}$ is grouped together by $\bar{n}$-collinear gauge transformation

- n-collinear-soft gauge transformation:

  Groups together $V_n^\dagger X_n$

  $$W_n^\dagger \xi_n \to W_n^\dagger \xi_n\,, \qquad S_n \to S_n\,, \qquad \boxed{V_n \to U_{ncs} V_n\,, \qquad X_n \to U_{ncs} X_n}$$

  Similarly $X_{\bar{n}}^\dagger V_{\bar{n}}$ is grouped together by $\bar{n}$-collinear-soft gauge transformation

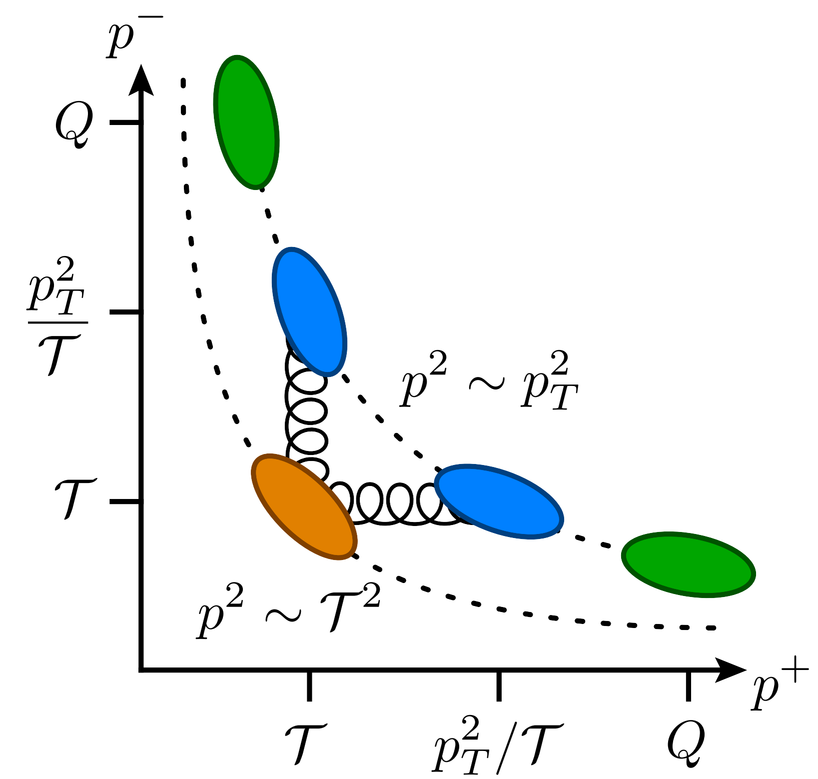
- soft gauge transformation:

  $$\begin{aligned} W_n^\dagger \xi_n &\to W_n^\dagger \xi_n\,, \\ \bar{\xi}_{\bar{n}} W_{\bar{n}} &\to \bar{\xi}_{\bar{n}} W_{\bar{n}}\,, \end{aligned} \qquad \boxed{\begin{array}{lll} S_n \to U_s S_n\,, & V_n \to U_s V_n U_s^\dagger\,, & X_n \to U_s X_n U_s^\dagger \\ S_{\bar{n}} \to U_s S_{\bar{n}}\,, & V_{\bar{n}} \to U_s V_{\bar{n}} U_s^\dagger\,, & X_{\bar{n}} \to U_s X_{\bar{n}} U_s^\dagger \end{array}}$$

  Fixes the remaining ordering

Effective theory framework

II. BPS field redefinition



- At this point the soft fields still interact with the collinear-soft fields
- Performing an analog to the BPS field redefinition: Bauer, Pirjol, Stewart, '02

$$V_n \to S_n V_n S_n^\dagger\,, \qquad X_n \to S_n X_n S_n^\dagger\,,$$
$$V_{\bar{n}} \to S_{\bar{n}} V_{\bar{n}} S_{\bar{n}}^\dagger\,, \qquad X_{\bar{n}} \to S_{\bar{n}} X_{\bar{n}} S_{\bar{n}}^\dagger$$

- Finally:

$$\bar{\Psi}\,\Gamma\,\Psi = C(Q^2,\mu)\,\bar{\xi}_{\bar{n}} W_{\bar{n}} X_{\bar{n}}^\dagger V_{\bar{n}}\, S_{\bar{n}}^\dagger \Gamma\, S_n V_n^\dagger X_n W_n^\dagger \xi_n$$

- No interaction between various modes anymore
  $\to$ Derive factorisation theorems

Factorisation theorems: SCET I

Stewart, Tackmann, Waalewijn, '09;
Jain, Procura, Waalewijn, '11

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\,\mathrm{d}Y\,\mathrm{d}p_T^2\,\mathrm{d}\mathcal{T}} = \sum_q \hat{\sigma}_q^0\, H(Q^2) \int \mathrm{d}t_1\,\mathrm{d}t_2 \int \mathrm{d}^2\vec{k}_{1\perp}\,\mathrm{d}^2\vec{k}_{2\perp} \int \mathrm{d}k^+\, S(k^+)$$
$$\times \left[B_q(t_1, x_1, \vec{k}_{1\perp})\, B_{\bar{q}}(t_2, x_2, \vec{k}_{2\perp}) + (q \leftrightarrow \bar{q})\right]$$
$$\times\, \delta\Big(\mathcal{T} - \frac{e^{-Y}t_1 + e^{Y}t_2}{Q} - k^+\Big)\, \delta\big(p_T^2 - |\vec{k}_{1\perp} + \vec{k}_{2\perp}|^2\big)$$

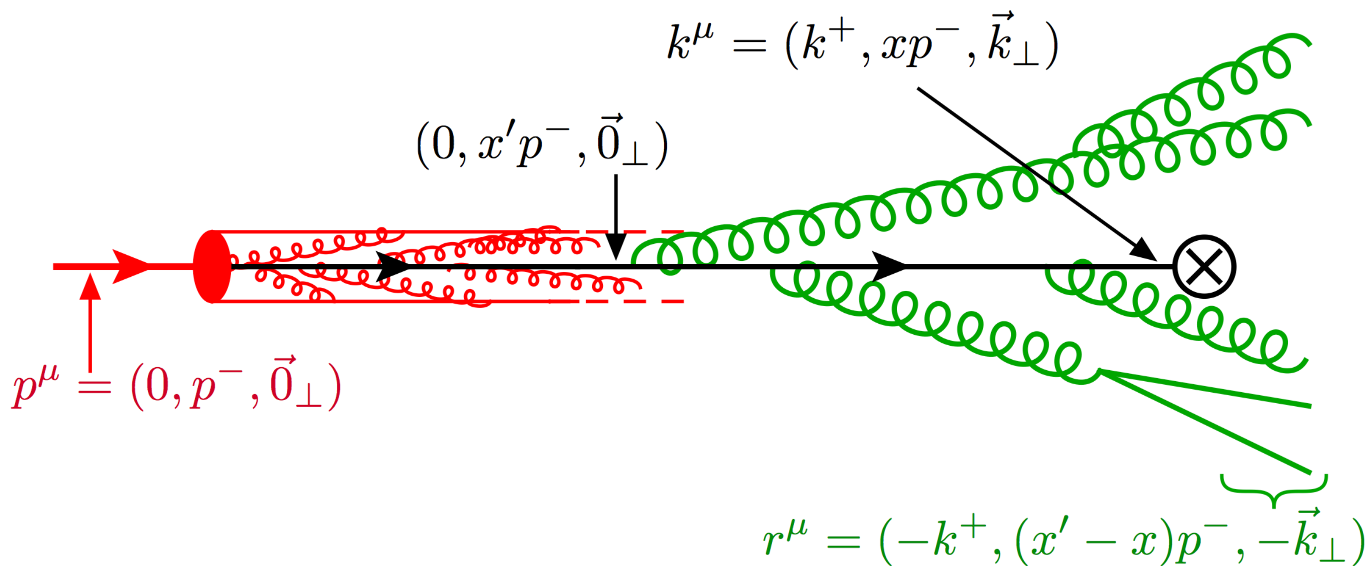
- <u>Ingredients:</u>

  FU beam function

$\mathbf{P}$ operator returns momentum of intermediate state

$\chi_n = W_n^\dagger \xi_n$

$$\boxed{B_q(t, x, \vec{k}_\perp) = \Big\langle p_n(p^-)\Big|\bar{\chi}_n(0)\,\frac{\bar{n}\!\!\!/}{2}\,\big[\delta(k^- - p^- + \mathbf{P}^-)\;\delta(t - k^-\mathbf{P}^+)\,\delta^2(\vec{k}_\perp - \vec{\mathbf{P}}_\perp)\,\chi_n(0)\big]\Big|p_n(p^-)\Big\rangle}$$



$\hat{\sigma}_q^0$: Born cross section

$H(Q^2, \mu) = |C(Q^2, \mu)|^2$: Hard function

$-t_i = k_i^- k_i^+ \; (i = 1, 2)$: Transverse virtuality

$x_i = Q/E_{\mathrm{cm}}\, e^{\pm Y} \; (i = 1, 2)$: Momentum fraction,

Y = rapidity

Factorisation theorems: SCET I

Stewart, Tackmann, Waalewijn, '09;
Jain, Procura, Waalewijn, '11

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\,\mathrm{d}Y\,\mathrm{d}p_T^2\,\mathrm{d}\mathcal{T}} = \sum_q \hat{\sigma}_q^0\, H(Q^2) \int \mathrm{d}t_1\,\mathrm{d}t_2 \int \mathrm{d}^2\vec{k}_{1\perp}\,\mathrm{d}^2\vec{k}_{2\perp} \int \mathrm{d}k^+\, S(k^+)$$
$$\times \left[B_q(t_1, x_1, \vec{k}_{1\perp})\, B_{\bar{q}}(t_2, x_2, \vec{k}_{2\perp}) + (q \leftrightarrow \bar{q}) \right]$$
$$\times\, \delta\Big(\mathcal{T} - \frac{e^{-Y}t_1 + e^{Y}t_2}{Q} - k^+\Big)\, \delta\big(p_T^2 - |\vec{k}_{1\perp} + \vec{k}_{2\perp}|^2\big)$$

- Ingredients:

  Soft function

Time ordering

$$\boxed{S(k^+) = \frac{1}{N_c} \langle 0|\mathrm{Tr}\left[\overline{\mathbf{T}}(S_n^\dagger(0)S_{\bar{n}}(0))\,\delta(k^+ - \mathbf{P}_1^+ - \mathbf{P}_2^-)\,\mathbf{T}(S_{\bar{n}}^\dagger(0)S_n(0))\right]|0\rangle}$$

$\mathbf{P}_1$ operator returns momentum of soft radiation in hemisphere 1 ($p^+ < p^-$)

$\hat{\sigma}_q^0$: Born cross section

$H(Q^2,\mu) = |C(Q^2,\mu)|^2$: Hard function

$-t_i = k_i^- k_i^+ \;(i=1,2)$: Transverse virtuality

$x_i = Q/E_{\mathrm{cm}}\, e^{\pm Y} \;(i=1,2)$: Momentum fraction, $Y =$ rapidity

Factorisation theorems: SCET II

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\,\mathrm{d}Y\,\mathrm{d}p_T^2\,\mathrm{d}\mathcal{T}} = \sum_q \hat{\sigma}_q^0\, H(Q^2) \int \mathrm{d}^2\vec{k}_{1\perp}\,\mathrm{d}^2\vec{k}_{2\perp}\,\mathrm{d}^2\vec{k}_\perp \int \mathrm{d}k^+\, \delta\big(p_T^2 - |\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{k}_\perp|^2\big)$$
$$\times\, \delta\big(\mathcal{T} - k^+\big)\Big[B_q(x_1, \vec{k}_{1\perp})\, B_{\bar{q}}(x_2, \vec{k}_{2\perp}) + (q \leftrightarrow \bar{q})\Big]\, S(k^+, \vec{k}_\perp)$$

Extension of: Chiu, Jain, Neill, Rothstein, '12
See also: Becher, Neubert, '10

- Ingredients:

TMD beam function

$$\boxed{B_q(x, \vec{k}_\perp) = \Big\langle p_n(p^-)\Big|\bar{\chi}_n(0)\,\frac{\bar{n}\!\!\!/}{2}\big[\delta(k^- - p^- + \mathbf{P}^-)\,\delta^2(\vec{k}_\perp - \vec{\mathbf{P}}_\perp)\,\chi_n(0)\big]\Big|p_n(p^-)\Big\rangle}$$

FU soft function

$$S(k^+, \vec{k}_\perp) = \frac{1}{N_c}\,\langle 0|\mathrm{Tr}\big[\overline{\mathbf{T}}(S_n^\dagger(0)S_{\bar{n}}(0))\,\delta(k^+ - \mathbf{P}_1^+ - \mathbf{P}_2^-)$$
$$\delta^2(\vec{k}_\perp - \vec{\mathbf{P}}_\perp)\,\mathbf{T}(S_{\bar{n}}^\dagger(0)S_n(0))\big]|0\rangle$$

Factorisation theorems: SCET+

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\,\mathrm{d}Y\,\mathrm{d}p_T^2\,\mathrm{d}\mathcal{T}} = \sum_q \hat{\sigma}_q^0\,H(Q^2) \int \mathrm{d}^2\vec{k}_{1\perp}\,\mathrm{d}^2\vec{k}_{2\perp}\,\mathrm{d}^2\vec{k}_{1\perp}^{\mathrm{cs}}\,\mathrm{d}^2\vec{k}_{2\perp}^{\mathrm{cs}} \int \mathrm{d}k_1^+\,\mathrm{d}k_2^+\,\mathrm{d}k^+$$

$$\times\, S(k^+)\,B_q(x_1,\vec{k}_{1\perp})\,B_{\bar{q}}(x_2,\vec{k}_{2\perp})$$

$$\times\, \mathscr{S}(k_1^+,\vec{k}_{1\perp}^{\mathrm{cs}})\,\mathscr{S}(k_2^+,\vec{k}_{2\perp}^{\mathrm{cs}})\,\delta\big(\mathcal{T}-k_1^+-k_2^+-k^+\big)$$

$$\times\, \delta\big(p_T^2-|\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{1\perp}^{\mathrm{cs}}+\vec{k}_{2\perp}^{\mathrm{cs}}|^2\big)+(q\leftrightarrow\bar{q})$$

- Ingredients:
  Soft function → SCET I
  TMD beam function → SCET II

- In SCET+ we have a TMD beam function without a TMD soft function
  We cannot combine them as was done in Becher, Neubert, '11; Echevarria, Idilbi, Scimemi, '12

Factorisation theorems: SCET+

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\,\mathrm{d}Y\,\mathrm{d}p_T^2\,\mathrm{d}\mathcal{T}} = \sum_q \hat{\sigma}_q^0\, H(Q^2) \int \mathrm{d}^2\vec{k}_{1\perp}\,\mathrm{d}^2\vec{k}_{2\perp}\,\mathrm{d}^2\vec{k}_{1\perp}^{\mathrm{cs}}\,\mathrm{d}^2\vec{k}_{2\perp}^{\mathrm{cs}} \int \mathrm{d}k_1^+\,\mathrm{d}k_2^+\,\mathrm{d}k^+$$
$$\times\, S(k^+)\, B_q(x_1,\vec{k}_{1\perp})\, B_{\bar{q}}(x_2,\vec{k}_{2\perp})$$
$$\times\, \mathscr{S}(k_1^+,\vec{k}_{1\perp}^{\mathrm{cs}})\, \mathscr{S}(k_2^+,\vec{k}_{2\perp}^{\mathrm{cs}})\, \delta\big(\mathcal{T} - k_1^+ - k_2^+ - k^+\big)$$
$$\times\, \delta\big(p_T^2 - |\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{k}_{1\perp}^{\mathrm{cs}} + \vec{k}_{2\perp}^{\mathrm{cs}}|^2\big) + (q \leftrightarrow \bar{q})$$

- Ingredients:

  Collinear-soft functions (separately for n and $\bar{n}$ directions)

$$\boxed{\begin{aligned}\mathscr{S}(k^+,\vec{k}_\perp) &= \frac{1}{N_c}\langle 0|\mathrm{Tr}\big[\overline{\mathbf{T}}(X_n^\dagger(0)V_n(0))\,\delta(k^+ - \mathbf{P}^+)\,\delta^2(\vec{k}_\perp - \vec{\mathbf{P}}_\perp)\mathbf{T}(V_n^\dagger(0)X_n(0))\big]|0\rangle \\ &= \frac{1}{N_c}\langle 0|\mathrm{Tr}\big[\overline{\mathbf{T}}(V_{\bar{n}}^\dagger(0)X_{\bar{n}}(0))\,\delta(k^+ - \mathbf{P}^-)\,\delta^2(\vec{k}_\perp - \vec{\mathbf{P}}_\perp)\mathbf{T}(X_{\bar{n}}^\dagger(0)V_{\bar{n}}(0))\big]|0\rangle\end{aligned}}$$

- FU soft function and collinear-soft function look quite similar

  Difference: Collinear-soft radiation goes only into one hemisphere

  $\rightarrow$ Different treatment of the two hemispheres

Matching of the effective theories

- The SCET I, SCET+ and SCET II factorization theorems can be matched achieving a continuous cross section description

SCET I ← SCET+

beam function matching coefficients*

$$\mathcal{I}_{ij}(t,x,\vec{k}_\perp) = \int \mathrm{d}^2\vec{k}'_\perp\, \mathcal{I}_{ij}(x,\vec{k}'_\perp)\, \mathscr{S}(t/p^-,\vec{k}_\perp - \vec{k}'_\perp) + \text{power corrections}$$

$$S(k^+,\vec{k}_\perp) = \int \mathrm{d}^2\vec{k}'_\perp \int \mathrm{d}k'^+\, \mathrm{d}k''^+\, S(k^+ - k'^+ - k''^+)\, \mathscr{S}(k'^+,\vec{k}'_\perp)\mathscr{S}(k''^+,\vec{k}_\perp - \vec{k}'_\perp) + \text{power corrections}$$

SCET II ← SCET+

This holds for common scales: $\mu = \mu_B = \mu_{\mathscr{S}} = \mu_S$ and $\nu = \nu_B = \nu_{\mathscr{S}} = \nu_S$

- This follows from:
  - Switching off resummation, SCET I and SCET II produce fixed order cross section up to power corrections
  - SCET+ regime can be obtained by a further expansion of SCET I or SCET II

$$^*B_q(x,\vec{k}_\perp,\mu,\nu) = \sum_j \int_x^1 \frac{\mathrm{d}x'}{x'} \mathcal{I}_{qj}\Big(\frac{x}{x'},\vec{k}_\perp,\mu,\nu\Big) f_j(x',\mu)\Big[1 + \mathcal{O}\Big(\frac{\Lambda^2_{\mathrm{QCD}}}{\vec{k}^2_\perp}\Big)\Big]$$

Matching of the effective theories

- At NNLL one can show:

$$\mathcal{I}_{qq}^{(1)}(t,x,\vec{k}_\perp) = \delta(t)\,\mathcal{I}_{qq}^{(1)}(x,\vec{k}_\perp) + \delta(1-x)\,\mathscr{S}^{(1)}(t/p^-,\vec{k}_\perp)$$
$$\mathcal{I}_{qg}^{(1)}(t,x,\vec{k}_\perp) = \delta(t)\,\mathcal{I}_{qg}^{(1)}(x,\vec{k}_\perp)\,,$$
$$S^{(1)}(k^+,\vec{k}_\perp) = \frac{1}{\pi}\,\delta(\vec{k}_\perp^{\,2})S^{(1)}(k^+) + 2\mathscr{S}^{(1)}(k^+,\vec{k}_\perp)$$

- Patch together the NNLL cross section

$$\begin{aligned}\frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\,\mathrm{d}Y\,\mathrm{d}p_T^2\,\mathrm{d}\mathcal{T}} = {} & \sum_q \hat{\sigma}_q^0\, H(Q^2)\int \mathrm{d}t_1\,\mathrm{d}t_2 \int \mathrm{d}^2\vec{k}_{1\perp}\,\mathrm{d}^2\vec{k}_{2\perp}\,\mathrm{d}^2\vec{k}_{1\perp}^{\mathrm{cs}}\,\mathrm{d}^2\vec{k}_{2\perp}^{\mathrm{cs}}\,\mathrm{d}^2\vec{k}_\perp \int \mathrm{d}k_1^+\,\mathrm{d}k_2^+\,\mathrm{d}k^+ \\ & \times \Big[B_q(t_1,x_1,\vec{k}_{1\perp}) - \mathscr{S}^{(1)}(t_1 e^{-Y}/Q,\vec{k}_{1\perp})\Big]\mathscr{S}(k_1^+,\vec{k}_{1\perp}^{\mathrm{cs}}) \\ & \times \Big[B_{\bar{q}}(t_2,x_2,\vec{k}_{2\perp}) - \mathscr{S}^{(1)}(t_2 e^{Y}/Q,\vec{k}_{2\perp})\Big]\mathscr{S}(k_2^+,\vec{k}_{2\perp}^{\mathrm{cs}}) \\ & \times \Big[S(k^+,\vec{k}_\perp) - 2\mathscr{S}^{(1)}(k^+,\vec{k}_\perp)\Big]\delta\Big(\mathcal{T} - \frac{e^{-Y}t_1 + e^{Y}t_2}{Q} - k_1^+ - k_2^+ - k^+\Big) \\ & \times \delta\big(p_T^2 - |\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{k}_{1\perp}^{\mathrm{cs}} + \vec{k}_{2\perp}^{\mathrm{cs}} + \vec{k}_\perp|^2\big) + (q \leftrightarrow \bar{q})\end{aligned}$$

Non-global logarithms

- To what extend can our framework be used to calculate non-global logarithms, arising when different restrictions are applied in different regions of phase space?

- Consider: Instead of measuring p_T of Z boson, measure p_T of ISR it recoils against (ISR in <u>one</u> hemisphere)

- Factorization theorem:

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\,\mathrm{d}Y\,\mathrm{d}p_{T,\mathrm{ISR}}^2\,\mathrm{d}\mathcal{T}} = \sum_q \hat{\sigma}_q^0\, H(Q^2,\mu) \int \mathrm{d}t_2 \int \mathrm{d}^2\vec{k}_{1\perp}\,\mathrm{d}^2\vec{k}_{1\perp}^{\mathrm{cs}} \int \mathrm{d}k_1^+\,\mathrm{d}k^+\, S(k^+,\mu)$$
$$\times B_q(x_1,\vec{k}_{1\perp},\mu,\nu)\, B_{\bar{q}}(t_2,x_2,\mu)\, \mathscr{S}\big(k_1^+,\vec{k}_{1\perp}^{\mathrm{cs}},\mu,\nu\big)$$
$$\times\,\delta\Big(\mathcal{T}-k_1^+-\frac{e^Y t_2}{Q}-k^+\Big)\,\delta\big(p_{T,\mathrm{ISR}}^2-|\vec{k}_{1\perp}+\vec{k}_{1\perp}^{\mathrm{cs}}|^2\big)+(q\leftrightarrow\bar{q})$$

- This does not address the problem arising when the soft function contains multiple scales (e.g. when beam thus measurement would be restricted to one hemisphere)