NNLO dijet production in DIS with the antenna subtraction method

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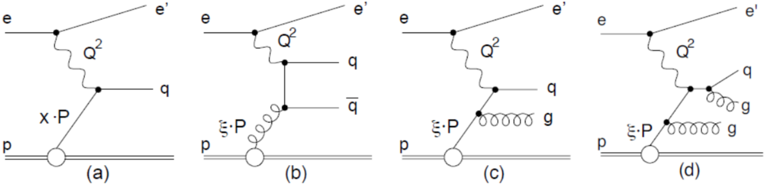
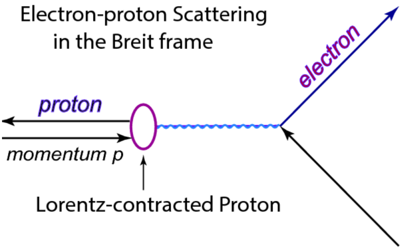
Overview

Topics

1. DIS and dijet observables
2. Antenna formalism at NNLO
3. Preliminary NNLO results
4. Outlook

Deep Inelastic Scattering





Breit-frame

Frame in which the photons momentum is completely space like → two jets of same P_T @LO.

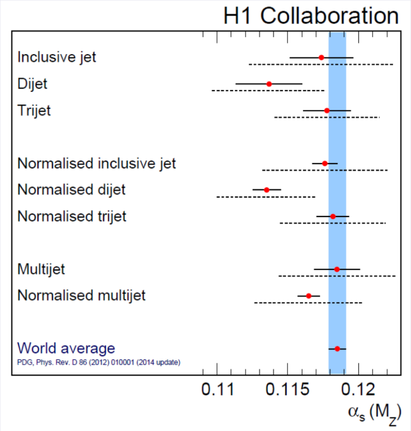
HERA I

Applications

HERA measurements can be used to:

- determine α_S:
  Experimental (solid line) is much smaller than theoretical error (dashed line)
  $\rightarrow$ **NNLO calculation needed!**
- constrain gluon PDFs.

HERA α_S measurement



HERA II

Observables

The H1 collaboration focused on distributions measured in [arXiv:1406.4709]:

- $\xi_2 = x_{bj}(1 + M_{12}/Q^2)$,
- $\langle P_T \rangle_2 = \left(P_T^1 + P_T^2\right)/2$,

and trijet observables:

- $\xi_3 = x_{bj}(1 + M_{123}/Q^2)$,
- $\langle P_T \rangle_3 = \left(P_T^1 + P_T^2 + P_T^3\right)/3$.

Cuts in Breit frame

- $5\ \mathrm{GeV} < \mathrm{p}_{\mathrm{jet}}^{\mathrm{T}} < 50\ \mathrm{GeV}$
- $M_{12} > 16\ \mathrm{GeV}$

Cuts in HERA frame

- $-1.0 < \eta_{\mathrm{lab}}^{\mathrm{jet}} < 2.5$

Jet Cross Sections

Cross sections at different orders:

$$d\sigma_{\mathrm{LO}} = \int_{d\Phi_m} d\sigma_{\mathrm{B}},$$

$$d\sigma_{\mathrm{NLO}} = \int_{d\Phi_{m+1}} d\sigma_{\mathrm{NLO}}^{\mathrm{R}} + \int_{d\Phi_m} d\sigma_{\mathrm{NLO}}^{\mathrm{V}} + \int_{d\Phi_m} d\sigma_{\mathrm{NLO}}^{\mathrm{MF}},$$

$$\begin{aligned} d\sigma_{\mathrm{NNLO}} &= \int_{d\Phi_{m+2}} d\sigma_{\mathrm{NNLO}}^{\mathrm{RR}} + \int_{d\Phi_{m+1}} d\sigma_{\mathrm{NNLO}}^{\mathrm{RV}} + \int_{d\Phi_m} d\sigma_{\mathrm{NNLO}}^{\mathrm{VV}} \\ &+ \int_{d\Phi_m} d\sigma_{\mathrm{NNLO}}^{\mathrm{MF},1} + \int_{d\Phi_{m+1}} d\sigma_{\mathrm{NNLO}}^{\mathrm{MF},2}. \end{aligned}$$

Subtraction method I

Adding a zero

$$\overbrace{\mathrm{d}\sigma_{\mathrm{NNLO}}}^{\text{IR finite}} = \int_{\mathrm{d}\Phi_{m+2}} \overbrace{\left[\mathrm{d}\sigma^{\mathrm{RR}}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{\mathrm{RR,S}}_{\mathrm{NNLO}}\right]}^{\text{IR finite}}$$

$$+ \int_{\mathrm{d}\Phi_{m+1}} \underbrace{\left[\mathrm{d}\sigma^{\mathrm{RV}}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{\mathrm{RV,T}}_{\mathrm{NNLO}}\right]}_{\text{IR finite}}$$

$$+ \int_{\mathrm{d}\Phi_{m}} \underbrace{\left[\mathrm{d}\sigma^{\mathrm{VV}}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{\mathrm{VV,U}}_{\mathrm{NNLO}}\right]}_{\text{IR finite}} .$$

Subtraction method II

Factorisation of IR divergences

Colour ordered matrix elements follow universal IR divergence factorisation at NLO:

- Soft radiation, $p_j \to 0$:
  $|\mathcal{M}^0_{m+1}(\cdots, p_i, p_j, p_k, \cdots)|^2 \to S_{ijk} |\mathcal{M}^0_m(\cdots, p_i, p_k, \cdots)|^2$
- Collinear splitting $p_i || p_j$:
  $|\mathcal{M}^0_{m+1}(\cdots, p_i, p_j, p_k, \cdots)|^2 \to \frac{P_{ij \to I}}{s_{ij}} |\mathcal{M}^0_m(\cdots, p_{i+j}, p_k, \cdots)|^2$

Same idea at NNLO where up to two particles can be unresolved:

- Triple collinear splitting $p_i || p_j || p_k$:
  $|\mathcal{M}^0_{m+2}(\cdots, p_i, p_j, p_k, p_l, \cdots)|^2 \to \frac{P_{ijk \to I}}{s_{ijk}} |\mathcal{M}^0_m(\cdots, p_{i+j+k}, p_l, \cdots)|^2$

Subtraction method III

Factorisation of phase space

Phase space factorises for appropriate linear mappings $p_i, p_j, p_k \to p_I, p_K$:

$$\mathrm{d}\Phi_{m+1}(p_1, \cdots, p_{m+1}; q) = \underbrace{\mathrm{d}\Phi_m(p_1, \cdots, p_I, p_K, \cdots, p_{m+1}; q)}_{\text{reduced phase space}} \times \underbrace{\mathrm{d}\Phi_3(p_i, p_j, p_k; p_I + p_K)}_{\text{NLO antenna phase space}}$$

- Mapping conserves 4 momentum.
- p_I and p_K 'on mass-shell'.
- Factorization into 4-particle antenna phase space for double unresolved mapping.

Subtraction method IV

Construction principle NLO example

Using momentum map from $\{p_{m+1}\} \to \{\widetilde{p_m}\}$ with $\{p_X\} \subset \{p_{m+1}\}$, construct subtraction terms according to factorisation:

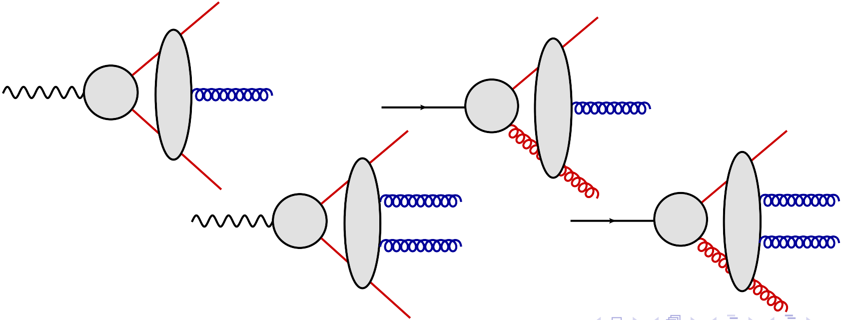
$$\mathrm{d}\sigma^{RR,S}_{NNLO} \approx \underbrace{X(\{p_X\})}_{\text{antenna}} \mathrm{d}\Phi_3(\{p_X\}) \times \overbrace{|\mathcal{M}(\{\widetilde{p_m}\})|^2}^{\text{reduced ME}} \mathrm{d}\Phi_m(\{\widetilde{p_m}\}) \times \underbrace{\mathcal{J}(\{\widetilde{p_m}\})}_{\text{jet function}}$$

All integrated antennae known analytically
$\to$ IR divergences as explicit ϵ poles.

Subtraction method V

Antenna functions

- Antennae have two hard radiators.
- Antennae mimic all soft/collinear divergences of real emissions.
- Reproduce ϵ-poles of virtual corrections.
- Have all been analytically integrated over antenna phase space!



Identity Changing Limits

Gluon initiated example

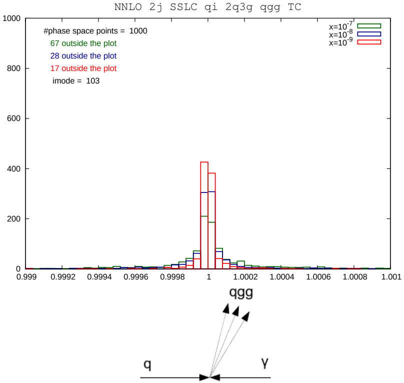
$$
\begin{aligned}
\mathcal{M} &= B_5^0(k_q, 1_g, i_g, j_g, l_{\bar{q}}) \\
\mathrm{d}\sigma^{\mathrm{RR,S}} &= + \cdots \\
&- D_4^0(l_{\bar{q}}, \overbrace{j_g, i_g, 1_g}^{j_g||i_g||1_g}) B_3^0(k_q, \bar{1}_g, \widetilde{(lji)}_{\bar{q}}) \\
&+ \cdots \\
&+ A_4^0(k_q, j_g, 1_g, l_{\bar{q}}) B_3^0(\widetilde{(klj)}_q, \bar{1}_g, i_{\bar{q}})
\end{aligned}
$$

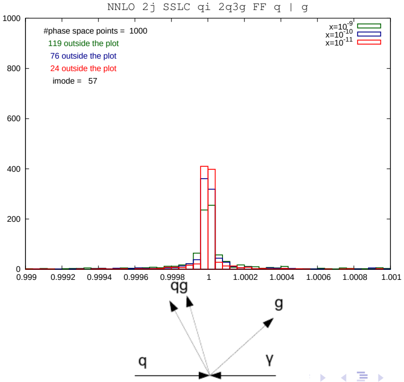
- D40 antenna has $i_g||1_g||l_{\bar{q}}$ and $j_g||i_g||1_g$ triple collinear limits
  $\rightarrow$ reduced matrix element can factor onto q or g initiated process
  $\rightarrow$ in this case correct with A40.
- Issue can be solved by appropriate combination of antennae.

Validation I

Consistency checks

- Subtraction gives right limits in all singular regions / spike plots,
- $\sigma^{RR,RV} + \sigma^{S,T}$ is stable under variation of technical phase space cut,
- Analytic pole cancellation against virtual matrix elements.





Validation II

SHERPA

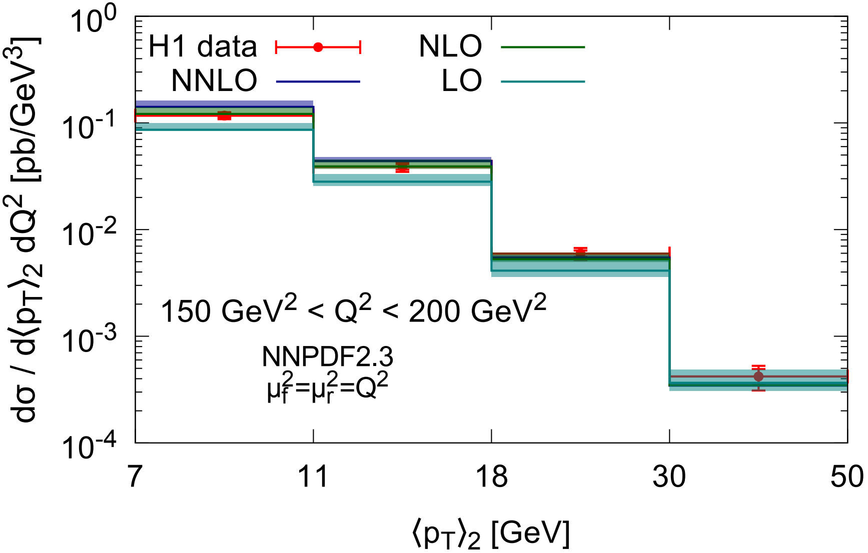
Following observables were validated against SHERPA using the H1 analysis cuts:

1. LO 2,3,4-jet, NLO 2,3-jet total cross sections,
2. Differential distributions in $\langle P_T \rangle_2$,$\langle P_T \rangle_3$, ξ_2,ξ_3.

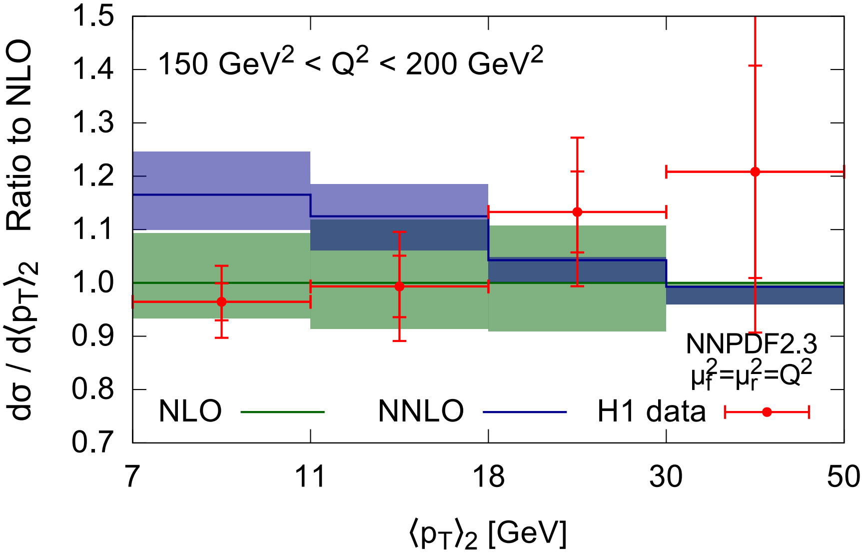
Scale variation

Dependency on the scales of the total cross section were checked against analytic expressions.

Preliminary Results I



Preliminary Results II



Discussion

HERA cuts

- H1 uses $P^T_{\text{jet}} > 5$ GeV on all jets.
- $M_{\text{jj}} > 16$ GeV cuts into first bin.
- Phase space opens up at NLO
  $\rightarrow$ large NLO/LO factor!
  $\rightarrow$ large NNLO correction $+$ large NNLO scale uncertainty.
- Should use asymmetric P^T_{jet} cuts for leading and subleading jet and no M_{jj} cut!

Outlook

The program is ready now and can be used for:

- The recent H1 analysis for high Q^2,
- Future H1 analysis for low Q^2 ranges.

To:

1. Extract α_S from DIS jet data.
2. Incorporate disjets into full NNLO PDF fits
   $\rightarrow$ This may have a significant effect on some PDF fits.

Work in progress!!!