Two-Loop integrals for precision Higgs boson phenomenology

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Based on work in collaboration with
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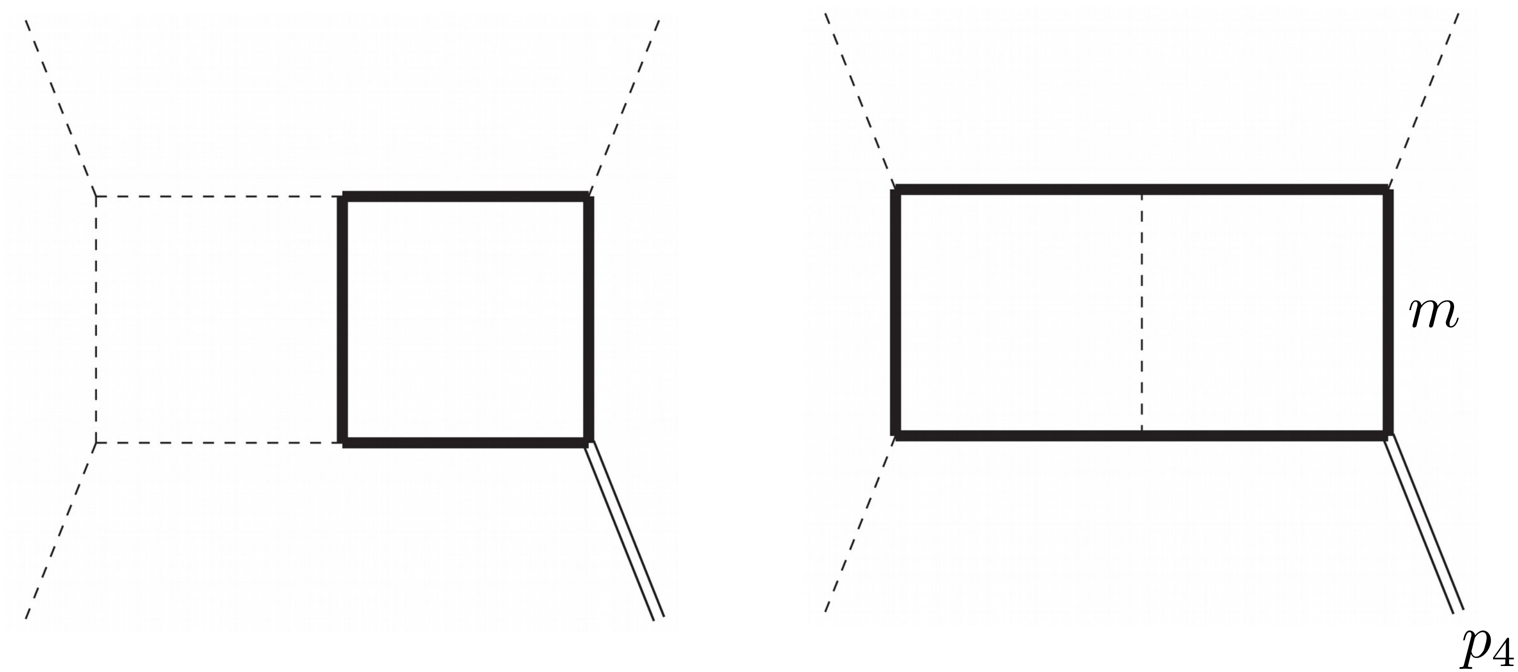
Loops and Legs
Leipzig, 26 April 2016

Integrals in this talk



$$(s, t, p_4^2, m^2)$$

Motivations

- NLO QCD corrections to Higgs decay width in a massive boson and a photon, with exact top quark mass
  [Bonciani, Del Duca, Frellesvig, Henn, FM, Smirnov, 2015]
  [Gehrmann et al, 2015]
- Higgs + jet at NLO, with exact top quark mass

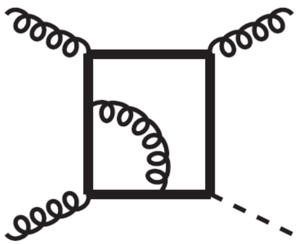
$gg \to Hg$

2 Loop QCD corrections with exact top mass

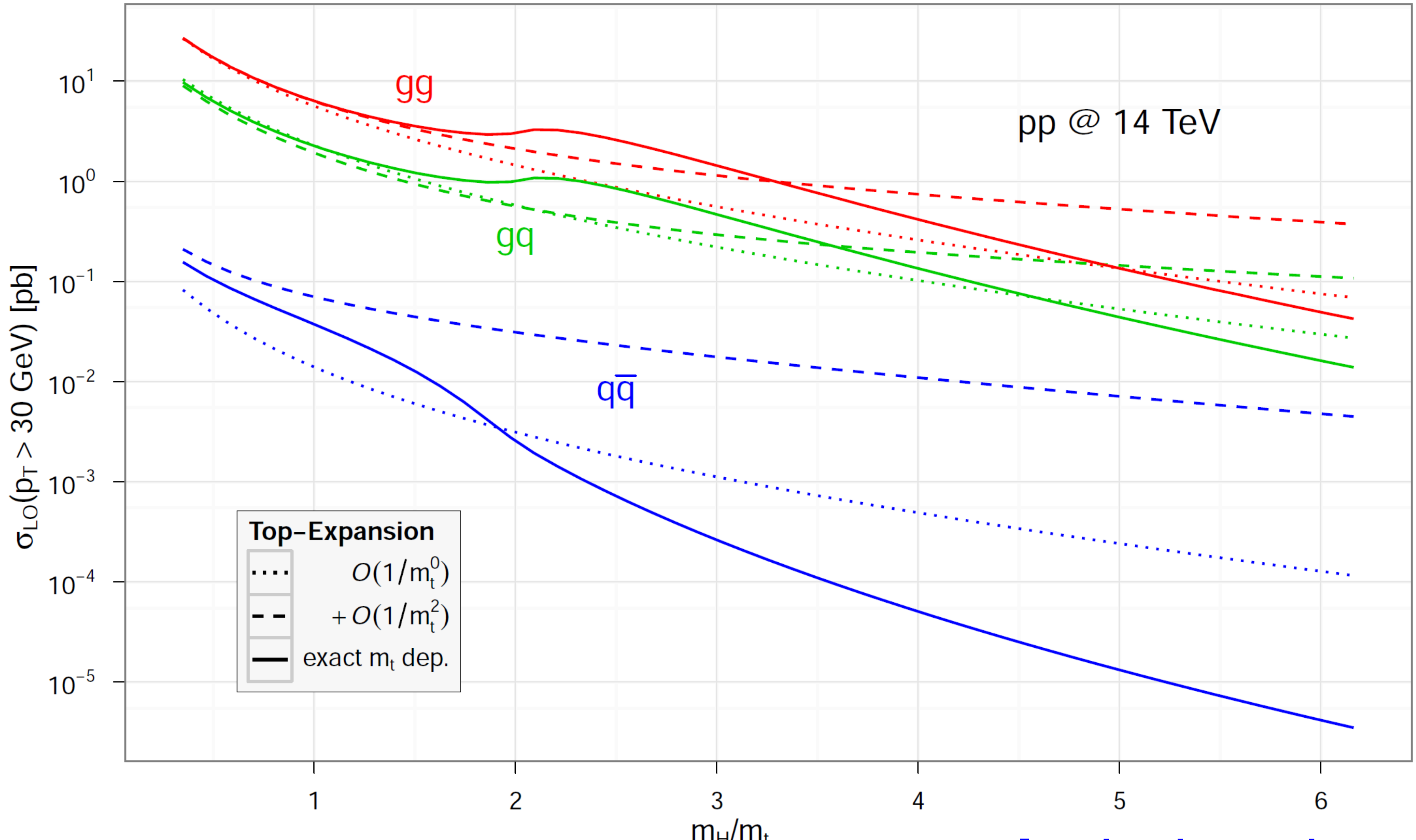
Theoretical uncertainties in this channel are among the largest errors of Higgs analysis ! **[Heinemeyer et al,2013]**

High transverse Higgs momentum (p_T) distribution has been shown to be very sensitive to new physics effects in many BSM models **[Azatov et al,2013][Grojean et al,2014]....**

Known analytically at NLO only in the infinite top mass limit **[Gehrmann et al,2012]**



Finite top mass effects in H+jet



[Harlander et al, 2012]
[Frederix et al, 2016]

$H \to Z\gamma$

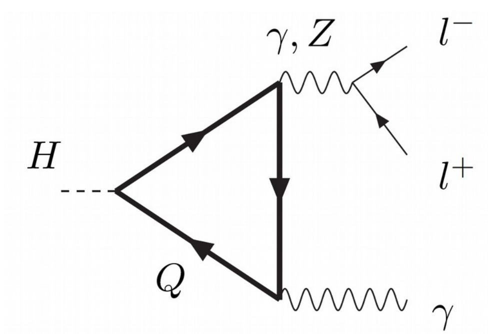
2 Loop QCD , EW light quark corrections with exact top mass

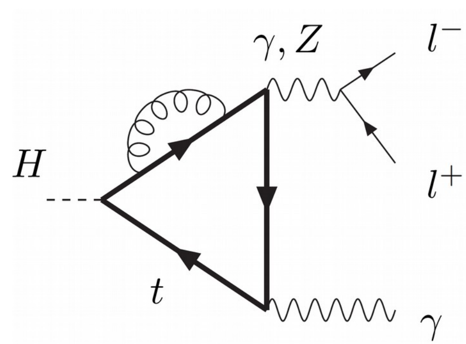
Master integrals are a subset of gg -> Hg

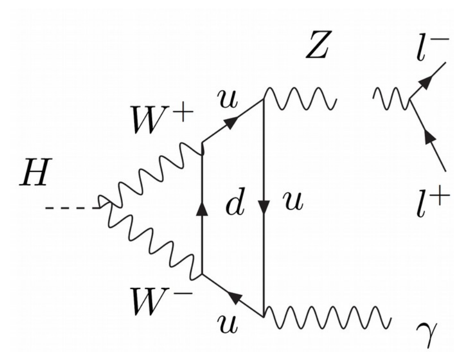
This channel is suitable for precision experiments

Lepton pair disentangled from the QCD background

[CMS coll., 2013]







err ~ 10%

err ~ 5 %

[Denner et al 2011]

Outline of the talk

- **Differential equations**
- **Canonical form**
- **Different representations of the solution**

  Chen iterated integrals

  (DE cannot be directly integrated in terms of GPL, even if not elliptic)

- **Analytic continuation**

  Automatic analytic continuation

Differential Equations (notation)

- **Compute the derivative of (n) Feynman integrals** [Kotikov, 1991] [Remiddi, 1997] [Gehrmann, Remiddi, 2000]

$$\partial_x f(x,\epsilon) = A(x,\epsilon) f(x,\epsilon)$$

- **Each integral satisfies an n-th order DE**
- **Equivalent to a 1-st order n*n system of DE**

Canonical form

- Direct solution is not feasible
- How do we decouple it ?
- An effective approach is decoupling the different eps orders of the solution [Henn, 2013]

$$\partial_x f(x)_i = \epsilon\, A(x) f(x)_{i-1}$$

- Solution in terms of Chen iterated integrals [Chen, 1977] (path encodes boundaries)

$$f_i(x) = \epsilon \int_C dA(x) f_{i-1}$$

Canonical form - algorithms

- Uniform weight Integrals satisfy canonical DE
(Q-linear combinations of transcendental functions of same weight)

- Integrals with constant leading singularities should have uniform weight (Maximal cut)
[Arkani-Hamed, Bourjaily, Cachzao, Trnka, 2010] [Arkani-Hamed et al, 2012]

- Univariate problems in Fuchsian form (algorithmic)
[Lee, 2014]

- Linear DE (algorithmic)[Argeri et al, 2014]

$$\partial_x f(x) = (A_0(x) + \epsilon\, A_1(x))f(x)$$

- Block diagonal linear DE [Gehrmann et al, 2014]

Chen integrals, polylogarithms

- N fold integrals

$$f_i(x) = \epsilon \int_C dA(x) f_{i-1}$$

- Solve the DE in terms of Goncharov polylogarithms: numeric evaluation [Vollinga et al., 2005], Hopf algebra structure [Goncharov, 2005] (only if DE are in Fuchsian form)

constant matrices

$$\partial_x f(x)_i = \epsilon \sum_k \frac{a_k}{x - c_k} f(x)_{i-1} \qquad f_i = \epsilon \sum_k a_k \int_C d\log(x - c_k)\, f_{i-1}$$

independent of x

- Are we always this lucky ? No (of course)

Higgs -> Zeta Gamma

- D-log (trivial algebraic functions)

$$\sqrt{x(x-4)}$$

- Landau variable makes Fuchsian form manifest (Goncharov Polylogarithms)

$$x(y) = -\frac{(1-y)^2}{y}$$

Higgs + jet

- **6 independent square roots:**

$$\sqrt{s(s-4m^2)},\quad \sqrt{m^4\left(p_4^2-s\right)^2+2m^2st\left(p_4^2-s\right)+st^2\left(s-4m^2\right)},$$
$$\sqrt{t(t-4m^2)},\quad \sqrt{st\left(4m^2p_4^2-4m^2(s+t)+st\right)},$$
$$\sqrt{p_4^2(p_4^2-4m^2)},\quad \sqrt{\left(p_4^2-s\right)\left(-4m^2+p_4^2-s\right)}.$$

- **"No change of variables removes them" !**
- **No Fuchsian form**
- **We cannot integrate DE in terms of Goncharov polylogarithms**

From DE to Symbols

- **Thanks to the very peculiar form of DE we are still able to find a solution in terms of polylogarithms**

DE in d-log form

$$df(x)_i = \epsilon \sum_k a_k \, d\log(\beta_k(x)) \, f(x)_{i-1}$$

constant matrices → a_k logarithms (alphabet) → $\beta_k(x)$

"Chen integral solution = Symbol" !

$$f_i = \epsilon \sum_k a_k \int_C d\log(\beta_k(x)) \, f_{i-1}$$

$$S(f_i) = \epsilon \sum_k a_k \, \beta_k(x) \otimes f_{i-1}$$

Higgs + jet (polylogarithms)

- **We can use algorithms to find the corresponding polylogarithmic functions [Goncharov et al. 1998-2010][Duhr et al 2011]**

$$S(f_i) \to \bar{f}_i$$

- **We are still missing terms in the kernel of the symbol**

- **We can extract the information from DE**

free coefficients — lower w. func.

$$\partial_x f(x)_i = \epsilon\, dA(x) f(x)_{i-1} \;\to\; f_i = \bar{f}_i + \sum_k c_k g_k$$

- **Pure constants are fixed imposing boundary conditions**

$$f_i = \bar{f}_i + \textstyle\sum_k c_k g_k + b$$

pure constant

Higgs + jet (polylogarithms)

- The alphabet of H+jet is large (~ 50 letters)
- Finding the function matching a given symbol amounts to constructing a general ansatz (polylogarithms arguments)

[Duhr et al 2011]

$$\text{Li}_n(a),\ \text{Li}_{22}(a,b) \quad a,b = \prod_i \beta_i^{m_i} \quad (m_i \in \mathbb{N})$$

DE logarithms arguments (alphabet)

$$1-a, 1-b = \prod_i \beta_i^{m'_i},\ 1-ab = \prod_i \beta_i^{m'_i}$$

arguments factorize over the alphabet

- The combinatorics is very large, solving for the coefficients of the ansatz can be challenging!

Higgs + jet (PL+Chen integrals)

- **Up to weight 2 solution in terms of polylogarithms**

- **Weight 3 and 4 are 1 fold integrals (Chen integrals), they can be computed numerically in many frameworks (e.g. Mathematica, Cquad)**

$$f_3 = \sum_k \beta_k \int d\log(b_k) f_2$$

$$f_4 = \sum_{k'} \alpha'_k \int d\log(a'_k) \sum_k \beta_k \int d\log(b_k) f_2$$

Analytic continuation

- We compute the integrals in the euclidean region (s_ij<0)
- The physical region is reached by analytic continuation (Feynman prescription)
- The analytic continuation is in general complicated because of spurious branch cuts of the functional basis (symbol blinds to cuts of individual functions) [Goncharov et al., 2010]

rational numbers

$$\mathcal{I} \stackrel{?}{=} \sum_k c_k \mathcal{B}_k$$

Feynman Integral

(pure) basis functions

basis has branch cuts in the physical region (identity might not hold everywhere because of "spurious" branch cuts)

- We can solve the differential equations directly in the physical region (effectively circumventing the difficult task of analytic continuation!)

Solving in the physical region

- Physical Boundary conditions
- Analytic basis functions

$$\mathcal{I} = \sum_k c_k \mathcal{A}_k$$

analytic basis in the physical region (no spurious branch cuts)

- Identity holds in the entire physical region

Boundary conditions

- It is natural to expand wrt the (large) top mass
- When the external invariants approach 0, the expansion is exact (boundary point s_ij=0)
- Large mass expansion

For a given scalar graph depending on a set of momenta q and small masses m, the expansion in the large mass M limit is given by [V. Smirnov]

$$F_\Gamma(q, M, m, \epsilon) \overset{M\to\infty}{\sim} \sum_\gamma F_{\Gamma/\gamma}(q, m, \epsilon)\, T_{q^\gamma, m^\gamma} F_\gamma(q, M, m, \epsilon)$$

- The boundary point s_ij=0 is both physical and euclidean !

Analytic functions

- Basis (freedom to rotate to a preferred basis)

$$\mathrm{Li}_n(x) = -G(0_{n-1}, 1, x), \quad Li_{2,2}(x,y) = G\left(0, \frac{1}{y}, 0, \frac{1}{xy}, 1\right)$$

- Choose polylogarithms with no branch cuts in the physical region (choose proper arguments)
- Li_n(x), x<1
- Li_22(x,y), y<1 and x y <1
- Caveat, in principle functions with these requirements may not be a basis [Bonciani, Del Duca, Frellesvig, Henn, FM, Smirnov, 2015]

Analytic functions

- Analytic continuation of weight 3 and 4 is elementary

- Resulting Chen integrals requires just the continuation of logarithms and algebraic functions

$$f_4 = \sum_{k'} \alpha'_k \int d\log(a'_k) \sum_k \beta_k \int d\log(b_k) f_2$$

analytic in the entire physical region (by construction)

Summary and outlook

- New results for integrals of gg->Hg (only 4 integrals missing)
- The integrals can be computed using a basis of UT integrals
- The solution is represented in terms of a minimal polylogs basis (w=1,w=2), and 1 fold integrals (w=3,w=4)

- The numeric evaluation is still quite slow, O(min)/(physical point), implementing it in c (Cquad) will significantly improve the evaluation speed
- Solution in terms of polylogarithms up to w4 (combinatorics!)
- Integrate the DE in terms of GPL. Introducing a (non-physical) parameter such that the DE wrt it are in Fuchsian form. [Papadopulous 2014]