

# PRECISION MEASUREMENT OF $\sin^2 \theta_w$ AT MESA

Loops & Legs 2016

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Electron proton / ion scattering at low and high  $Q^2$

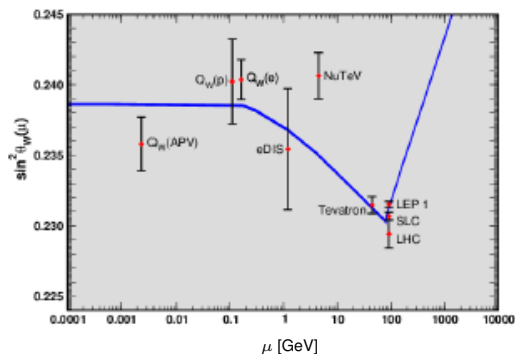
Goals:

- High-precision measurements of the nucleon structure
- Search for new physics
- Electroweak physics:
  - Test of the standard model, and after LHC discoveries
  - Test of the standard model extended with new physics
- Key parameter of the standard model:  $\sin^2 \theta_w$

This talk:

- The MESA project at Mainz University
- Theory for precision measurements of  $\sin^2 \theta_w$  in low-energy elastic  $ep$  scattering

# The Running Weak Mixing Angle: Present Status



## PDG 2014

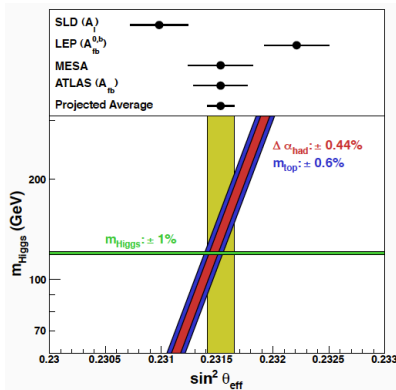
- $Q_W(APV)$ : atomic parity violation (Cs)
- $Q_W(p)$ :  $Q_{Weak}$
- $Q_W(e)$ : Moller scattering
- NuTeV: Neutrino scattering (re-analysis needed)

## Z-pole measurements:

- LEP1 and SLC
- Tevatron
- LHC: CMS and ATLAS

- Most precise single measurements disagree ( $3\sigma$ )
- ➔ very different implications for new physics

## Standard Model Relation: Higgs Boson Mass versus $\sin^2 \theta_W$




Combination of precision measurements at the Z-pole

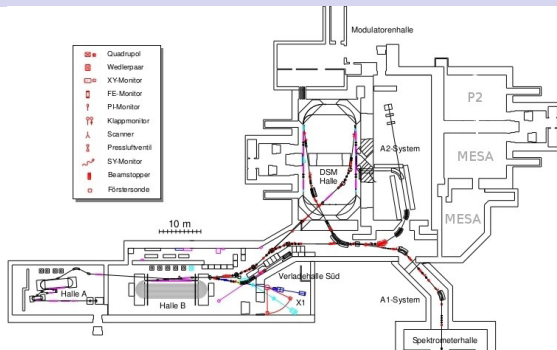
→  $M_{Higgs} - \sin^2 \hat{\theta}_W(\mu)$  SM relation (red-blue band)

Precision measurement of  $\sin^2 \hat{\theta}_W(\mu)$  has provided indirect evidence for the allowed range of  $M_{Higgs}$

Combination of measurements provide strong tests of the SM, ... and maybe evidence for new physics

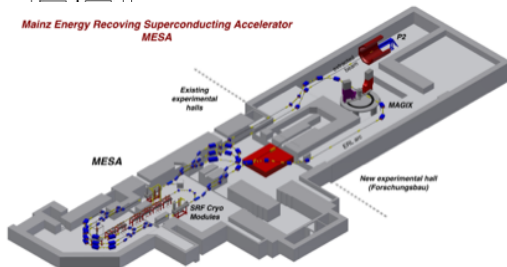
- **MESA =**  
**Mainz Energy-recovering Superconducting Accelerator**  
A small superconducting accelerator for particle and nuclear physics
- Funded by PRISMA - Cluster of Excellence and Collaborative Research Center 1044  
German Science Foundation (DFG)  
 PRISMA  
SFB  
THE COLLABORATIVE PROJECT  
OF THE STANDARD MODEL
- **P2** (Project Precision 2):  
Parity-violating electron proton scattering
- Other Projects: Search for a dark photon,  
Nuclear physics program, proton radius
- Commissioning planned for 2019
- Main competition from Qweak at JLAB

# MESA Layout



MAMI (Mainzer Mikrotron)  
and MESA

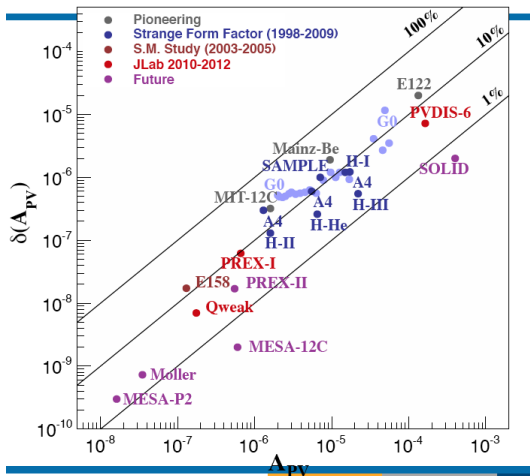
Mainz Energy Recovering Superconducting Accelerator  
MESA



# MESA Parameters

Parameter	P2 (Mainz)
Beam energy	155 MeV
Central scattering angle $\theta_e$	35°
Detector acceptance for $\theta_e$	25° to 45°
Azimuthal detector acceptance $\phi_e$	$2\pi$
Central $Q^2$	$\simeq 0.0071 \text{ (GeV/c)}^2$
Averaged $Q^2$	$\simeq 0.0045 \text{ (GeV/c)}^2$
Polarization	$(85 \pm 0.5) \%$
Beam current	$150 \mu\text{A}$
$\ell$ H <sub>2</sub> Target length	60 cm
$A_{exp}$	-28.35 ppb
$\Delta A(G_A)$ (0.4 %)	$\pm 0.11$ ppb
$\Delta A(\gamma Z \text{ box})$ (0.3 %)	$\pm 0.09$ ppb
$\Delta A_{stat}$ (1.3 %)	$\pm 0.38$ ppb
$\Delta A_{sys}$ (0.6 %)	$\pm 0.17$ ppb
$\Delta A_{tot}$ (1.5 %)	$\pm 0.44$ ppb
$\sin^2 \theta_W$	0.238
$\Delta \sin^2 \theta_W$	$3.1 \cdot 10^{-4}$
$\Delta \sin^2 \theta_W / \sin^2 \theta_W$	0.13 %

## PVeS Experiment Summary



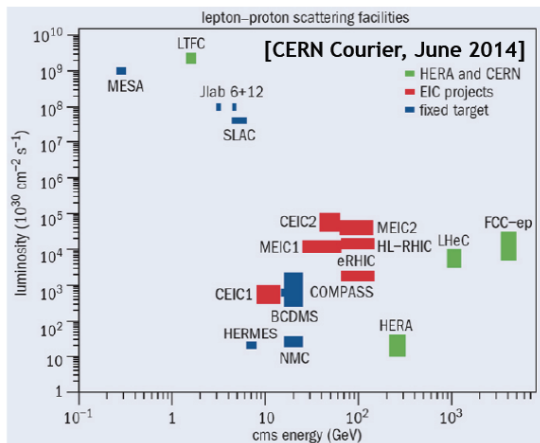


## LHeC / FCC-he Context

(10,000 h)  
data taking



$$\int \mathcal{L} dt \simeq 8.6 \text{ ab}^{-1}$$



P. Newman

Measure the (tiny) difference between cross sections for electrons with positive and negative helicity to filter out the weak interaction

$$A_{LR} = \frac{\sigma(e_{\downarrow}) - \sigma(e_{\uparrow})}{\sigma(e_{\downarrow}) + \sigma(e_{\uparrow})} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \left( Q_W(\mathcal{N}) - F(Q^2) \right)$$

Weak charge of the proton (SM at LO):

$$Q_W(p) = 1 - 4 \sin^2 \theta_W$$

$$\frac{\Delta \sin^2 \theta_W}{\sin^2 \theta_W} = \frac{1 - 4 \sin^2 \theta_W}{4 \sin^2 \theta_W} \frac{\Delta Q_W(p)}{Q_W(p)}$$

1.5 % precision for  $Q_W(p)$  corresponds to 0.13 % precision for  $\sin^2 \theta_W$

Measurement errors from: statistics, polarization ( $A_{exp} = P_e A_{LR}$ ),  
systematic effects and required hadronic physics: form factors

Higher-order corrections

$$F(Q^2) = F_{\text{EMFF}}(Q^2) + F_{\text{Axial}}(Q^2) + F_{\text{Strangeness}}(Q^2)$$

$$A_{LR} = A_{Q_{\text{weak}}} + A_{\text{EMFF}} + A_{\text{Axial}} + A_{\text{Strangeness}}$$

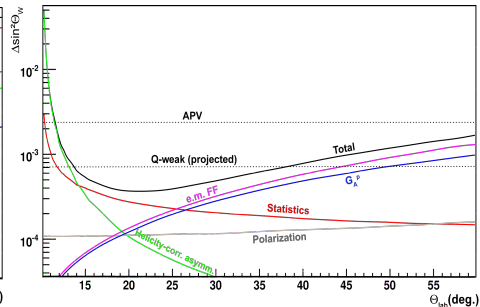
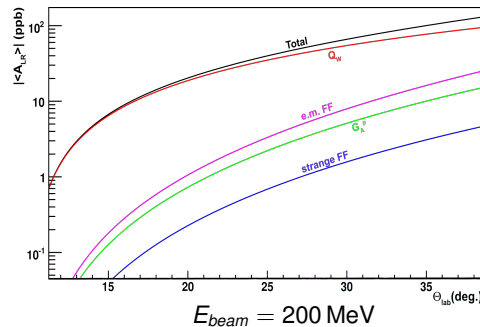
$$F_{\text{EMFF}}(Q^2) = -\frac{\epsilon G_E^p G_E^n + \tau G_M^p G_M^n}{\epsilon (G_E^p)^2 + \tau (G_M^p)^2},$$

$$F_{\text{Axial}}(Q^2) = -\frac{(1 - 4 \sin^2 \theta_W) \sqrt{1 - \epsilon^2} \sqrt{\tau(1 + \tau)} G_M^p G_A^p}{\epsilon (G_E^p)^2 + \tau (G_M^p)^2},$$

$$F_{\text{Strangeness}}(Q^2) = -\frac{\epsilon G_E^p G_E^s + \tau G_M^p G_M^s}{\epsilon (G_E^p)^2 + \tau (G_M^p)^2} - \frac{\epsilon G_E^p G_E^{ud} + \tau G_M^p G_M^{ud}}{\epsilon (G_E^p)^2 + \tau (G_M^p)^2},$$

$$\epsilon = [1 + 2(1 + \tau) \tan^2(\theta/2)]^{-1}, \quad \tau = Q^2/4m_p^2$$

# Contributions to the PV Asymmetry and Expected Error



→ Optimal measurement for  $E = 155 \text{ MeV}$ ,  $\theta_e = 35^\circ \pm 10^\circ$

$$\langle Q \rangle = 0.067 \text{ GeV}$$

SM prediction:  $A_{LR} = -2.8 \times 10^{-8}$ , precision goal: 1.5 %

$$\Delta \sin^2 \theta_W = \pm 0.00031, \text{ i.e. } 0.13 \%$$

Polarization asymmetry including higher-order corrections:

$$A_{LR} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \left( Q_W(\mathcal{N})(1 + \delta_1) - \tilde{F}(Q^2) \right)$$

$$Q_W(\mathcal{N})(1 + \delta_1) = (\rho_{\text{NC}} + \Delta_e) (1 - 4\kappa \sin^2 \theta_W + \Delta'_e) + \delta_{\text{Box}}$$

Universal corrections:  $\rho_{\text{NC}}$  and  $\kappa$  from loop diagrams:

scale-dependence ( $\mu \rightarrow Q$ )  $\rightarrow$

$$\sin^2 \theta_{\text{eff}}(\mu^2) = \kappa(\mu^2) \sin^2 \theta_W$$

and scheme-dependent

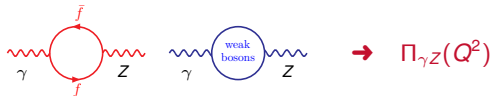
$\delta_{\text{Box}}$  from box graphs for  $WW$ ,  $ZZ$ ,  $Z\gamma$  exchange,

non-universal vertex correction  $\Delta_e$ ,  $\Delta'_e$ , QED corrections

Match complete 1-loop corrections to the definition of  $\sin^2 \theta_W$

# One-loop Corrections: Scale-Dependent $\sin^2 \theta_W$

- Photon-Z mixing



can be absorbed into **effective**, running, **scale-dependent weak mixing angle**

Definitions of the weak mixing angle,  $\kappa(Q^2) \sin^2 \theta_W$ :

- On-shell definition:  $\cos \theta_W = \frac{m_W}{m_Z}$  (fixed)
- $\sin^2 \theta_{\text{eff}}(Q^2)$**  absorbs  $\Pi_{\gamma Z}(Q^2)$ , usually together with parts of vertex corrections (e.g. Czarnecki, Marciano for Moller scattering)
- $\overline{\text{MS}}$  scheme:  $\sin \hat{\theta}_W(\mu)$**  (via  $\tan \hat{\theta}_W(\mu) = g_1/g_2$  and RGE)  
less sensitive to  $m_{\text{top}}$ , suited for comparisons with extensions of the SM

→ Relation

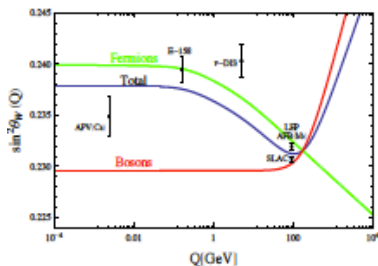
$$\sin^2 \hat{\theta}_W(\mu = M_Z) = \left( 1 + \frac{\rho_t}{\tan^2 \theta_W} + \dots \right) \sin^2 \theta_W$$

$$\text{with } \rho_t = 3G_\mu m_{\text{top}}^2 / 8\sqrt{2}\pi^2 = 0.00939 (m_{\text{top}}/173 \text{ GeV})^2$$

→ Additional uncertainty:  $\Delta \sin^2 \hat{\theta}_W \simeq \pm 0.0006$  for a 1 % error on  $m_{\text{top}}$

## $\sin^2 \hat{\theta}_W(Q)$ : Uncertainties

- Use running  $\sin^2 \hat{\theta}_W(Q)$  to compare different measurements
- Match definition of  $\sin^2 \hat{\theta}_W(Q)$  with the complete 1-loop corrections



Czarnecki-Marciano, Jegerlehner

- Scheme dependence, partly compensated by  $\delta_{non-universal}$
- Include higher orders  
→ 2-loop corrections
- Parameter dependence?  $m_t, m_H, \dots$
- Hadronic contribution!  
need data and models (e.g. ChPT)

Prescriptions are known, with small uncertainties . . .

# Higher-Order Corrections: Hadronic Contributions

$\Pi^{\gamma\gamma}$  and  $\Pi^{\gamma Z}$  are sensitive to low-scale hadronic physics

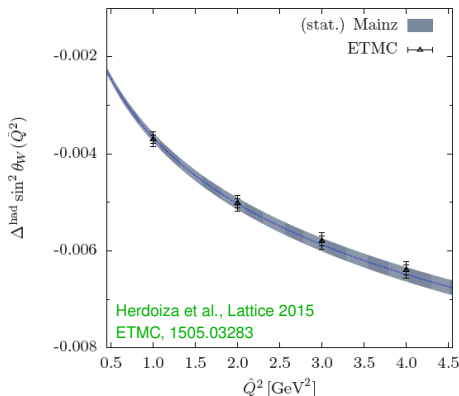
→ Use dispersion relation, e.g.

$$\Delta\alpha(q^2) = \frac{q^2}{12\pi^2} \int_{4m_\pi^2}^{\infty} \frac{ds}{s} \frac{R^{\gamma\gamma}(s)}{s - q^2} \quad \text{with} \quad R^{\gamma\gamma}(s) = \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})}{4\pi\alpha^2/3s}$$

→ Similar approach for  $\Pi^{\gamma Z}$  requires data for  $\sigma_{\text{tot}}(\nu\bar{\nu} \rightarrow \text{hadrons})$  or use flavor-separated  $e^+e^-$  data, isospin symmetry and OZI-rule

→ Use lattice techniques

First results available,  
errors start to be competitive





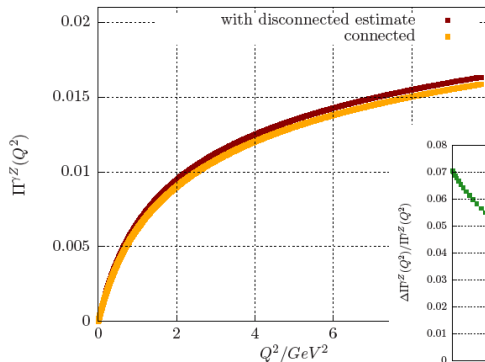
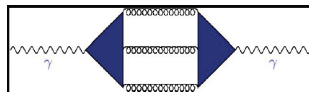
# Higher-Order Corrections: Hadronic Contributions

→ OZI-rule violating contributions?

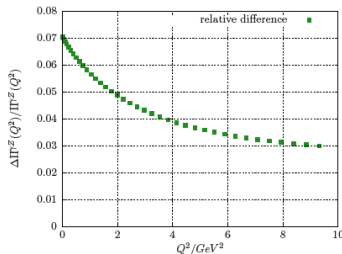
Lattice QCD

still with large statistical errors

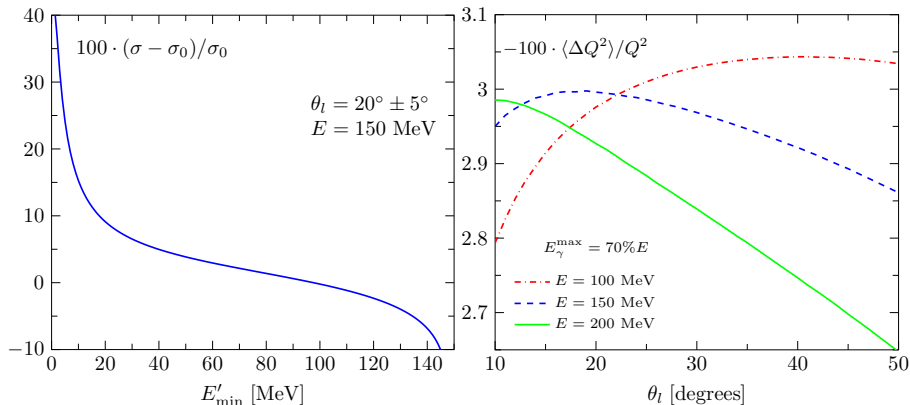
but obtain an upper limit



Gülpers et al., Lattice 2015

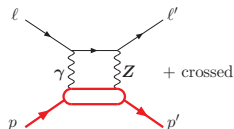


## Higher-Order Corrections: QED



- QED does not violate parity symmetry, but real photon emission leads to a shift of  $Q^2$
- Straightforward to calculate, but need flexible MC simulation

## Higher-Order Corrections: Box Graphs

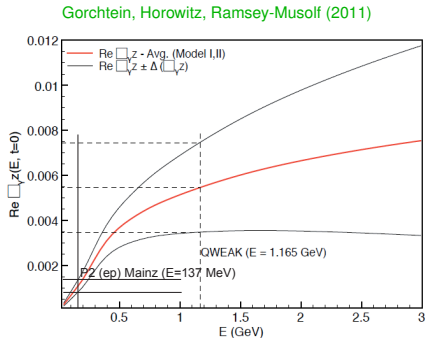


## $\gamma Z$ box graphs

Sensitivity to hadronic physics at low  $Q^2 \rightarrow$  an important source of error

Status  $\sim$  3 years ago:  
3 groups with independent analyses  
agree in size, but disagree on errors

Hall et al.; Carlson and Rislow;  
Gorchtein et al.



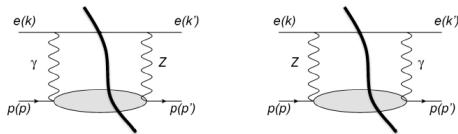
For Qweak at JLAB,  $E = 1.165$  GeV:  $7\sigma(\text{theory})$  effect

Advantage at P2@MESA: low energy  $E = 0.155$  GeV

$$\Delta A_{LR}^{box} / A_{LR} = \pm 0.4\%$$

More work needed to reduce the error ...

# Box Graphs: Dispersion Relations



Optical theorem and dispersion relations:

$$\text{Im}\square_{\gamma Z}(E) = \frac{\alpha}{(s - M_Z^2)^2} \int_{W_\pi^2}^s dW^2 \int_0^{Q_{\max}^2} dQ^2 \frac{M_Z^2}{Q^2 + M_Z^2} \\ \times \left\{ F_1^{\gamma Z}(x, Q^2) + A F_2^{\gamma Z}(x, Q^2) + \frac{g_V^e}{g_A^e} B F_3^{\gamma Z} \right\}$$

Separated into vector and axial-vector parts of the proton current:

$$\text{Re}\square_{\gamma Z}^V(E) = \frac{2E}{\pi} \int_{\nu_\pi}^\infty \frac{dE'}{E'^2 - E^2} \text{Im}\square_{\gamma Z}^V(E'), \quad \text{Re}\square_{\gamma Z}^V(E=0) = 0 \\ \text{Re}\square_{\gamma Z}^A(E) = \frac{2}{\pi} \int_{\nu_\pi}^\infty \frac{E' dE'}{E'^2 - E^2} \text{Im}\square_{\gamma Z}^A(E'), \quad \text{Re}\square_{\gamma Z}^A(E=0) \neq 0$$

## Box Graphs: Sensitivity to Hadronic Input

Different calculations due to different assumptions about the structure function input (regions, parametrizations)

$Q^2_{\max} \backslash W_{\max}$	2 GeV	4 GeV	$\infty$
$\infty$	1%	1%	3%
2 GeV <sup>2</sup>	3%	2%	1%
1 GeV <sup>2</sup>	76%	10%	2%

Contributions to  $\text{Re}\Box_{\gamma Z}^V$

$$\text{Re}\Box_{\gamma Z}^V = 0.00107$$

from recent 2015 update:

M. Gorchtein, HS, X. Zhang

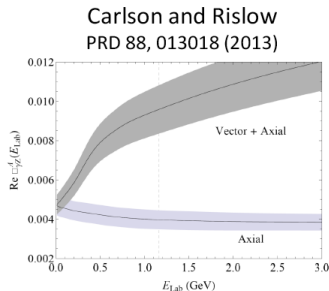
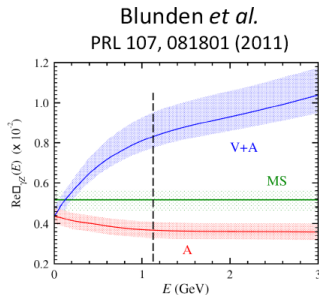
$Q^2_{\max} \backslash W_{\max}$	2 GeV	4 GeV	$\infty$
$\infty$	0.5%	0.5%	4.6%
2 GeV <sup>2</sup>	6.7%	6.7%	0.5%
1 GeV <sup>2</sup>	60%	20%	0.5%

Contributions to the uncertainty of  $\text{Re}\Box_{\gamma Z}^V$

$$\Delta\text{Re}\Box_{\gamma Z}^V = 0.00018$$

$$(Q_W(p) = 0.0712 \pm 0.0007, \\ E = 150 \text{ MeV}, \theta_e = 0)$$

## Axial Box Calculations



$\text{Re}\Omega_{\gamma Z}^A(E = 1.165 \text{ GeV})$	
$(3.7 \pm 0.4) \times 10^{-3}$	$(4.0 \pm 0.5) \times 10^{-3}$

And more:  $\gamma\gamma$ -box graph corrections with parity violation in the proton:  
**Anapole moment**, vanishes at  $E = 0$ ,  $\Omega_{\gamma\gamma}^{PV} \sim \frac{\alpha}{\pi} EQ^2$  (M. Gorchtein, HS, to appear)

Box graphs are energy dependent

→ Formal definition of the nucleon's weak charge

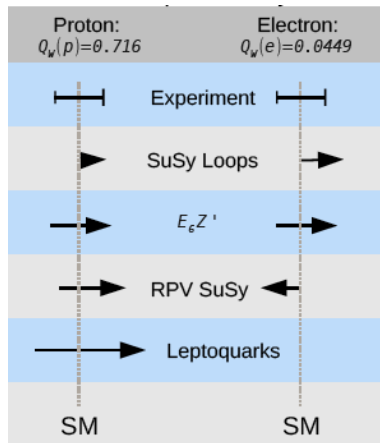
Conventional (old) formulation (assuming  $F(Q^2) = -Q^2 B(Q^2)$ ):

$$A_{exp} = A_0 \left( Q_W(\mathcal{N}) + Q^2 B(Q^2) \right) \quad \rightarrow \quad Q_W(\mathcal{N}) = \lim_{Q^2 \rightarrow 0} \frac{A_{exp}}{A_0}$$

New: including  $E$ -dependent box-graph corrections:

$$A_{exp} = A_0 \left( Q_W(\mathcal{N}) + Q^2 B(Q^2) + \square(E) \right) \quad \rightarrow \quad Q_W(\mathcal{N}) = \lim_{Q^2, E \rightarrow 0} \frac{A_{exp}}{A_0}$$

# Beyond the Standard Model: New Physics in $Q_W(p)$



Searches based on

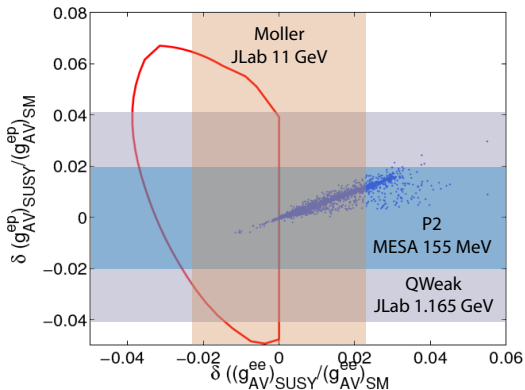
- model-dependent analyses
- effective quark couplings
- contact interactions

Characteristic shifts of  $Q_W$  predicted by extensions of the Standard Model

**Complementarity** between elastic  $ep$  ( $Q_W(p)$ ) and Moller ( $Q_W(e)$ ) scattering



# Supersymmetric Models and the Weak Charge



Erlar, Su, 2013

Example: supersymmetric models with and without  $R$ -parity violation

Also precision measurements at low-energy are sensitive to TeV-scale physics

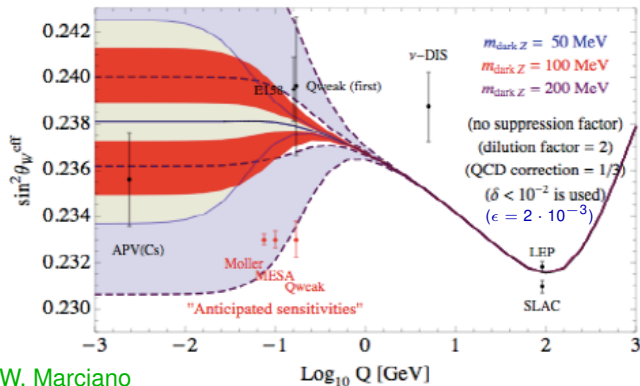
Perspective will shift after LHeC discoveries

# Dark Photon and $\sin^2 \hat{\theta}_W(Q)$

Dark matter and extra  $U(1)$  symmetry with kinetic mixing (B. Holdom)

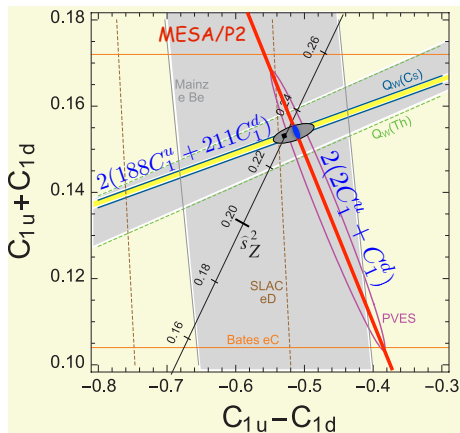
→ Dark photon with very small mass:  $m_D = O(50)$  MeV

- Free mixing parameter  $\epsilon = O(10^{-3})$ , possibly generated by loop effects
- Model with **parity violation**, like ordinary  $Z$ , but suppressed by  $\epsilon Z$
- Combine kinetic and mass mixing → shift  $\Delta \sin^2 \theta_w(Q^2) \simeq 0.42 \epsilon \frac{\delta m_Z^2}{Q^2 + m_D^2}$



W. Marciano

# Effective Low-Energy Couplings



Conventionally used at low-energy:  
Effective 4-fermion interaction

$$C_{1q}: 2a_e \otimes v_q, C_{2q}: 2v_e \otimes a_q$$

Low-energy experiments probe

$$C_{1q} = -I_q^{(3)} + 2Q_q \sin^2 \theta_W$$

i.e., quark vector couplings

Parity-violating electron scattering:  
PVES at JLAB and MAMI  
and at MESA (red)

Atomic PV (Cs, yellow)

SM prediction (black square)

$$\mathcal{L}_{eq} = \left( \frac{G_F}{\sqrt{s}} g_{VA}^{eq}(SM) + \frac{g^2}{\Lambda^2} \right) \bar{e} \gamma_\mu e \bar{q} \gamma^\mu q$$

Convention:  $g^2 = 4\pi$

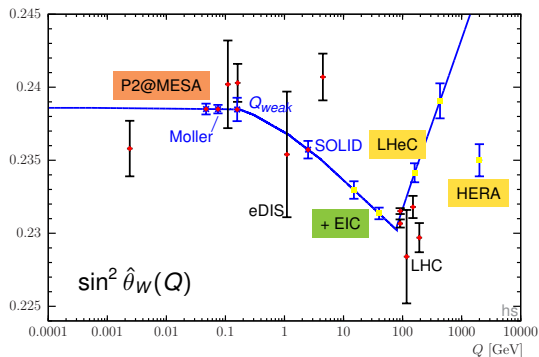
P2@MESA probes  
 $\Lambda$  up to  $\simeq 50$  TeV

comparable with  
 LHC ( $300 \text{ fb}^{-1}$ )

	precision	$\Delta \sin^2 \bar{\theta}_W(0)$	$\Lambda_{\text{new}}$ (expected)
APV Cs	0.58 %	0.0019	32.3 TeV
E158	14 %	0.0013	17.0 TeV
Qweak I	19 %	0.0030	17.0 TeV
Qweak final	4.5 %	0.0008	33 TeV
PVDIS	4.5 %	0.0050	7.6 TeV
SoLID	0.6 %	0.00057	22 TeV
MOLLER	2.3 %	0.00026	39 TeV
P2	2.0 %	0.00036	49 TeV
PVES $^{12}\text{C}$	0.3 %	0.0007	49 TeV

J. Erler

# Scale Dependence of $\sin^2 \hat{\theta}_W(Q)$ : Present and Future



Future additions to the PDG

Expected from low-energy:

- Mainz: P2@MESA
- Moller at JLAB
- $Q_{weak}$  at JLAB
- SOLID at JLAB

HERA estimate  
(see DIS2016)

LHeC: from  $A_{LR}$  and  $\sigma_{NC}/\sigma_{CC}$ ,  
energy range 10 - 1000 GeV

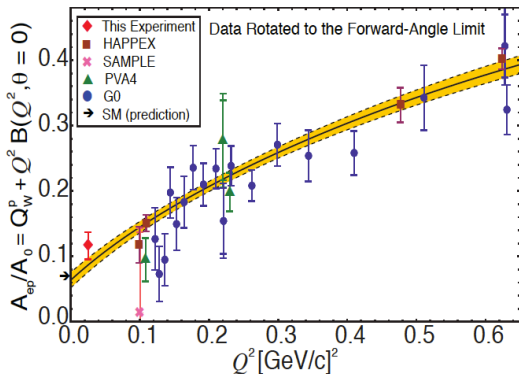
future EIC

- Present measurements of the weak mixing angle need improvement
- MESA, a new high-precision measurement of  $\sin^2 \theta_w$  from parity-violating electron scattering
- Combined with possible future measurements at low energies, EIC, LHeC:
- ➔ Precision measurements will cover a wide range of energy scales
- Uncertainties for PVES
  - short-range, perturbative contributions, 1-loop under control
  - long-range effects in  $\Box_{\gamma Z}^A$  suppressed by  $Q_W(e)$  (main limitation at P2@MESA)
  - long-range effects in  $\Box_{\gamma Z}^V$  suppressed by low  $E$  at P2@MESA
  - form factor corrections  $F(Q^2)$  suppressed by low  $Q^2$
- Still interesting theory work to be completed ...



## Extra Slides

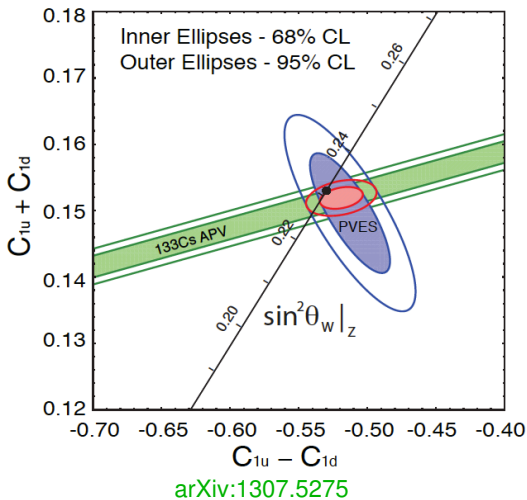




arXiv:1307.5275 - PRL111(2013)

## First results from $Q_{weak}$ at JLAB

- $E = 1.165 \text{ GeV}$ ,  
 $Q^2 = 0.025 \text{ GeV}^2$
- Analyzed 4% of data  
(expect factor 5 improved errors with full data taken)
- Used previous data to extrapolate to  $Q^2 = 0$
- $Q_W^p = 0.064 \pm 0.012$   
( $Q_W^p(SM) = 0.0710 \pm 0.0007$ )



First results from  $Q_{weak}$

- 2-dim fit to quark couplings
- Best fit at  $\sin^2 \theta_w = 0.23116$

## Dark matter and $U(1)$ symmetry $\rightarrow$

- Kinetic mixing  $B_{\mu\nu} D^{\mu\nu}$  (parameter  $\epsilon$ ) (B. Holdom)
- $\rightarrow$  Dark photon, interacts via  $\epsilon e D^\mu J_\mu^{em}$ , like  $e A^\mu J_\mu^{em}$
- Very small mass:  $m_D = O(50)$  MeV
- $\epsilon = O(10^{-3})$ , possibly generated by loop effects
- Negligible effect at the  $Z$  pole
- May provide solution to the  $g - 2$  discrepancy
- Model with parity violation, like ordinary  $Z$ , but suppressed by  $\epsilon Z$   
W. Marciano