Higgs production at N3LO

In collaboration with
Claude Duhr, Falko Dulat, Elisabetta Furlan, Thomas Gehrmann, Franz Herzog, Achilleas Lazopoulos, Bernhard Mistlberger

Babis Anastasiou, ETH Zurich

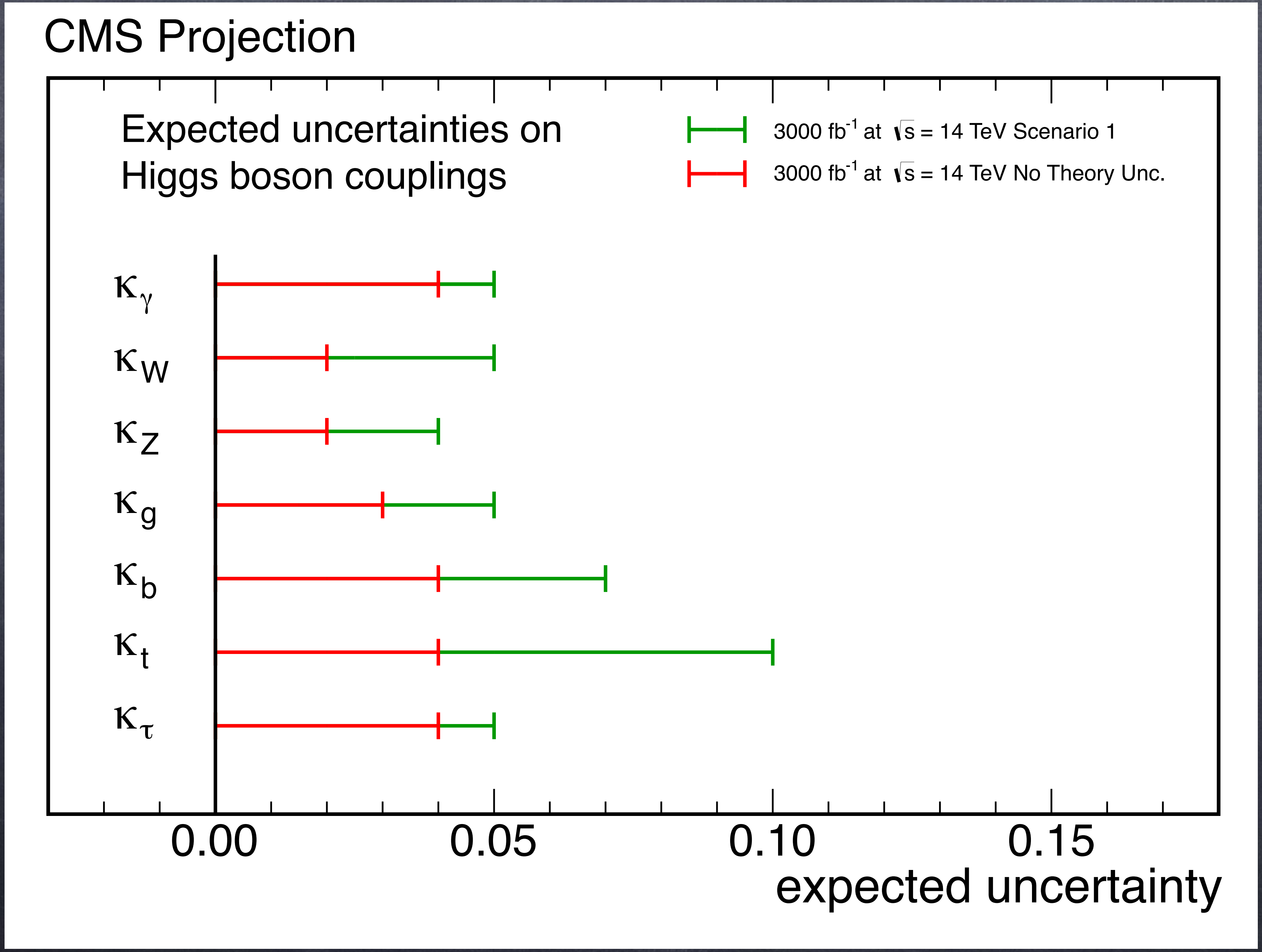
How many Higgs bosons at the LHC?

- Important test of the Standard Model Higgs sector
- Theoretical input needed for Higgs coupling extractions
- Differential measurements per decay channel will be very precise.

Inclusive cross-section: rough, idealised but also crucial...

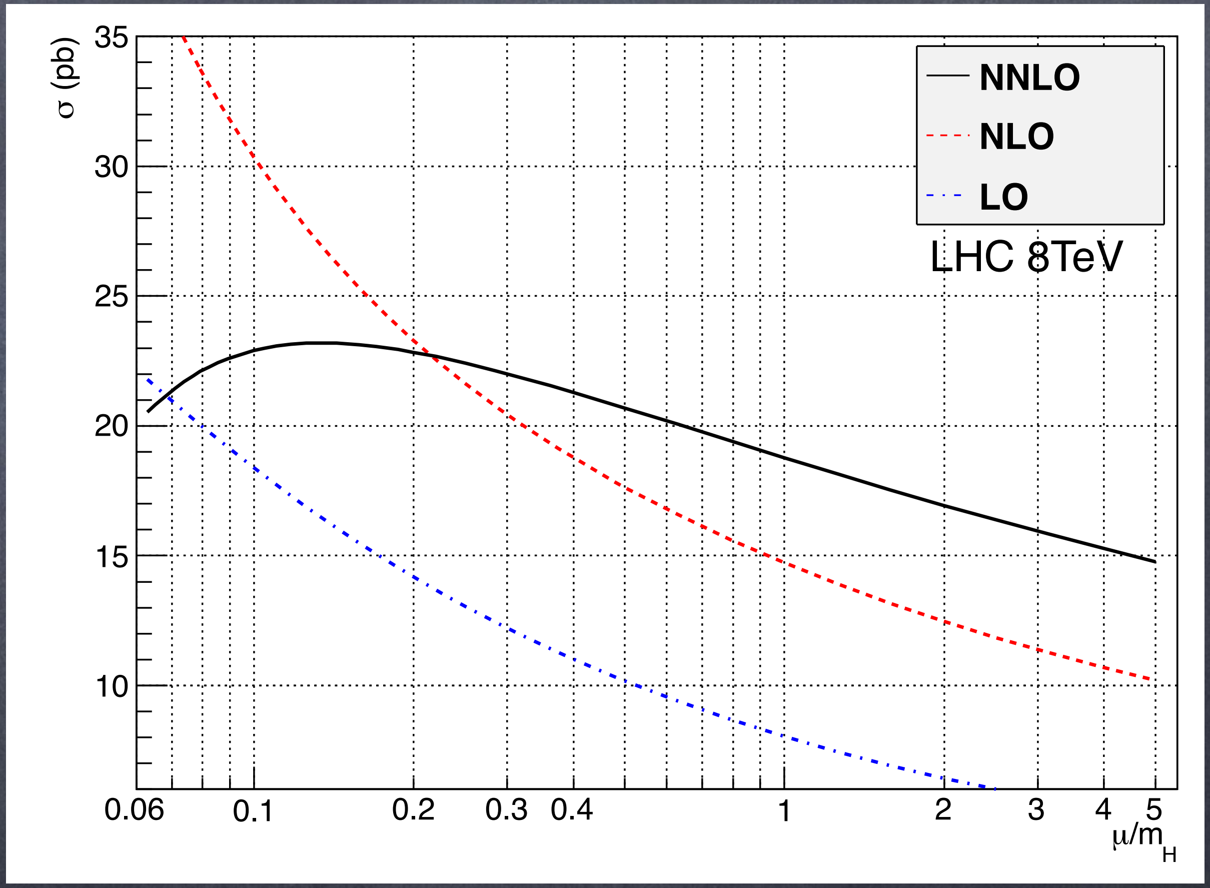
- Does not correspond to a directly measured cross-section...
- but it is the reference number that the experiments extrapolate to
- Normalization for detailed Monte-Carlo simulations
- Traditional first step towards fully differential cross-sections at NLO and NNLO.
- Theoretical laboratory for perturbative QCD

N3LO will have a very important impact in Higgs coupling measurements



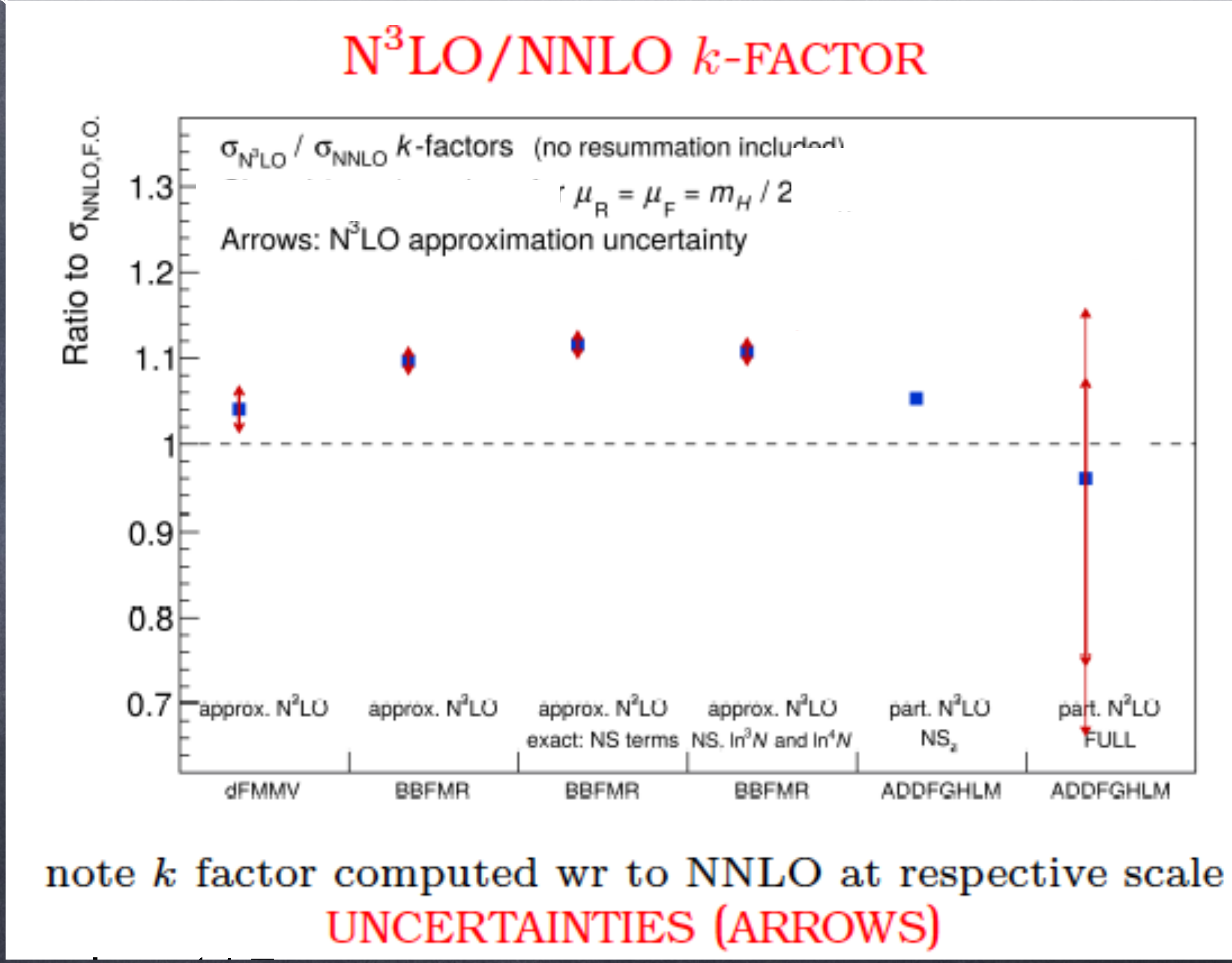
NNLO

- Convergence through NNLO is slow...
- but acceptable with a judicious scale choice (mu=mh/2).
- O(10%) scale uncertainty
- Indications that corrections beyond NNLO are small from some flavours of resummation, but...



...and estimates beyond

- some estimates of beyond NNLO corrections were large.
- N3LO necessary not only to reduce scale variation
- but to also prove the validity of perturbation theory

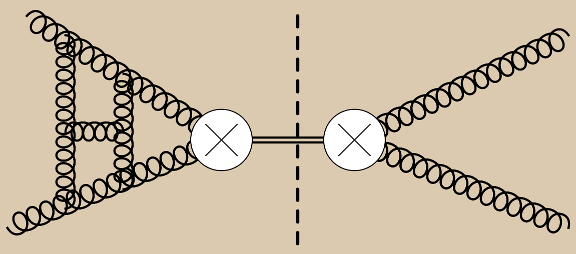


HXSWG-2015

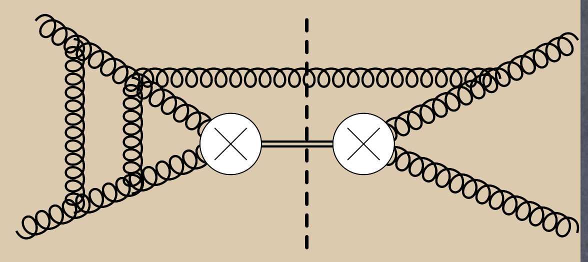
From NNLO to N3LO

- going one order higher in perturbation theory is a big challenge
- NNLO has been a big challenge on its own, not very far in the past...
- ...strategy and division of the problem is crucial!

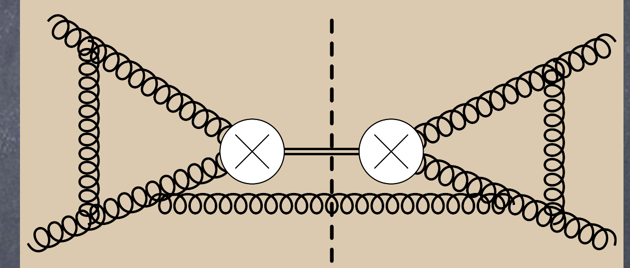
A natural division



VVV



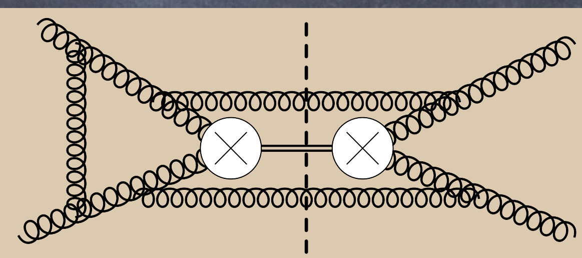
RVV



(RV)x(RV)

$$\frac{P_{gg}^{(1)}}{\epsilon} \otimes \sigma^{\mathrm{NNLO}}(\epsilon)$$

IR+UV



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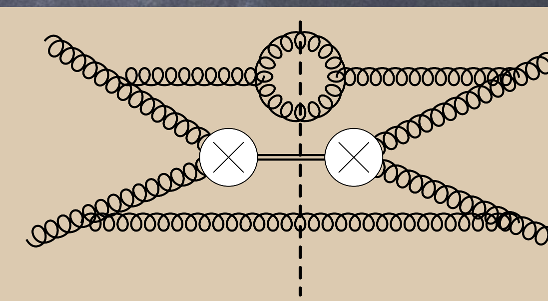

RRR

THE CALCULATION OF THE SECOND ORDER SOFT AND VIRTUAL CONTRIBUTIONS TO THE DRELL–YAN CROSS SECTION

T. MATSUURA, S.C. van der MARCK and W.L. van NEERVEN

Instituut-Lorentz, University of Leiden, P.O.B. 9506, 2300 RA Leiden, The Netherlands

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A COMPLETE CALCULATION OF THE ORDER α_s^2 CORRECTION TO THE DRELL–YAN *K*-FACTOR

R. HAMBERG and W.L. van NEERVEN*

Instituut–Lorentz, University of Leiden, P.O.B. 9506, 2300 RA Leiden, The Netherlands

T. MATSUURA**

II. Institut für Theoretische Physik, Universität Hamburg, D-2000 Hamburg 50, Germany

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(Revised 13 February 1991)

Soft and virtual corrections to proton proton ---> H + x at NNLO
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Detailed record - Cited by 680 records 500+

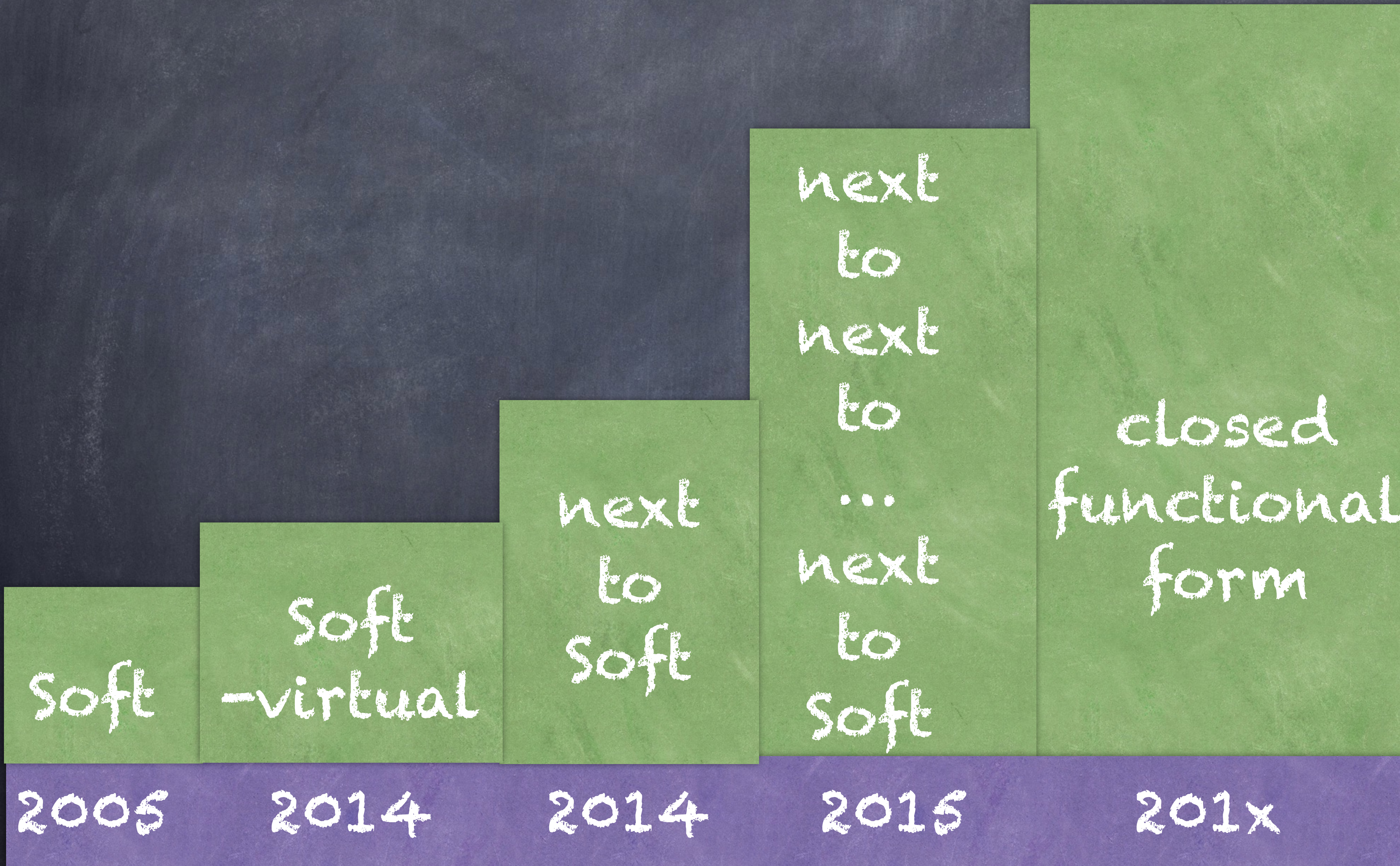
From NNLO to N3LO

- learn from the experience at NNLO and do a "soft expansion" for the partonic cross-sections first

$$\sigma = \sum_n (1-z)^n C_n$$

with $z = \frac{m_H^2}{S}$

From NNLO to N3LO



From NNLO to N3LO

- Wilson coefficient
- Three-loop splitting functions
- Collinear and UV counterterms
- Triple virtual
- Soft expansion for triple real
- Exact (real-virtual)^2
- Exact real-virtual-virtual
- Soft expansion real-real-virtual
- Expansion using the differential equation method
- Exact quark channels
- Exact real-real-virtual

Schroder, Steinhauser
Chetyrkin, Kuhn, Sturm
Moch, Vermaseren, Vogt
Hoeschele, Hoff, Pak, Steinhauser, Ueda
Bueller, Lazopoulos
CA, Buehler, Duhr, Herzog
Baikov, Chetyrkin, Smirnov, Steinhauser
Gehrmann, Glover, Huber, Ikizlerli, Studerus
CA, Duhr, Dulat, Mistlberger
CA, Duhr, Dulat, Furlan, Herzog, Mistlberger
Kilgore
Gehrmann, Jaquier, Glover, Koukoutsakis
Duhr, Gehrmann
Dulat, Mistlberger
Ye Li, Hua Xing Zhu
CA, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger
Ye Li, von Manteuffel, Schwinger, Hua Xing Zhu
CA, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger
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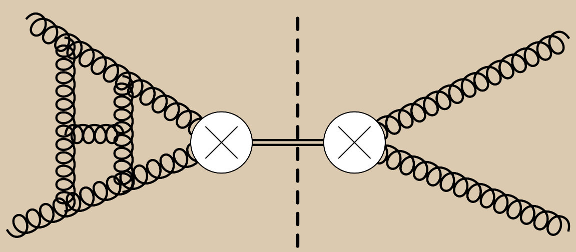
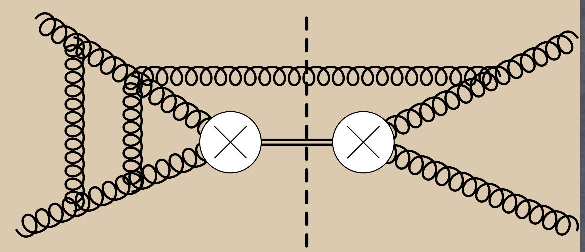
CA, Duhr, Dulat, Herzog, Mistlberger
CA, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger
Anzai, Kasselhuhn, Hoff, Kilgore, Steinhauser, Ueda
Duhr, Dulat, Mistlberger
Mistlberger et al

What is now known for the N3LO correction



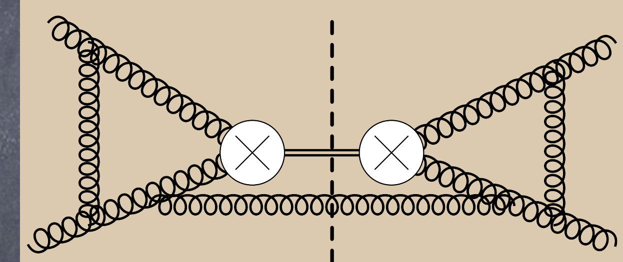
VVV

exact



RVV

exact



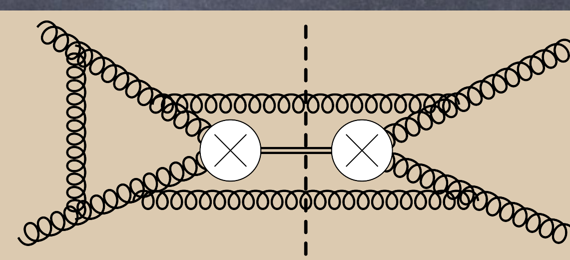
(RV)x(RV)

exact

$$\frac{P_{gg}^{(1)}}{\epsilon} \otimes \sigma^{\mathrm{NNLO}}(\epsilon)$$

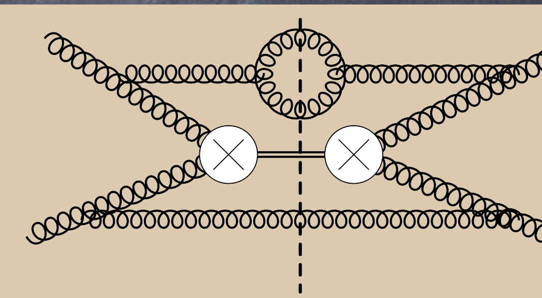
IR+UV

exact



RRV

exact



RRR

expansion

What is now known for the N3LO correction

$$\sigma(z) = \delta(1-z)\, C_\delta + \sum_{\alpha=0}^{5} \log^\alpha(1-z)\, f_\alpha(z)$$

$C_\delta, f_5(z), \ldots, f_1(z)$	$f_0(z)$
Exact	Expansion ~37 terms

Why now?

- Since the NNLO computations in 2002 a lot has changed.
- Could this computation had happened earlier?
- Some techniques and ideas have been present for quite some time now
- but, recent progress in the field of loop computations and new ideas were also crucial.

Old and new

- Reverse unitarity: map phase space integrals on loop integrals with Cutkosky rules:

$$2\pi\,\delta(p_h^2 - m^2) \rightarrow \frac{i}{p_h^2 - m}$$

- expand around the threshold limit:
"Cutkosky rules can be differentiated with respect to masses and kinematic parameters"

$$2\pi\,\delta(p_h^2 - m^2) \rightarrow \frac{i}{p_h^2 - m^2} = \sum_{n=0}^{\infty} \frac{\left[2(p_1+p_2)\cdot p_g\right]^n}{\left[s - m_h^2\right]^{n+1}}$$

Old and new

- Laporta algorithm: Gauss elimination of linearly dependent integrals and reduction of amplitudes to master integrals.
- New implementation of the algorithm with great efficiency optimisations.

Old and new

- Dimensional shifts, Mellin-Barnes, multi-dimensional integrations, polylogarithms
- New criteria to chose the order of integrations
- Clever representations of phase-space integrals
- From Mellin-Barnes to Euler type representations
- Symbol/coproduct and algebraic techniques for iterative integrations

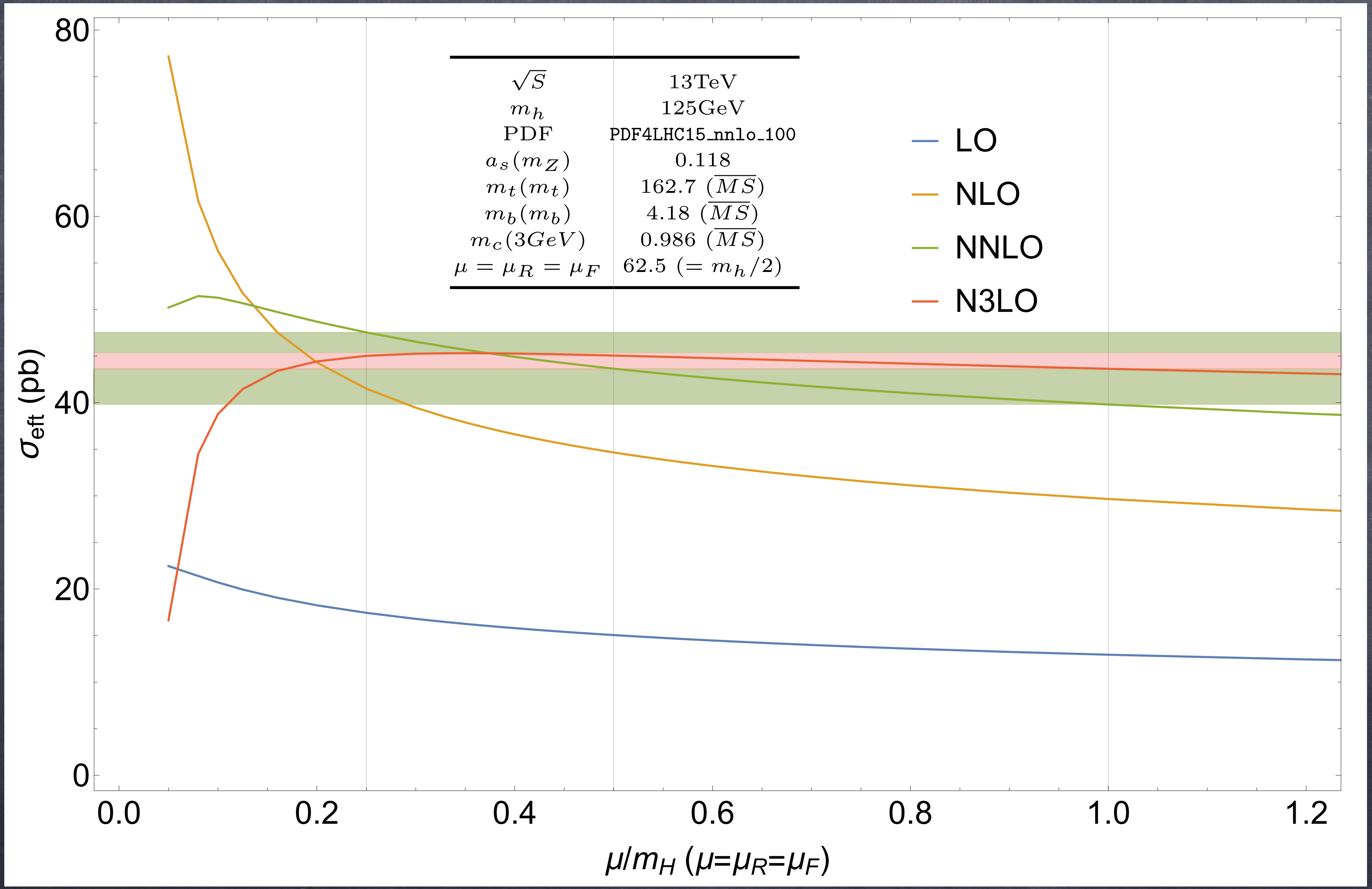
Old and new

- Differential equations method
- Finding Henn canonical forms
- Strategy of regions to determine boundary conditions
- Expansion of differential equations around the threshold limit turning their solution into an algebraic problem

How tough of a problem?

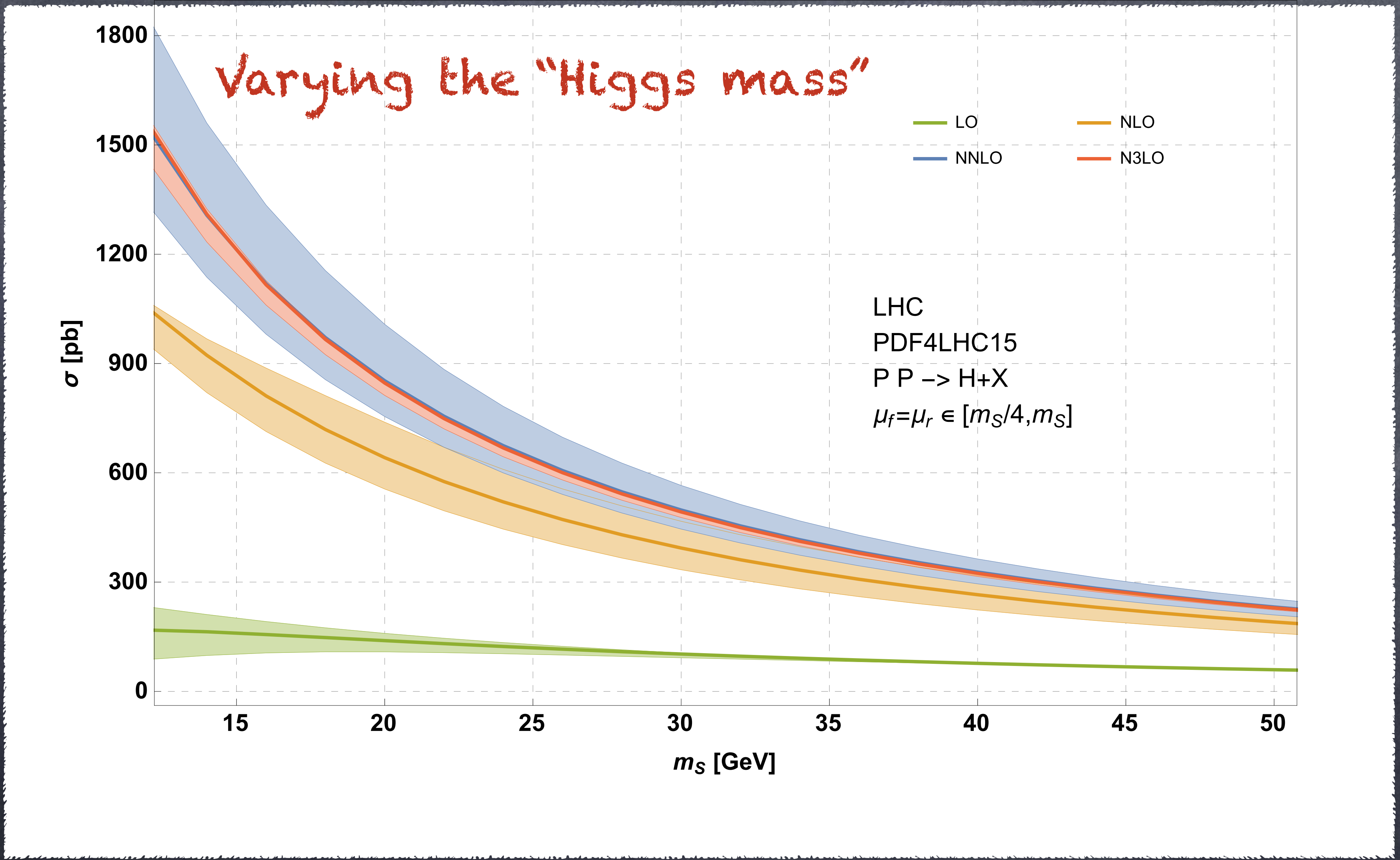
- Two orders of magnitude more Feynman diagrams than NNLO
- 1028 N3LO master integrals (27 at NNLO)
- 72 boundary conditions for the N3LO master integrals (5 at NNLO)

From NNLO to N3LO



N3LO result is very precise and within the NNLO scale variation.

From NNLO to N3LO



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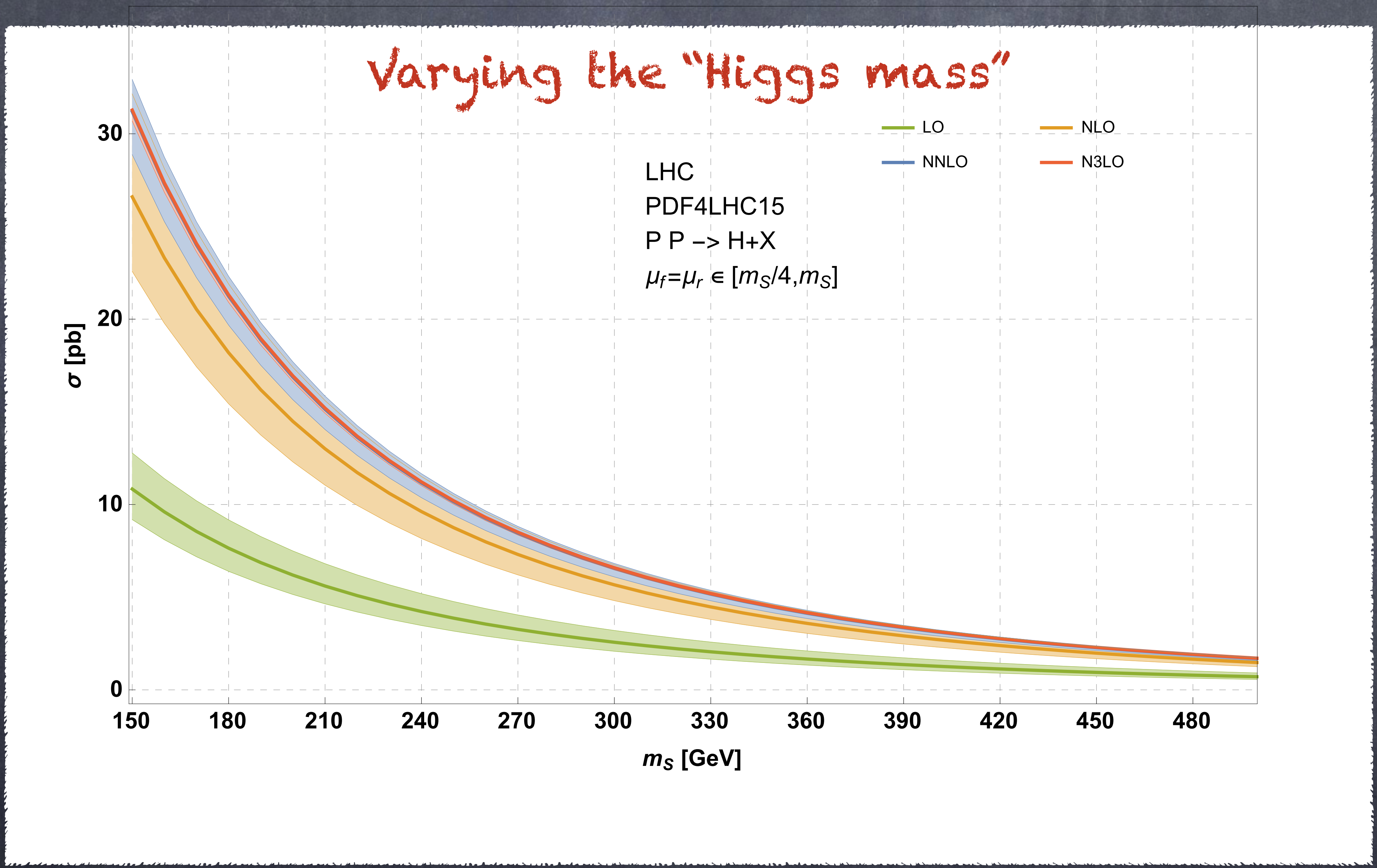
From NNLO to N3LO



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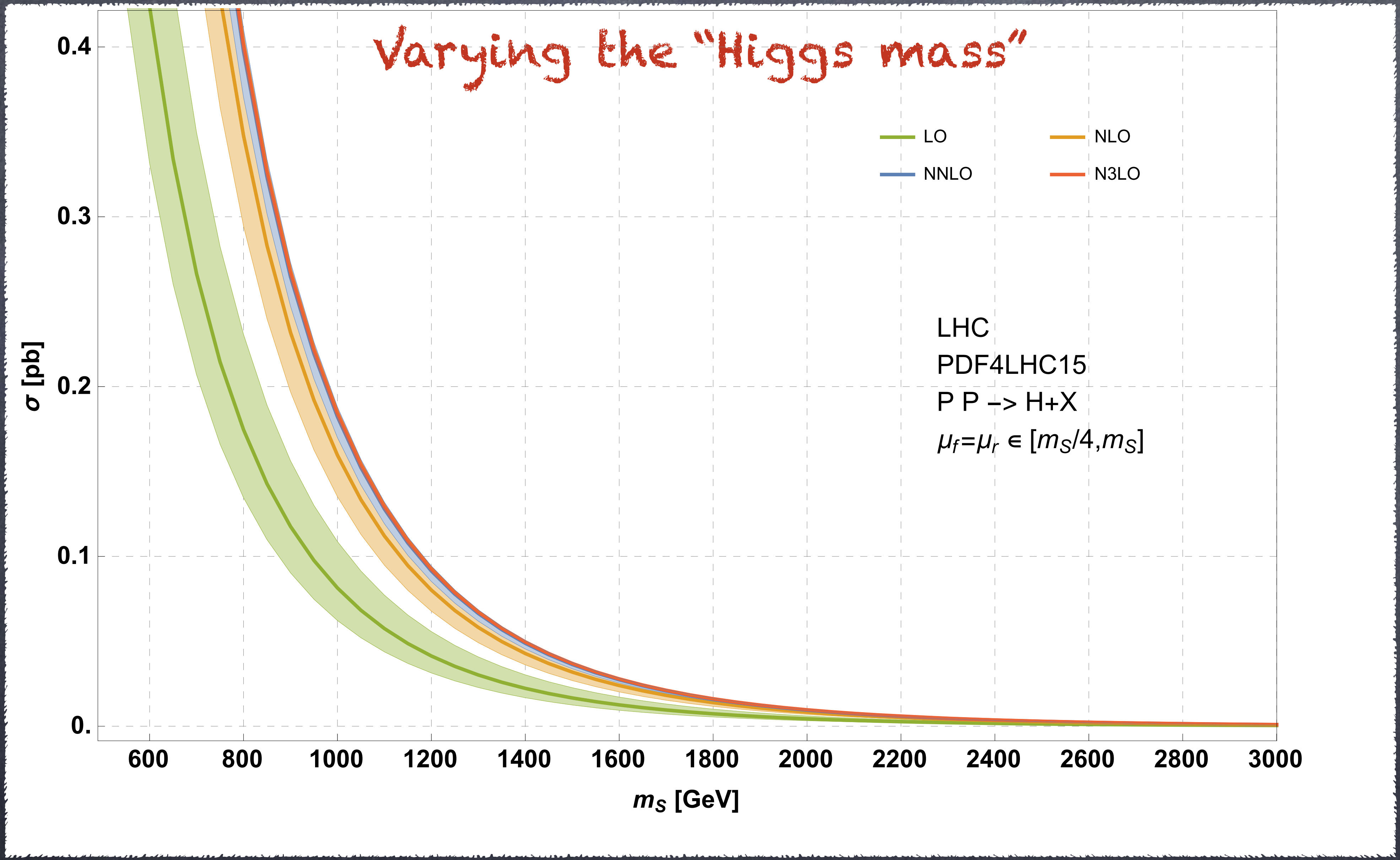
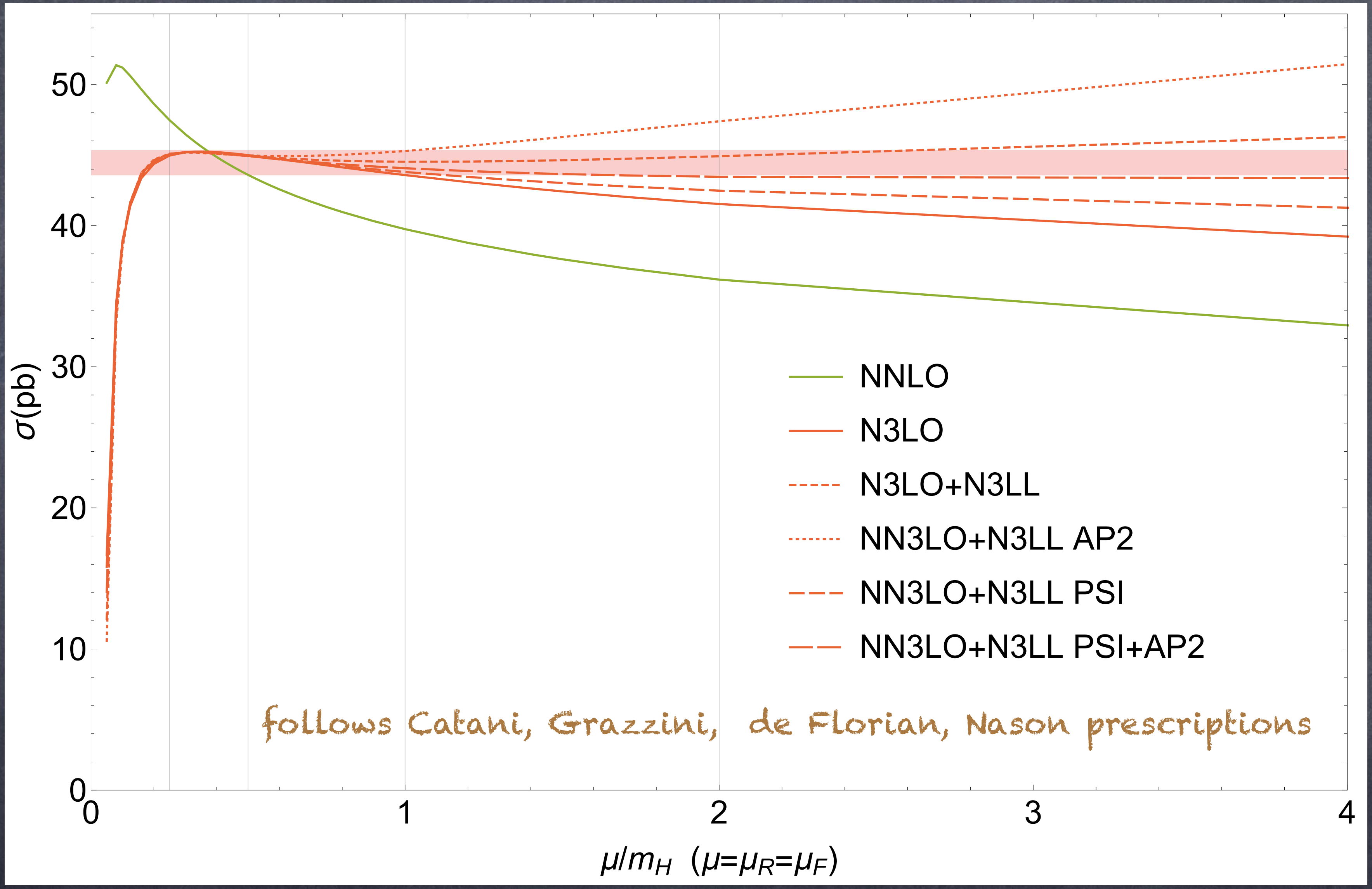
From NNLO to N3LO



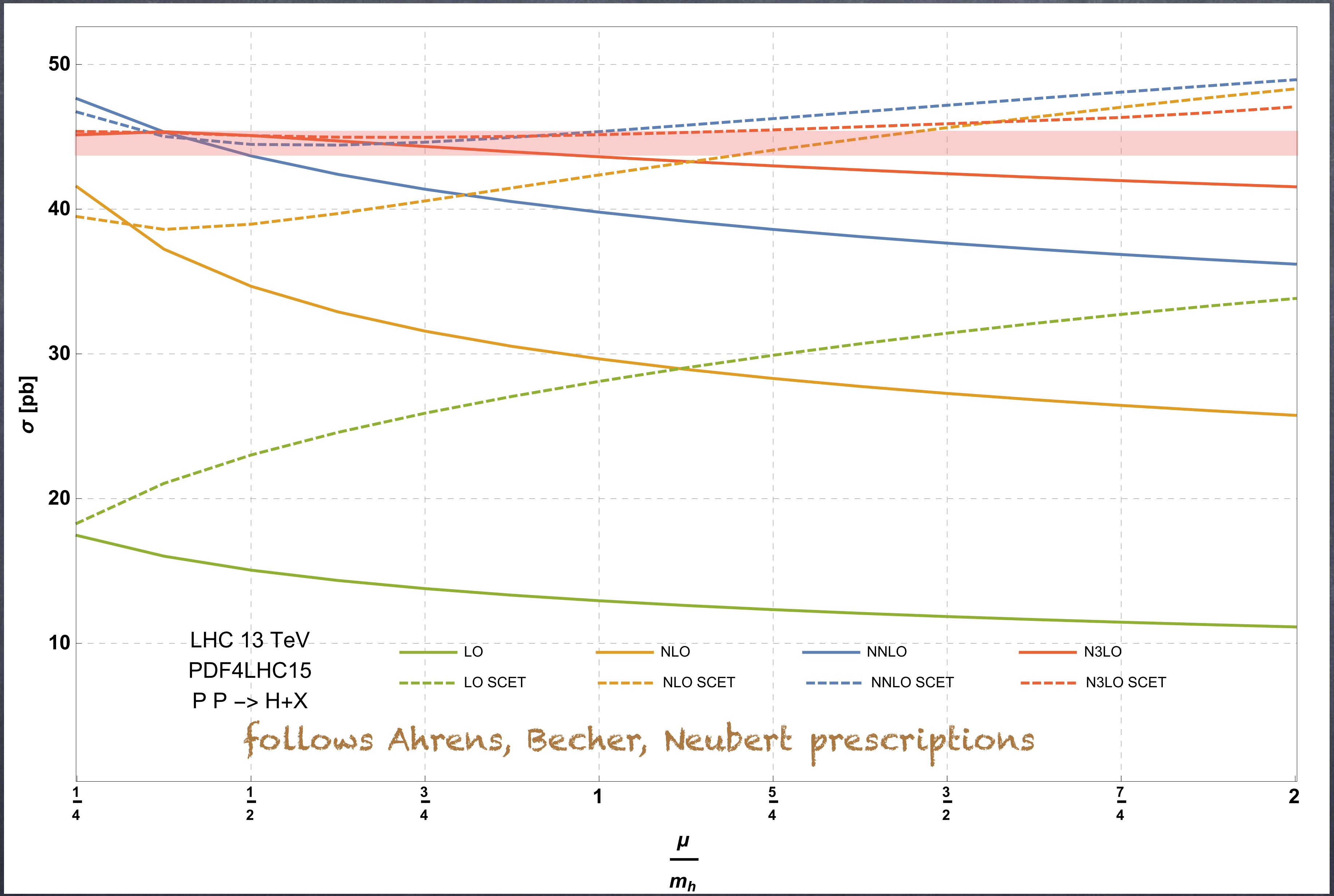
N3LO result is very precise and within the NNLO scale variation.

Resummation (I)



Traditional QCD threshold resummation agrees with N3LO

Resummation (II)



SCET renormalisation group improvement agrees with N3LO

Composition of the inclusive cross-section

$$
\begin{aligned}
48.58\,\text{pb} = \;& 16.00\,\text{pb} && (+32.9\%) && (\text{LO, rEFT}) \\
&+20.84\,\text{pb} && (+42.9\%) && (\text{NLO, rEFT}) \\
&-\;\;2.05\,\text{pb} && (-4.2\%) && ((t,b,c),\ \text{exact NLO}) \\
&+\;\;9.56\,\text{pb} && (+19.7\%) && (\text{NNLO, rEFT}) \\
&+\;\;0.34\,\text{pb} && (+0.7\%) && (\text{NNLO}, 1/m_t) \\
&+\;\;2.40\,\text{pb} && (+4.9\%) && (\text{EW, QCD-EW}) \\
&+\;\;1.49\,\text{pb} && (+3.1\%) && (\text{N}^3\text{LO, rEFT})
\end{aligned}
$$

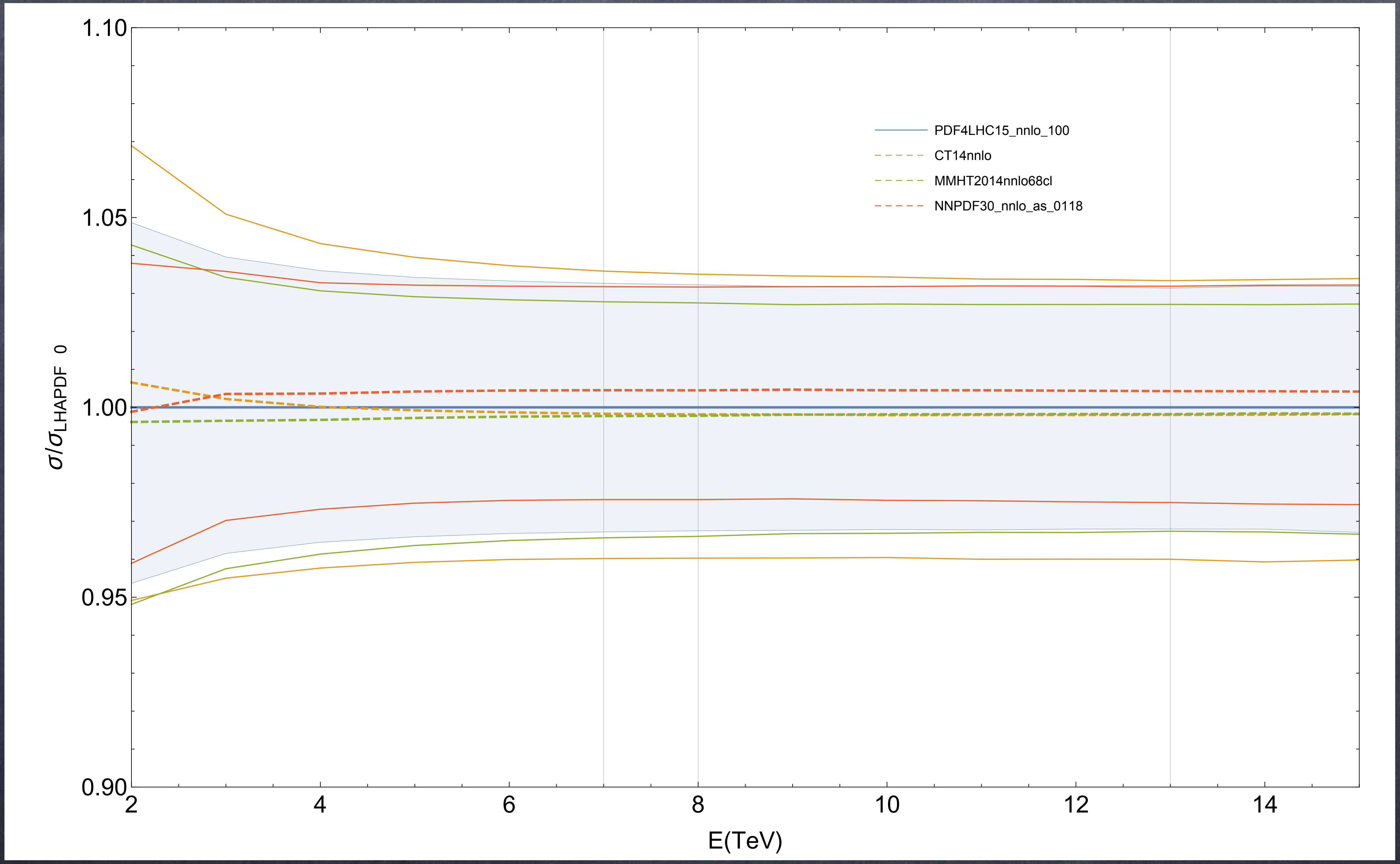
- N3LO QCD for infinite Mtop limit — CA, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger
- Finite quark-mass corrections at
  - NLO exact — Dawson; Djouadi, Gtaudenz, Spira, Zerwas; Harlander, Kant; CA,Beerli, Bucherer, Daleo, Kunszt; Bonciani, Degrassi, Vicini
  - NNLO 1/mtop expansion — Harlander, Mantler, Marzani, Ozeren; Pak, Rogal, Steinhauser
- Two-loop electroweak corrections — Actis, Passarino, Sturm, Uccirati; Aglietti, Bonciani, Degrassi, Vicini
- Mixed QCD-electroweak corrections — CA, Boughezal, Petriello

Theoretical Uncertainties

$\delta(\text{scale})$	$\delta(\text{trunc})$	$\delta(\text{PDF-TH})$	$\delta(\text{EW})$	$\delta(t,b,c)$	$\delta(1/m_t)$
$^{+0.10\text{ pb}}_{-1.15\text{ pb}}$	± 0.18 pb	± 0.56 pb	± 0.49 pb	± 0.40 pb	± 0.49 pb
$^{+0.21\%}_{-2.37\%}$	$\pm 0.37\%$	$\pm 1.16\%$	$\pm 1\%$	$\pm 0.83\%$	$\pm 1\%$

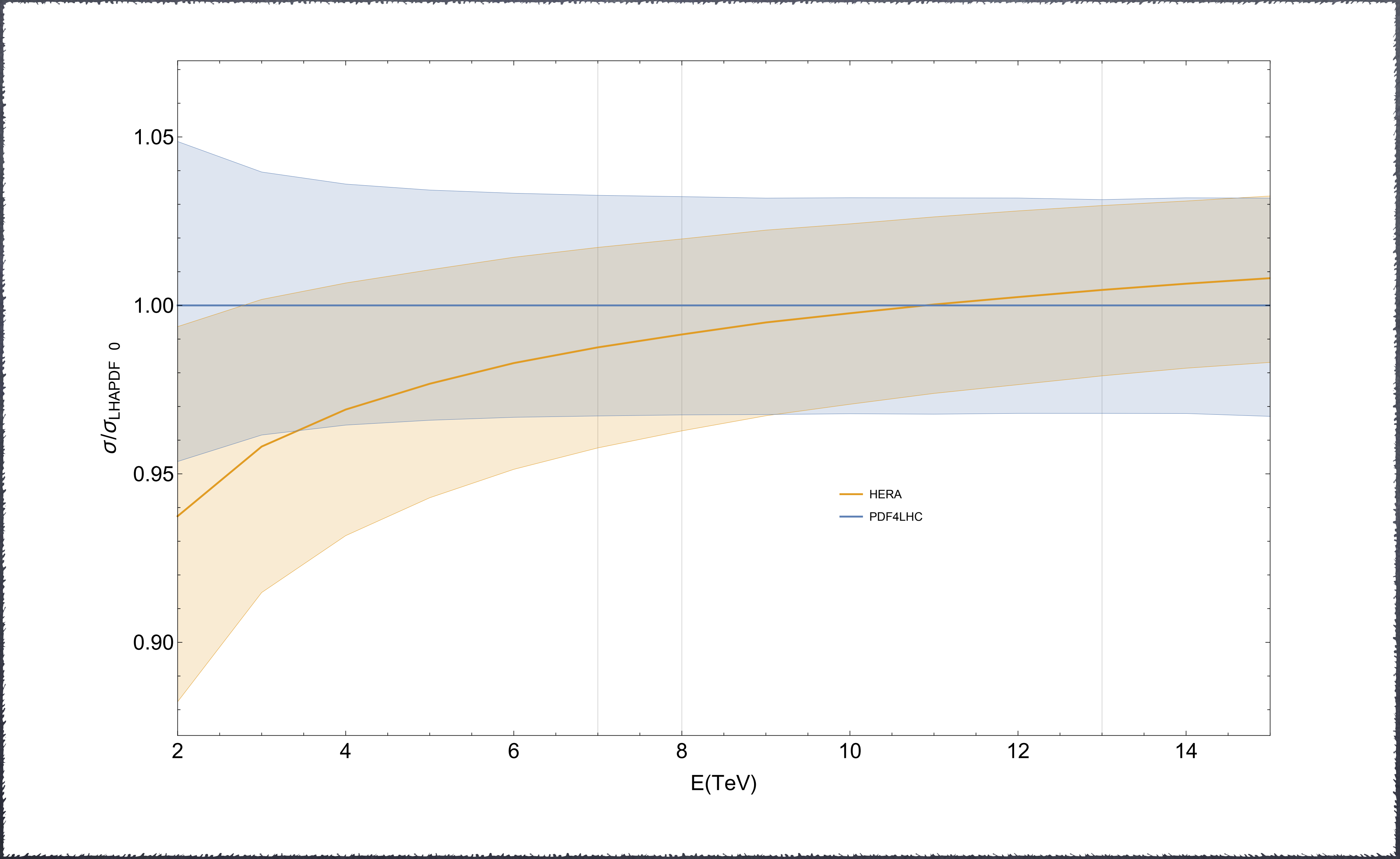
- Small uncertainties O(1% - 2%)...but quite a few of them
- missing N3LO pdfs
- missing exactly computed mixed QCD+EWK
- missing N3LO partonic cross-sections in closed functional form
- missing top-bottom interference effects at NNLO

PDF uncertainties



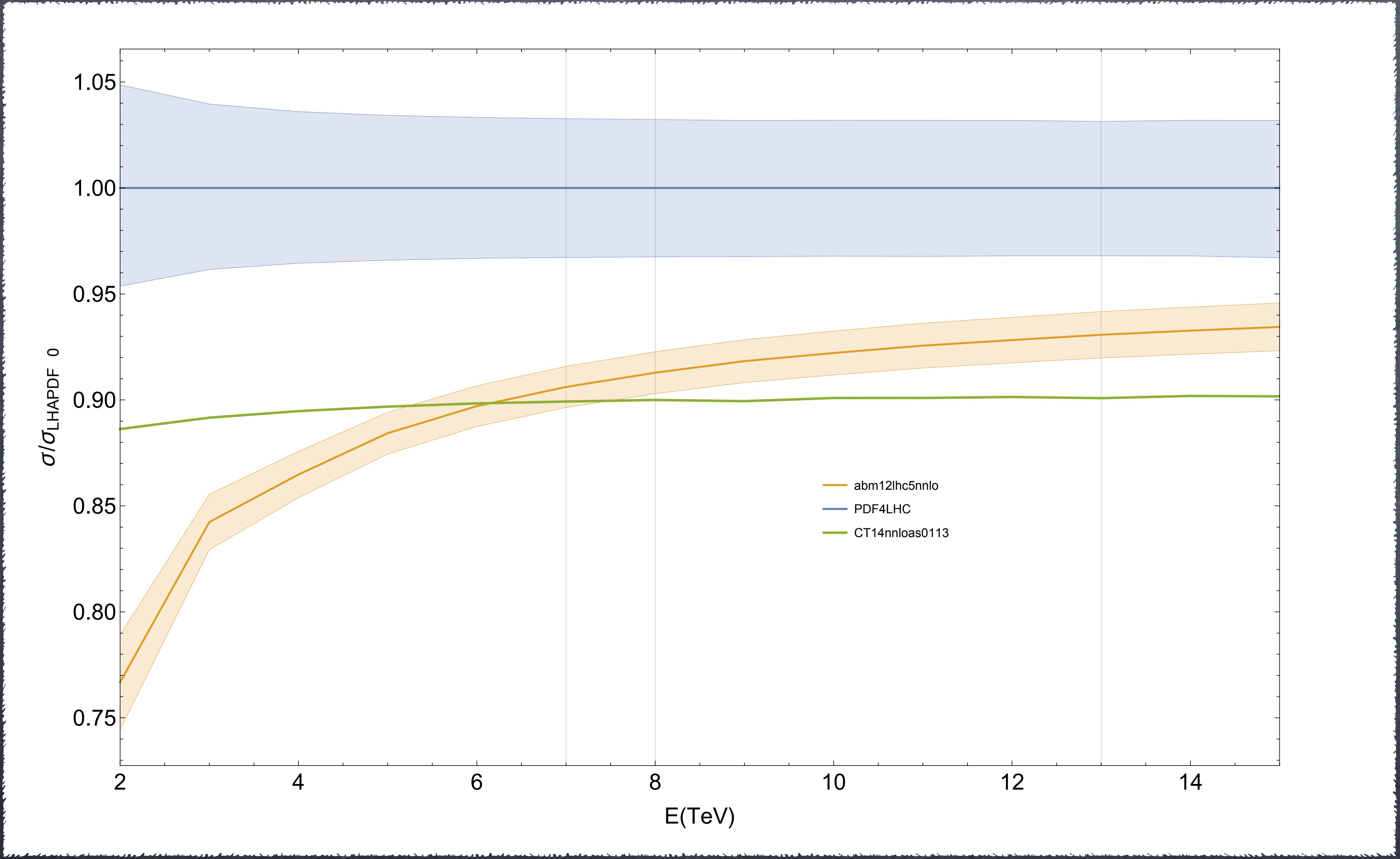
members of PDF4LHC comparison

PDF uncertainties



HERA vs PDF4LHC comparison

PDF uncertainties



ABM vs CTEQ comparison

From NNLO to N3LO

$$\sigma^{NNLO} = 47.02 \text{ pb } {}^{+5.13 \text{ pb } (10.9\%)}_{-5.17 \text{ pb } (11.0\%)} \text{ (theory)}{}^{+1.48 \text{ pb } (3.14\%)}_{-1.46 \text{ pb } (3.11\%)} \text{ (PDF+}\alpha_s\text{)}$$

A doubling of the "theory" precision...

$$\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)} \text{ (theory)} \pm 1.56\,\text{pb}\,(3.20\%) \text{ (PDF+}\alpha_s\text{)}$$

PDF Uncertainties

$$\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)}\ (\text{theory}) \pm 1.56\,\text{pb}\,(3.20\%)\ (\text{PDF}+\alpha_s)$$

Discrepancies between PDFs exist.

$$\sigma_{\text{ABM12}} = 45.07\,\text{pb}^{+2.00\,\text{pb}\,(+4.43\%)}_{-2.88\,\text{pb}\,(-6.39\%)}\ (\text{theory}) \pm 0.52\,\text{pb}\,(1.17\%)\ (\text{PDF}+\alpha_s)$$

Cross-section with ABM pdfs and alphas differs from PDF4LHC beyond the level of the quoted accuracy.

Conclusions/ Outlook

- First N3LO computation for a hadron collider process
- Results to the most precise determination of the Higgs production rate.
- Further improvents can come with further cutting edge calculations: exact quark mass dependence at NLO, exact EWK-QCD corrections, more NNLO and N3LO processes for PDF fits
- Tempting next theoretical challenge: can we do differential distributions?

Every particle physicist of our lucky generation has a story to tell about the Higgs boson.

I described a story of precision Higgs physics

As with many other Higgs stories, the end of our little fairy tale has not been written yet… watch out for the next chapter!