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Differential distributions for top-quark pair production at NNLO

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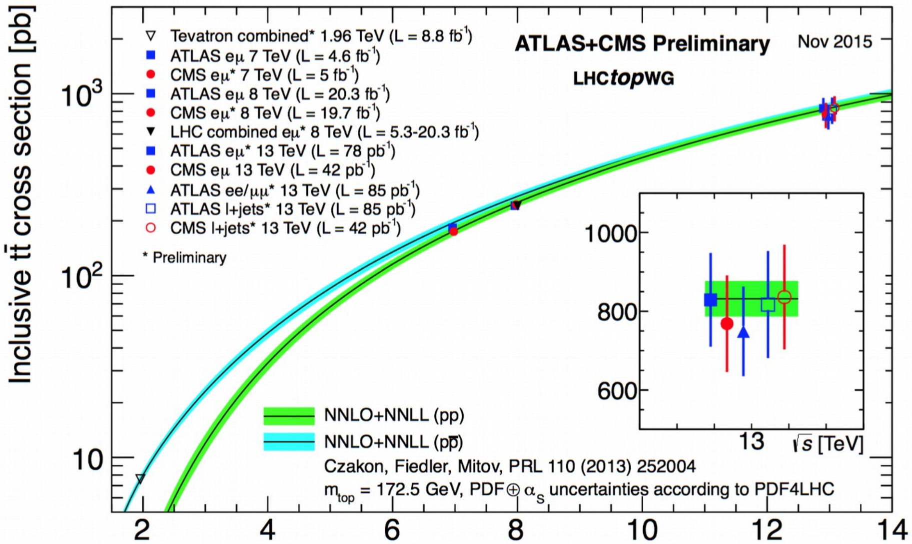
Top-quark pairs at the LHC

- Total cross section measured (~ 7 %)

$$\sigma_{t\bar{t}}(13\,\mathrm{TeV}) \approx (803 \pm 50)\mathrm{pb}$$

- Uncertainty of the prediction
  - LO (30 %) → NLO (15%) → NNLO + NNLL (5%)



[Czakon,Fiedler, Mitov; 2013] $\sqrt{s}$ [TeV]

→ Percent - level precision required

- Precision tests of the Standard Model
- Background for many searches and processes (Higgs, New Physics,...)
- Constrain gluon PDF at high x
- ...

NNLO – Cross section

$$\sigma_{h_1 h_2}(P_1, P_2) = \sum_{ab} \iint_0^1 \mathrm{d}x_1 \mathrm{d}x_2\, f_{a/h_1}(x_1, \mu_\mathrm{F}^2)\, f_{b/h_2}(x_2, \mu_\mathrm{F}^2)\, \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2;\, \alpha_s(\mu_\mathrm{R}^2),\, \mu_\mathrm{R}^2,\, \mu_\mathrm{F}^2)$$

- Partonic cross section expansion in α_s at NNLO

$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \hat{\sigma}_{ab}^{(1)} + \hat{\sigma}_{ab}^{(2)}$$

R: Real radiation
V: Virtual
C: Collinear Factorization

- NNLO contribution

$$\hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{\mathrm{RR}} + \hat{\sigma}_{ab}^{\mathrm{RV}} + \hat{\sigma}_{ab}^{\mathrm{VV}} + \hat{\sigma}_{ab}^{\mathrm{C1}} + \hat{\sigma}_{ab}^{\mathrm{C2}} \Rightarrow$$

Sum is Finite

Real IR - poles

Virtual IR - poles

- Subtraction/slicing scheme is needed to consistently cancel IR - singularities

Subtraction/slicing schemes at NNLO

- Antenna subtraction — Talk by A. Huss, Talk by J. Niehues
  - e+e- → 3 jets, pp → 2 jets, qq → tt, H + jet

[Gehrmann, GehrmannDeRidder, Glover, Heinrich '05 - '08]
[Weinzierl '08, '09]
[Currie, Gehrmann, GehrmannDeRidder, Glover, Pires, '13 -'14]
[Bernreuther, Bogner, Dekkers, '11,'14]
[Abelof, (Dekkers), GermannDeRidder, '11-'15]
[Abelof, GehrmannDeRidder, Maierhofer, Pozzorini, '14]
[Chen, Gehrmann, Glover, Jaquier, '15]

- Colorful subtraction — Talk by G. Somogyi, Talk by A. Kardos
  - H → bb

[DelDuca, Somogyi, Troscsanyi, '05 - '13]
[DelDuca, Duhr, Somogyi, Tramontano, Trocsanyi, '15]

- qT – slicing
  - pp → H, pp → V, pp → H + V, pp → VV, qq → tt (flavour off-diagonal)

[Catani, Grazzini, '07]
[Ferrera, Grazzini, Tramontano, '11]
[Catani, Cieri, DeFlorian, Ferrera, Grazzini, '12]
[Gehrmann, Grazzini, Kallweit, Maierhofer, Manteuffel, Rathlev, Torre, '14 -'15]
[Bonciani, Catani, Grazzini, Sargsyan, Torre; '14. '15]

- N-jettiness slicing — Talk by K. Ellis
  - pp → H + jet, pp → W + jet, pp → Z + jet, pp → H + V, pp → γγ

[Gaunt, Stahlhofen, Tackmann, Walsh, '15]
[Boughezal, Focke, Giele, Liu, Petriello, '15-'16]
[Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello, '15]
[Campbell, Ellis, Williams, '16]

Sector improved residue subtraction (STRIPPER)

SecToR Improved Phase sPacE for real Radiation

- Local subtraction scheme for NNLO (no approximations)
- First formulation [Czakon, '10, '11]
  - pp → tt (total cross section, A_{FB} at Tevatron, distributions at Tevatron) [Czakon, Fiedler, Mitov; '13, '15] [Czakon, Fiedler, DH, Mitov; '16]
  - pp → H + jet , Z → e+ e-, Muon – decay, b – decay, top – decay, single top production [Boughezal, Caola, Melnikov, Petriello, Schulze, '13 '14] [Boughezal, Melnikov, Petriello, '11] [Caola, Czernecki, Liang, Melnikov, Szafron, '14] [Brucherseifer, Caola, Melnikov, '13, '13, '14]
- Generalization to 4 dimensions [Czakon, DH, '14]

STRIPPER – Main Idea

- Numerical cancellation of IR – poles between NNLO contributions
- Example: Double real radiation (most complicated)

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \frac{1}{N_{ab}} \int \mathrm{d}\mathbf{\Phi}_{n+2} \, \langle \mathcal{M}_{n+2}^{(0)} | \mathcal{M}_{n+2}^{(0)} \rangle \, \mathrm{F}_{n+2}$$

1) Use selector functions to split phase space into triple and double collinear sectors

2) Use physical parametrization (angles, energies) [Frixione, Kunszt, Signer (FKS); '95]

3) Physical sector decomposition: Factorization of non-commuting singularities [Binoth, Heinrich; '00]

4) Generate subtraction terms

5) Laurent series in ϵ (→ numerical integration of all coefficients)

STRIPPER – General formulation

[Czakon, DH; '14]

$$\hat{\sigma}^{(2)}_{ab} = \hat{\sigma}^{\mathrm{RR}}_{ab} + \hat{\sigma}^{\mathrm{RV}}_{ab} + \hat{\sigma}^{\mathrm{VV}}_{ab} + \hat{\sigma}^{\mathrm{C1}}_{ab} + \hat{\sigma}^{\mathrm{C2}}_{ab}$$

- Each contribution is a Laurent series in ϵ
- Separation of independently finite contributions (check number of unresolved particles)
  - Finite contribution (all particles resolved)
  - Single unresolved $|\mathcal{M}^{(0)}_{n+1}\rangle$
  - Double unresolved $|\mathcal{M}^{(0)}_{n}\rangle$
  - Finite Remainder $|\mathcal{F}^{(1)}_n\rangle = |\mathcal{M}^{(1)}_n\rangle - \mathbf{Z}^{(1)}|\mathcal{M}^{(0)}_n\rangle$

$$\hat{\sigma}^{\mathrm{RR}}_{\mathrm{F}}\,, \quad \hat{\sigma}^{\mathrm{RV}}_{\mathrm{F}}\,, \quad \hat{\sigma}^{\mathrm{VV}}_{\mathrm{F}}\,, \quad \hat{\sigma}_{\mathrm{FR}} = \hat{\sigma}^{\mathrm{RV}}_{\mathrm{FR}} + \hat{\sigma}^{\mathrm{VV}}_{\mathrm{FR}} + \hat{\sigma}^{\mathrm{C2}}_{\mathrm{FR}}\,,$$

$$\hat{\sigma}_{\mathrm{SU}} = \hat{\sigma}^{\mathrm{RR}}_{\mathrm{SU}} + \hat{\sigma}^{\mathrm{RV}}_{\mathrm{SU}} + \hat{\sigma}^{\mathrm{C1}}_{\mathrm{SU}}\,, \quad \hat{\sigma}_{\mathrm{DU}} = \hat{\sigma}^{\mathrm{RR}}_{\mathrm{DU}} + \hat{\sigma}^{\mathrm{RV}}_{\mathrm{DU}} + \hat{\sigma}^{\mathrm{VV}}_{\mathrm{DU}} + \hat{\sigma}^{\mathrm{C1}}_{\mathrm{DU}} + \hat{\sigma}^{\mathrm{C2}}_{\mathrm{DU}}$$

- Make sure that SU and DU cancel independently ($\rightarrow$ resolved particles in 4 dimensions)

STRIPPER - Implementation

- General purpose event generator for NNLO computation
- Based on four-dimensional formulation of the subtraction scheme
- Complete independent implementation
- SM tree-level matrix elements are included [vanHameren, Bury; '09, '15]
- **Process independent:** User has to interface the one-loop and two-loop **finite** contributions
- **Speed:** Monte Carlo over processes and polarizations
- Simultaneous computation of:
  - Different PDF sets (LHAPDF interface)
  - Different renormalization and factorizations scales
  - Different observables

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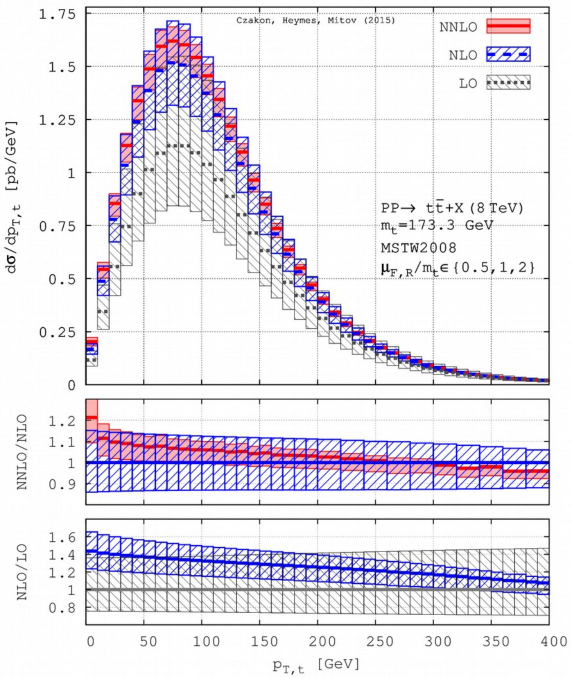
Differential top-quark pair production at NNLO

[Czakon, DH, Mitov; published 2015]
[Czakon, DH, Mitov; in preparation]

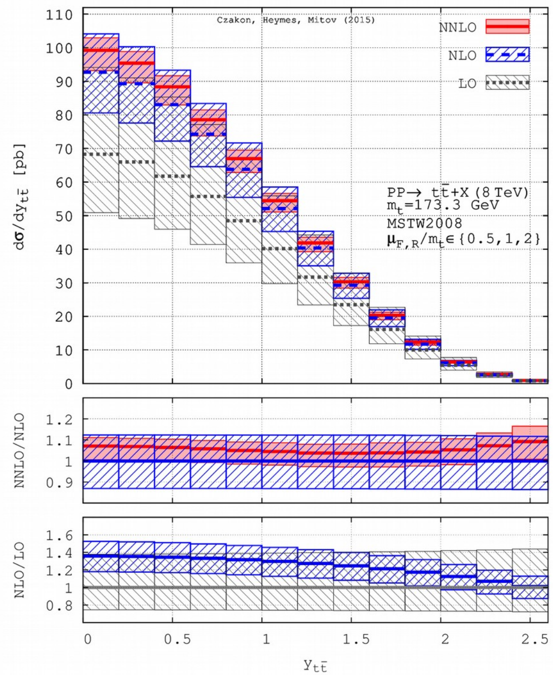
Differential Distributions @ 8 TeV

[Czakon, DH, Mitov; published 2015]

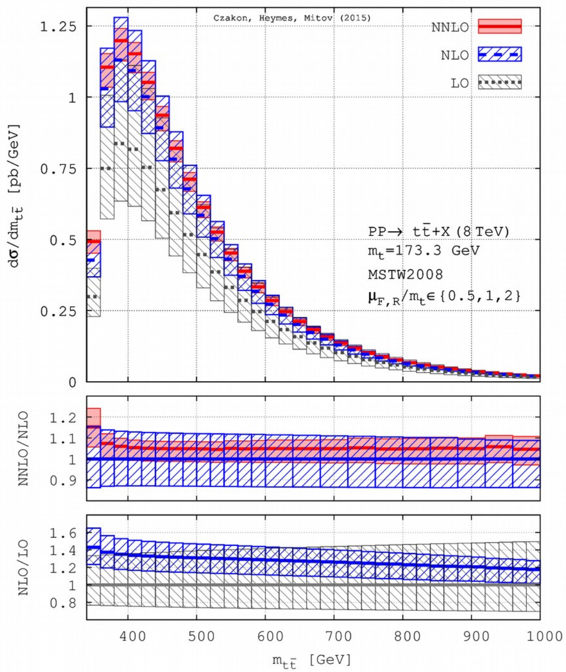
P_T ot the top



Rapidity of the top-pair



Invariant mass



- NNLO has important impact (Good perturbative convergence)
- Good agreement with data → [CMS 2015, ATLAS 2015]
- **However: Results with fixed scales applicable only to limited kinematical range**

Differential Distributions @ 8 TeV

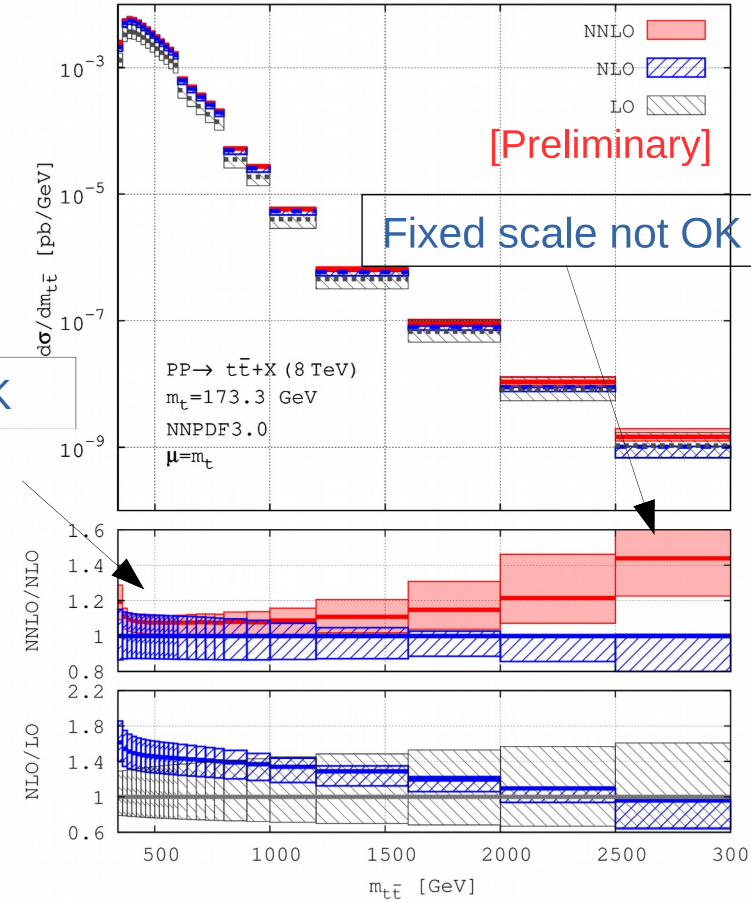
- Indeed: Direct interpolation of fixed scales to higher values leads to worse convergence

- But: Boosted regime, 13 TeV

  → wider kinematic range required

- 2 main Improvements
  - Technical: Enhance statistics in the Monte Carlo
  - Conceptual: Choose the “right” dynamical scale to maintain perturbative convergence

Fixed scale OK



What is the “right” scale?

$$\sigma_{h_1h_2}(P_1,P_2) = \sum_{ab} \iint_0^1 \mathrm{d}x_1\mathrm{d}x_2\, f_{a/h_1}(x_1,\mu_\mathrm{F}^2)\, f_{b/h_2}(x_2,\mu_\mathrm{F}^2)\, \hat{\sigma}_{ab}(x_1P_1,x_2P_2;\, \alpha_s\boxed{(\mu_\mathrm{R}^2)},\, \boxed{\mu_\mathrm{R}^2},\, \boxed{\mu_\mathrm{F}^2})$$

Scale dependence

[Czakon, DH, Mitov; in preparation]

Problem

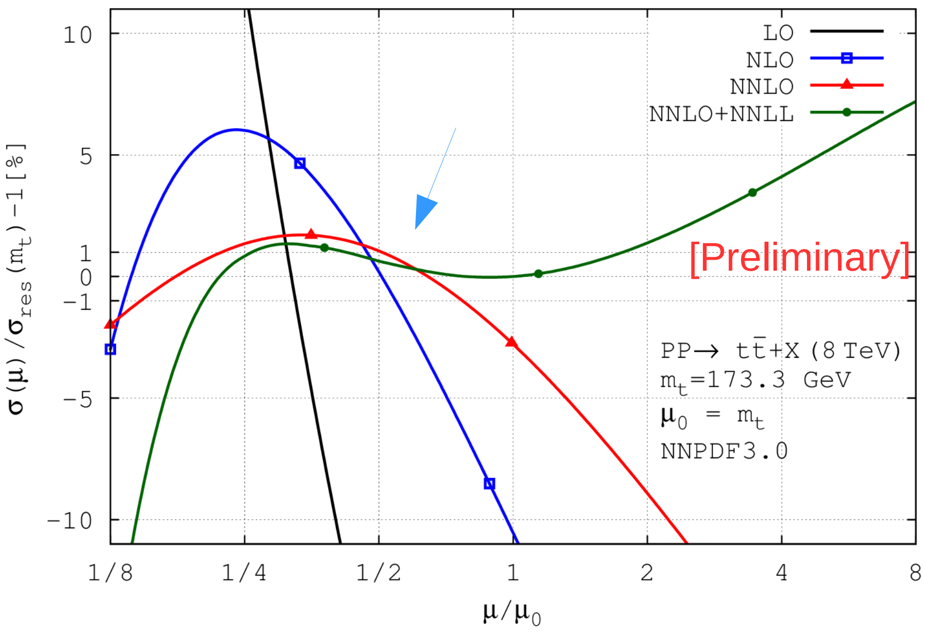
- A number of dynamical scales have been used in the past (NLO, resummation)
- However: picking a dynamical scale would not solve the problem because:
  - Difference between different dynamical scales could be as large as difference between dynamical scale and fixed scale
  - Effect never been studied before
- Needed: Comparative study of perturbative convergence based on different scales

Plan for action

- Selection of the “correct” scale is based on the following criteria:
  - Perturbative convergence for both total and differential cross section
  - Limiting behavior: Low p_T (m_{tt}): ~ m_{top} ↔ High p_T (m_{tt}): ~ p_T
  - Restriction to simple functional forms studied in the past (H_T, m_{tt}, ...)

Scale dependence – Total cross section

[Czakon, DH, Mitov; in preparation]



- Look for convergence
  - Scale value which minimizes difference
    - NLO → NNLO → (NNLO + NNLL)
  - Best convergence: $\mu_0 < m_{top}$
  - Little dependence on PDFset at NNLO

- Value of NNLO cross section at point of best convergence equals the NNLO+NNLL at the usual canonical scale $\mu_0 = m_{top}$

  → Therefore: Resummation has negligible impact on the total cross section at the point of fastest convergence

Scale dependence – Differential Distributions

[Czakon, DH, Mitov; in preparation]

- Main guidance is perturbative convergence to discriminate between scales

  - Invariant mass distribution

$$\mu_0 = H_T/4$$

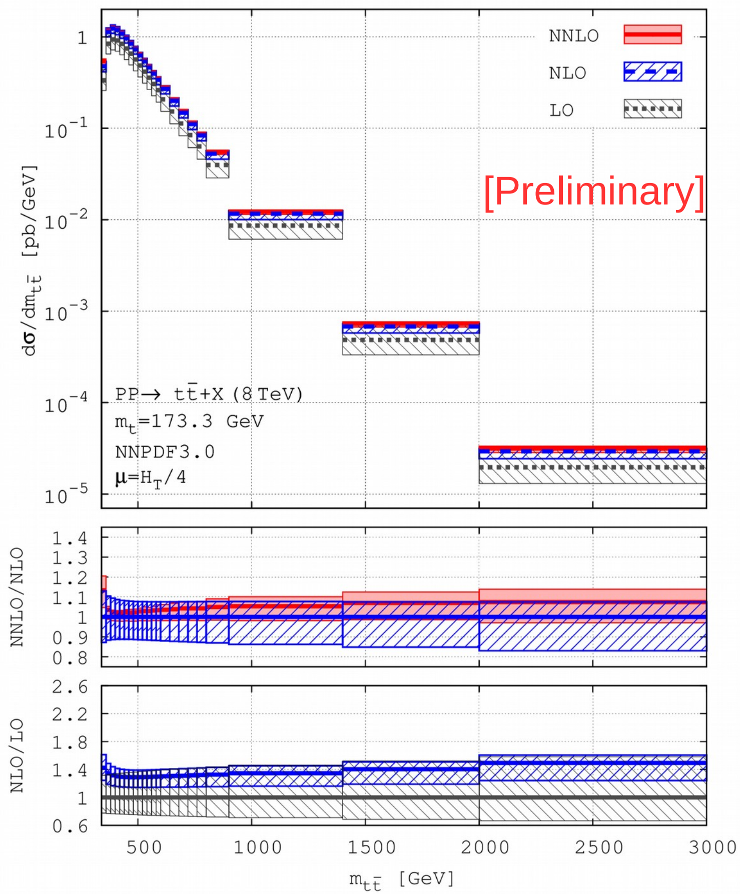
$$H_T = \sqrt{m_t^2 + p_{\mathrm{T}t}^2} + \sqrt{m_t^2 + p_{\mathrm{T}\bar{t}}^2}$$

  - Limiting behaviour

$$\mu_0(p_{\mathrm{T}} \to 0) \quad \to \quad m_t/2$$
$$\mu_0(p_{\mathrm{T}} \to \infty) \quad \to \quad (p_{\mathrm{T},t} + p_{\mathrm{T},\bar{t}})/4$$

  - Scales based on the invariant mass itself

$$\mu \propto m_{t\bar{t}}$$



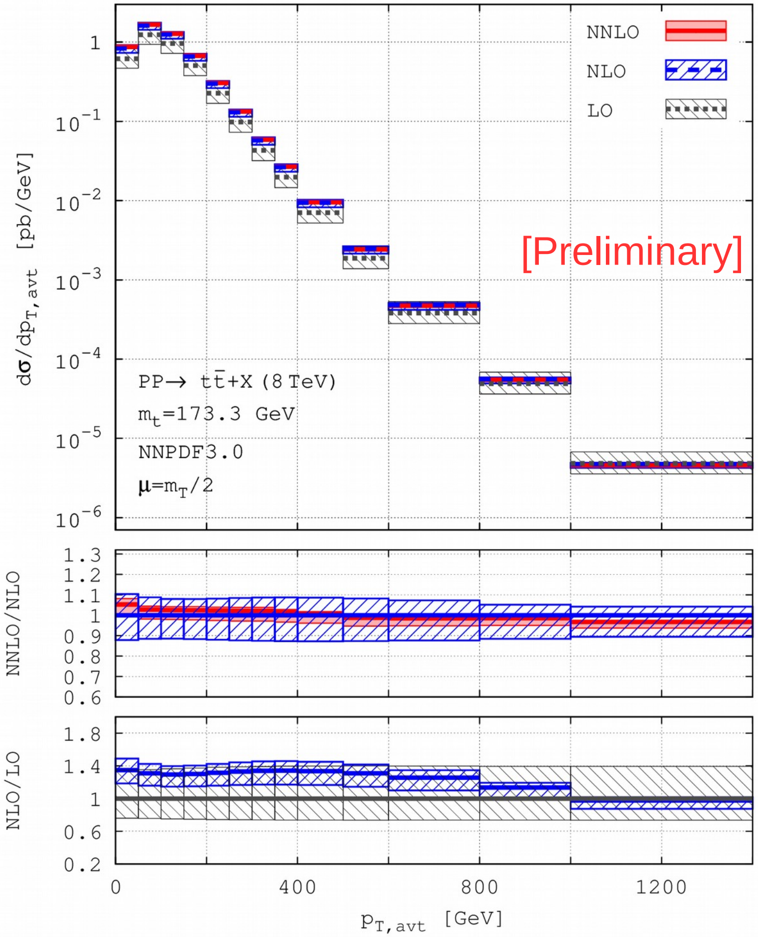
Scale dependence – Differential Distributions

[Czakon, DH, Mitov; in preparation]

- Main guidance is perturbative convergence to discriminate between scales
  - Choose individual scales for top and antitop p_T
  - Transverse mass scale

$$\mu_0 = \frac{1}{2} m_T(t/\bar{t}) = \frac{1}{2}\sqrt{m_t^2 + p_{T,t/\bar{t}}^2}$$

  - Average distributions afterwards

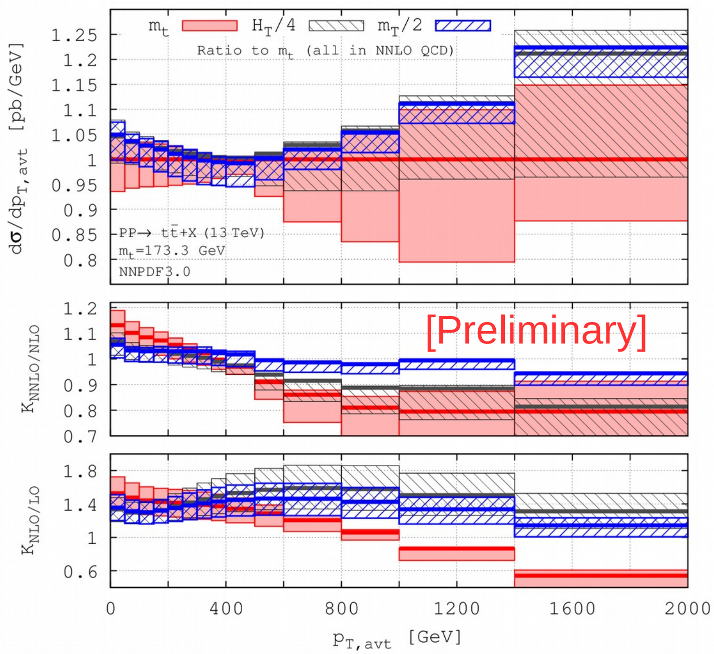


→ Different scale choices for different observables

Scale dependence – Differential Distributions

[Czakon, DH, Mitov; in preparation]

- Comparison between different scale choices
  - Difference within uncertainty
  - Main impact on scale dependence at high values and the K-factor



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Differential top-quark pair production @ 13 TeV

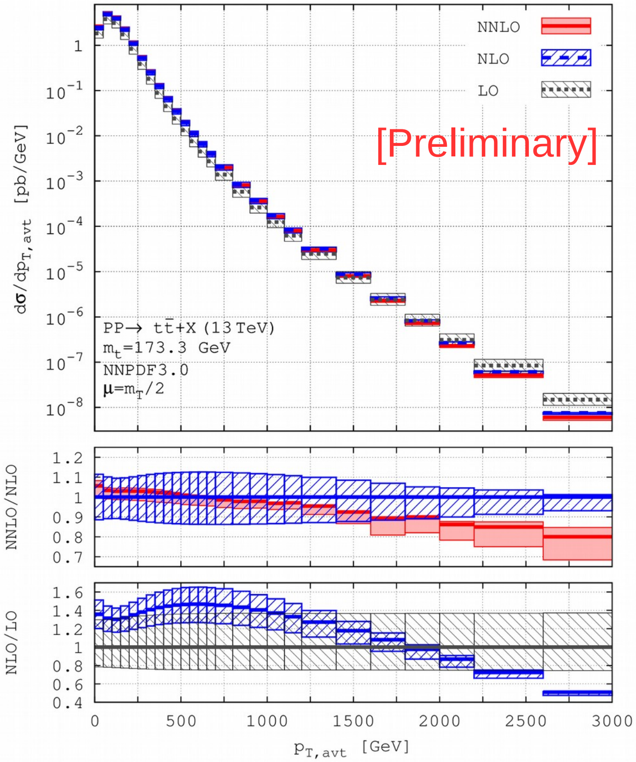
- Dynamical scales
- Covers multi-TeV range

[Czakon,DH, Mitov; in preparation]

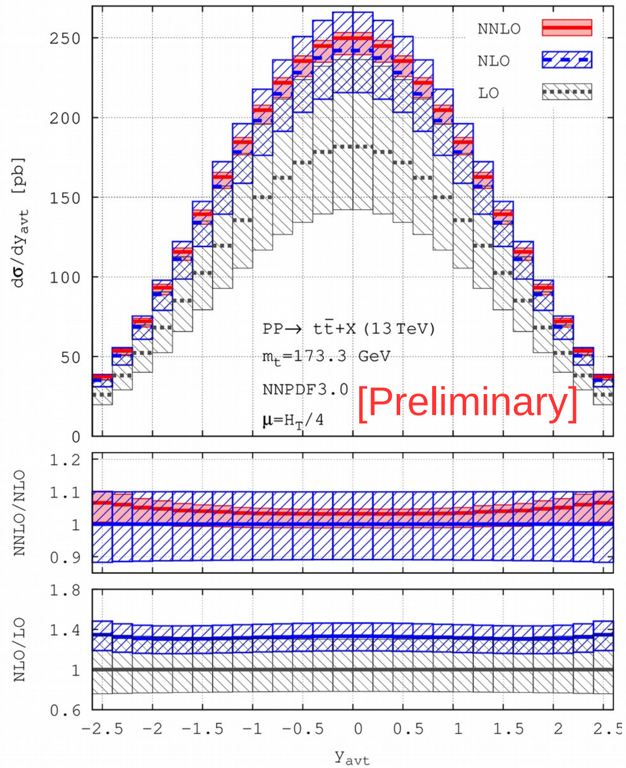
Differential Distributions @ 13 TeV

[Czakon, DH, Mitov; in preparation]

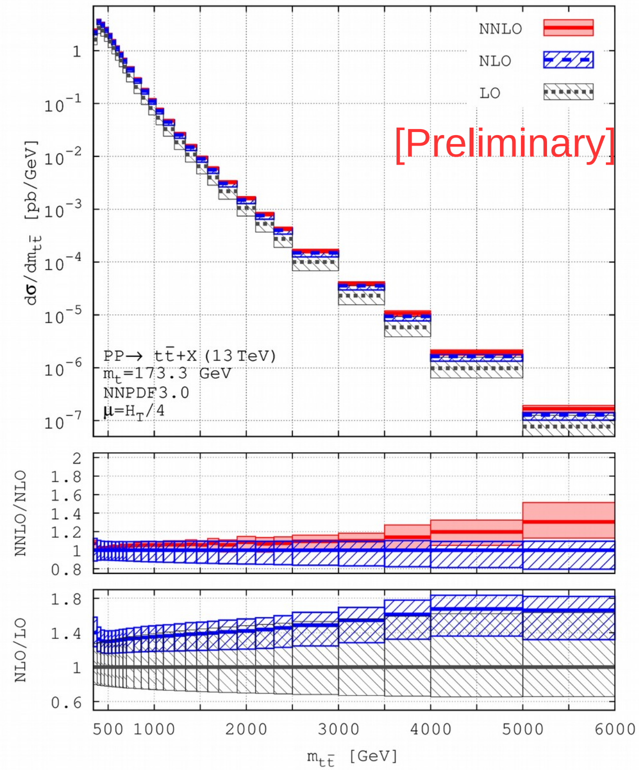
PT ot the top



Rapidity of the top



Invariant mass

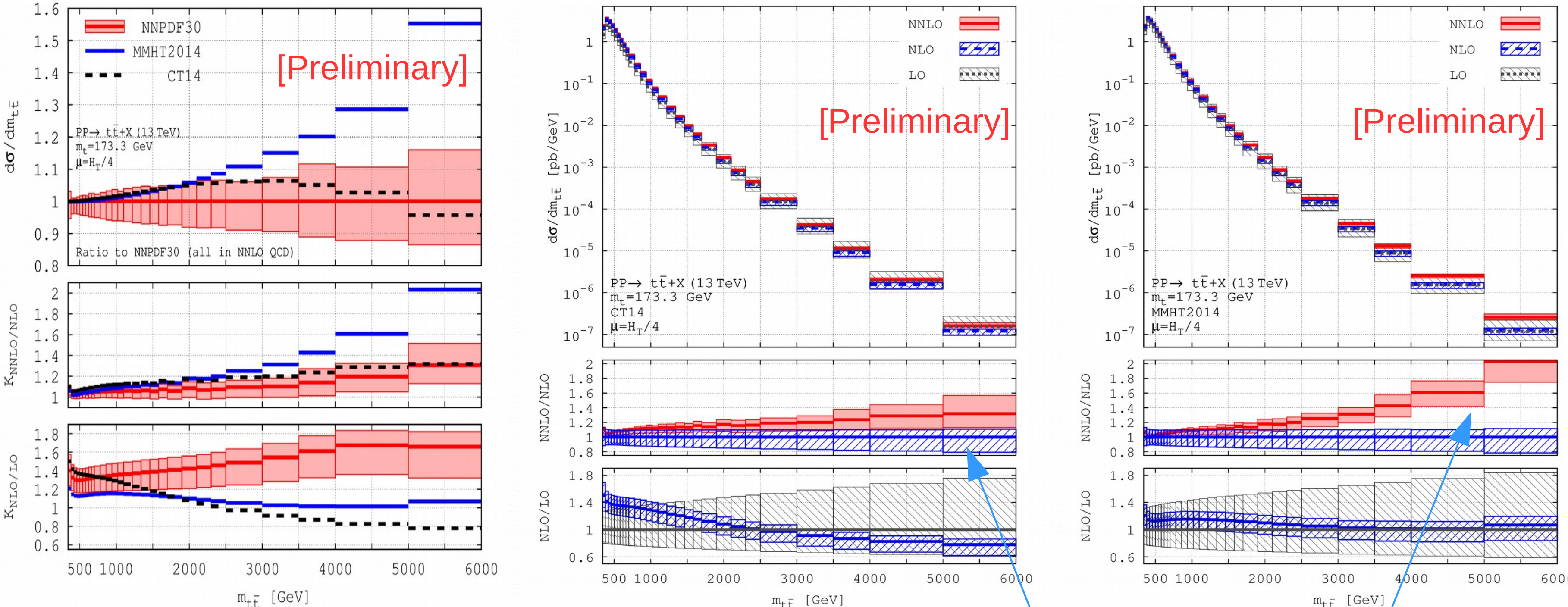


Dynamical scales → extended kinematical regime

- Preliminary comparison with data → Good agreement

Differential Distributions @ 13 TeV

- PDF - dependence



Large PDF error in the multi TeV region expected

- Useful to constrain PDF sets

Summary and Outlook

- **General** implementation of the Sector improved residue subtraction (STRIPPER)
- Applied to differential top-quark pair productions (no approximations)
  - Agreement with data at 8 TeV and 13 TeV
- Outlook
  - Combine NNLO-QCD with NLO-EW
  - Include top-decays