

How to exploit canonical bases for $gg \rightarrow H$ at NNNLO

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Outline

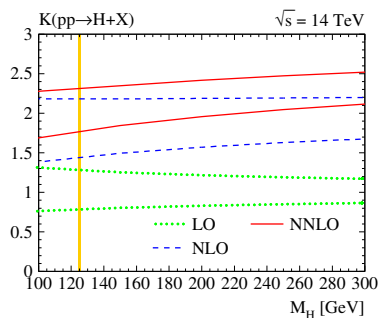
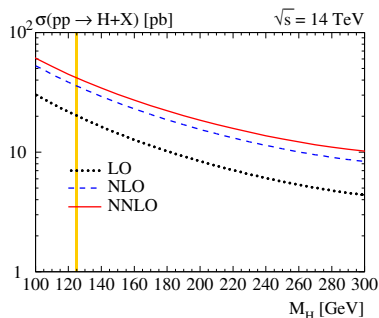
- ▶ Why $gg \rightarrow H$ at NNNLO?
- ▶ Computational methods
- ▶ Canonical bases and their relation to boundary conditions
- ▶ Analytic results of the qq' -channel
- ▶ Mathematical structures in the results
- ▶ Conclusions

Single Higgs production – why NNNLO?

determine Higgs boson properties

- ▶ known: $m_H = 125\text{GeV}$, spin 0
- ▶ wanted: couplings to bosons and fermions (currently $O(10 - 20\%)$)
→ precise theory predictions needed

Our interest: total inclusive cross section for Higgs production



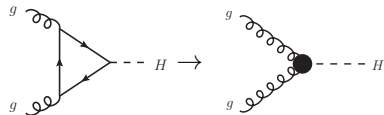
[Harlander/Kilgore; '02]

Effective theory for $m_t \rightarrow \infty$

Dominant channel: gluon-gluon fusion

▶ exact calculations known up to NLO [Dawson 1991 NPB,]

▶ effective theory of $m_t \rightarrow \infty$, i.e.



$$\mathcal{L} = -\frac{H}{4v} C_1 G_{\mu\nu} G^{\mu\nu} + \mathcal{L}_{\text{QCD}}^{(5)}$$

▶ 1%-level accurate at NNLO

[Harlander/Ozeren (et al.), Pak/Rogal/Steinhauser 2009/10]

→ completely analytic calculation at parton level in the EFT

Known QCD corrections

LO...NLO...NNLO...There are approximate NNNLO-results...

NNNLO exact in m_H^2/s :

- ▶ effective Higgs coupling
[Chetyrkin/Kniehl/Steinhauser 97, Schröder/Steinhauser 06, Chetyrkin/Kühn/Sturm 06]
- ▶ LO,NLO,NNLO to orders $\epsilon^3, \epsilon^2, \epsilon^1$
[Höchele/Hoff/Steinhauser/Ueda 13, Buehler/Laszopoulos 13]
- ▶ convolutions with splitting functions
[Höchele/Hoff/Steinhauser/Ueda 13-14, Buehler/Laszopoulos 13]
- ▶ 3-loop form factor
[Baikov/Chetyrkin/Smirnov²/Steinhauser 09, Gehrmann/Glover/Huber/Ikizerli/Studerus 10]
- ▶ 2-loop single soft current [Duhr/Gehrmann 13, Hua/Zhu 13]
- ▶ real-virtual-virtual [Dulat/Mistlberger 14, Duhr/Gehrmann/Jaquier 14]
- ▶ (1-loop single-real)² [Anastasiou/Duhr/Dulat/Herzog/Mistlberger 13, Kilgore 13]

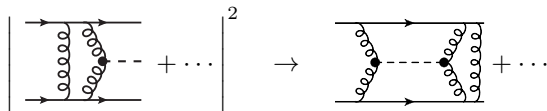
Known contributions

- ▶ soft limit for $H + 2$ partons
[Anastasiou/Duhr/Dulat/Furlan/Gehrmann/Herzog/Mistlberger 14, Li/von Manteuffel/Schabinger/Zhu 14]
- ▶ soft triple real contr. for $gg \rightarrow H$ [Anastasiou/Duhr/Dulat/Mistlberger 13]
- ▶ double-real-virtual contr. [Anastasiou/Duhr/Dulat/Furlan/Herzog/Mistlberger 15]
- ▶ leading terms in threshold exp.
[Anastasiou/Duhr/Dulat/Furlan/Gehrmann/Herzog/Mistlberger 14, Li/von Manteuffel/Schabinger/Zhu]
- ▶ > 30 terms in threshold exp. [Anastasiou/Duhr/Dulat/Herzog/Mistlberger 15]
→ sufficient for phenomenology! → Babis Anastasiou's talk

→ cross check!

Computation — unitarity and IBP

use unitarity & the optical theorem to relate



phase-space integrals and loop integrals [Melnikov, Anastasiou 02],

$$\delta_+(p^2) \leftrightarrow \frac{1}{p^2 + i\epsilon} - \frac{1}{p^2 - i\epsilon}$$

i.e. phase-space int \leftrightarrow cut loop integrals

- ▶ integration by parts (IBP) relations \rightarrow reduction to master integrals
- ▶ differential equations \rightarrow compute master integrals

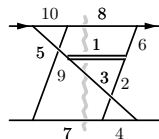
\rightarrow cuts act as filters on the right

Diagrams \rightarrow Master Integrals

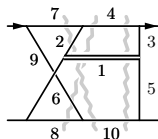
computation of qq' -channel:

- ▶ Diagram generation with QGRAF [Nogueira] + filtering for available cuts
 \rightarrow 220 diagrams
- ▶ (IBP) reduction to master integrals (TopoID, rows [Pak,Hoff])
 \rightarrow 332 master integrals (17 families)
- ▶ minimization across families
 \rightarrow 111 master integrals

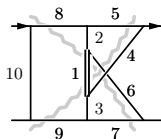
\rightarrow need 108 “Laporta master integrals” in 16 families:



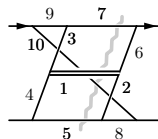
BT1



BT2



BT3



BT4

...

Check: general gauge parameter ξ vanishes

Complexity of the calculation

Number of diagrams at N³LO

Channel	Number of diagrams (fermionic loops)			
qq'	#220	= #216	+ #4 n_l	
qq	#404	= #396	+ #8 n_l	
$q\bar{q}$	#4 889	= #4 438	+ #445 n_l	+ #6 n_l^2
gg	#9 591	= #8 976	+ #612 n_l	+ #3 n_l^2
gg_{virt}	#9 538	= #7 266	+ #2 180 n_l	+ #92 n_l^2
gg_{real}	#150 246	= #128 676	+ #21 196 n_l	+ #374 n_l^2
Σ	#174 938	= #150 014	+ #24 449 n_l	+ #475 n_l^2

reduction:

Number of	LO	NLO	NNLO	N ³ LO
Diagrams	1	50	2 946	174 938
Topologies	0	1	11	188
Master integrals	0	1	20	$\mathcal{O}(300)$

Canonical bases

transform (“Laporta”) master integrals into a canonical basis [Henn 13]:

$$\partial_x \tilde{f}(x, \varepsilon) = \tilde{A}(x, \varepsilon) \tilde{f}(x, \varepsilon) \quad \rightarrow \quad \partial_x f(x, \varepsilon) = \varepsilon A(x) f(x, \varepsilon)$$

using methods developed by [Cachazo 08, Arkani-Hamed et al. 12, Henn et al. 13-14, Höschele/Hoff/Ueda 14], one finds for qq' :

$$A(x) = \frac{a}{1-x} + \frac{b}{1+x} + \frac{c}{x} + \frac{d}{1+4x} + \frac{e}{x\sqrt{1+4x}}$$

→ generic solution:

$$f(x, \varepsilon) = \mathbf{P} e^{\varepsilon \int dx A(x)} f_0(\varepsilon)$$

with boundary condition $f_0 \rightarrow$ taken from soft limit ($x \rightarrow 1$).

relations among the boundary values

The (canonical) DE implies relations among boundary values.
Explicit computation of the soft limit ($y := (1 - x) \rightarrow 0$) gives

$$\tilde{f}(x, \varepsilon) = \sum_{m=0}^{\infty} \varepsilon^m \sum_{n=0}^{\infty} y^n \left[y^{a_1 \varepsilon} \tilde{c}_{mn}^{(1)} + y^{a_2 \varepsilon} \tilde{c}_{mn}^{(2)} + \dots \right]$$

due to change of basis:

soft limit \rightarrow canon. basis \rightarrow matching \rightarrow reduction basis

\rightarrow relations among boundary conditions

- ▶ for some masters $\tilde{f}_i(x, \varepsilon) \xrightarrow{x \rightarrow 1} (\text{free of } \tilde{c}_{imn}^{(k)})$
 \rightarrow simple replacements
- ▶ simple relations among different orders
 \rightarrow higher orders in ε in terms of lower orders
 \rightarrow higher orders in y in terms of lower orders

Resummation of $\log(y)$ -terms

- ▶ necessary for cancelling soft poles in (real + virtual)
- ▶ useful to derive additional relations [Dulat, Mistlberger '14]
- ▶ idea: solve soft part to all orders

$$\partial_x f = \frac{\varepsilon A}{1-x} f + O((1-x)^0); \quad A = R J R^{-1}$$

$$f = R y^{-\varepsilon \Lambda} \exp\left\{-(J - \Lambda)\varepsilon \log(y)\right\} f_R; \quad \Lambda = \text{diag}(\lambda_1, \dots, \lambda_n)$$

where f_R is regular in the limit $y \rightarrow 0$.

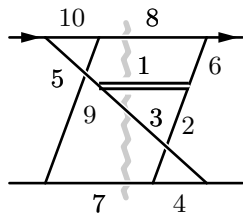
→ canonical basis guarantees: $(1-x)^{-1}$ is lowest order in soft DE

Boundary conditions

Boundary conditions are taken from the soft limit: $x = \frac{m_H^2}{s} \rightarrow 1$
Need soft expansion of the (3- and 4-particle) phase space integrals

method for 4-particle cut diagrams from [\[Anastasiou et al. 13\]](#):

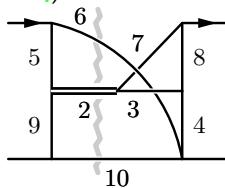
- ▶ reversed unitarity \rightarrow expansion of phase space measure
- ▶ higher powers of propagators \rightarrow IBP reduction (FIRE [\[Smirnov²\]](#))
- ▶ 11 master integrals (10 are known [\[Anastasiou et al. 13\]](#))
- ▶ introduce Mellin-Barnes integrals to simplify denominators
- ▶ perform energy and angular integrals \rightarrow more Mellin-Barnes integrals
- ▶ simplify using barnesroutines [\[Kosower\]](#)
- ▶ expand in ε using MB [\[Czakon\]](#)



Boundary conditions

method for 3-particle cut diagrams (sim. to 4-pc):

- ▶ perform loop-integral in terms of Mellin-Barnes representations
- ▶ parameterization in terms of energies and angles
- ▶ soft limit $x \rightarrow 1$ (MBasymptotics [Czakon])
- ▶ introduce Mellin-Barnes integrals to simplify denominators
- ▶ perform energy and angular integrals \rightarrow more Mellin-Barnes integrals
- ▶ simplify using barnesroutines [Kosower], MB [Czakon] and standardizations of contours
- ▶ residue theorem \rightarrow hypergeometric series
- ▶ convert to nested sums (algorithm by [Anzai/Sumino 12])
 \rightarrow zeta values



Results

$$\begin{aligned}\tilde{\sigma}_{qq'}^{(3)} = & \frac{1}{54}(x-1)(400x^2 - 1019x + 35945)H_1(x)H_0^2(x) \\ & - \frac{35}{9}(x-2)^2H_{0,-1,s4}(x)H_0^2(x) + \frac{35}{216}(118x+85)\sqrt{4x+1}H_0^2(x) \\ & + \frac{8}{9}(13x^2+31x+30)\zeta_3H_0^2(x) - \frac{64}{3}(x-2)^2H_{0,1,0,1,-1}(x) + \dots\end{aligned}$$

- ▶ $\zeta_2, \zeta_3, \zeta_5, \log(2), \text{Li}_4\left(\frac{1}{2}\right)$
- ▶ HPLs up to weight 5
- ▶ polynomials in $x, \sqrt{1+4x}$
- ▶ Iterated integrals over $\left\{ \frac{1}{x}, \frac{1}{1+x}, \frac{1}{x} \left(\frac{1}{\sqrt{1+4x}} - 1 \right) \right\}$
0 -1 s4

Iterated Integrals

Packages used for iterated integrals: HPL [Maitre], HarmonicSums [Ablinger]

There are iterated integrals with square roots, e.g.

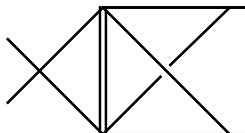
$$H_{0,-1,s4}(x) = \int_0^x dy_1 \frac{1}{y_1} \int_0^{y_1} dy_2 \frac{1}{1+y_2} \int_0^{y_2} dy_3 \frac{1}{y_3} \left(\frac{1}{\sqrt{1+4y_3}} - 1 \right)$$

representable as mult. polylogs with $e_3 = e^{2i\pi/3}$

$$\begin{aligned} H_{0,-1,s4}(x) = & -8H_1 \left(\frac{2x - \sqrt{4x+1} + 1}{2x} \right) \operatorname{Re} \left[H_{1,e_3} \left(\frac{2x - \sqrt{4x+1} + 1}{2x} \right) \right] \\ & + 4 \operatorname{Re} \left[H_{0,e_3,1} \left(\frac{2x - \sqrt{4x+1} + 1}{2x} \right) \right] + 16 \operatorname{Re} \left[H_{1,1,e_3} \left(\frac{2x - \sqrt{4x+1} + 1}{2x} \right) \right] \\ & - 4H_{0,1,1} \left(\frac{2x - \sqrt{4x+1} + 1}{2x} \right) + \frac{4}{3} H_1^3 \left(\frac{2x - \sqrt{4x+1} + 1}{2x} \right) \end{aligned}$$

Origin of the square roots

common subtopology:



Anticipate from DE in the reduction basis:

- ▶ compute matrix residue of A at $x_0 = -\frac{1}{4}$
$$\partial_x M = \dots \frac{A_{-1}(\varepsilon)}{1+4x} + O((1-x)^0)$$
- ▶ diagonalize the residue: $U^{-1}A_{-1}U = \text{diag}(\dots, \frac{1}{2} - 4\varepsilon)$
- ▶ entries $\frac{1}{2}$ imply the occurrence of $\frac{1}{\sqrt{x-x_0}}$
- ▶ helps in finding variable transformations

\Rightarrow canonical basis makes the letter explicit.

Conclusions

- ▶ Higgs production by gluon fusion for $m_t \rightarrow \infty$ is important and interesting.
- ▶ Missing analytical pieces: real-real-virtual, triple-real
- ▶ We have calculated the NNNLO contribution from the qq' -channel, analytically, with exact dependence on $x = m_H^2/s$.
- ▶ Methods were presented to exploit canonical bases also for the computation of some boundary conditions.
- ▶ HPLs are not enough to represent the result \rightarrow square root letters.