Factorization for Massive Quark Initiated Jets
and
The Pythia Top Quark Mass Parameter

André H. Hoang

University of Vienna

Loops and Legs 2016, Leipzig, April 28, 2016

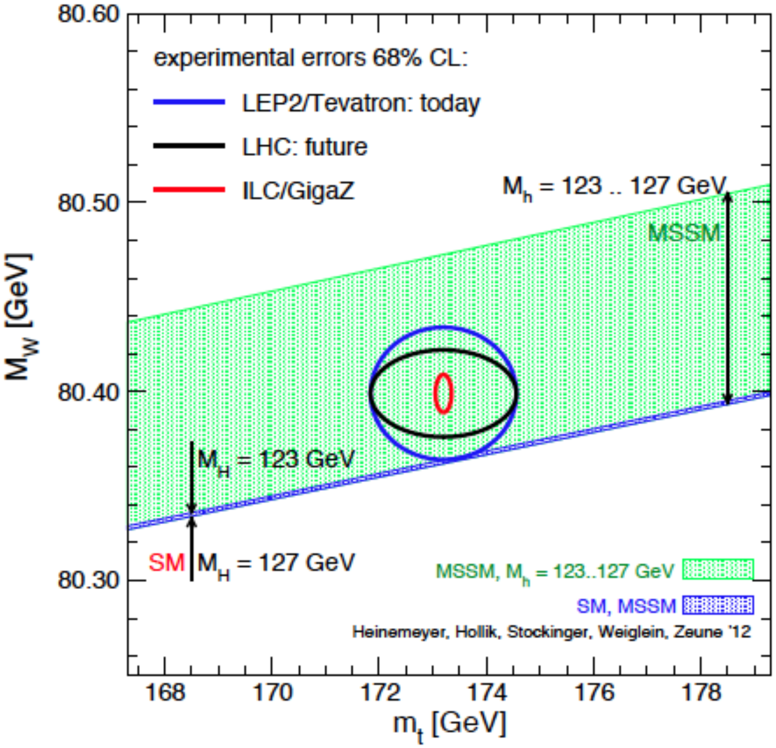
Outline

- Introduction
- Top mass and Renormalization Schemes
- Basic methods for top mass measurements
- Monte Carlo generators and the top quark mass
- Calibration of the Monte Carlo top mass parameter
- Preliminary results of first serious analysis

In collaboration with:
M. Butenschön
B. Dehnadi,
V. Mateu,
M. Preisser
I. Stewart

Why Precision Top Mass?

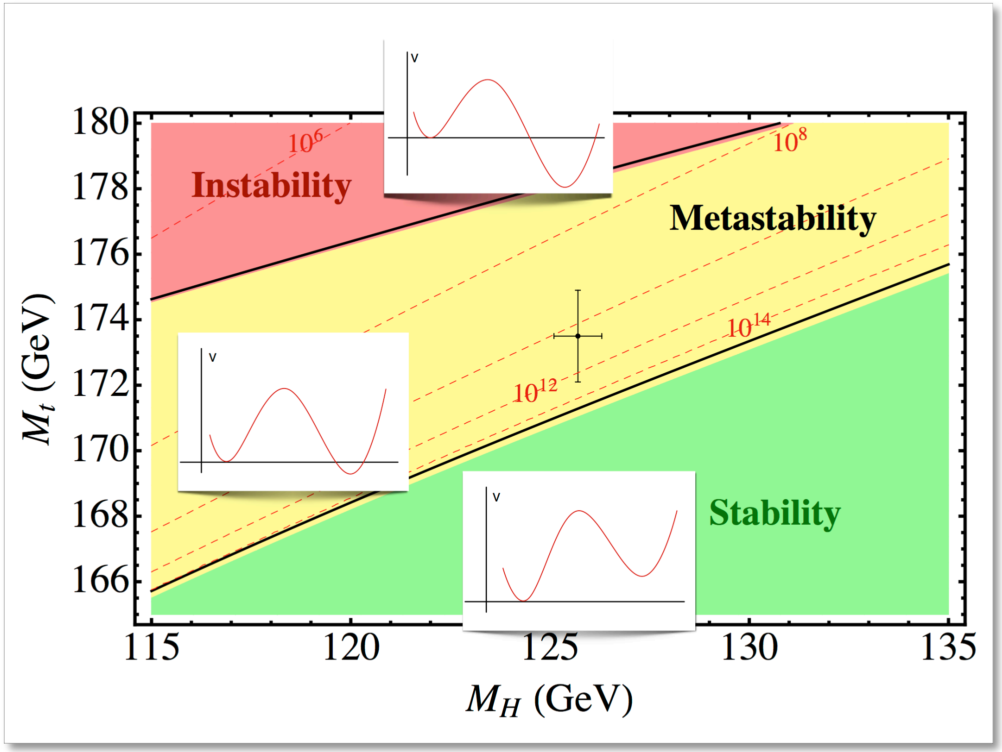
Indirekt search for new physics:



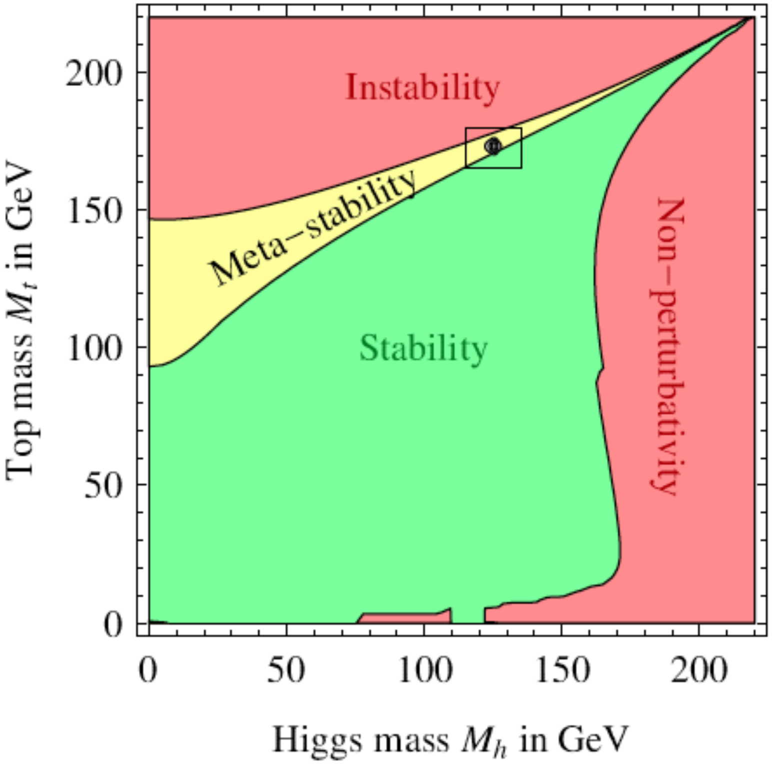
- Top quark mass crucial ingredient for global fits within the Standard Model
- Largest impact on indirect evidence for the Higgs
- Relations among electroweak precision observables put stringent constraints on BSM models

Why Precision Top Mass?

Role in the fate of our universe (?):



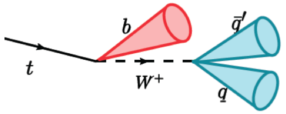
- Strong dependence of SM Higgs potential on top quark mass
- Existence of a lower energy vacuum state (if calculations apply)
- Dependence on higher-dimension Higgs interactions



Top Mass Measurements Methods

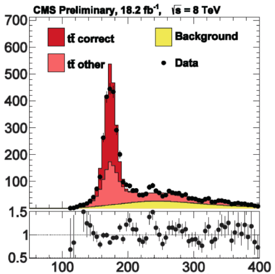
LHC+Tevatron

Direct Reconstruction:

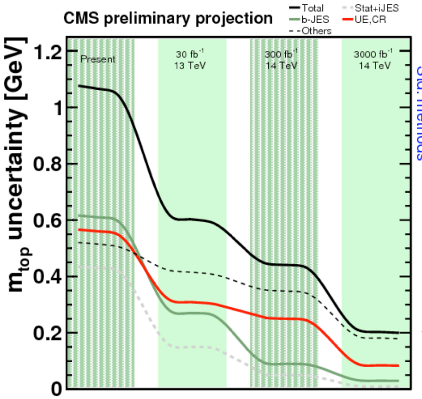


kinematic mass
determination

CMS-PAS-TOP-14-002



4356 events



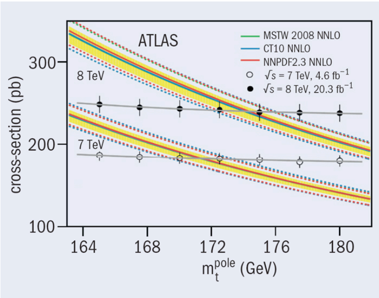
- ⊕ High top mass sensitivity
- ⊖ Precision of MC ?
- ⊖ Meaning of m_t^{MC} ?

Δ m_t ~ 0.5 GeV

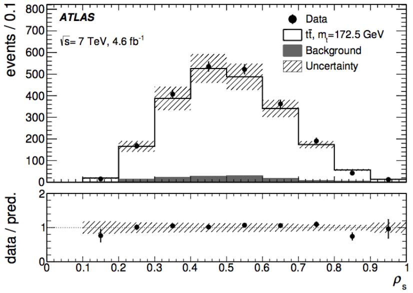
Δ m_t ~ 200 MeV (projection)

Indirect Mass Fit:

global mass
dependence



Total cross section



tt+jet invariant mass

- ⊕ pQCD calculations dominate
- ⊕ Control of mass scheme
- ⊖ Lower top mass sensitivity
- ⊖ High sensitivity to norm errors

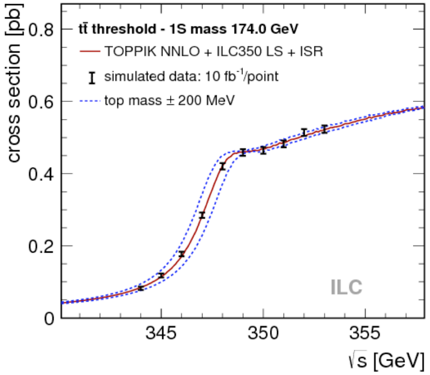
Δ m_t ~ 1-2 GeV

Future Linear Collider:

Top Pair Threshold:

kinematic mass
determination

perturbative
toponium



- ⊕ High top mass sensitivity
- ⊕ pQCD calculations dominate
- ⊕ Control of mass scheme

Δ m_t ~ 100 MeV

Top Mass Measurements Methods

LHC+Tevatron

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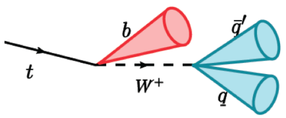
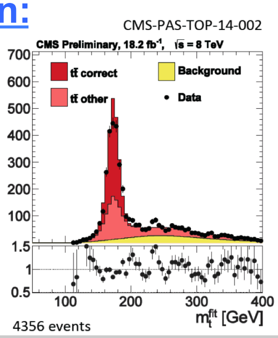
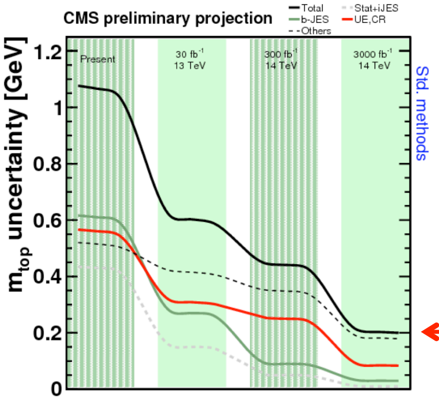


kinematic mass determination





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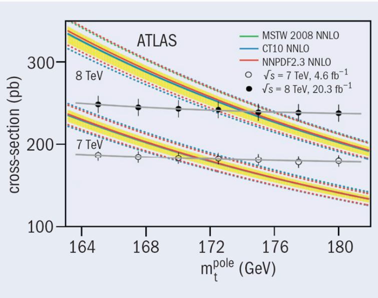
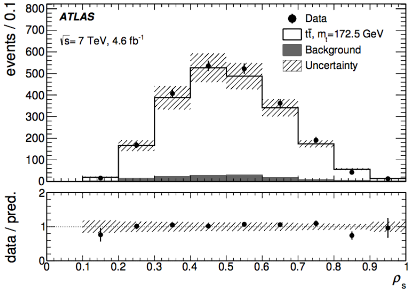


Total cross section



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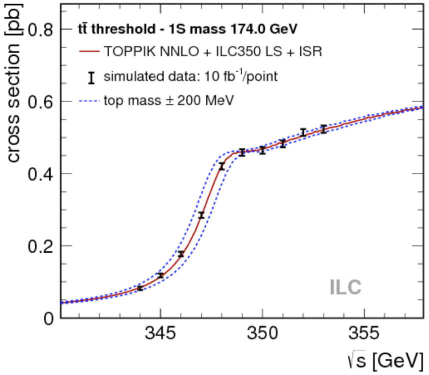

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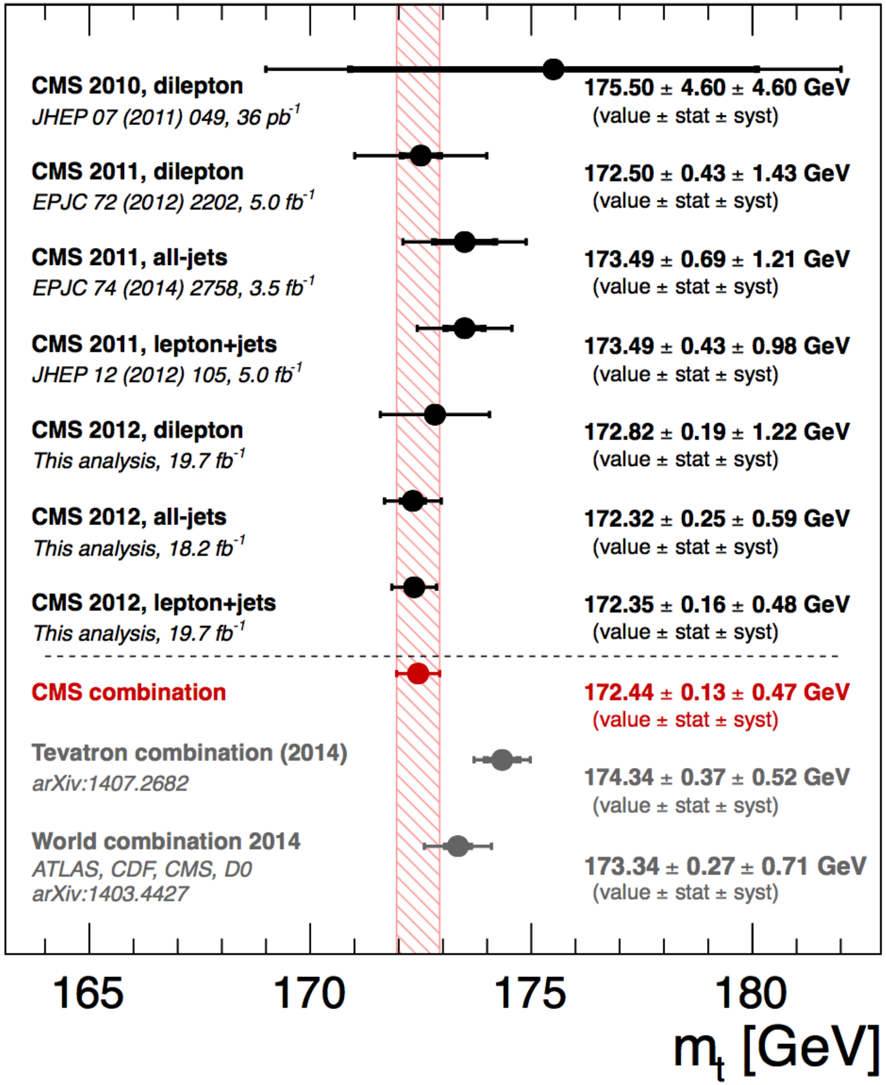
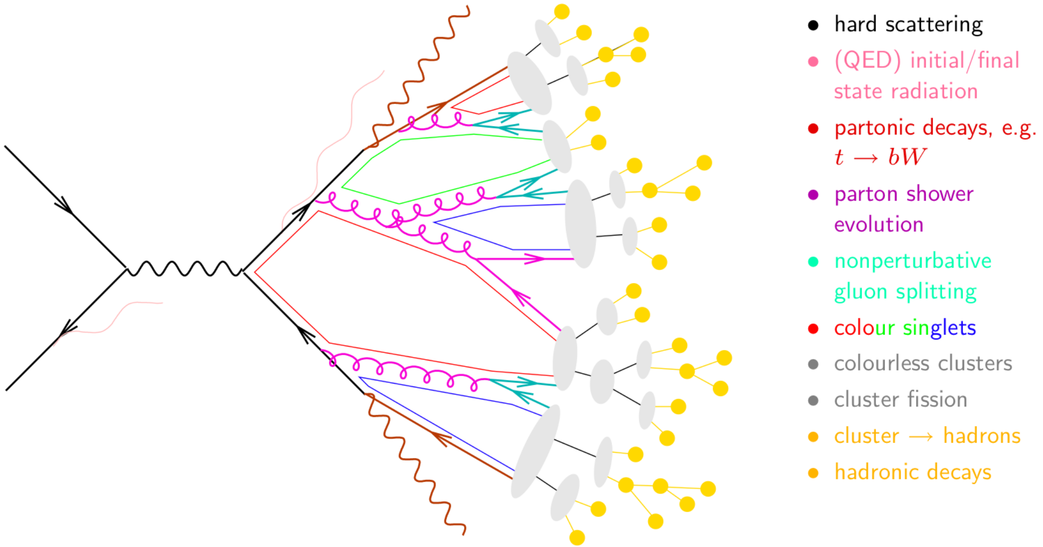
- ⊕ Control of mass scheme

Δ m_t ~ 100 MeV

Most Recent CMS Reconstruction Result



Monte-Carlo Event Generators



- Full simulation of all processes (all experimental aspects accessible)
- QCD-inspired: partly first principles QCD ⇔ partly model
- Description power of data better than intrinsic accuracy. (But how precise?)
- Top quark: treated like a real particle ($m_t^{MC} \approx m_t^{pole}$ +?).

m_t^{MC} has different meaning for different types of observables (σ_{tot} vs. $d\sigma/dM_{rec}$)

But pole mass ambiguous by O(1 GeV) due to confinement.

Better mass definition needed.

MC Top Quark Mass

AHH, Stewart 2008
AHH, 2014

$$m_t^{\rm MC} = m_t^{\rm MSR}(R=1\ {\rm GeV}) + \Delta_{t,{\rm MC}}(R=1\ {\rm GeV})$$

$$\Delta_{t,{\rm MC}}(1\ {\rm GeV}) \sim \mathcal{O}(1\ {\rm GeV})$$

- small size of $\Delta_{t,\rm MC}$
- Renormalon-free
- little parametric dependence on other parameters

MSR Mass Definition

MS Scheme: $(\mu > \overline{m}(\overline{m}))$

$$\overline{m}(\overline{m}) - m^{\rm pole} = -\overline{m}(\overline{m})\,[0.42441\,\alpha_s(\overline{m}) + 0.8345\,\alpha_s^2(\overline{m}) + 2.368\,\alpha_s^3(\overline{m}) + \ldots]$$

$\Downarrow$

MSR Scheme: $(R < \overline{m}(\overline{m}))$

$$m_{\rm MSR}(R) - m^{\rm pole} = -R\,[0.42441\,\alpha_s(R) + 0.8345\,\alpha_s^2(R) + 2.368\,\alpha_s^3(R) + \ldots]$$

$$m_{\rm MSR}(m_{\rm MSR}) = \overline{m}(\overline{m})$$

$\Rightarrow$ $m_{\rm MSR}(R)$ Short-distance mass that smoothly interpolates all R scales

universität wien

Calibration of the MC Top Mass

Method:

✓ 1) Strongly mass-sensitive observable (must be closely related to reconstructed invariant mass distribution !)

✓ 2) Accurate analytic hadron level QCD predictions at ≧ NLL/NLO with full control over the quark mass scheme dependence.

✓ 3) QCD masses as function of m_t^{MC} from fits of observable.

4) Cross check observable independence (e^+e^-, DIS, pp)

$$m_t^{\rm MC} = m_t^{\rm MSR}(R = 1~\text{GeV}) + \Delta_{t,\rm MC}(R = 1~\text{GeV})$$

$$\Delta_{t,\rm MC}(1~\text{GeV}) = \bar{\Delta} + \delta\Delta_{\rm MC} + \delta\Delta_{\rm pQCD} + \delta\Delta_{\rm param}$$

Monte Carlo errors:

- different tunings
- parton showers
- color reconnection
- Intrinsic error, …

QCD errors:

- perturbative error
- scale uncertainties
- electroweak effects

Parametric errors:

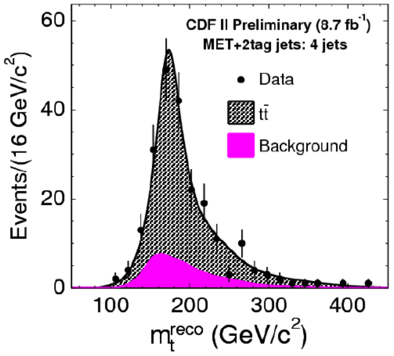
- strong coupling α_s
- Non-perturbative parameters

Thrust Distribution

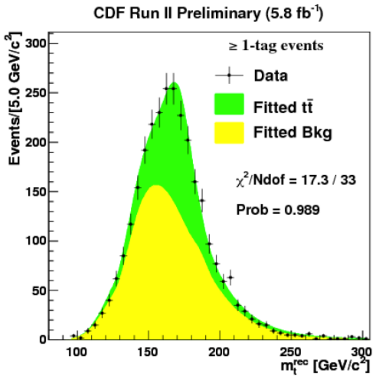
Observable: 2-jettiness in e+e- for $Q \sim p_T \gg m_t$ (boosted tops)

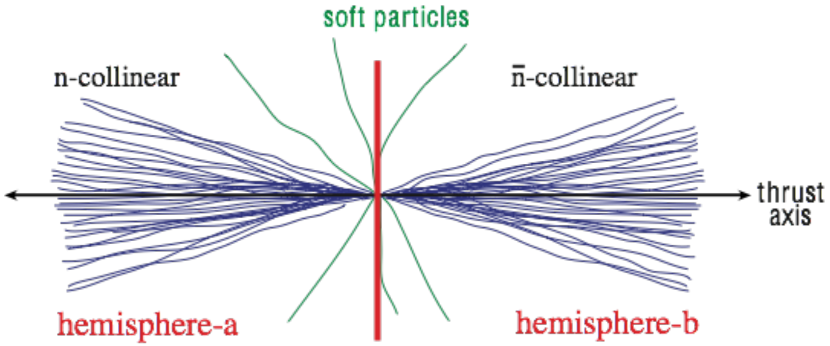
$$\tau \;=\; 1 - \max_{\vec{n}} \frac{\sum_i |\vec{n}\cdot\vec{p}_i|}{Q}$$

$$\overset{\tau\to 0}{\approx} \; \frac{M_1^2 + M_2^2}{Q^2}$$

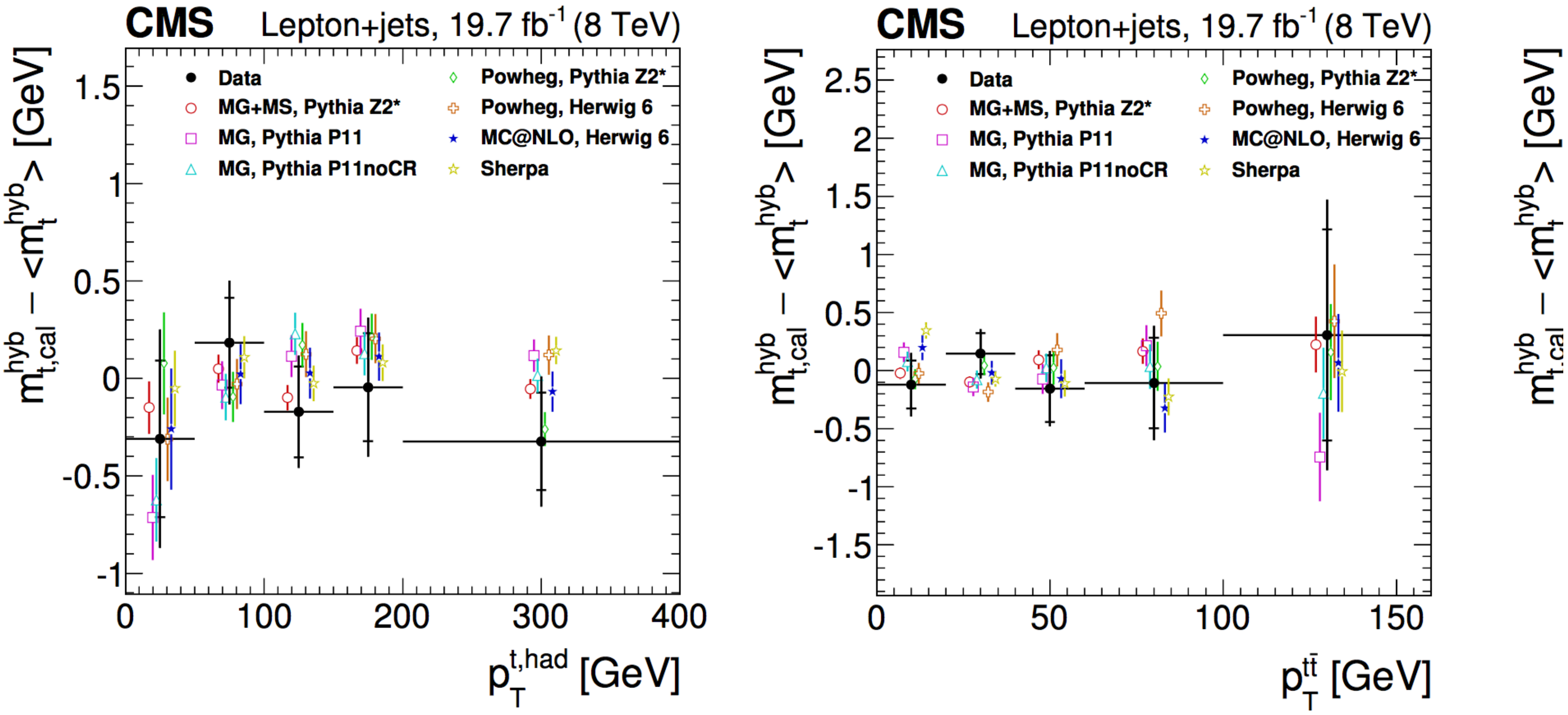


Invariant mass distribution in the resonance region of wide hemisphere jets → "event-shapes"



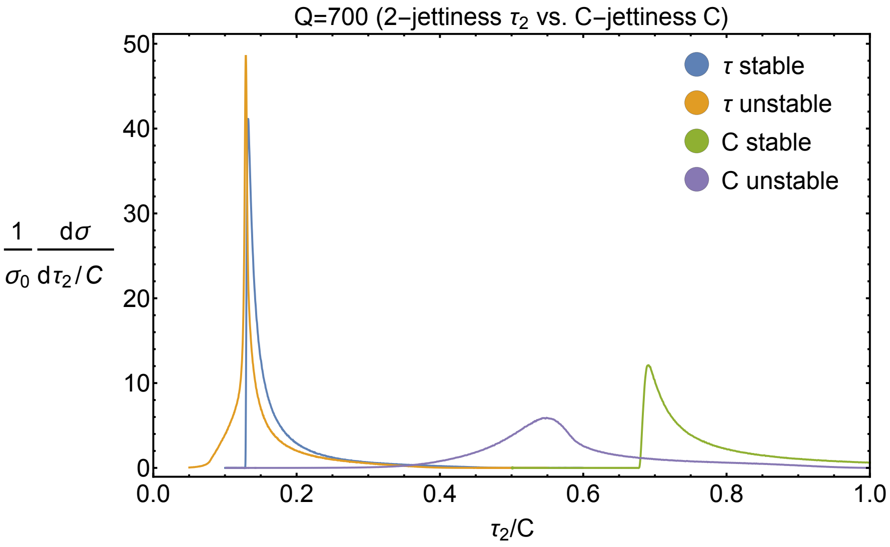


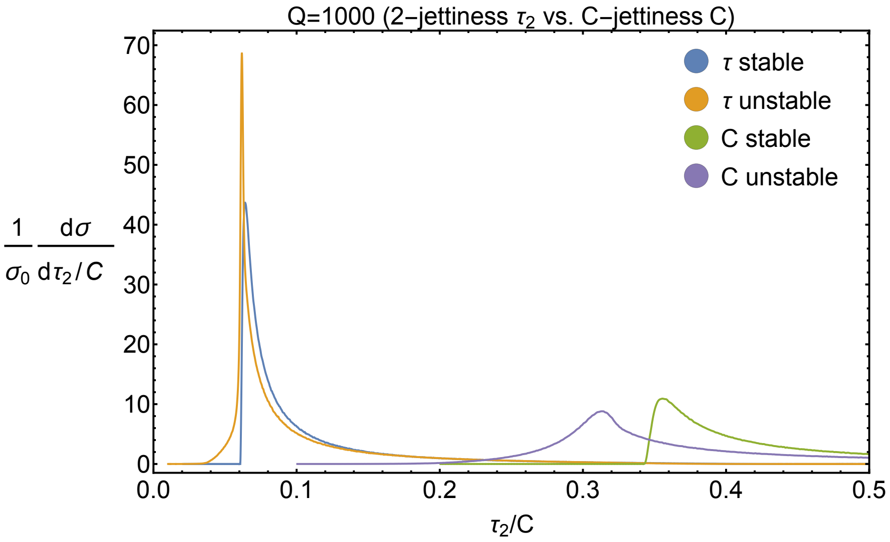
Boosted Top Mass Measurements at CMS

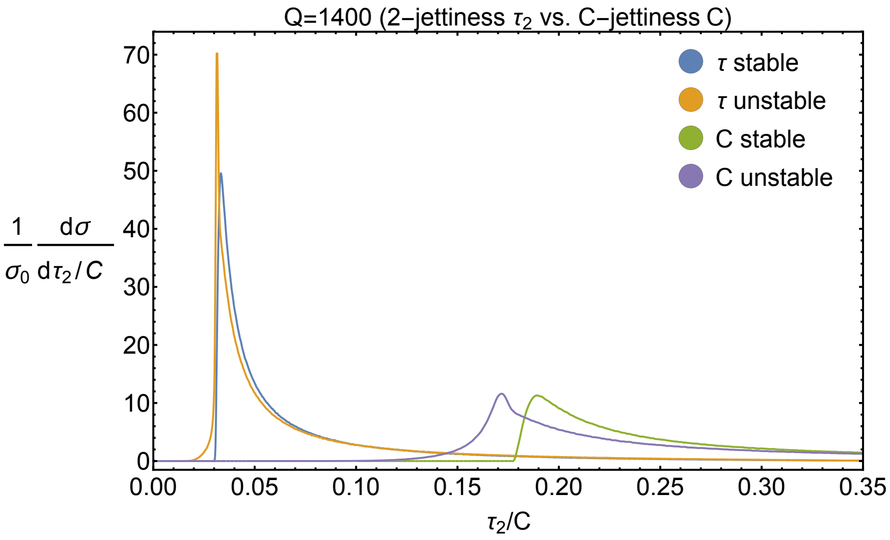


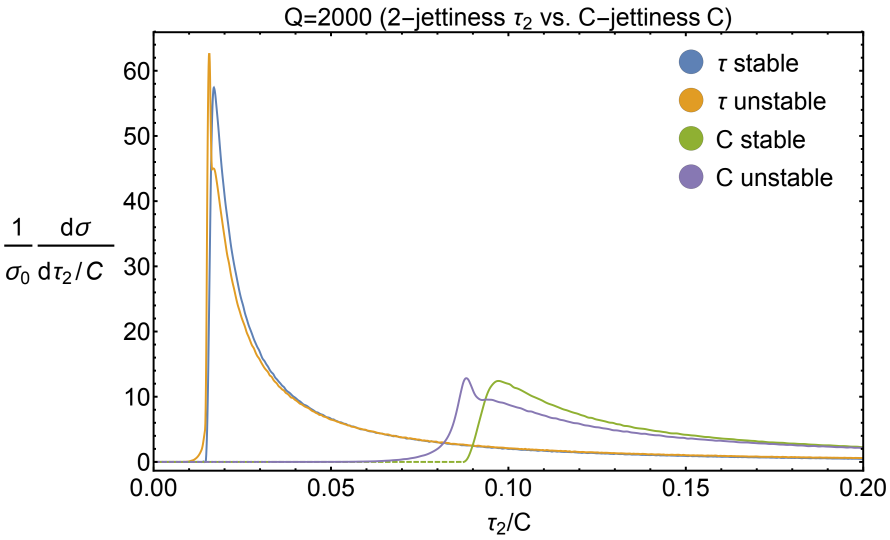
- Top mass from reconstruction of boosted tops consistent with low p_T results.
- More precise studies possible with more statistics from Run2.

Event Shape Distributions (Pythia 8.2)









Factorization for Event Shapes

$$\frac{d\sigma}{d\tau} = Q^2\sigma_0 H_0(Q,\mu)\int d\ell\; J_0(Q\ell,\mu)\, S_0\,(Q\tau-\ell,\mu)$$

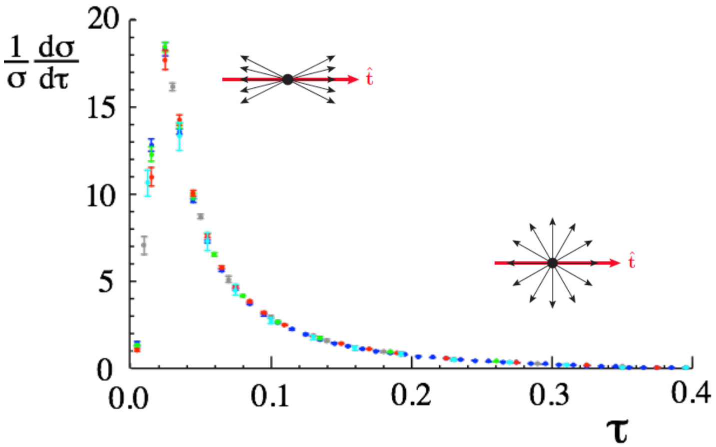
Massless quarks:

Korshemski, Sterman 1995-2000

Bauer, Fleming, Lee, Sterman (2008)

Becher, Schwartz (2008)

Abbate, AHH, Fickinger, Mateu, Stewart 2010



Extension to massive quarks:

- VFNS for final state jets (with massive quarks): log summation incl. mass
- Boostet fat top jets

Fleming, AHH, Mantry, Stewart 2007

Gritschacher, AHH, Jemos, Mateu Pietrulewicz 2013-2014

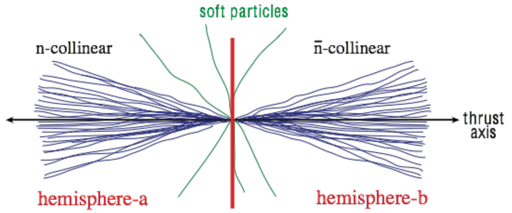
Butenschön, Dehnadi, AHH, Mateu 2016 (to appear soon)

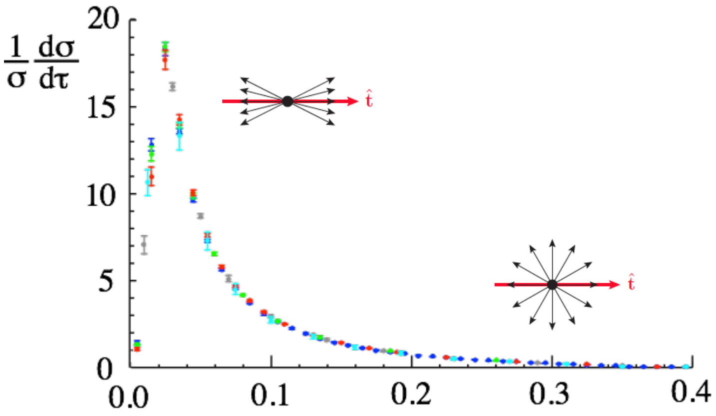
➡ NNLL + NLO + non-singular + hadronization + renormalon-subtraction

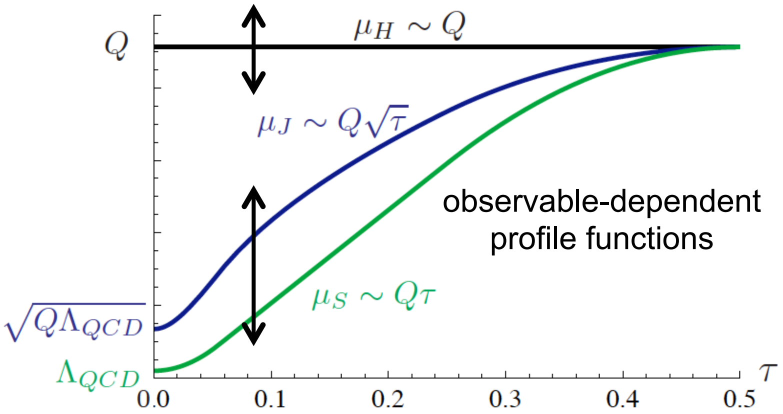
Factorization for Massless Quarks

$$\frac{d\sigma}{d\tau} = Q^2 \sigma_0 H_0(Q,\mu) \int d\ell \; J_0(Q\ell,\mu)\, S_0\left(Q\tau - \ell, \mu\right)$$

Korshemski, Sterman
Schwartz
Fleming, AH, Mantry, Stewart
Bauer, Fleming, Lee, Sterman





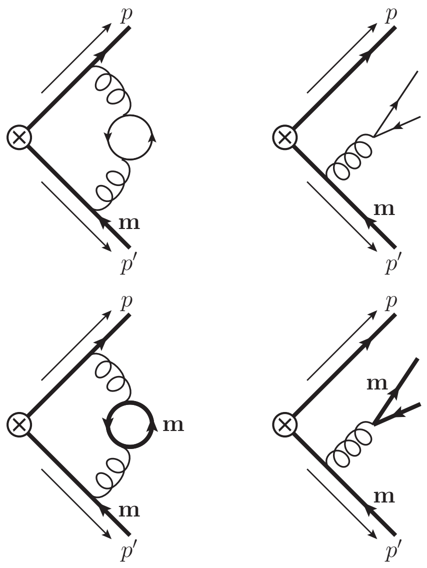


observable-dependent
profile functions

Abbate, AH, Fickinger, Mateu,
Stewart

$$\left(\frac{d\sigma}{d\tau}\right)^{\text{sing}}_{\text{part}} \sim \sigma_0 \, H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int d\ell d\ell' \, U_J(Q\tau - \ell - \ell', \mu_Q, \mu_s)\, J_T(Q\ell', \mu_j)\, S_T(\ell - \Delta, \mu_s)$$

VFN Scheme: Primary Massive Quarks

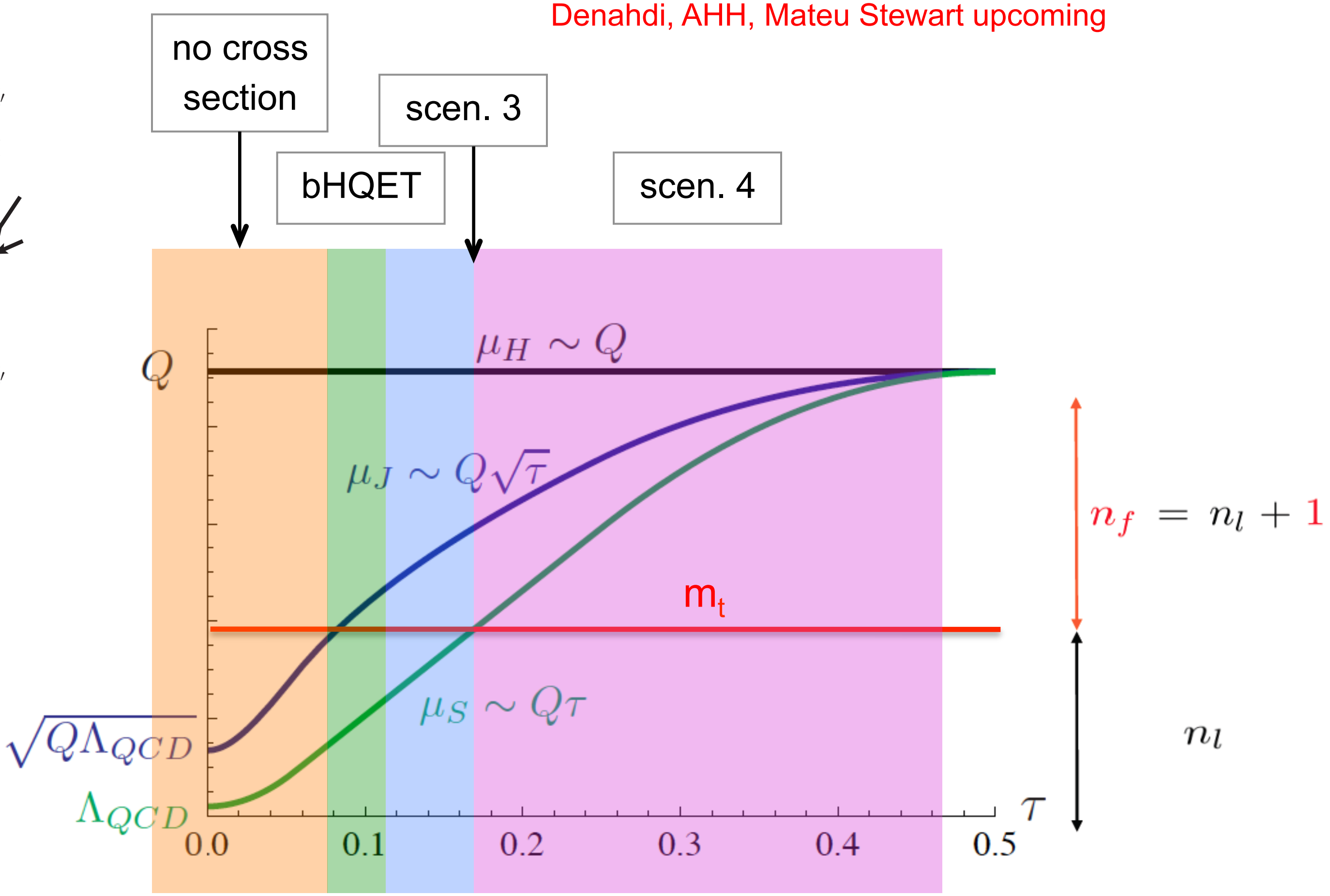


→ bHQET-type theory when
the jet scale approaches the quark mass

Fleming, AHH, Mantry, Stewart 2007

→ two SCET-type theories

Denahdi, AHH, Mateu Stewart upcoming

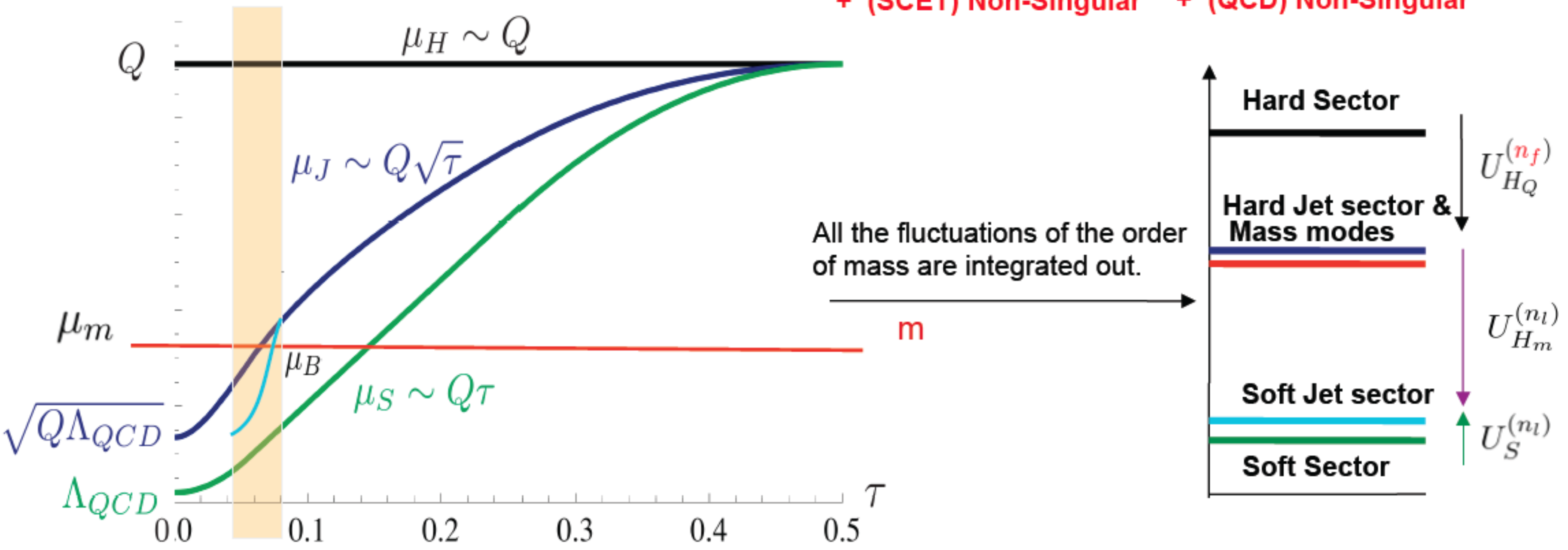


b(oosted)HQET

$$\left|\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}(\tau)}{\mathrm{d}\tau}\right|^{\mathrm{bHQET}} = Q\,H_Q^{(n_f)}(Q,\mu_Q)\,U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_m)\,\boxed{H_m^{(n_f)}(\overline{m}^{(n_f)},\mu_m)}\,U_{H_m}^{(n_l)}\Big(\frac{Q}{\overline{m}^{(n_l)}},\mu_m,\mu_B\Big) \qquad n_f = n_l + 1$$

$$\int \mathrm{d}s \int \mathrm{d}k\; B^{(n_l)}\Big(\frac{s}{m_J^{(n_l)}},\mu_B, m_J^{(n_l)}\Big)\,U_S^{(n_l)}(k,\mu_B,\mu_S)\,S_{\mathrm{part}}^{(n_l)}\Big(Q\tau - Q\tau_{\mathrm{MIN}} - \frac{s}{Q} - k,\mu_S\Big)$$

+ (SCET) Non-Singular **+ (QCD) Non-Singular**



All the fluctuations of the order of mass are integrated out.

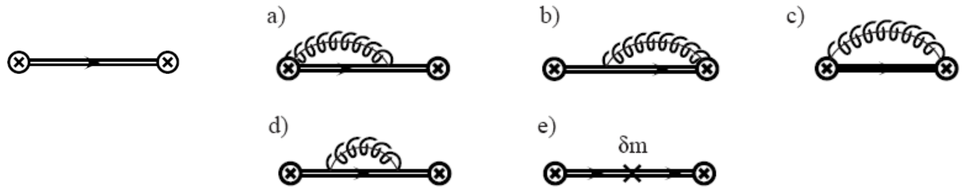
- Matching coefficient of SCET and bHQET have a large log from secondary corrections.

Reconstructed Top Jets (ILC)

bHQET jet function:

$$B_+(2v_+\cdot k) \;=\; \frac{-1}{8\pi N_c m}\,\mathrm{Disc}\int d^4x\, e^{ik\cdot x}\,\langle 0|\mathrm{T}\{\bar{h}_{v_+}(0)\overbrace{W_n(0)W_n^\dagger(x)}^{\text{Color singlet !}}h_{v_+}(x)\}|0\rangle$$

- perturbative, <u>any mass scheme</u>
- depends on m_t, Γ_t
- Breit-Wigner at tree level



$$B_\pm(\hat{s},\Gamma_t) = \frac{1}{\pi m_t}\frac{\Gamma_t}{\hat{s}^2+\Gamma_t^2} \qquad \hat{s} = \frac{M^2-m_t^2}{m_t}$$

- Describes soft cross talk of the top (and its decay b quark) with the anti-top (and its decay anti-b quark) in the <u>top rest frame</u>
- Soft function describes soft radiation in the <u>lab frame</u>

Jet function identical for boosted tops at hadron collisions (but differences in soft radiation in the lab frame)

Scenario III (SCET)

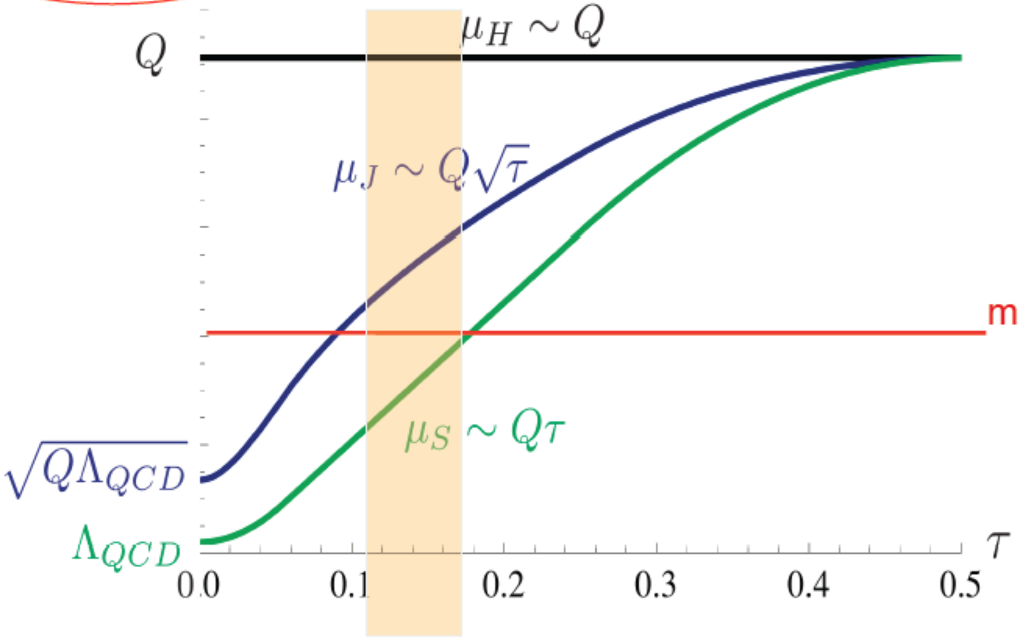
$$\left|\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}(\tau)}{\mathrm{d}\tau}\right|^{\mathrm{SCET-III}} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_J)\int \mathrm{d}s \int \mathrm{d}k\, \mathrm{d}k'\, \mathrm{d}k''\, J^{(n_f)}(s,\mu_J,\overline{m}^{(n_f)}(\mu_J))\, U_S^{(n_f)}(k,\mu_J,\mu_m)$$

$n_f = n_l + 1$

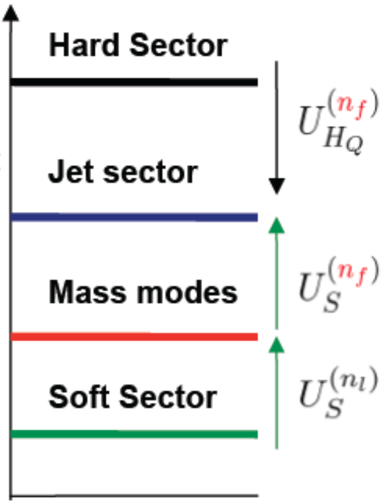
$$\boxed{\mathcal{M}_S^{(n_f)}(k'-k,\overline{m}^{(n_f)}(\mu_m),\mu_m,\mu_s)}\, U_S^{(n_l)}(k''-k',\mu_m,\mu_S)\, S_{\mathrm{part}}^{(n_l)}\Big(Q\tau - Q\tau_{\min} - \frac{s}{Q} - k'',\mu_S\Big)$$

large rapidity logs
$\alpha_s^2 \log \sim \alpha_s$

+ (QCD) Non-Singular



mass modes enter in the jet and hard sectors.


Hard Sector

$U_{H_Q}^{(n_f)}$

Jet sector

Mass modes

$U_S^{(n_f)}$

Soft Sector

$U_S^{(n_l)}$

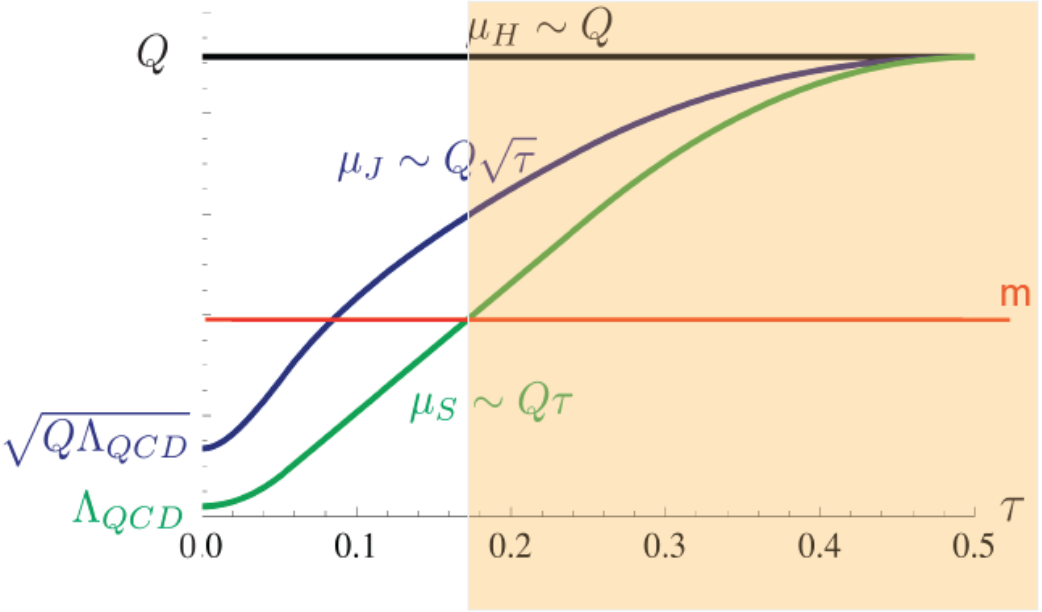
- Soft mass-mode matching: **integrating in** the **mass-mode (secondary)** effects in the **evolution of the soft function** (top-down resummation). $\mathcal{O}(\alpha_s^2)$

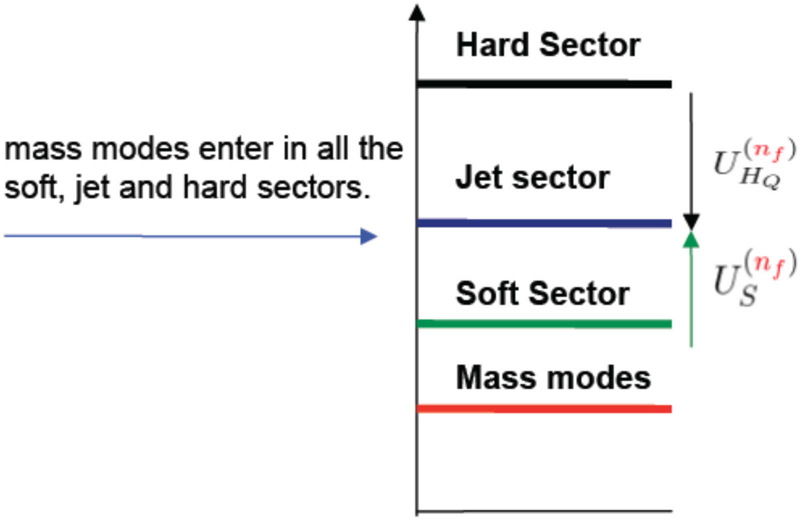
Scenario IV (SCET)

$$n_f = n_l + 1$$

$$\left|\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}(\tau)}{\mathrm{d}\tau}\right|^{\mathrm{SCET-IV}} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_J) \int \mathrm{d}s \int \mathrm{d}k\, J^{(n_f)}(s,\mu_J,\overline{m}^{(n_f)}(\mu_J))$$
$$U_S^{(n_f)}(k,\mu_J,\mu_S)\, S_{\mathrm{part}}^{(n_f)}\left(Q\tau - Q\tau_{\mathrm{min}} - \frac{s}{Q} - k,\mu_S\right)$$

+ (QCD) Non-Singular



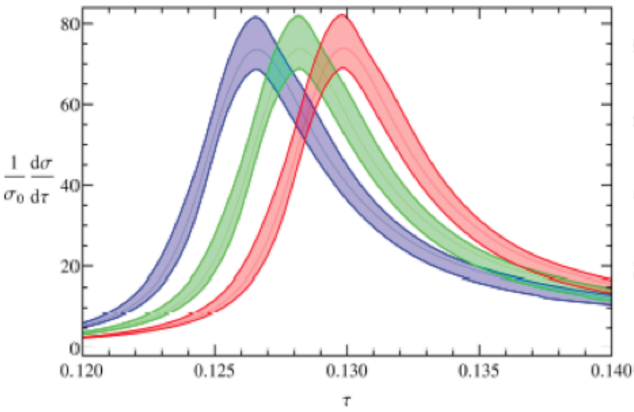

mass modes enter in all the soft, jet and hard sectors.

- ✓ (QCD) No-singular → Non-singular + Sub-leading singular contributions

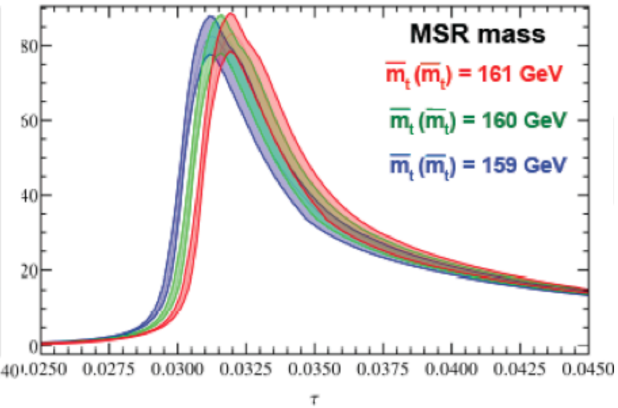
2-Jettiness for Top Production (QCD)

$$\frac{d\sigma}{d\tau_2} = f(\underbrace{m_t^{\mathrm{MSR}}(R)}_{\text{any scheme possible}}, \alpha_s(M_Z), \underbrace{\Omega_1, \Omega_2, \ldots}_{\text{Non-perturbative}}, \underbrace{\mu_h, \mu_j, \mu_s, \mu_m}_{\text{renorm. scales}}, \underset{\text{finite lifetime}}{R, \Gamma_t})$$

Q=700 GeV



Q=1400 GeV



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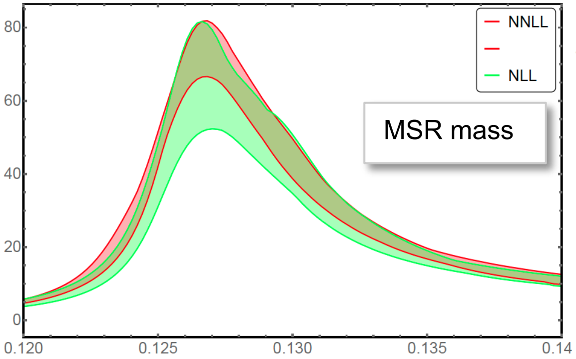

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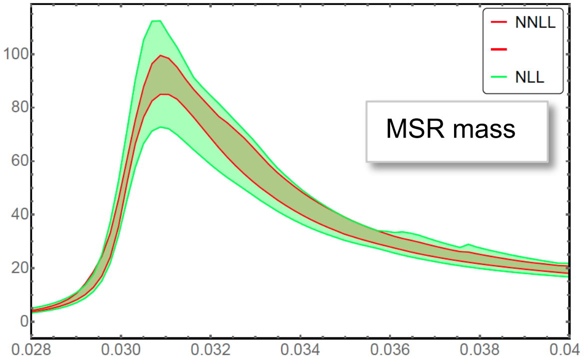


- Higher mass sensitivity for lower Q (p_T)
- Finite lifetime effects included
- Dependence on non-perturbative parameters
- Convergence: $\Omega_{1,2,\ldots}$

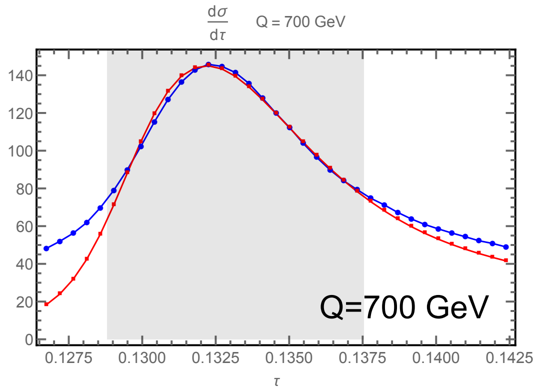
- Good convergence
- Reduction of scale uncertainty (NLL to NNLL)
- Control over whole distribution

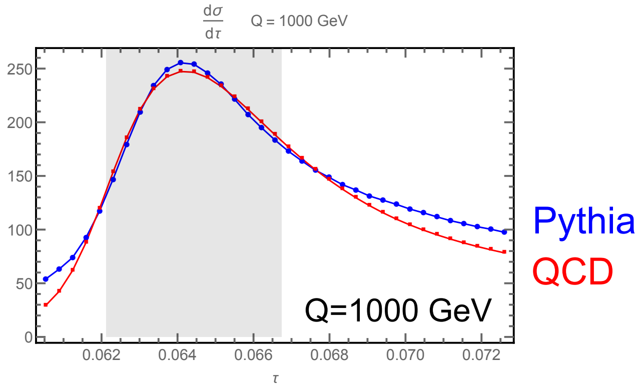
Fit Procedure Details

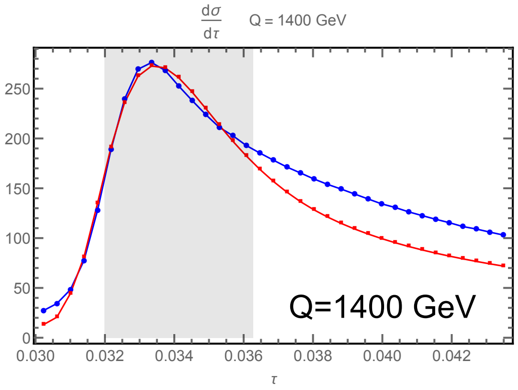
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_2} = f(\underbrace{m_t^{\mathrm{MSR}}(R)}_{\text{any scheme possible}}, \alpha_s(M_Z), \underbrace{\Omega_1, \Omega_2, \ldots}_{\text{Non-perturbative}}, \underbrace{\mu_h, \mu_j, \mu_s, \mu_m}_{\text{renorm. scales}}, R, \underset{\text{finite lifetime}}{\Gamma_t})$$

- Fit parameters: $\overbrace{m_t^{\mathrm{MSR}}(R),\ \alpha_s(M_Z),\ \Omega_1,\ \Omega_2,\ \ldots,}^{\text{QCD parameters measured from Pythia}}$
- Perturbative error: fits for 500 randomly picked sets of renor. scales
- Tunings: 1, 3, 7 (default)
- Top quark width: $\Gamma_t = \mathrm{dynamical\ (default)},\ 0.7,\ 1.4,\ 2.0\,\mathrm{GeV}$
- External smearing (Detector effects): $\Omega_{1,\mathrm{smear}} = 0,\ 0.5,\ \ldots,\ 3.0,\ 3.5,\ \mathrm{GeV}$
- Pythia masses: $m_t^{\mathtt{Pythia}} = 170,\ \ldots\ ,\ 175\,\mathrm{GeV}$
- Fit possible for any mass scheme

Preliminary Peak Fits





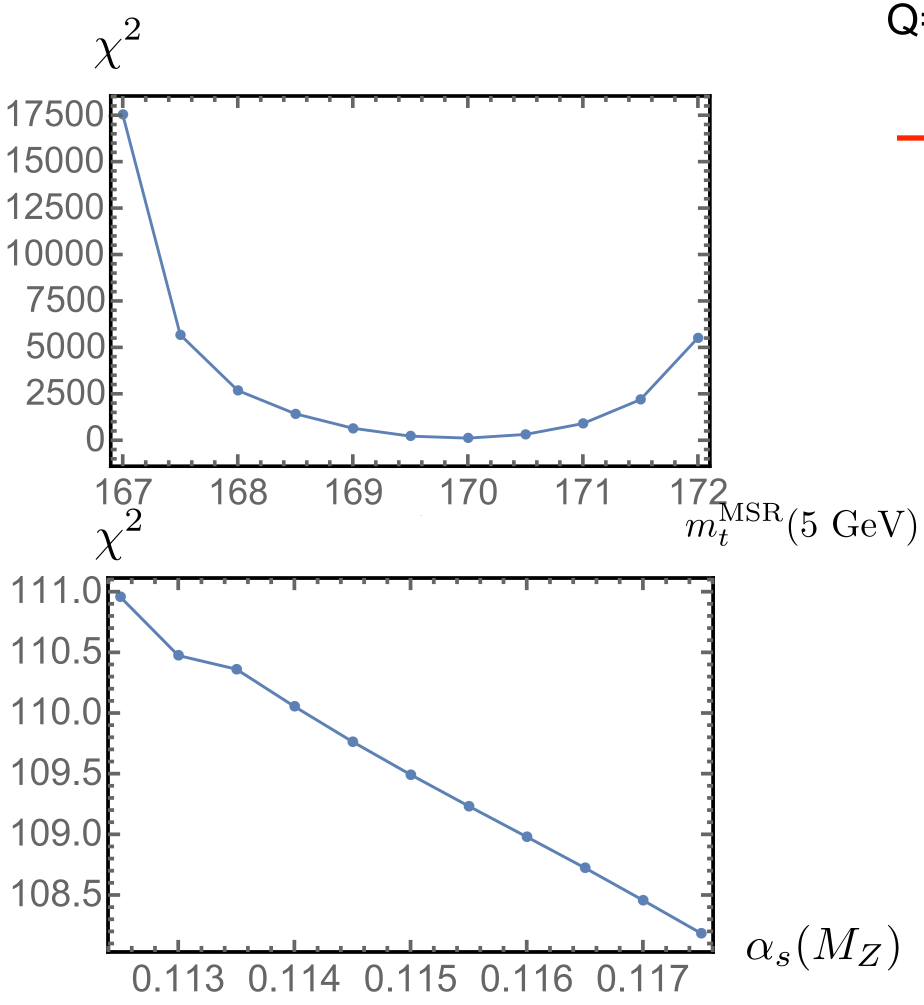


Default renormalization scales; Γ_t=1.4 GeV, tune 7, $\Omega_{1,smear}$=2.5 GeV, m_t^{Pythia}=171 GeV, Q={700, 1000, 1400} GeV, peak fit (60/80)%

- Good agreement of Pythia 8.2 with NNLL+NLO QCD description
- Pythia statistics: 10^6 events
- Discrepancies in distribution tail and for higher energies (Pythia is less reliable where fixed-order results valid, well reliable in soft-collinear limit)
- Excellent sensitivity to the top quark mass.

Preliminary

Peak Fits



Default renormalization scales; Γ_t=1.4 GeV, tune 7, $\Omega_{1,\mathrm{smear}}$=2.5 GeV, m_t^{Pythia}=171 GeV, Q={700, 1000, 1400} GeV, peak fit (60/80)%

→ χ^2_{min} ~ O(100)

- Very strong sensitivity to m_t
- Low sensitivity to strong coupling

- Take strong coupling as input
- χ^2_{min} and $\delta m_t^{\mathrm{stat}}$ do not have any physical meaning

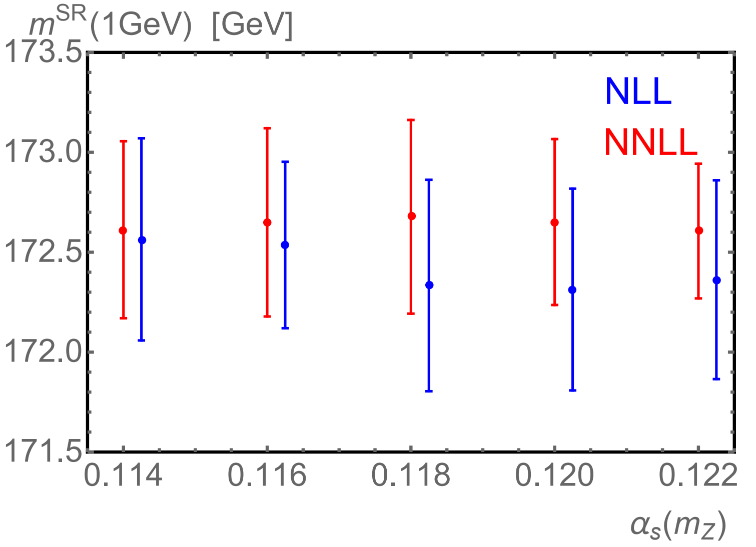
- We use rescaled χ^2/dof (PDG prescreption) to defind "intrinsic MC compatibility uncertainty"

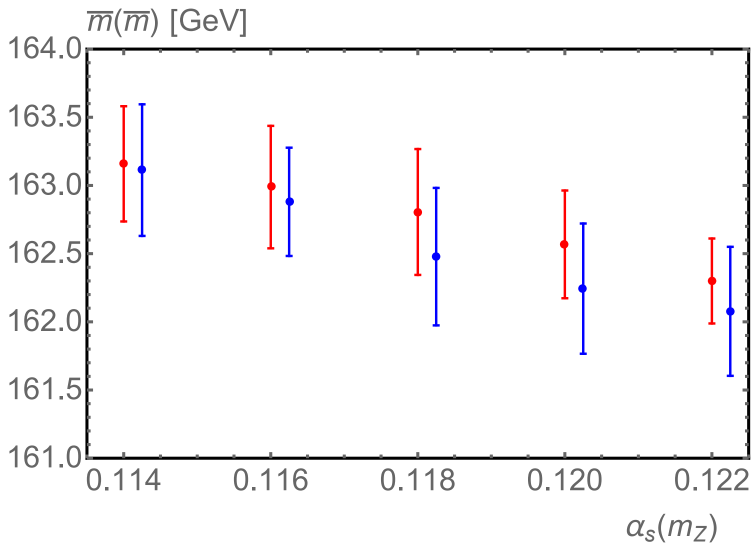
Preliminary

Peak Fits

First serious run: Γ_t=1.4 GeV, tunes 1, 3, 7,
$\Omega_{1,smear}$=1.5, 2.0, 2.5, 3.0, 3.5 GeV,
Q={700, 1000, 1400} GeV, peak fit (60/80)%
m_t^{Pythia}=173 GeV,
NLL: 177 scan survivors, NNLL: 254 scan survivors

Preliminary





- Very low sensitivity of m_t^{MSR}(5GeV) on $\alpha_s(M_z)$. ✔
- Large sensitivity of MSbar mass on $\alpha_s(M_z)$. ✔

MC top mass indeed closely related to m_t^{MSR}(R~1 GeV) !!

Peak Fits

First serious run: Γ_t=1.4 GeV, tunes 1, 3, 7,

$\Omega_{1,\mathrm{smear}}$=1.5, 2.0, 2.5, 3.0, 3.5 GeV,

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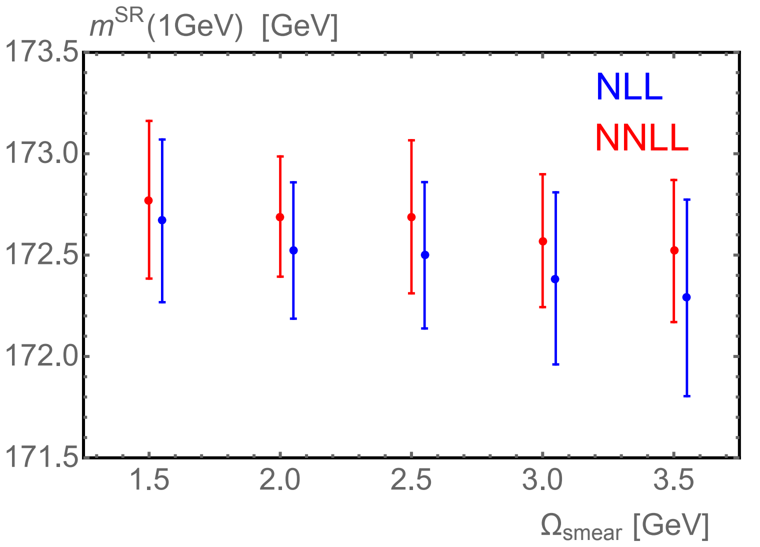
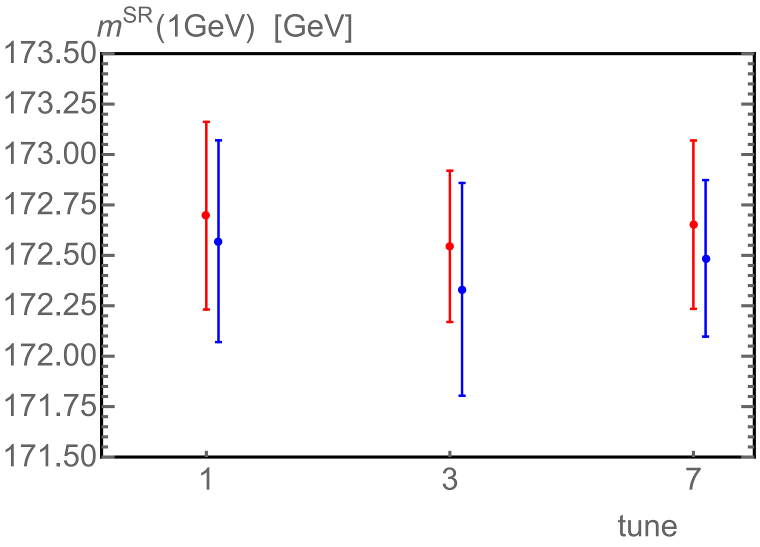
Preliminary





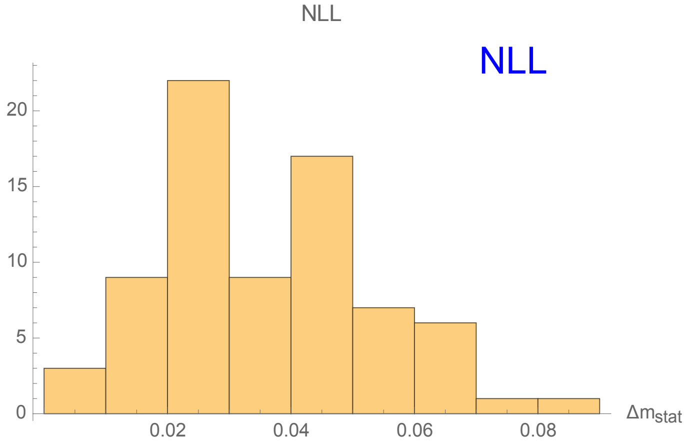
- "Detector effects" (~100 MeV) << perturbative uncertainty ($\lesssim$ 500 MeV). ✔
- MC tune dependence ($\lesssim$ 100 MeV) << perturbative uncertainty ($\lesssim$ 500 MeV). ✔

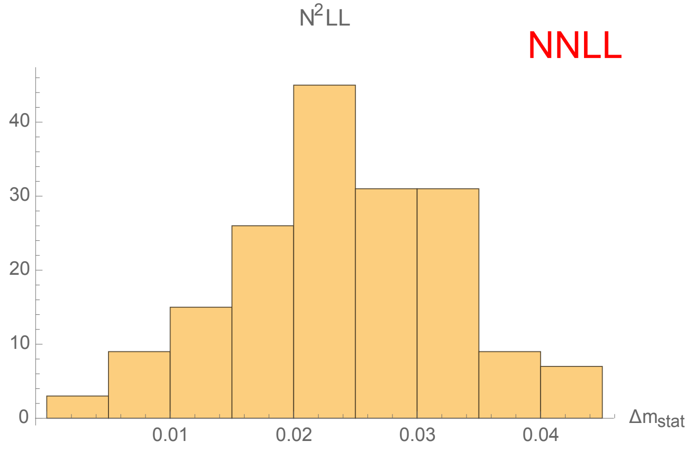
MC top mass indeed closely related to m_t^{MSR}(R~1 GeV) !!

Peak Fits

First serious run: Γ_t=1.4 GeV, tunes 1, 3, 7,
$\Omega_{1,\text{smear}}$=1.5, 2.0, 2.5, 3.0, 3.5 GeV,
Q={700, 1000, 1400} GeV, peak fit (60/80)%
m_t^{Pythia}=173 GeV,
NLL: 177 scan survivors, NNLL: 254 scan survivors

Preliminary





- "MC compatibility error" ~ tuning error ~ detector effect error ✔
- Effects are O(100) MeV. (Maybe represents for ultimate precision)

Peak Fits

First serious run: Γ_t=1.4 GeV, tunes 1, 3, 7,

$\Omega_{1,\text{smear}}$=1.5, 2.0, 2.5, 3.0, 3.5 GeV,

Q={700, 1000, 1400} GeV, peak fit (60/80)%

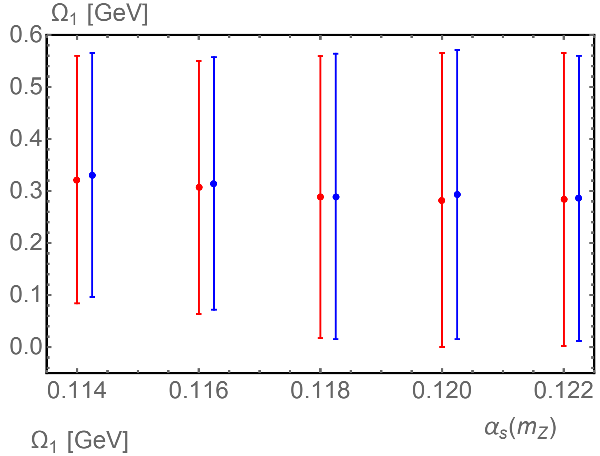
m_t^{Pythia}=173 GeV,

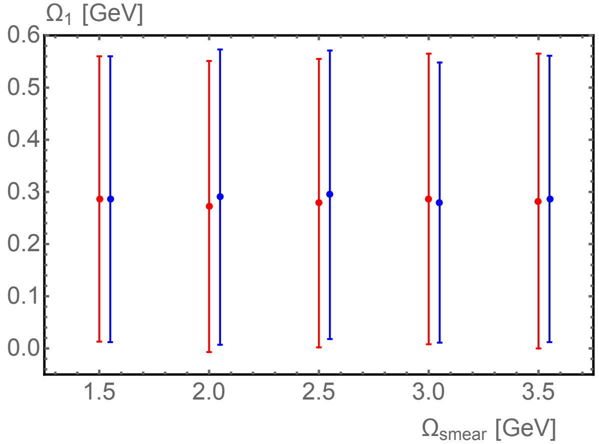
NLL: 177 scan survivors, NNLL: 254 scan survivors

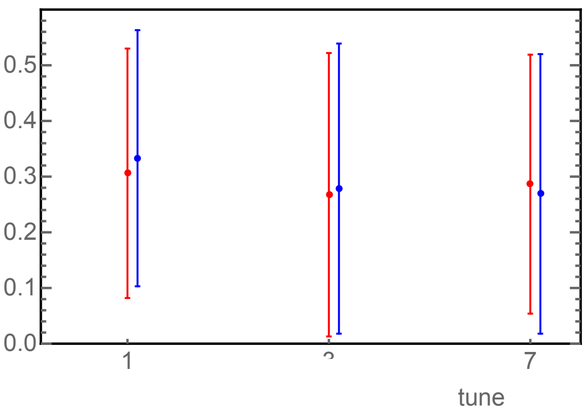
Preliminary

NLL

NNLL







- Reliable determination of non-perturbative matrix element Ω_1 (hadronization effects)
- Expected: $\delta m_t \sim \delta\Omega_1$ ✔
- Compatible with α_s-fits to e^+e^- data tail fits (Abbate etal, AHH etal.), larger err.

Peak Fits

First serious run: Γ_t=1.4 GeV, tunes 1, 3, 7,

$\Omega_{1,\mathrm{smear}}$=1.5, 2.0, 2.5, 3.0, 3.5 GeV,

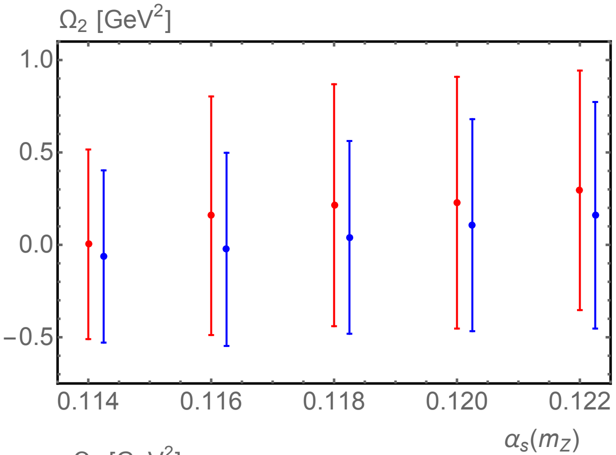
Q={700, 1000, 1400} GeV, peak fit (60/80)%

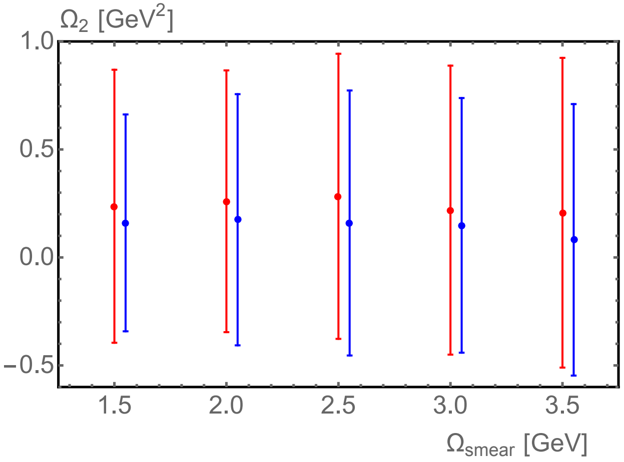
m_t^{Pythia}=173 GeV,

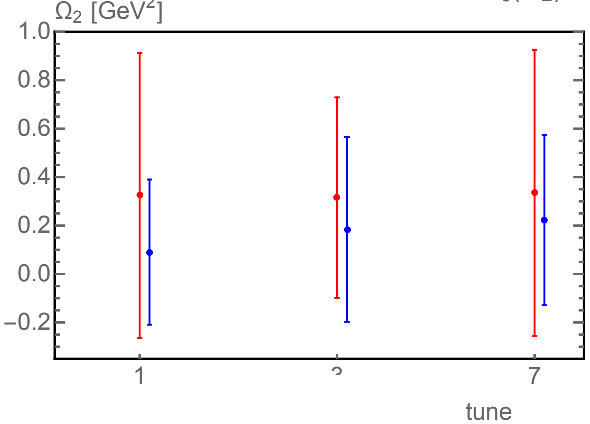
NLL: 177 scan survivors, NNLL: 254 scan survivors

NLL

NNLL







- Reliable determination of non-perturbative matrix element Ω_2 (hadronization effects)
- Found to be have huge error as expected due to little sensitivity ✔

Peak Fits

First serious run: Γ_t=1.4 GeV, tunes 1, 3, 7,

$\Omega_{1,\mathrm{smear}}$=1.5, 2.0, 2.5, 3.0, 3.5 GeV,

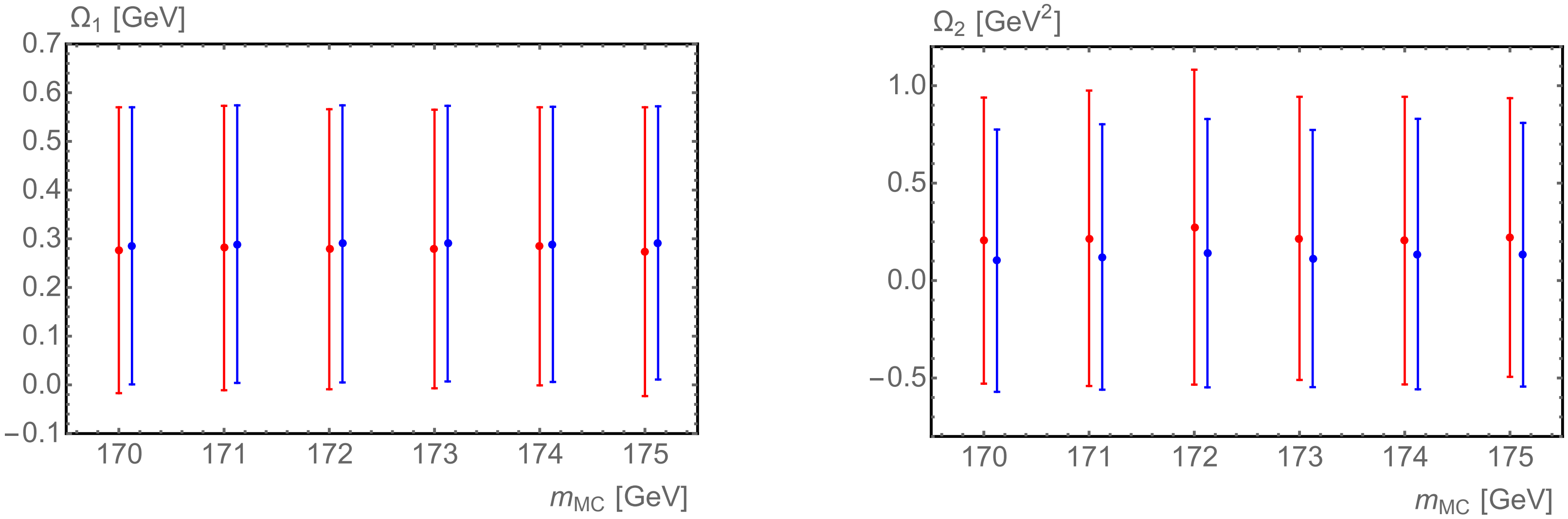
Q={700, 1000, 1400} GeV, peak fit (60/80)%

m_t^{Pythia}=170,171, 172, 173, 174, 175 GeV

NLL: 177 scan survivors, NNLL: 254 scan survivors

Preliminary

NLL
NNLL



- Non-pert. matrix elements $\Omega_{1,2}$ independent of top mass. ✔

Peak Fits

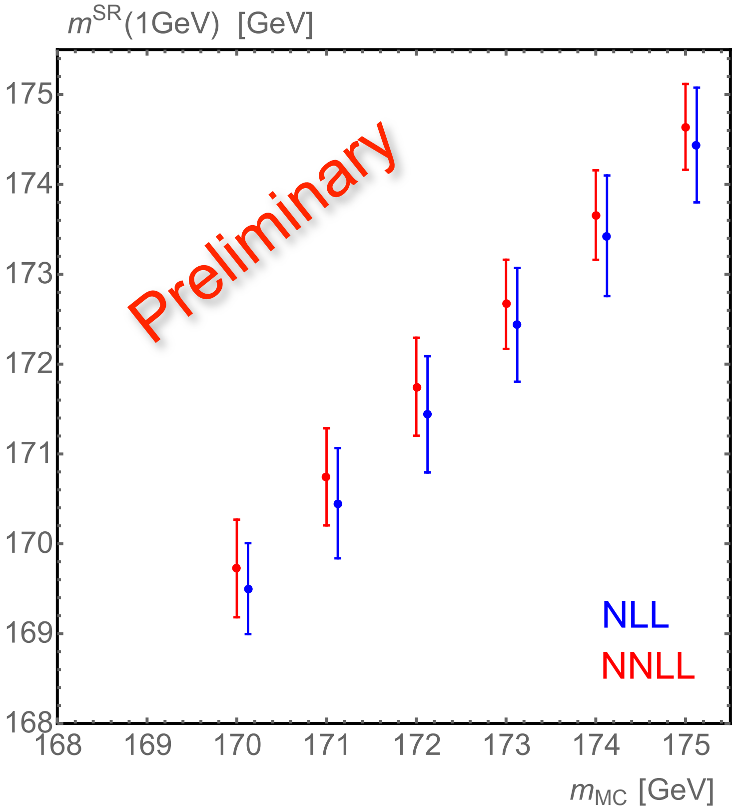
First serious run: Γ_t=1.4 GeV, tunes 1, 3, 7,

$\Omega_{1,\mathrm{smear}}$=1.5, 2.0, 2.5, 3.0, 3.5 GeV,

Q={700, 1000, 1400} GeV, peak fit (60/80)%

m_t^{Pythia}=170,171, 172, 173, 174, 175 GeV

NLL: 177 scan survivors, NNLL: 254 scan survivors



- Many more cross checks to be done.
- Calibration error: 0.5 GeV seems feasible at NNLL !

Conclusions & Outlook

- First serious precise MC top quark mass calibration based on e^+e^- 2-jettiness (large p_T): preliminary results.
- NNLL+NLO QCD calculations based on an extension of the SCET approach concerning massive quark effects (all large logs incl. Ln(m)'s summed systematically).
- The Monte Carlo top mass calibration in terms of MSR mass with perturbative error O(500 MeV) appears feasible at NNLL+NLO
- Intrinsic MC error seems O(100 MeV).

Outlook:

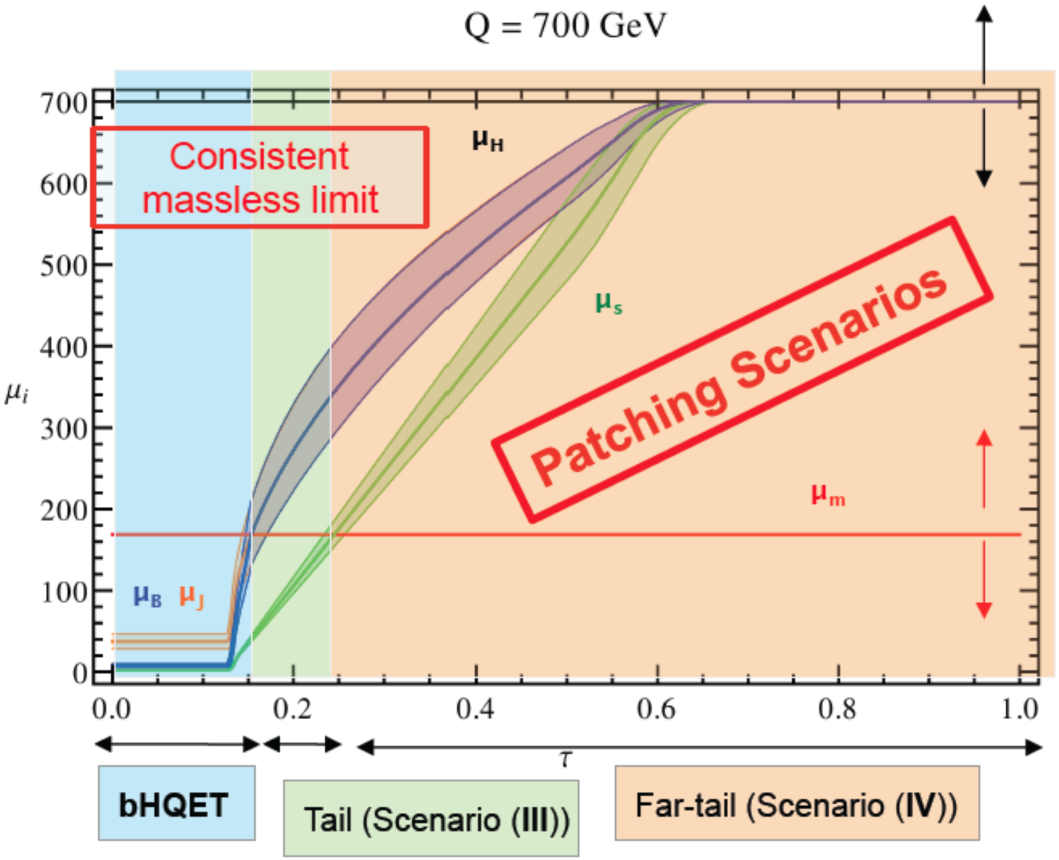
- Full verified error analysis @ NNLL+NLO on the way
- Calibration for other MC generators
- Heavy jet mass, C-parameter (NNLL), pp-2 jettiness analysis (NLL) w.i.p.
- NNNLL+NNLO (2jettiness for e^+e^-) w.i.p
- Mass (+ Yukawa coupling) conversions w. QCD + electroweak

Backup Slides

Profile Functions

Profile functions should sum up large logarithms and achieve smooth transition between the peak, tail and far-tail.

$$\log\left(\frac{Q}{\mu_H}\right) \quad \log\left(\frac{m_J}{\mu_m}\right) \quad \log\left(\frac{\mu_J^2}{Q\mu_s}\right) \quad \log\left(\frac{m_J\mu_B}{Q\mu_s}\right) \quad \log\left(\frac{Q(\tau-\tau_{\min})+2\Lambda_{\rm QCD}}{\mu_s}\right)$$



Scales Variation

- ✓ **Generalized to arbitrary mass values**
- ✓ **Compatible with massless profiles**

Proper scale variations are essential in reliable estimation of missing higher order terms.

Masses Loop-Theorists Like to use

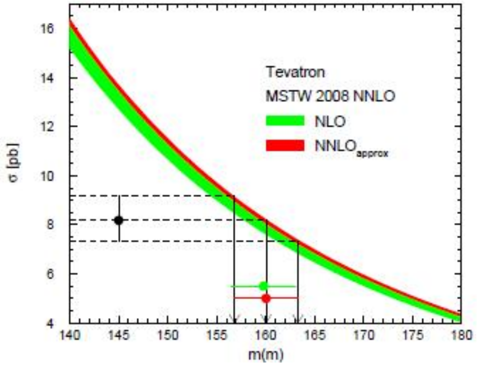
Total cross section (LHC/Tev):

$$m_t^{\mathrm{MSR}}(R=m_t) = \overline{m}_t(\overline{m}_t)$$

- more inclusive
- sensitive to top production mechanism (pdf, hard scale)
- indirect top mass sensitivity
- large scale radiative corrections

$$M_t \;=\; M_t^{(O)} \;+\; M_t(0)\alpha_s \;+\; \ldots$$

Langenfeld, Moch, Uwer



Threshold cross section (ILC):

$$m_t^{\mathrm{MSR}}(R\sim 20\ \mathrm{GeV})\,,\;\; m_t^{\mathrm{1S}}\,,\;\; m_t^{\mathrm{PS}}(R)$$

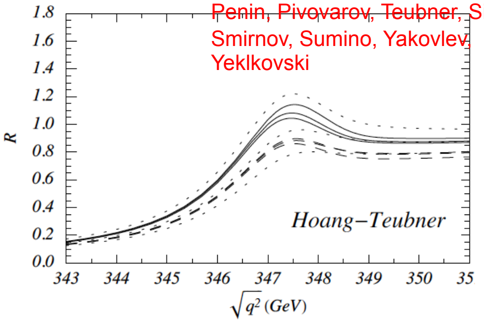
$$M_t \;=\; M_t^{(O)} \;+\; \langle p_{\mathrm{Bohr}}\rangle\alpha_s \;+\; \ldots$$

$$\langle p_{\mathrm{Bohr}}\rangle \;=\; 20\,\mathrm{GeV}$$

Mass schemes related to different computational methods

Relations computable in perturbation theory

Beneke, AH, Melnikov, Nagano, Penin, Pivovarov, Teubner, Signer, Smirnov, Sumino, Yakovlev, Yeklkovski



Inv. mass reconstruction (ILC/LHC):

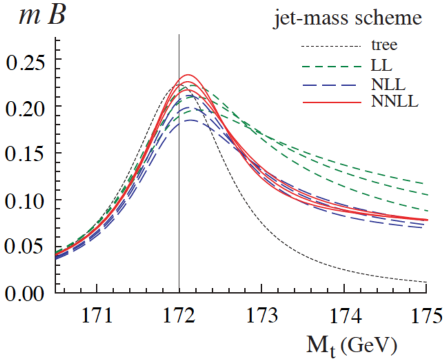
$$m_t^{\mathrm{MSR}}(R\sim\Gamma_t)\,,\;\; m_t^{\mathrm{jet}}(R)$$

$$M_t \;=\; M_t^{(O)} \;+\; \Gamma_t\alpha_s \;+\; \ldots$$

$$\Gamma_t \;=\; 1.3\,\mathrm{GeV}$$

- more exclusive
- sensitive to top final state interactions (low scale)
- direct top mass sensitivity
- small scale radiative corrections

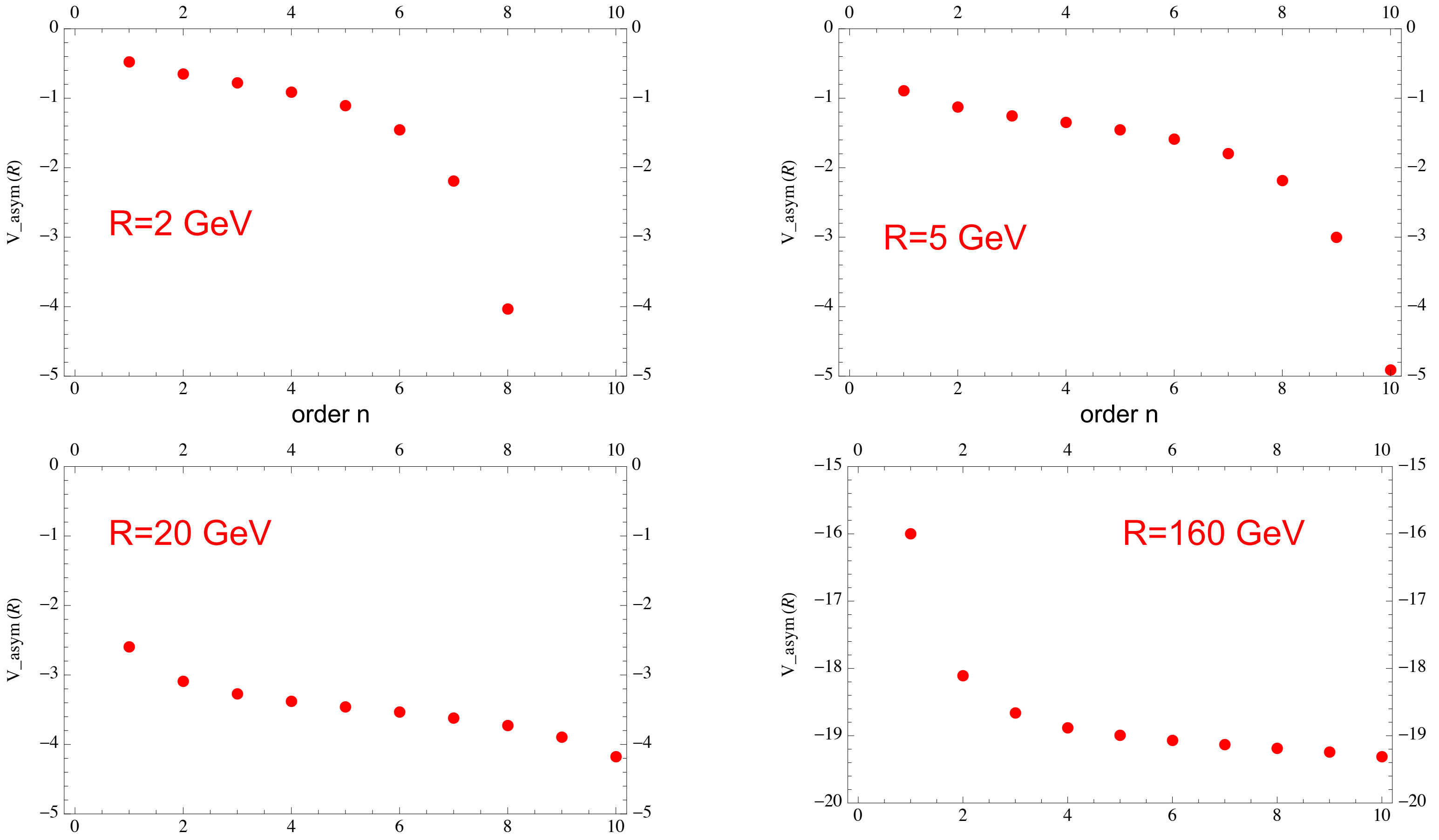
Fleming, AH, Mantry, Stewart



Loops and Legs 2016, Leipzig, April 28, 2016

Series with a Renormalon

- → Behavior depends on the typical scale R of the observable ?
- → Series for large R converge longer, but size of corrections at lower orders are large
- → Formal ambiguity always the same: $\Lambda_{\rm QCD} \approx 0.5\ {\rm GeV}$



NLL Numerical Analysis

Double differential invariant mass distribution:

$$Q = 5 \times 172 \text{ GeV}$$

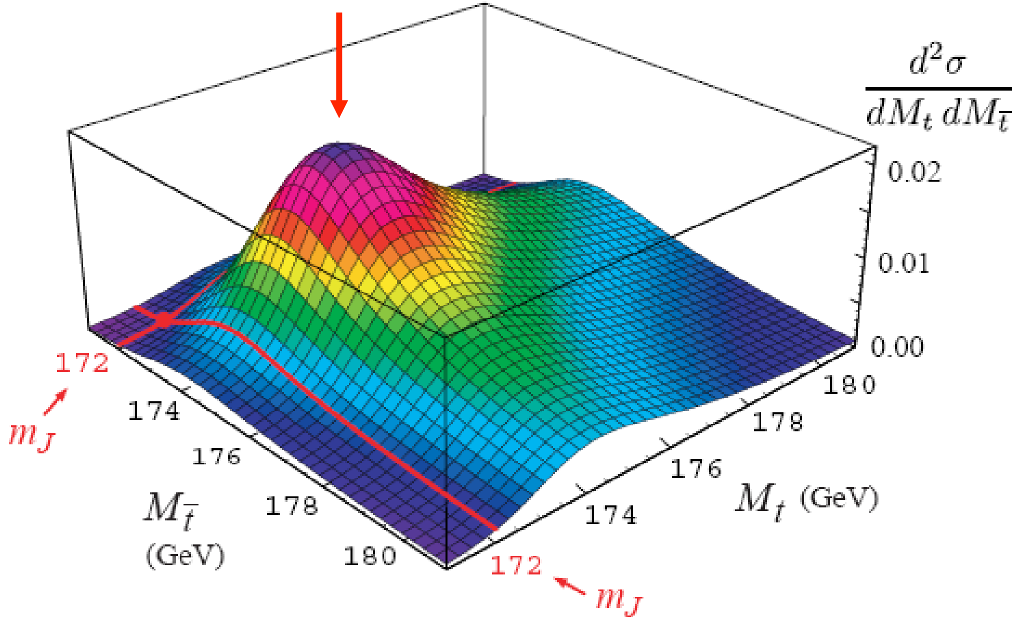
$$\Gamma = 1.43 \text{ GeV}$$

$$m_J(2\,\text{GeV}) = 172 \text{ GeV}$$

$$\mu_\Gamma = 5 \text{ GeV}$$

$$\mu_\Lambda = 1 \text{ GeV}$$

$$a = 2.5\,, \quad b = -0.4$$
$$\Lambda = 0.55 \text{ GeV}$$



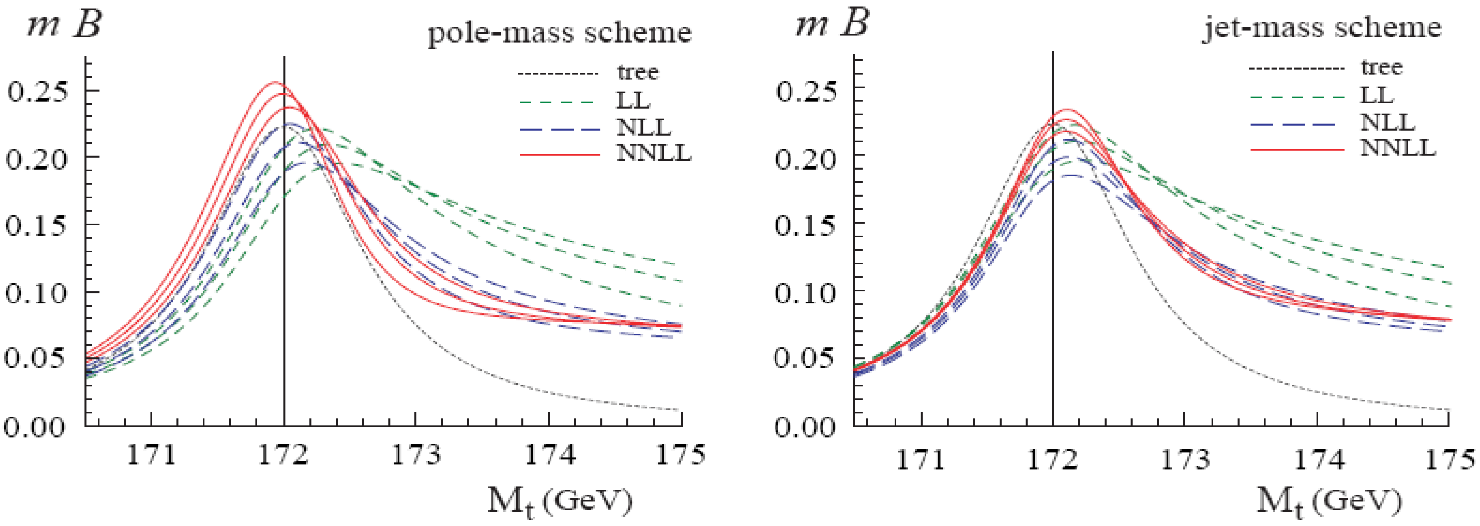
Non-perturbative effects shift the peak by +2.4 GeV and broaden the distribution.

Reconstructed Top Jets (ILC)

→ Jet function has an $\mathcal{O}(\Lambda_{\rm QCD})$ renormalon in the pole mass scheme

$$\mathcal{B}_\pm(\hat s,0,\mu,\delta m) = -\frac{1}{\pi m}\,\frac{1}{\hat s+i0}\left\{1+\frac{\alpha_s C_F}{4\pi}\left[4\ln^2\left(\frac{\mu}{-\hat s-i0}\right)+4\ln\left(\frac{\mu}{-\hat s-i0}\right)+4+\frac{5\pi^2}{6}\right]\right\} - \frac{1}{\pi m}\,\frac{2\delta m}{(\hat s+i0)^2}$$

$$\delta m = m_t^{\rm scheme} - m_t^{\rm pole}$$



Jain, Scimemi, Stewart

PRD77, 094008(2008)

$$m_{\rm pole} = m_J(\mu) + e^{\gamma_E} R\,\frac{\alpha_s(\mu)\,C_F}{\pi}\left[\ln\frac{\mu}{R}+\frac{1}{2}\right] + \mathcal{O}(\alpha_s^2)$$

$$\boxed{R \sim \Gamma_t}$$

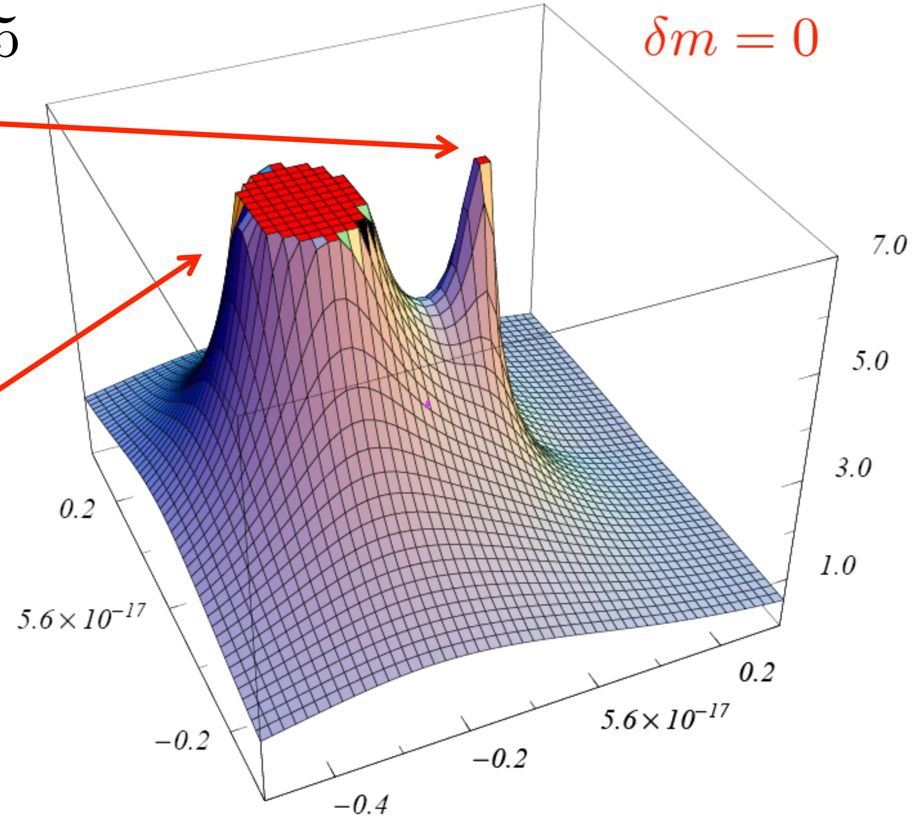
Reconstructed Top Jets (ILC)

Why is the pole mass not visible?

$$\hat{s} = \frac{M_t^2 - m_t^2}{m_t}$$

$$\Gamma = \Gamma_t/5$$

pole mass peak



$$|\mathcal{B}_\pm(\hat{s}, \Gamma_t, \mu)|^2$$

$$\delta m = 0$$

observable peak

→ jet mass is observable

- Located at the visible peak
- Short-distance mass

QCD Factorization

$$\left(\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m,\frac{Q}{m},\mu_m,\mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat s_t - \frac{Q\ell^+}{m},\Gamma,\mu\right) B_-\left(\hat s_{\bar t} - \frac{Q\ell^-}{m},\Gamma,\mu\right) S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

Jet functions:

$$B_+(\hat s,\Gamma_t,\mu) = {\rm Im}\left[\frac{-i}{12\pi m_J}\int d^4x\, e^{ir.x}\,\langle 0|\,T\,\{\bar h_{v_+}(0) W_n(0)\, W_n^\dagger(x) h_{v_+}(x)\,\}\,|0\rangle\right]$$

- perturbative
- dependent on mass, width, color charge

$$B_\pm^{\rm Born}(\hat s,\Gamma_t) = \frac{1}{\pi m_t}\frac{\Gamma_t}{\hat s^2+\Gamma_t^2} \qquad \hat s = \frac{M^2-m_t^2}{m_t}$$

Soft function:

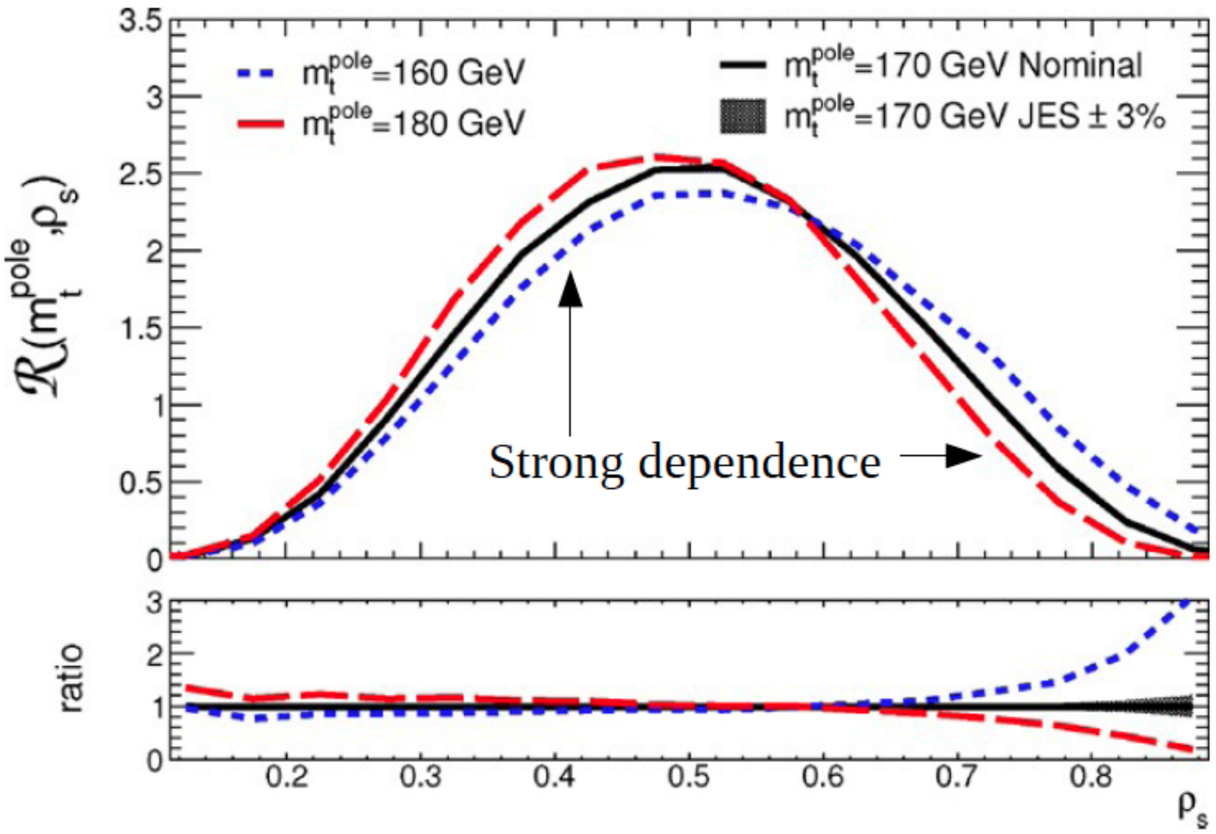
$$S_{\rm hemi}(\ell^+,\ell^-,\mu) = \frac{1}{N_c}\sum_{X_s}\delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\langle 0|\overline{Y}_{\bar n}\, Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\,\overline{Y}_{\bar n}^\dagger(0)|0\rangle$$

- non-perturbative
- analogous to the pdf's
- dependent on color charge, kinematics

Independent of the mass !

Indirect ways to determine the top quark mass

Top-antitop+Jets invariant mass @ LHC:

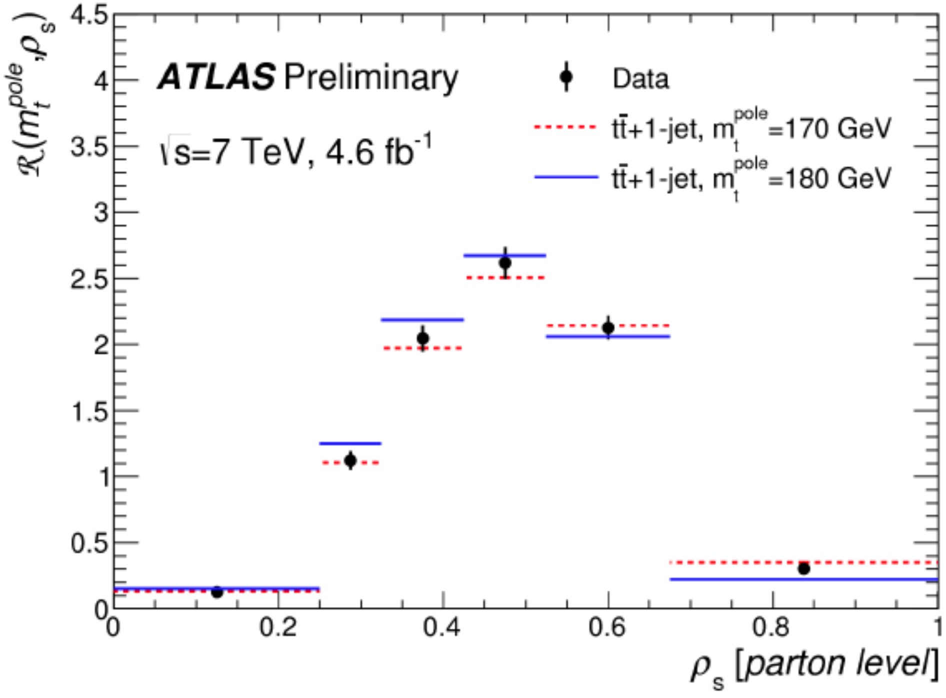


- NLO calculations (pole mass scheme currently)
- Distribution has a intrinsic peak in the distribution
- Mass determination less sensitive to PDF (< 1 GeV)

Indirect ways to determine the top quark mass

Top-antitop+Jets invariant mass @ LHC:

ATLAS-CONF-2014-053



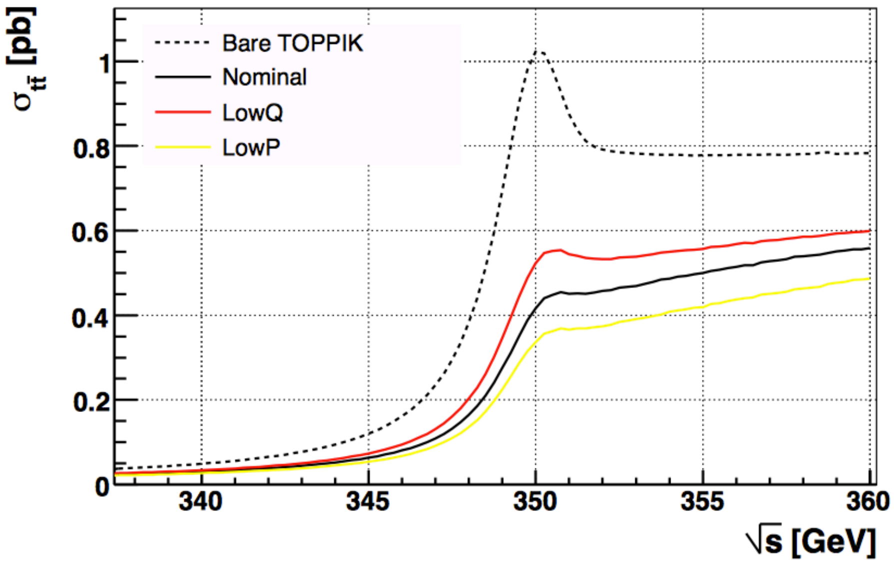
Fit with tt+jet NLO+PS theory

Is this the pole mass? Yes!
Scheme fixed in NLO calculation
(difference NLO vs. NLO+PS ~ 300 MeV)

Top Reconstruction + Total Cross Section

Top pair total cross section at a lepton collider:

$$\sigma(e+e- \to t\bar{t} + X) \text{ at } E_{cm} \approx 2m_t$$



Principle: **m_t from $\sigma_{tt}(m_t)$**

Advantages:

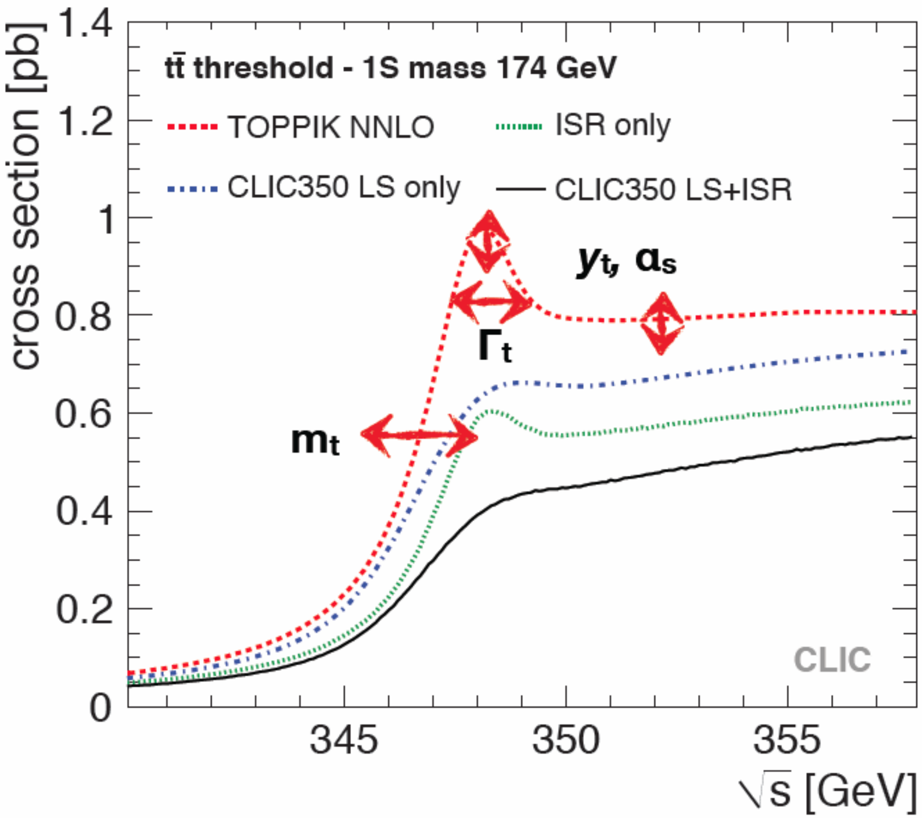
- ▹ count number of $t\bar{t}$ events
- ▹ color singlet state
- ▹ background is non-resonant
- ▹ physics well understood (renormalons, summations)

➢ Top decay protects from non-pert effects

- Remnant of a topionium resonance (“postronium of QCD”)
- Crucial to control e+e- luminosity spectrum
- Binding energy about twice the top quark width: $E_{\rm bind} \approx \dfrac{\alpha_s^2 m_t}{2} \approx 2\Gamma_t$
- Can be calculated in pQCD (nonrelativistic expansion)
- True final state: WWbb (includes single-top + nonresonant background)

Top Reconstruction + Total Cross Section

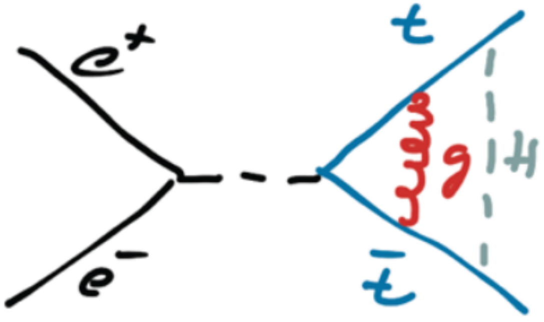
Experimental Studies:



- Effects of some parameters are correlated; dependence on Yukawa coupling rather weak - precise external α_s helps

The cross-section around the threshold is affected by several properties of the top quark and by QCD

- Top mass, width, Yukawa coupling
- Strong coupling constant



Frank Simon

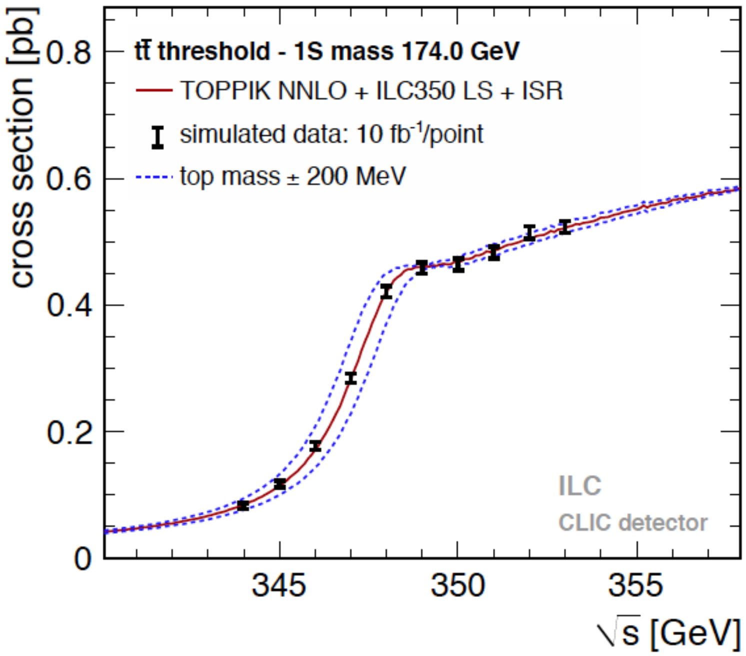
Top Reconstruction + Total Cross Section

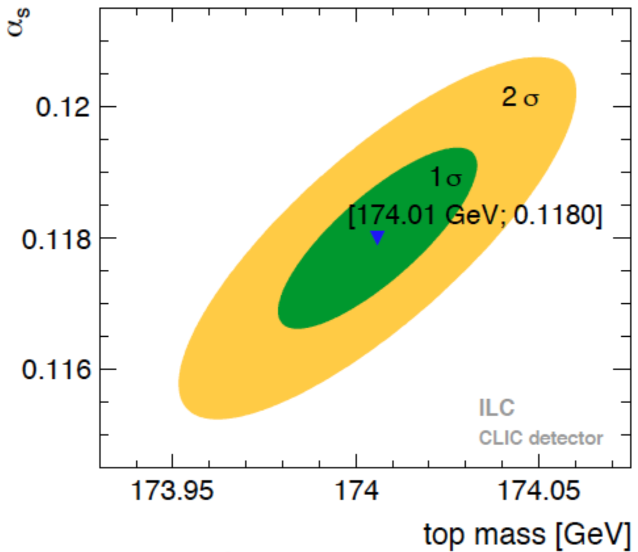
Frank Simon

Experimental Studies:

- The sensitivity of a ttbar threshold scan at Linear Colliders to top quark properties has been studied be a variety of groups at different points in time - with different degrees of realism
  - Here: Focus on the most recent studies

EPJ C73, 2540 (2013)





Combined "2D" fit: m_t and α_s:

$\Delta m_t = 27$ MeV (stat), $\Delta\alpha_s = 0.0008$

Mass alone: 18 MeV (stat)

➡ $$(\delta\overline{m}_t(\overline{m}_t))^{\text{total}} \approx 40\ \text{MeV}$$