Completely local fully differential subtractions at NNLO

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Outline

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2. The problem
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5. Cancellation of poles
6. Applications: $H \to b\bar{b}$ and $e^+e^- \to 3$ jets
7. Conclusions and outlook

Why precision?

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Why Precision?

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1. Introduction

Precision matters. Any progress in the exact sciences relies both on precise measurements and highly accurate theoretical calculations. Many of the fundamental laws of physics had unavoidably to be found whenever precise data were described by theoretical concepts, often within a new framework of relations. The Rudolphine Tables of the late Tycho Brahe [1] led J. Kepler to derive his laws [2] and later I. Newton the law of gravity [3]. A. Michelson's experiments [4] led A. Einstein to Special Relativity [5], with numerous experimental confirmations in flat space-time. [1] The

The problem

The problem

Consider the NNLO correction to a generic *m*-jet observable

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \int_{m+2} {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} + \int_{m+1} {\rm d}\sigma^{\rm RV}_{m+1} J_{m+1} + \int_{m} {\rm d}\sigma^{\rm VV}_{m} J_{m}\,.$$

- matrix elements for $\sigma^{\rm RR}_{m+2}$ (tree) and $\sigma^{\rm RV}_{m+1}$ (1-loop) known for many processes
- $\sigma^{\rm VV}_{m}$ (2-loop) know for 4 parton, V+3 parton processes, higher multiplicities are on the horizon
- the three contributions are separately infrared divergent in $d = 4$ dimensions

Double real

- kin. singularities as one or two partons unresolved: up to $\mathrm{O}(\epsilon^{-4})$ poles from PS integration
- no explicit ϵ poles

Real-virtual

- kin. singularities as one parton unresolved: up to $\mathrm{O}(\epsilon^{-2})$ poles from PS integration
- explicit ϵ poles up to $\mathrm{O}(\epsilon^{-2})$

Double virtual

- kin. singularities screened by jet function: PS integration finite
- explicit ϵ poles up to $\mathrm{O}(\epsilon^{-4})$

The problem

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KLN theorem

Infrared singularities cancel between real and virtual quantum corrections at the same order in perturbation theory, for sufficiently inclusive (i.e. IR safe) observables.

However

How to make this cancellation explicit, so that the various contributions can be computed numerically? Need a method to deal with implicit poles.

Several approaches – why this one?

Several approaches

- sector decomposition
- antenna subtraction
- STRIPPER
- non-local $q_\perp$, τ_N slicing
- CoLoRFulNNLO

CoLoRFulNNLO is built around the idea that a solution should

- give the exact perturbative result, at least in principle
- be mathematically well-defined
- lead to general and explicit expressions

At NLO, such solutions are known: local subtraction schemes (dipole, FKS, ...)

How to build a local subtraction scheme?

Repeat what already worked at NLO!

1. Compute relevant IR factorization formulae
2. Use those to construct general, explicit, local subtractions
3. Integrate subtractions once and for all, check cancellation of poles
4. Apply scheme to specific processes

Message: can be systematically done at NNLO

The recipe

Use known ingredients

Collinear and soft factorization of QCD matrix elements at NNLO known

- Tree level 3-parton splitting functions and double soft gg and $q\bar{q}$ currents

(Campbell, Glover 1997; Catani, Grazzini 1998; Del Duca, Frizzo, Maltoni 1999; Kosower 2002)

- One-loop 2-parton splitting functions and soft gluon current

(Bern, Dixon, Dunbar, Kosower 1994; Bern, Del Duca, Kilgore, Schmidt 1998-9; Kosower, Uwer 1999; Catani, Grazzini 2000; Kosower 2003)

Use known ingredients

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Are these useful for NNLO? – Yes!

Structure of the NNLO correction

Rewrite the NNLO correction as a sum of three terms

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

each integrable in four dimensions

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \Big\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \Big[{\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \Big] \Big\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \Big\{ \Big[{\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big] J_{m+1} - \Big[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \Big(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big)^{\rm A_1} \Big] J_m \Big\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \Big\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \Big[{\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \Big] + \int_1 \Big[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \Big(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big)^{\rm A_1} \Big] \Big\} J_m$$

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Rewrite the NNLO correction as a sum of three terms

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each integrable in four dimensions

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \left[\mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \right] \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left[\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right] J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] J_m \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left[\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} \right] + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

1. $\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2}$ regularizes the double unresolved limits of $\mathrm{d}\sigma^{\mathrm{RR}}_{m+2}$

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4. $\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1}$ regularizes the single unresolved limits of $\mathrm{d}\sigma^{\mathrm{RV}}_{m+1}$

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4. ${\rm d}\sigma^{\rm RV,A_1}_{m+1}$ regularizes the single unresolved limits of ${\rm d}\sigma^{\rm RV}_{m+1}$
5. $\left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2}\right)^{\rm A_1}$ regularizes the single unresolved limit of $\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2}$

CoLoRFulNNLO

CoLoRFulNNLO: Completely Local subtRactions for Fully differential predictions at NNLO

Subtractions built from universal IR limit formulae

- Altarelli-Parisi splitting functions, soft currents (tree and one-loop, triple AP functions)
- simple and general procedure for matching of limits using physical gauge
- extension based on momentum mappings that can be generalized to any number of unresolved partons

Completely local in color $\otimes$ spin space, fully differential in phase space

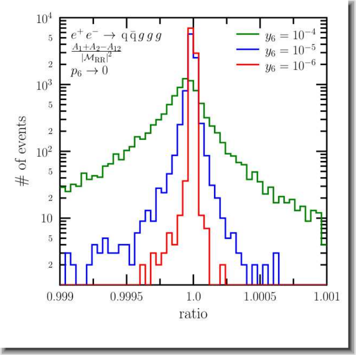
- no need to consider the color decomposition of real emission ME's
- azimuthal correlations correctly taken into account in gluon splitting
- can check explicitly that the ratio of the sum of counterterms to the real emission cross section tends to unity in any IR limit

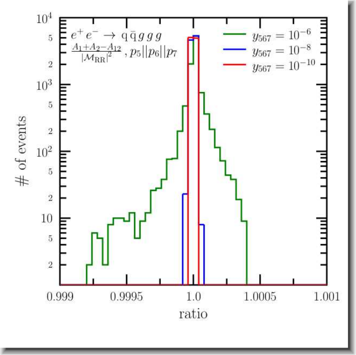
Given explicitly for any process with colorless initial state

Kinematic singularities cancel in RR

Ratio of the RR matrix element to sum of all subtractions goes to one for all IR limits

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \left[\mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \right] \right\}$$



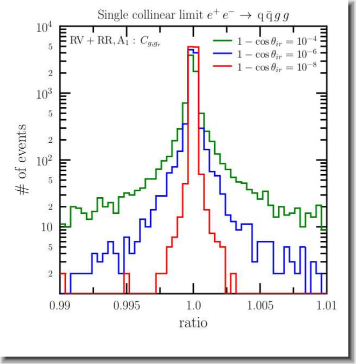


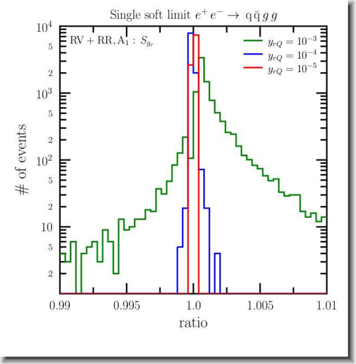
ratio = subtractions/RR

Kinematic singularities cancel in RV

Ratio of the RV matrix element to sum of all subtractions goes to one for all IR limits

$$\sigma_{m+1}^{\rm NNLO} = \int_{m+1} \left\{ \left[{\rm d}\sigma_{m+1}^{\rm RV} + \int_1 {\rm d}\sigma_{m+2}^{\rm RR,A_1} \right] J_{m+1} - \left[{\rm d}\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 {\rm d}\sigma_{m+2}^{\rm RR,A_1} \right)^{\rm A_1} \right] J_m \right\}$$





ratio = subtractions/(RV + RR,A$_1$)

Integrating the subtractions

Integrating the subtractions

Momentum mappings used to define the counterterms

$$\{p\}_{n+p} \xrightarrow{R} \{\tilde{p}\}_n \quad \Rightarrow \quad \mathrm{d}\phi_{n+p}(\{p\}; Q) = \mathrm{d}\phi_n(\{\tilde{p}\}_n^{(R)}; Q)[\mathrm{d}p_{p,n}^{(R)}]$$

- implement exact momentum conservation, recoil distributed democratically (can be generalized to any p)
- different collinear and soft mappings (R labels precise limit)
- exact factorization of phase space

Counterterms are products (in color and spin space) of

- factorized ME's independent of variables in $[\mathrm{d}p_{p,n}^{(R)}]$
- singular factors (AP functions, soft currents), to be integrated over $[\mathrm{d}p_{p,n}^{(R)}]$

$$\mathcal{X}_R(\{p\}_{n+p}) = \left(8\pi\alpha_{\mathrm{s}}\mu^{2\epsilon}\right)^p \mathit{Sing}_R(p_p^{(R)}) \otimes |\mathcal{M}_n^{(0)}(\{\tilde{p}\}_n^{(R)})|^2$$

Can compute once and for all the integral over unresolved partons

$$\int_p \mathcal{X}_R(\{p\}_{n+p}) = \left(8\pi\alpha_{\mathrm{s}}\mu^{2\epsilon}\right)^p \left\{ \int [\mathrm{d}p_{p,n}^{(R)}]\, \mathit{Sing}_R(p_p^{(R)}) \right\} \otimes |\mathcal{M}_n^{(0)}(\{\tilde{p}\}_n^{(R)})|^2$$

List of basic integrals

Int	status
$\mathcal{I}_{1\mathcal{C},0}^{(k)}$	✔
$\mathcal{I}_{1\mathcal{C},1}^{(k)}$	✔
$\mathcal{I}_{1\mathcal{C},2}^{(k)}$	✔
$\mathcal{I}_{1\mathcal{C},3}^{(k)}$	✔
$\mathcal{I}_{1\mathcal{C},4}^{(k)}$	✔
$\mathcal{I}_{1\mathcal{C},5}^{(k,l)}$	✔
$\mathcal{I}_{1\mathcal{C},6}^{(k,l)}$	✔
$\mathcal{I}_{1\mathcal{C},7}^{(k)}$	✔
$\mathcal{I}_{1\mathcal{C},8}$	✔

Int	status
$\mathcal{I}_{1\mathcal{S},0}$	✔
$\mathcal{I}_{1\mathcal{S},1}$	✔
$\mathcal{I}_{1\mathcal{S},2}$	✔
$\mathcal{I}_{1\mathcal{S},3}^{(k)}$	✔
$\mathcal{I}_{1\mathcal{S},4}$	✔
$\mathcal{I}_{1\mathcal{S},5}$	✔
$\mathcal{I}_{1\mathcal{S},6}$	✔
$\mathcal{I}_{1\mathcal{S},7}$	✔

Int	status
$\mathcal{I}_{1\mathcal{CS},0}$	✔
$\mathcal{I}_{1\mathcal{CS},1}$	✔
$\mathcal{I}_{1\mathcal{CS},2}^{(k)}$	✔
$\mathcal{I}_{1\mathcal{CS},3}$	✔
$\mathcal{I}_{1\mathcal{CS},4}$	✔

Int	status
$\mathcal{I}_{12\mathcal{C},1}^{(k,l)}$	✔
$\mathcal{I}_{12\mathcal{C},2}^{(k,l)}$	✔
$\mathcal{I}_{12\mathcal{C},3}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{C},4}^{(k,l)}$	✔
$\mathcal{I}_{12\mathcal{C},5}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{C},6}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{C},7}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{C},8}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{C},9}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{C},10}^{(k)}$	✔

Int	status
$\mathcal{I}_{2\mathcal{S},1}$	✔
$\mathcal{I}_{2\mathcal{S},2}$	✔
$\mathcal{I}_{2\mathcal{S},3}$	✔
$\mathcal{I}_{2\mathcal{S},4}$	✔
$\mathcal{I}_{2\mathcal{S},5}$	✔
$\mathcal{I}_{2\mathcal{S},6}$	✔
$\mathcal{I}_{2\mathcal{S},7}$	✔
$\mathcal{I}_{2\mathcal{S},8}$	✔
$\mathcal{I}_{2\mathcal{S},9}$	✔
$\mathcal{I}_{2\mathcal{S},10}$	✔
$\mathcal{I}_{2\mathcal{S},11}$	✔
$\mathcal{I}_{2\mathcal{S},12}$	✔
$\mathcal{I}_{2\mathcal{S},13}$	✔
$\mathcal{I}_{2\mathcal{S},14}$	✔
$\mathcal{I}_{2\mathcal{S},15}$	✔
$\mathcal{I}_{2\mathcal{S},16}$	✔
$\mathcal{I}_{2\mathcal{S},17}$	✔
$\mathcal{I}_{2\mathcal{S},18}$	✔
$\mathcal{I}_{2\mathcal{S},19}$	✔
$\mathcal{I}_{2\mathcal{S},20}$	✔
$\mathcal{I}_{2\mathcal{S},21}$	✔
$\mathcal{I}_{2\mathcal{S},22}$	✔
$\mathcal{I}_{2\mathcal{S},23}$	✔

Int	status
$\mathcal{I}_{12\mathcal{S},1}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{S},2}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{S},3}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{S},4}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{S},5}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{S},6}$	✔
$\mathcal{I}_{12\mathcal{S},7}$	✔
$\mathcal{I}_{12\mathcal{S},8}$	✔
$\mathcal{I}_{12\mathcal{S},9}$	✔
$\mathcal{I}_{12\mathcal{S},10}$	✔
$\mathcal{I}_{12\mathcal{S},11}$	✔
$\mathcal{I}_{12\mathcal{S},12}$	✔
$\mathcal{I}_{12\mathcal{S},13}$	✔

Int	status
$\mathcal{I}_{12\mathcal{CS},1}^{(k)}$	✔
$\mathcal{I}_{12\mathcal{CS},2}$	✔
$\mathcal{I}_{12\mathcal{CS},3}$	✔

Int	status
$\mathcal{I}_{2\mathcal{C},1}^{(j,k,l,m)}$	✔
$\mathcal{I}_{2\mathcal{C},2}^{(j,k,l,m)}$	✔
$\mathcal{I}_{2\mathcal{C},3}^{(j,k,l,m)}$	✔
$\mathcal{I}_{2\mathcal{C},4}^{(j,k,l,m)}$	✔
$\mathcal{I}_{2\mathcal{C},5}^{(-1,-1,-1,-1)}$	✔
$\mathcal{I}_{2\mathcal{C},6}^{(k,l)}$	✔

Int	status
$\mathcal{I}_{2\mathcal{CS},1}^{(k)}$	✔
$\mathcal{I}_{2\mathcal{CS},2}^{(k)}$	✔
$\mathcal{I}_{2\mathcal{CS},2}^{(2)}$	✔
$\mathcal{I}_{2\mathcal{CS},3}^{(k)}$	✔
$\mathcal{I}_{2\mathcal{CS},4}^{(k)}$	✔
$\mathcal{I}_{2\mathcal{CS},5}^{(k)}$	✔

✔ poles and logs of the finite parts known fully analytically, regular pieces of finite parts known numerically

Solving the integrals

Strategy for computing the master integrals

1. write phase space in terms of angles and energies
2. angular integrals in terms of Mellin-Barnes representations
3. resolve the ϵ poles by analytic continuation
4. MB integrals to Euler-type integrals, pole coefficients are finite parametric integrals

1. choose explicit parametrization of phase space
2. write the parametric integral representation in chosen variables
3. resolve the ϵ poles by sector decomposition
4. pole coefficients are finite parametric integrals

5. evaluate the parametric integrals in terms of multiple polylogs
6. simplify result (optional)

Cancellation of poles

Integrated approximate cross sections

Recall the NNLO correction is a sum of three terms

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

each integrable in four dimensions

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \Big\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \Big[{\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \Big] \Big\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \Big\{ \Big[{\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big] J_{m+1} - \Big[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \Big(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big)^{\rm A_1} \Big] J_m \Big\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \Big\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \Big[{\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \Big] + \int_1 \Big[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \Big(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big)^{\rm A_1} \Big] \Big\} J_m$$

Integrated approximate cross sections

- After summing over unobserved flavors, all integrated approximate cross sections can be written as products (in color space) of various insertion operators with lower point cross sections.
- Can be computed once and for all (though admittedly lots of tedious work).
- Poles and logs of finite part are computed analytically, regular pieces of finite part numerically.

Poles cancel: VV contribution finite

After adding all integrated approximate cross sections the double virtual contribution must be finite in ϵ.

$$\sigma_m^{\rm NNLO} = \int_m \left\{ {\rm d}\sigma_m^{\rm VV} + \int_2 \left[{\rm d}\sigma_{m+2}^{\rm RR,A_2} - {\rm d}\sigma_{m+2}^{\rm RR,A_{12}} \right] + \int_1 \left[{\rm d}\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 {\rm d}\sigma_{m+2}^{\rm RR,A_1} \right)^{\rm A_1} \right] \right\} J_m$$

- Have checked the cancellation of the $\frac{1}{\epsilon^4}$ and $\frac{1}{\epsilon^3}$ poles analytically for any number of jets (i.e., with m symbolic).
- Have checked $m = 2$ ($e^+e^- \to q\bar{q}$, $H \to b\bar{b}$) explicitly and we find that all poles cancel.
- Have checked $m = 3$ ($e^+e^- \to q\bar{q}g$) explicitly and we find that all poles cancel.

Example: $H \to b\bar{b}$

The double virtual contribution has the following pole structure ($\mu^2 = m_H^2$)

$$\mathrm{d}\sigma^{\mathrm{VV}}_{H\to b\bar{b}} = \left(\frac{\alpha_{\mathrm{s}}(\mu^2)}{2\pi}\right)^2 \mathrm{d}\sigma^{\mathrm{B}}_{H\to b\bar{b}} \Bigg\{ \frac{2C_{\mathrm{F}}^2}{\epsilon^4} + \left(\frac{11\,C_{\mathrm{A}}\,C_{\mathrm{F}}}{4} + 6\,C_{\mathrm{F}}^2 - \frac{C_{\mathrm{F}}\,n_{\mathrm{f}}}{2}\right)\frac{1}{\epsilon^3}$$
$$+ \left[\left(\frac{8}{9} + \frac{\pi^2}{12}\right) C_{\mathrm{A}}\,C_{\mathrm{F}} + \left(\frac{17}{2} - 2\pi^2\right) C_{\mathrm{F}}^2 - \frac{2\,C_{\mathrm{F}}\,n_{\mathrm{f}}}{9}\right]\frac{1}{\epsilon^2}$$
$$+ \left[\left(-\frac{961}{216} + \frac{13\zeta_3}{2}\right) C_{\mathrm{A}}\,C_{\mathrm{F}} + \left(\frac{109}{8} - 2\pi^2 - 14\zeta_3\right) C_{\mathrm{F}}^2 + \frac{65\,C_{\mathrm{F}}\,n_{\mathrm{f}}}{108}\right]\frac{1}{\epsilon}\Bigg\}$$

(Anastasiou, Herzog, Lazopoulos, JHEP **1203** (2012) 035)

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(Anastasiou, Herzog, Lazopoulos, JHEP **1203** (2012) 035)

The sum of the integrated approximate cross sections gives ($\mu^2 = m_H^2$)

$$\sum \int \mathrm{d}\sigma^{\mathrm{A}} = \left(\frac{\alpha_{\mathrm{s}}(\mu^2)}{2\pi}\right)^2 \mathrm{d}\sigma^{\mathrm{B}}_{H\to b\bar{b}} \left\{ \frac{-2C_{\mathrm{F}}^2}{\epsilon^4} + \left(-\frac{11 C_{\mathrm{A}} C_{\mathrm{F}}}{4} - 6C_{\mathrm{F}}^2 + \frac{C_{\mathrm{F}} n_{\mathrm{f}}}{2}\right)\frac{1}{\epsilon^3} + \left[\left(-\frac{8}{9} - \frac{\pi^2}{12}\right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(-\frac{17}{2} + 2\pi^2\right) C_{\mathrm{F}}^2 + \frac{2C_{\mathrm{F}} n_{\mathrm{f}}}{9}\right]\frac{1}{\epsilon^2} + \left[\left(\frac{961}{216} - \frac{13\zeta_3}{2}\right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(-\frac{109}{8} + 2\pi^2 + 14\zeta_3\right) C_{\mathrm{F}}^2 - \frac{65 C_{\mathrm{F}} n_{\mathrm{f}}}{108}\right]\frac{1}{\epsilon} \right\}$$

(Del Duca, Duhr, GS, Tramontano, Trócsányi,
JHEP **1504** (2015) 036)

Example: $e^+e^- \to 3$ jets

The double virtual contribution has the following pole structure ($\mu^2 = s$)

$$\mathrm{d}\sigma_3^{\mathrm{VV}} = \mathcal{P}oles\Big(A_3^{(2\times0)} + A_3^{(1\times1)}\Big) + \mathcal{F}inite\Big(A_3^{(2\times0)} + A_3^{(1\times1)}\Big)$$

where

$$\begin{aligned}\mathcal{P}oles\Big(A_3^{(2\times0)} + A_3^{(1\times1)}\Big) = 2\bigg[& -\Big(I_{q\bar{q}g}^{(1)}(\epsilon)\Big)^2 - \frac{\beta_0}{\epsilon} I_{q\bar{q}g}^{(1)}(\epsilon) \\ & + e^{-\epsilon\gamma}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\Big(\frac{\beta_0}{\epsilon} + K\Big) I_{q\bar{q}g}^{(1)}(2\epsilon) + H_{q\bar{q}g}^{(2)}\bigg] A_3^0(1_q, 3_g, 2_{\bar{q}}) \\ & + 2 I_{q\bar{q}g}^{(1)}(\epsilon) A_3^{1\times0}(1_q, 3_g, 2_{\bar{q}})\end{aligned}$$

with

$$\begin{aligned}H_{q\bar{q}g}^{(2)} = \frac{e^{\epsilon\gamma}}{4\epsilon\Gamma(1-\epsilon)}\bigg[& \Big(4\zeta_3 + \frac{589}{432} - \frac{11\pi^2}{72}\Big) N_{\mathrm{c}} + \Big(-\frac{1}{2}\zeta_3 - \frac{41}{54} - \frac{\pi^2}{48}\Big) \\ & + \Big(-3\zeta_3 - \frac{3}{16} + \frac{\pi^2}{4}\Big)\frac{1}{N_{\mathrm{c}}} + \Big(-\frac{19}{18} + \frac{\pi^2}{36}\Big) N_{\mathrm{c}} n_{\mathrm{f}} + \Big(-\frac{1}{54} - \frac{\pi^2}{24}\Big)\frac{n_{\mathrm{f}}}{N_{\mathrm{c}}} + \frac{5}{27} n_{\mathrm{f}}^2\bigg]\end{aligned}$$

(Gehrmann-De Ridder, Gehrmann, Glover, Heinrich,
JHEP **0711** (2007) 058)

Example: $e^+e^- \to 3$ jets

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Adding the sum of the integrated approximate cross sections gives

$$\mathcal{P}oles\Big(A_3^{(2\times0)} + A_3^{(1\times1)}\Big) + \mathcal{P}oles\sum\int \mathrm{d}\sigma^{\mathrm{A}} = 117\text{k terms}$$

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$$\mathcal{P}oles\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right) + \mathcal{P}oles\sum\int \mathrm{d}\sigma^{\mathrm{A}} = 117\text{k terms}$$

- zero numerically in any phase space point

In[35]:= `N[PolesVV3 /. {y13 → 2 / 10, y23 → 3 / 10}, 40]`

$$\text{Out[35]=} \frac{0.\times10^{-387} + \frac{0.\times10^{-388}}{\mathrm{Nc}^2} + 0.\times10^{-388}\,\mathrm{Nc}^2 + \frac{0.\times10^{-438}\,\mathrm{nf}}{\mathrm{Nc}} + 0.\times10^{-439}\,\mathrm{Nc}\,\mathrm{nf}}{\mathrm{e}^2} + \frac{1}{\mathrm{e}}\Big(\left(0.\times10^{-384} + 0.\times10^{-438}\,\mathrm{i}\right) + \frac{0.\times10^{-385}}{\mathrm{Nc}^2} + \left(0.\times10^{-384} + 0.\times10^{-438}\,\mathrm{i}\right)\mathrm{Nc}^2 + \frac{0.\times10^{-437}\,\mathrm{nf}}{\mathrm{Nc}} + 0.\times10^{-437}\,\mathrm{Nc}\,\mathrm{nf}\Big) + \mathrm{O}[\mathrm{e}]^0$$

- zero analytically after simplification using symbol technology (C. Duhr)

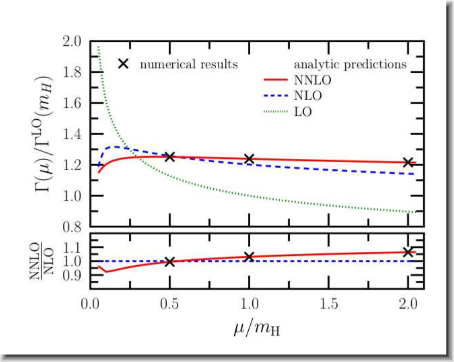
Applications

Higgs decay to b-quarks

Consider $H \to b\bar{b}$ decay at NNLO:

- admittedly the simplest case
- but this just amounts to having to sum less terms in general formulae

Inclusive decay rate



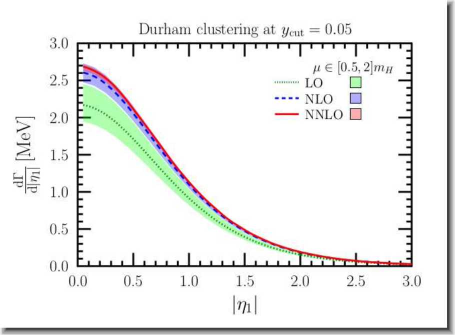
Higgs decay to b-quarks

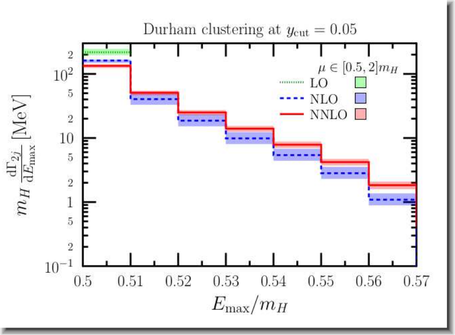
Consider $H \to b\bar{b}$ decay at NNLO:

- admittedly the simplest case
- but this just amounts to having to sum less terms in general formulae

Differential distributions

- pseudorapidity of highest energy jet (right) and leading jet energy (left)





Electron-positron annihilation into 3 jets

Why?

- relevant for extracting $\alpha_{\rm s}$ from data
- can compute new observables which may be better suited to such analysis
- the numerical accuracy of available results leaves something to be desired
- good testing ground for higher order technology

How?

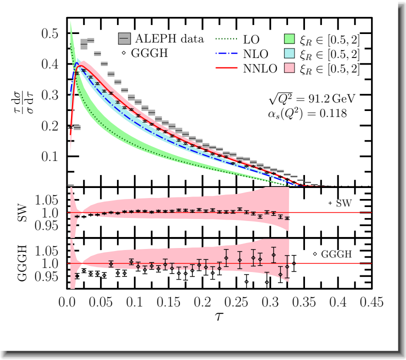
- replace $m = 2$ from previous example with $m = 3$
- the subtraction algorithm can and has been coded in general, one simply has to plug in the relevant matrix elements (Á. Kardos)
- disclaimer: the numerical computation of the regular finite parts of the insertion operators is geared towards two and three jets

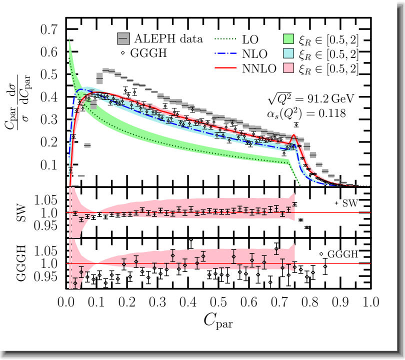
See Ádám Kardos' talk on Thursday for more details on the phenomenology and code.

Event shapes at NNLO

Physical predictions for thrust ($\tau = 1 - T$) and C-parameter at NNLO

(Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 2007;
Weinzierl 2009)





PRELIMINARY

Event shapes at NNLO

The NNLO coefficients

(Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 2007;
Weinzierl 2009)

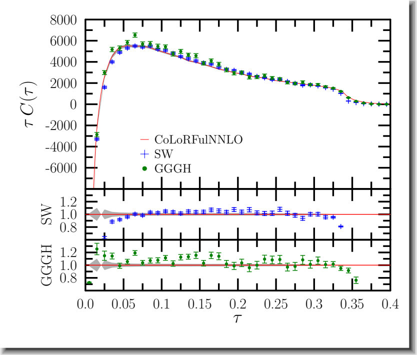
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}O} = \frac{\alpha_\mathrm{s}}{2\pi}A(O) + \left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^2 B(O) + \left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^3 C(O) + \mathcal{O}(\alpha_\mathrm{s}^4)$$

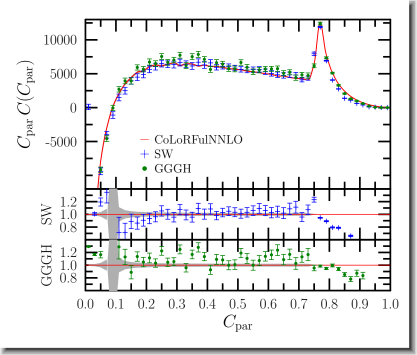
Event shapes at NNLO

The NNLO coefficients

(Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 2007;
Weinzierl 2009)

- overall reasonable agreement among predictions with some specific deviations
- very good numerical convergence of the code





PRELIMINARY

Jet cone energy fraction at NNLO

Physical prediction for jet cone energy fraction (JCEF) at NNLO

- JCEF is the energy deposited within a conical shell of the opening angle χ between a particle and the thrust axis $\vec{n}_T$

$$\mathrm{JCEF}(\chi) = \frac{1}{\sigma_{\mathrm{had}}} \sum_i \int \frac{E_i}{Q} \mathrm{d}\sigma_{e^+e^-\to i+X}\, \delta\left(\cos\chi - \frac{\vec{p}_i \cdot \vec{n}_T}{|\vec{p}_i|}\right)$$

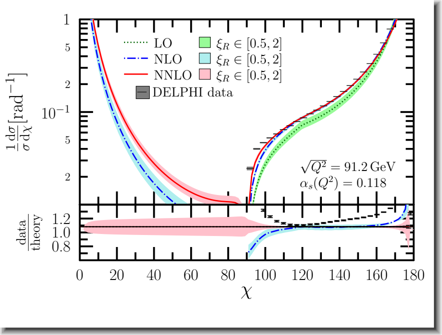
- a "particularly simple and excellent observable for the determination of α_{s}", since hadronization, detector and perturbative corrections are small

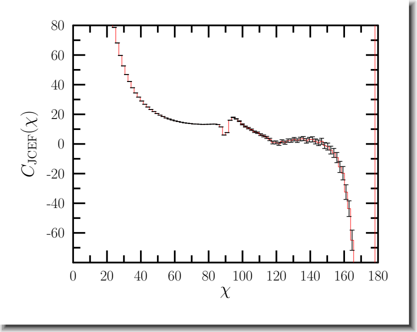
(DELPHI Collaboration Eur. Phys. J. **C14** (2000) 557)

Jet cone energy fraction at NNLO

Physical prediction for jet cone energy fraction (JCEF) at NNLO

- perturbative prediction completely stable w.r.t. including NNLO over large range
- NNLO scale dependence is tiny





PRELIMINARY

Conclusions and outlook

Conclusions and outlook

CoLoRFulNNLO framework

- Completely Local subtRactions for Fully differential predictions at NNLO
- construction of subtraction terms based on IR limit formulae
- analytic integration of subtraction terms is feasible with modern techniques
- demonstrated cancellation of ϵ poles for $m = 2$ and $m = 3$
- worked out in full detail for processes with no colored particles in the initial state

Applications

- Higgs boson decay into a b and anti-b quark
- $e^+e^- \to 2, 3$ jets
- code can compute any IR safe 3-jet observable
- very good numerical convergence of the code

Next steps

- extension to hadronic initial states on the way
- looking into full analytic computation of finite parts of integrated counterterms

Backup slides

Code performance

Double real contribution

- RR on one core: 10M phase space points in 9hrs
- RR smooth with 15B phase space points $\to \sim$ 45hrs on 300 cores

Real-virtual contribution

- RV on one core: 10M phase space points in 31hrs
- RV smooth with 1.5B phase space points $\to \sim$ 15hrs on 300 cores

Double virtual contribution

- VV is very quick $\to \sim$ 10mins on 300 cores

Constrained subtractions

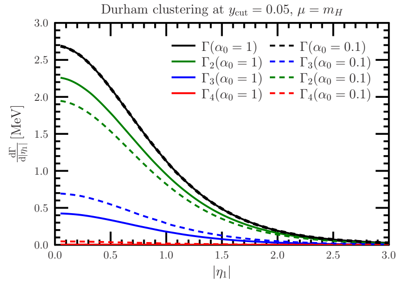
We can constrain subtractions to near singular regions: $\alpha_0 \in (0, 1]$

- poles cancel numerically ($\alpha_0 = 0.1$)

$$\mathrm{d}\sigma^{\mathrm{VV}}_{H\to b\bar{b}} + \sum \int \mathrm{d}\sigma^{\mathrm{A}} = \frac{5.4\times 10^{-8}}{\epsilon^4} + \frac{3.9\times 10^{-5}}{\epsilon^3} + \frac{3.3\times 10^{-3}}{\epsilon^2} + \frac{6.7\times 10^{-3}}{\epsilon} + \mathcal{O}(1)$$

$$Err\left(\sum \int \mathrm{d}\sigma^{\mathrm{A}}\right) = \frac{3.1\times 10^{-5}}{\epsilon^4} + \frac{5.0\times 10^{-4}}{\epsilon^3} + \frac{8.1\times 10^{-3}}{\epsilon^2} + \frac{7.7\times 10^{-2}}{\epsilon} + \mathcal{O}(1)$$

- results unchanged



Constrained subtractions

We can constrain subtractions to near singular regions: $\alpha_0 \in (0,1]$

- improved efficiency

α_0	1	0.1
timing (rel.)	1	0.40
$\langle N_{\rm sub} \rangle$	52	14.5

$\langle N_{\rm sub} \rangle$ is the average number of subtraction terms computed

Basic integrals: an example

The double soft subtraction term leads to the following integral, among others:

$$
\begin{aligned}
\mathcal{I}_{2\mathcal{S},2}(Y_{ik,Q};\epsilon,y_0,d_0') = &-\frac{4\Gamma^4(1-\epsilon)}{\pi\Gamma^2(1-\epsilon)}\,\frac{B_{y_0}(-2\epsilon,d_0')}{\epsilon}\,Y_{ik,Q}\int_0^{y_0}\mathrm{d}y\,y^{-1-2\epsilon}(1-y)^{d_0'-1+\epsilon} \\
&\times\int_{-1}^{1}\mathrm{d}(\cos\vartheta)\,(\sin\vartheta)^{-2\epsilon}\int_{-1}^{1}\mathrm{d}(\cos\varphi)\,(\sin\varphi)^{-1-2\epsilon}\big[f(\vartheta,\varphi;0)\big]^{-1}\big[f(\vartheta,\varphi;Y_{ik,Q})\big]^{-1} \\
&\times\big[Y(y,\vartheta,\varphi;Y_{ik,Q})\big]^{-\epsilon}{}_2F_1\big(-\epsilon,-\epsilon,1-\epsilon,1-Y(y,\vartheta,\varphi;Y_{ik,Q})\big)
\end{aligned}
$$

where

$$
f(\vartheta,\varphi;Y_{ik,Q}) = 1-2\sqrt{Y_{ik,Q}(1-Y_{ik,Q})}\sin\vartheta\cos\varphi-(1-2Y_{ik,Q})\chi\cos\vartheta
$$

$$
Y(y,\vartheta,\varphi;\chi) = \frac{4(1-y)Y_{ik,Q}}{[2(1-y)+y\,f(\vartheta,\varphi;0)][2(1-y)+y\,f(\vartheta,\varphi;Y_{ik,Q})]}
$$

Basic integrals: an example

This integral is equal to $(y_0 = 1,\ d_0' = 3 - 3\epsilon)$

$$
\begin{aligned}
&\mathcal{I}_{2\mathcal{S},2}(Y;\epsilon,1,3-3\epsilon) = \\
&= \frac{1}{2\epsilon^4} - \frac{1}{\epsilon^3}\Big[\ln(Y) - 3\Big] + \frac{1}{\epsilon^2}\Big[2\,\mathrm{Li}_2(1-Y) + \ln^2(Y) - \pi^2 - \Big(\frac{2}{1-Y} \\
&- \frac{1}{2(1-Y)^2} + \frac{9}{2}\Big)\ln(Y) + \frac{1}{2(1-Y)} + 16\Big] + \frac{1}{\epsilon}\Big[\frac{5}{3}\Big(\frac{18\,\mathrm{Li}_3(1-Y)}{5} + \frac{6\,\mathrm{Li}_3(Y)}{5} \\
&- \frac{6\,\mathrm{Li}_2(1-Y)\ln(Y)}{5} - \frac{2}{5}\ln^3(Y) + \frac{3}{5}\ln(1-Y)\ln^2(Y) + \pi^2\ln(Y) - \frac{78\zeta_3}{5}\Big) \\
&+ \Big(\frac{3}{1-Y} - \frac{3}{4(1-Y)^2} + \frac{15}{4}\Big)\Big(2\,\mathrm{Li}_2(1-Y) + \ln^2(Y)\Big) - 6\pi^2 - \Big(\frac{27}{2(1-Y)} \\
&- \frac{13}{4(1-Y)^2} + \frac{91}{4}\Big)\ln(Y) + \frac{19}{4(1-Y)} + \frac{163}{2}\Big] + \mathrm{O}(\epsilon^0)
\end{aligned}
$$

- Note the $Y \to 1$ limit is finite

$$
\lim_{Y\to 1}\mathcal{I}_{2\mathcal{S},2}(Y;\epsilon,1,3-3\epsilon) = \frac{1}{2\epsilon^4} + \frac{3}{\epsilon^3} + \frac{1}{\epsilon^2}\Big(\frac{71}{4} - \pi^2\Big) + \frac{1}{\epsilon}\Big(\frac{393}{4} - 6\pi^2 - 24\zeta_3\Big) + \mathrm{O}(\epsilon^0)
$$

Basic integrals: an example

Finite term is split into an asymptotic piece (which becomes singular as $Y \to 0$) and the remaining regular piece

$$\mathcal{F}in\Big(\mathcal{I}_{2\mathcal{S},2}(Y;\epsilon,1,3-3\epsilon)\Big) = \mathcal{F}in\Big(\mathcal{I}^{\mathrm{asy}}_{2\mathcal{S},2}(Y;\epsilon,1,3-3\epsilon)\Big) + \mathcal{F}in\Big(\mathcal{I}^{\mathrm{reg}}_{2\mathcal{S},2}(Y;\epsilon,1,3-3\epsilon)\Big)$$

- asymptotic expression for $Y \to 0$ computed analytically

$$\mathcal{F}in\Big(\mathcal{I}^{\mathrm{asy}}_{2\mathcal{S},2}(Y;\epsilon,1,3-3\epsilon)\Big) = \frac{\log^4(Y)}{3} - 4\log^3(Y) - \frac{4}{3}\pi^2\log^2(Y) + 33\log^2(Y) + 40\zeta_3\log(Y) + 8\pi^2\log(Y) - \frac{345\log(Y)}{2}$$

- regular part $\mathcal{F}in\Big(\mathcal{I}^{\mathrm{reg}}_{2\mathcal{S},2}(Y;\epsilon,1,3-3\epsilon)\Big)$ computed numerically