Jet cross sections with the CoLoRFulNNLO method

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in collaboration with
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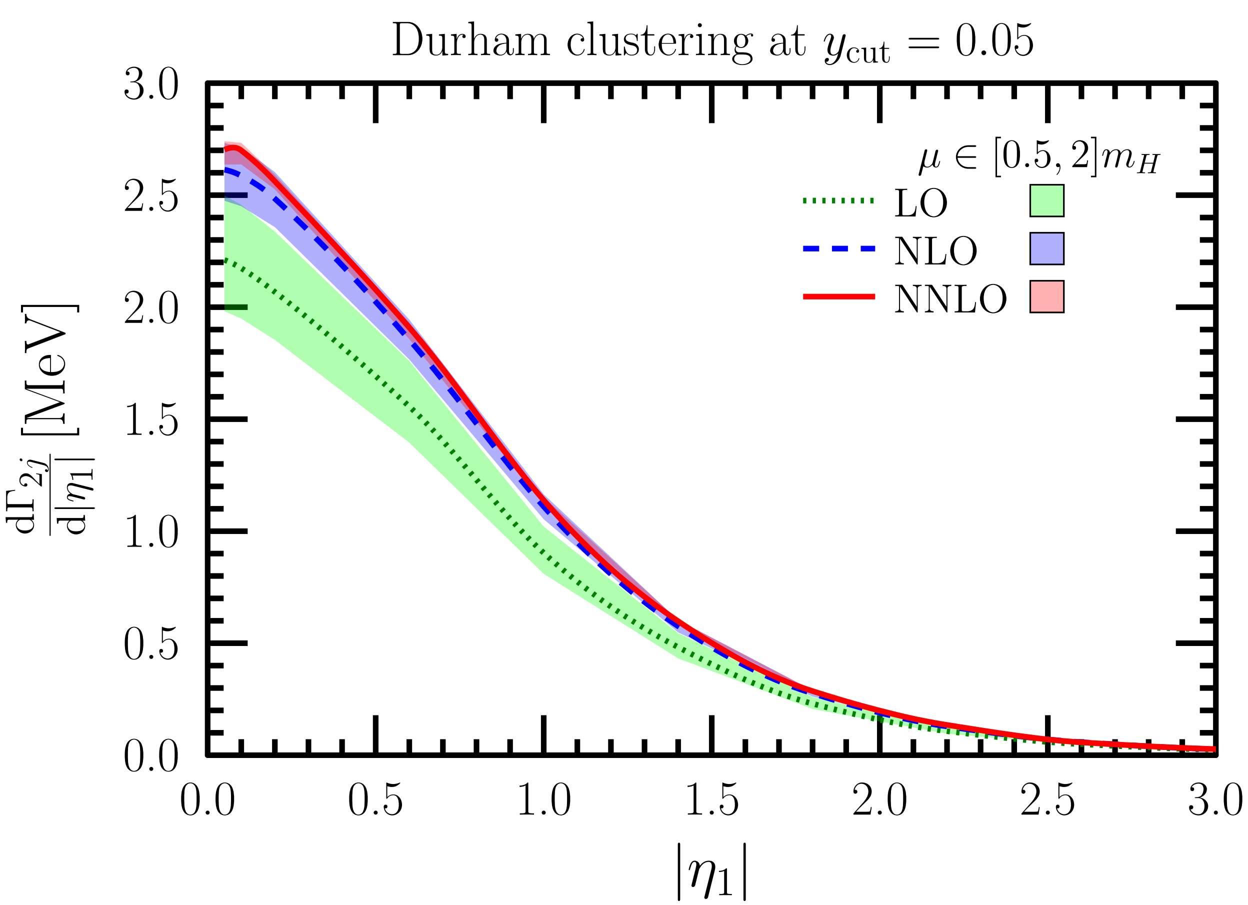
Why leptoproduction?

- Strong coupling: fundamental in SM and QCD. Can be extracted from event shapes in $e^+ e^-$ annihilation.
  - For α_S determination high quality predictions are needed (fitting).
  - Event shape with modest hadronization corrections.
- Ideal test bed for new methods: only FSR, in NNLO only FF, no IF nor II subtraction is needed.
- Up to 3 jets NNLO corrections are known for several event shape observables (comparison and validation).

Why NNLO?

- High precision experiments demand high precision predictions.
- Relatively large coupling ⇒ final state is QCD dominated.
- Key processes have irreducible QCD background and/or QCD corrections.



- Large scale uncertaint(y)(ies).

$H \to b\bar{b}$ decay @ NNLO
Del Duca et al. arXiv:1501.07226

Subtractions 101

Aim: Jet cross section for an m-jet observable (J_m):

$$\sigma_{\rm NNLO} = \sigma^{\rm LO} + \sigma^{\rm NLO} + \sigma^{\rm NNLO}$$

The NLO correction is:

$$\int_{m+1} {\rm d}\sigma^{\rm R}_{m+1} J_{m+1} + \int_m {\rm d}\sigma^{\rm V}_m J_m$$

Adding zero in a clever way:

$$\sigma^{\rm NLO} = \int_{m+1} \left[{\rm d}\sigma^{\rm R}_{m+1} J_{m+1} - {\rm d}\sigma^{\rm R\,,A_1}_{m+1} J_m \right]_{\varepsilon=0} + \int_m \left[{\rm d}\sigma^{\rm V}_m + \int_1 {\rm d}\sigma^{\rm R\,,A_1}_{m+1} \right]_{\varepsilon=0} J_m$$

Subtractions 201

The NNLO correction is:

$$\sigma_m^{\rm NNLO} + \sigma_{m+1}^{\rm NNLO} + \sigma_{m+2}^{\rm NNLO} = \int_{m+2} {\rm d}\sigma_{m+2}^{\rm RR} J_{m+2} + \int_{m+1} {\rm d}\sigma_{m+1}^{\rm RV} J_{m+1} + \int_m {\rm d}\sigma_m^{\rm VV} J_m$$

Have to add zero in a clever way:

The m+2 partonic contribution:

$$\sigma_{m+2}^{\rm NNLO} = \int_{m+2} \left\{ {\rm d}\sigma_{m+2}^{\rm RR} J_{m+2} - {\rm d}\sigma_{m+2}^{\rm RR,A_2} J_m - \left[{\rm d}\sigma_{m+2}^{\rm RR,A_1} J_{m+1} - {\rm d}\sigma_{m+2}^{\rm RR,A_{12}} J_m \right] \right\}_{\varepsilon=0}$$

The m+1 partonic contribution:

$$\sigma_{m+1}^{\rm NNLO} = \int_{m+1} \left\{ \left({\rm d}\sigma_{m+1}^{\rm RV} + \int_1 {\rm d}\sigma_{m+2}^{\rm RR,A_1} \right) J_{m+1} - \left[{\rm d}\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 {\rm d}\sigma_{m+2}^{\rm RR,A_1} \right)^{A_1} \right] J_m \right\}_{\varepsilon=0}$$

The m partonic contribution:

$$\sigma_m^{\rm NNLO} = \int_m \left\{ {\rm d}\sigma_m^{\rm VV} + \int_2 \left[{\rm d}\sigma_{m+2}^{\rm RR,A_2} - {\rm d}\sigma_{m+2}^{\rm RR,A_{12}} \right] + \int_1 \left[{\rm d}\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 {\rm d}\sigma_{m+2}^{\rm RR,A_1} \right)^{A_1} \right] \right\}_{\varepsilon=0} J_m$$

Subtractions 201

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Subtractions 201

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$$\sigma_m^{\mathrm{NNLO}} + \sigma_{m+1}^{\mathrm{NNLO}} + \sigma_{m+2}^{\mathrm{NNLO}} = \int_{m+2} \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} + \int_{m+1} \mathrm{d}\sigma_{m+1}^{\mathrm{RV}} J_{m+1} + \int_m \mathrm{d}\sigma_m^{\mathrm{VV}} J_m$$

Have to add zero in a clever way:

The m+2 partonic contribution:

$$\sigma_{m+2}^{\mathrm{NNLO}} = \int_{m+2} \left\{ \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} J_m - \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} J_{m+1} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} J_m \right] \right\}_{\varepsilon=0}$$

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The ColoRFulNNLO method

- For details see Gabor's talk...
- Completely local subtractions regularize kinematic singularities.
- Large number of subtraction terms: ~100 for $e^+ e^- \longrightarrow 3$jets.
  - ➡Manual implementation is impractical.
  - ➡Automation is needed.
- Method needs the standard SMEs.
  - ➡Automation is possible.

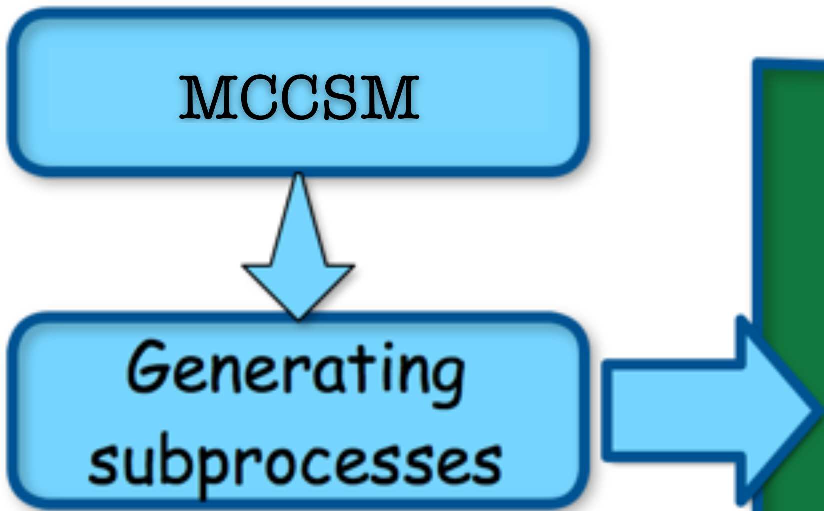
MCCSM

The MCCSM code

MCCSM

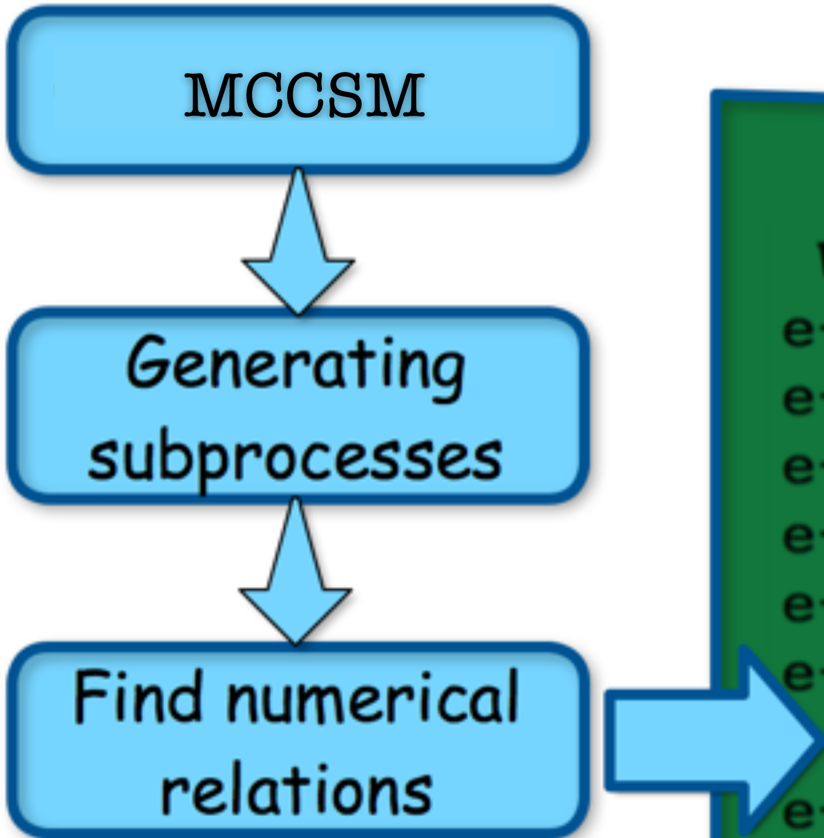
- It is a Monte Carlo implementing the CoLoRFulNNLO Subtraction Method $\longrightarrow$ MCCSM (AK)
- Written in standard Fortran90
- Exploits new features of F90: user types, operator overloading, screening through modules
- Fully automatic
- Highly flexible and tuneable
- MC integration done by MINT
- phase space is by PHASER (in-house multichannel PS generator)
- Histogram output in YODA format through an interface to YODA

MCCSM



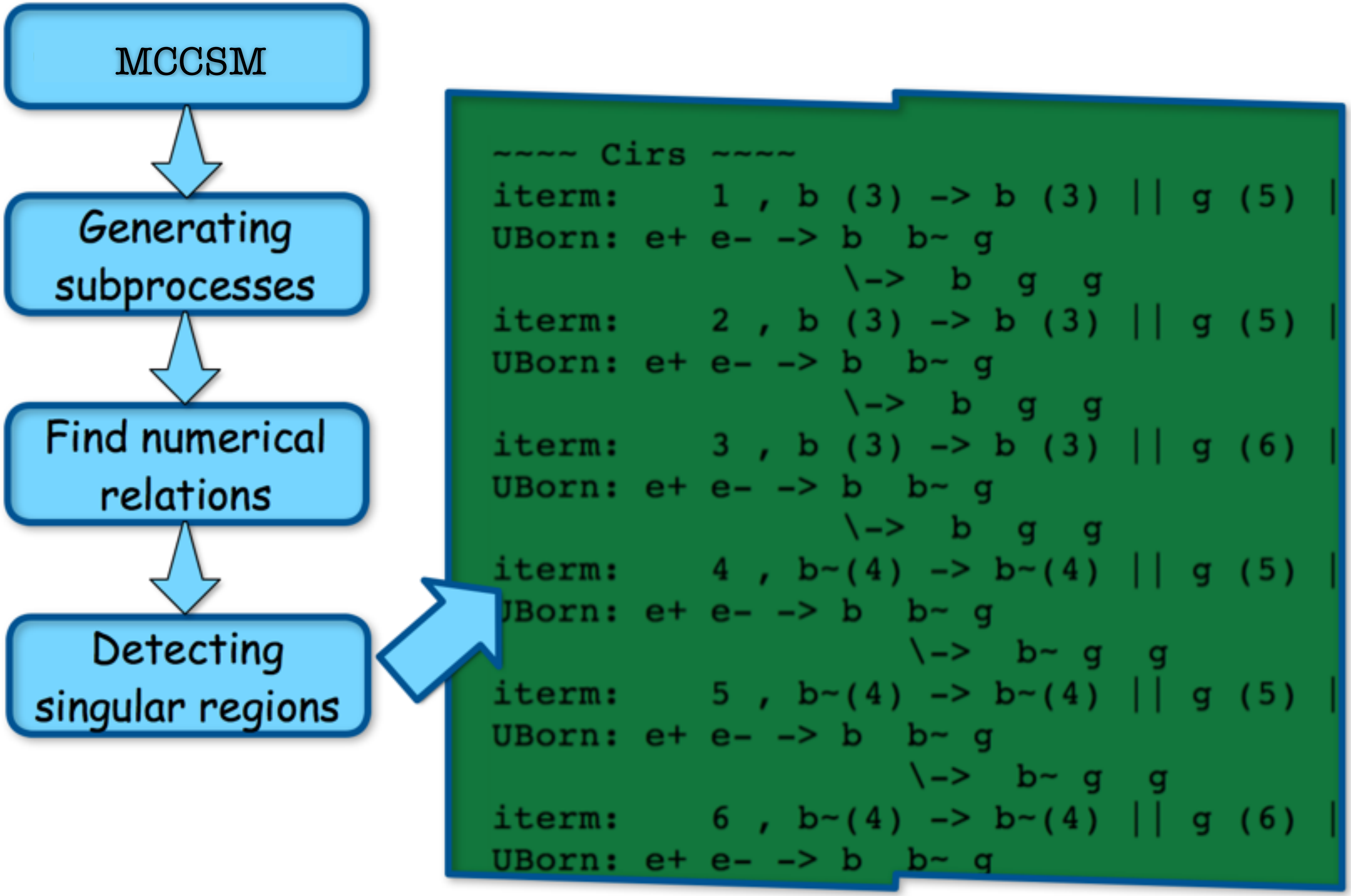

```
Number of subprocesses at the NNLO-
-11  11   5  -5   5  -5   0
-11  11   5  -5   4  -4   0
-11  11   3  -3   5  -5   0
-11  11   5  -5   2  -2   0
-11  11   1  -1   5  -5   0
-11  11   5  -5   0   0   0
-11  11   4  -4   4  -4   0
-11  11   3  -3   4  -4   0
-11  11   2  -2   4  -4   0
-11  11   1  -1   4  -4   0
-11  11   4  -4   0   0   0
-11  11   3  -3   3  -3   0
-11  11   3  -3   2  -2   0
-11  11   1  -1   3  -3   0
-11  11   3  -3   0   0   0
```

MCCSM

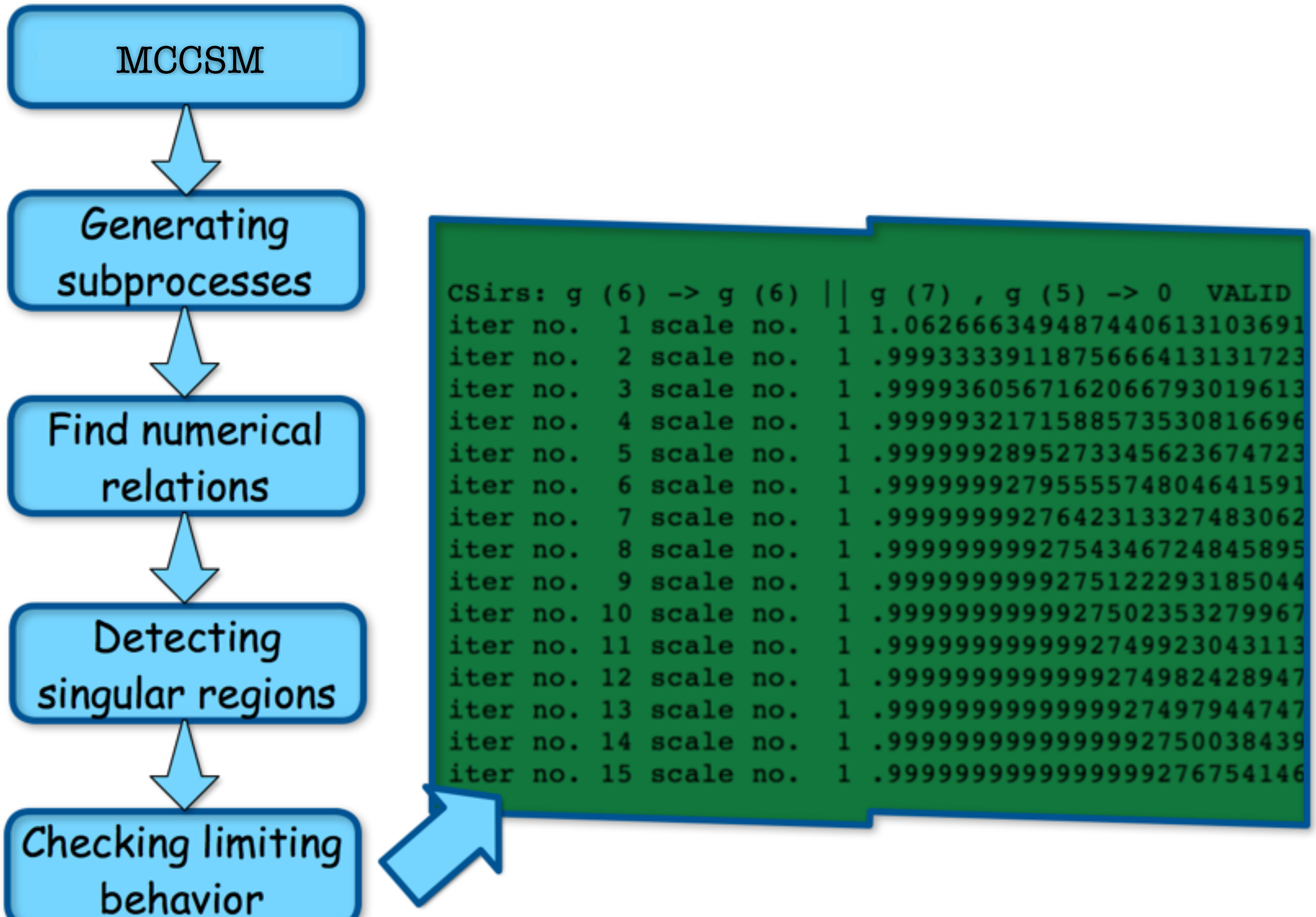



```
 We found the following relations for
e+ e- -> b  b~ b  b~ g   : Irreducibl
e+ e- -> b  b~ c  c~ g   : Irreducibl
e+ e- -> s  s~ b  b~ g   : Irreducibl
e+ e- -> b  b~ u  u~ g   ~ 1.0000 e+
e+ e- -> d  d~ b  b~ g   ~ 1.0000 e+
e+ e- -> b  b~ g  g  g   : Irreducibl
e+ e- -> c  c~ c  c~ g   ~ 4.0000 e+
e+ e- -> s  s~ c  c~ g   ~ 1.0000 e+
e+ e- -> u  u~ c  c~ g   ~ 4.0000 e+
e+ e- -> d  d~ c  c~ g   ~ 1.0000 e+
e+ e- -> c  c~ g  g  g   ~ 4.0000 e+
e+ e- -> s  s~ s  s~ g   ~ 1.0000 e+
e+ e- -> s  s~ u  u~ g   ~ 1.0000 e+
e+ e- -> d  d~ s  s~ g   ~ 1.0000 e+
e+ e- -> s  s~ g  g  g   ~ 1.0000 e+
```

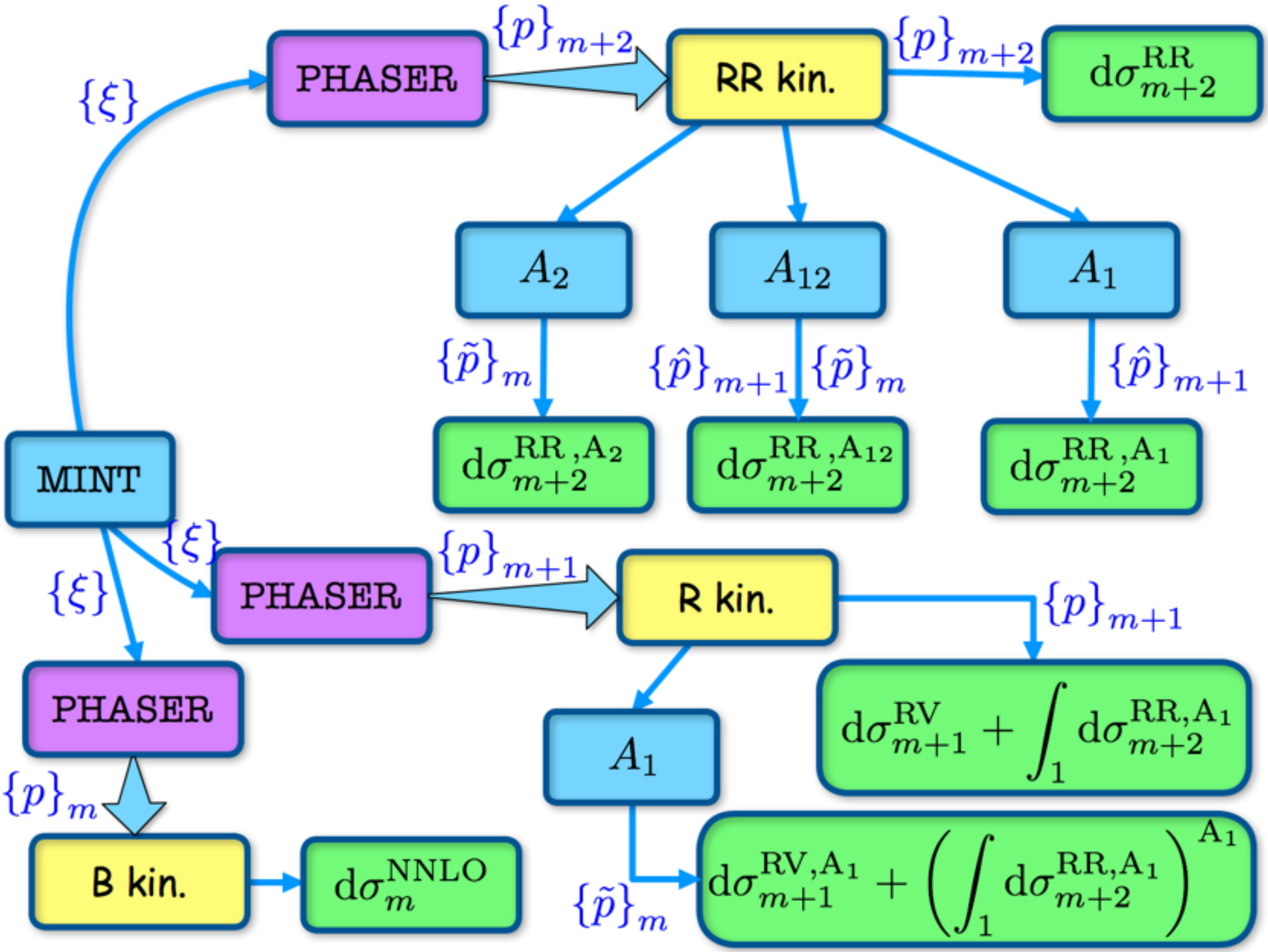
MCCSM


MCCSM
Generating subprocesses
Find numerical relations
Detecting singular regions
~~~~ Cirs ~~~~
iterm: 1 , b (3) -> b (3) || g (5) |
UBorn: e+ e- -> b b~ g
\-> b g g
iterm: 2 , b (3) -> b (3) || g (5) |
UBorn: e+ e- -> b b~ g
\-> b g g
iterm: 3 , b (3) -> b (3) || g (6) |
UBorn: e+ e- -> b b~ g
\-> b g g
iterm: 4 , b~(4) -> b~(4) || g (5) |
UBorn: e+ e- -> b b~ g
\-> b~ g g
iterm: 5 , b~(4) -> b~(4) || g (5) |
UBorn: e+ e- -> b b~ g
\-> b~ g g
iterm: 6 , b~(4) -> b~(4) || g (6) |
UBorn: e+ e- -> b b~ g
~~~~

MCCSM


MCCSM
Generating subprocesses
Find numerical relations
Detecting singular regions
Checking limiting behavior
CSirs: g (6) -> g (6) || g (7) , g (5) -> 0 VALID
iter no. 1 scale no. 1 1.0626663494874406131036 91
iter no. 2 scale no. 1 .99933339118756664131317 23
iter no. 3 scale no. 1 .99993605671620667930196 13
iter no. 4 scale no. 1 .99999321715885735308166 96
iter no. 5 scale no. 1 .99999928952733456236747 23
iter no. 6 scale no. 1 .99999992795555748046415 91
iter no. 7 scale no. 1 .99999999276423133274830 62
iter no. 8 scale no. 1 .99999999927543467248458 95
iter no. 9 scale no. 1 .99999999992751222931850 44
iter no. 10 scale no. 1 .99999999999275023532799 67
iter no. 11 scale no. 1 .99999999999927499230431 13
iter no. 12 scale no. 1 .99999999999992749824289 47
iter no. 13 scale no. 1 .99999999999999274979447 47
iter no. 14 scale no. 1 .99999999999999927500384 39
iter no. 15 scale no. 1 .99999999999999992767541 46

MCCSM



PHASER
$\{p\}_{m+2}$
RR kin.
$\{p\}_{m+2}$
$d\sigma^{RR}_{m+2}$
$\{\xi\}$
A_2
A_{12}
A_1
$\{\tilde{p}\}_m$
$\{\hat{p}\}_{m+1}$
$\{\tilde{p}\}_m$
$\{\hat{p}\}_{m+1}$
$d\sigma^{RR,A_2}_{m+2}$
$d\sigma^{RR,A_{12}}_{m+2}$
$d\sigma^{RR,A_1}_{m+2}$
MINT
$\{\xi\}$
$\{\xi\}$
PHASER
$\{p\}_{m+1}$
R kin.
$\{p\}_{m+1}$
PHASER
A_1
$d\sigma^{RV}_{m+1} + \int_1 d\sigma^{RR,A_1}_{m+2}$
$\{p\}_m$
B kin.
$d\sigma^{NNLO}_m$
$\{\tilde{p}\}_m$
$d\sigma^{RV,A_1}_{m+1} + \left(\int_1 d\sigma^{RR,A_1}_{m+2}\right)^{A_1}$

MCCSM

Available processes:

- $e^+ e^- \longrightarrow$ 2jets (Albers, AK, Somogyi)
- $e^+ e^- \longrightarrow$ 3jets (AK, Somogyi, Szőr, Tulipánt)
- ...

(In parentheses the people who did the implementation.)

Phenomenology

PRELIMINARY

$e^+ e^- \longrightarrow 3$jets

- Previously done @ NNLO QCD:
  - Gehrmann-De Ridder et al. (GGGH) JHEP 0712 (2007) 094
  - Weinzierl (SW) JHEP 0906 (2009) 041
- Comparisons were done for the six fundamental event shape variables
- Completely new NNLO QCD predictions are made for three further event shapes: oblateness, EEC and JCEF
- Comparison:
  - Weinzierl: w/ data presented in JHEP 0906 (2009) 041
  - GGGH: using data also used in the comparison in JHEP 0906 (2009) 041

$e^+ e^- \longrightarrow 3$ jets

New observables:

- Oblateness:

$$O = T_M - T_m$$

- Energy-energy correlation:

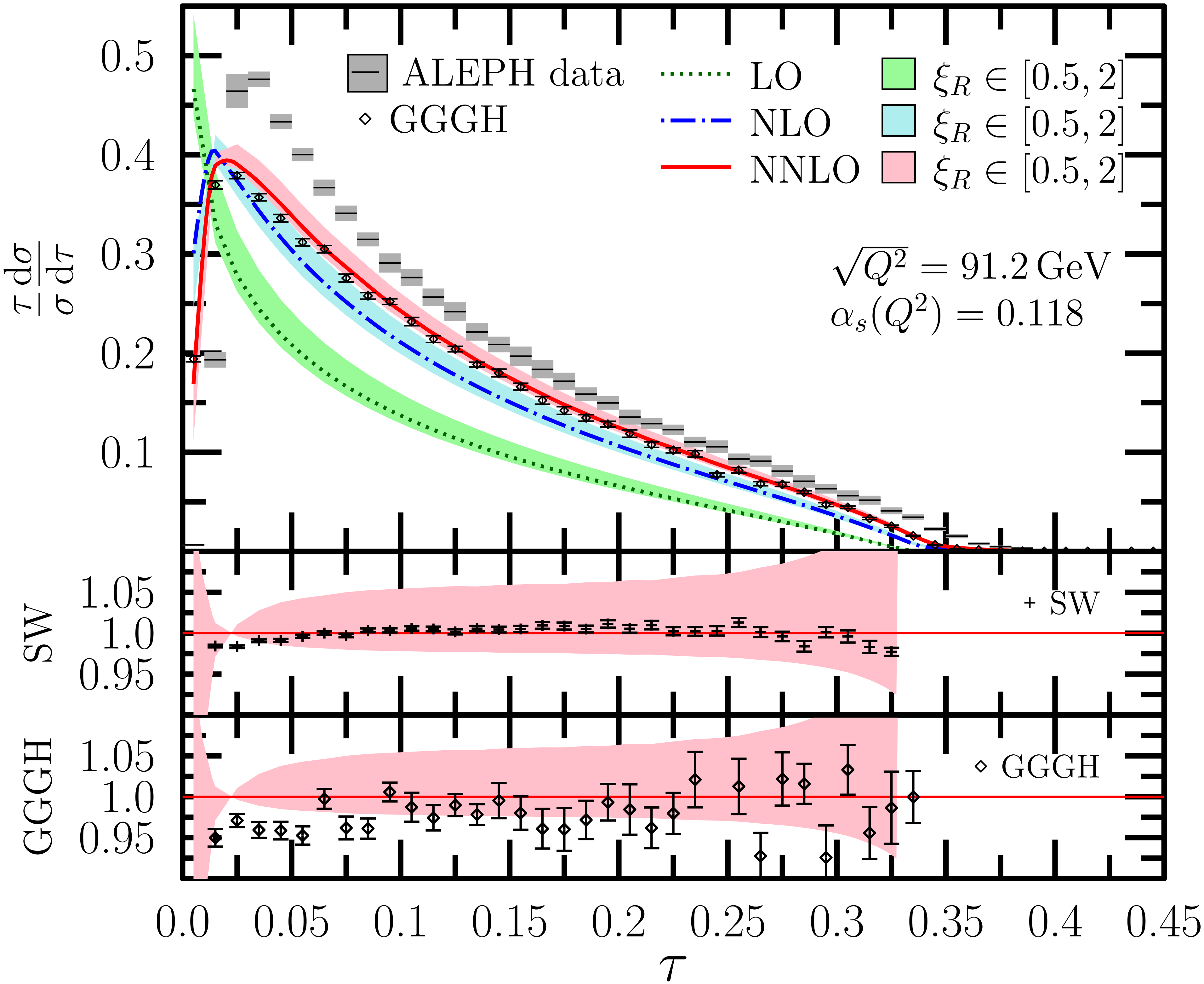
$$\mathrm{EEC}(\chi) = \frac{1}{\sigma_{\mathrm{had}}} \sum_{i,j} \int \mathrm{d}\sigma_{e^+ e^- \to i\, j + X} \frac{E_i E_j}{Q^2} \delta(\cos\chi + \cos\theta_{ij})$$

Already appeared in arXiv:1603.08927

- Jet cone energy fraction:

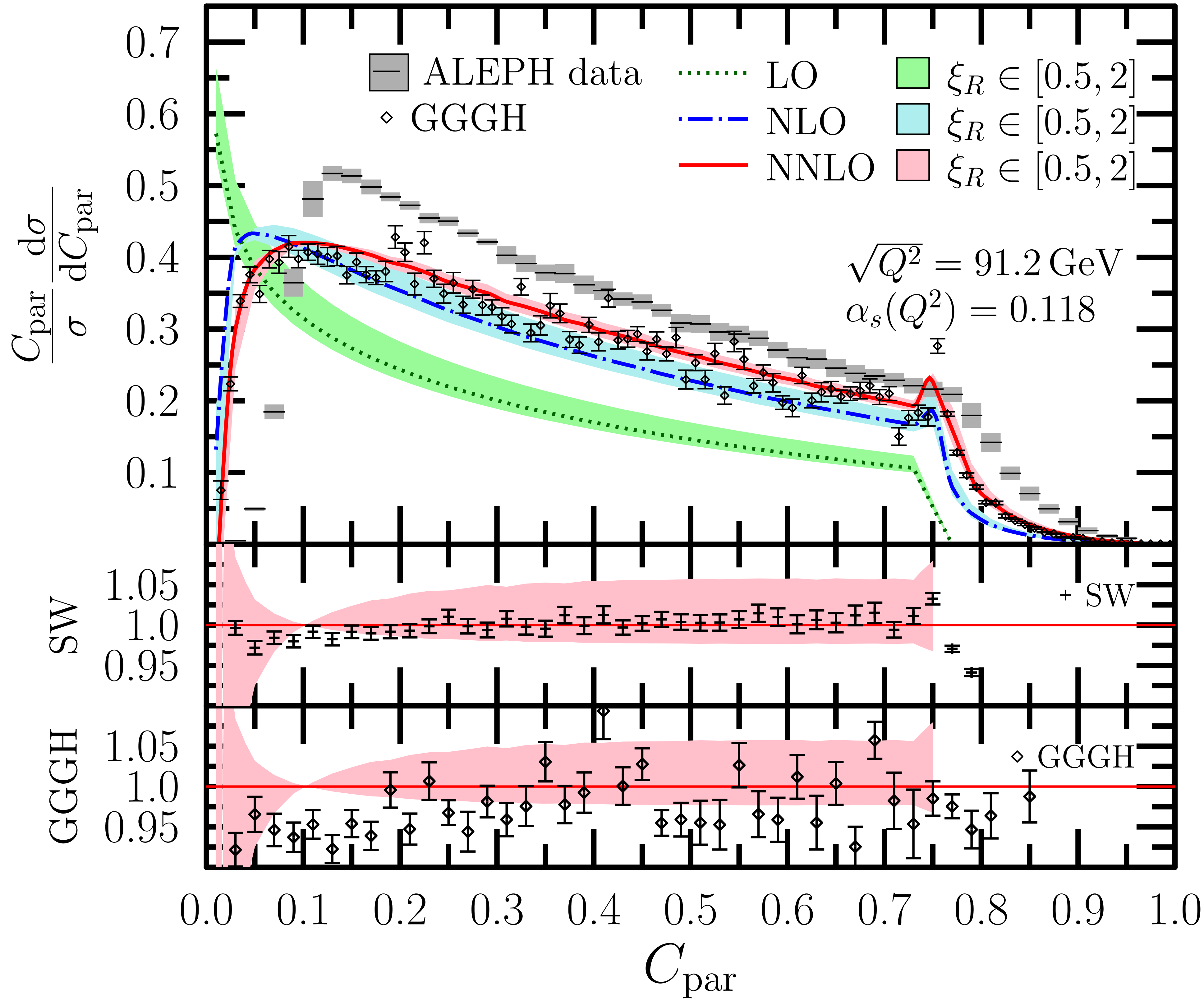
$$\mathrm{JCEF}(\chi) = \frac{1}{\sigma_{\mathrm{had}}} \sum_{i} \int \mathrm{d}\sigma_{e^+ e^- \to i + X} \frac{E_i}{\sqrt{Q^2}} \delta\left(\cos\chi - \frac{\boldsymbol{p}_i \cdot \boldsymbol{n}_T}{|\boldsymbol{p}_i|}\right)$$

$e^+ e^- \longrightarrow 3$jets



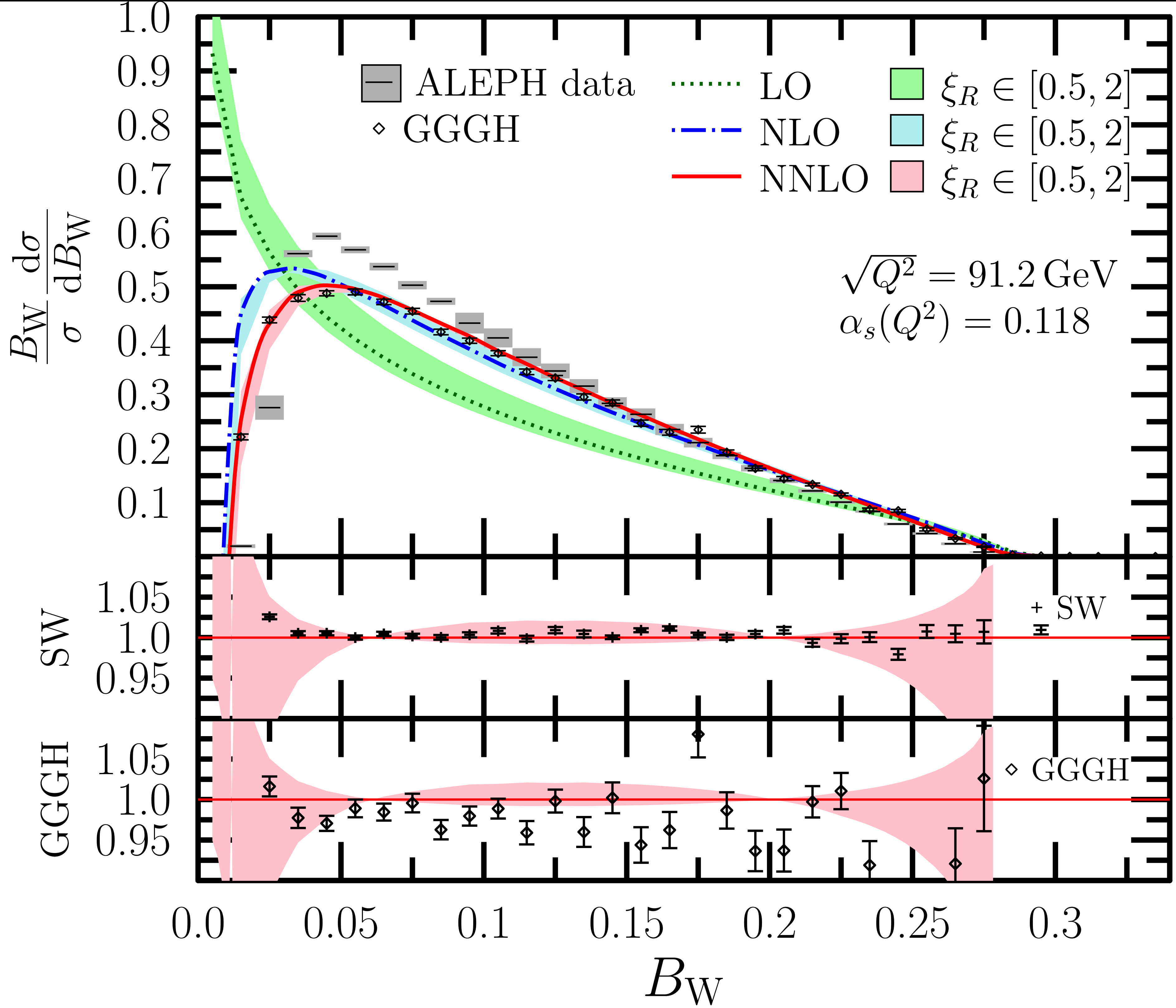
1 - Thrust distribution

$e^+ e^- \longrightarrow 3jets$



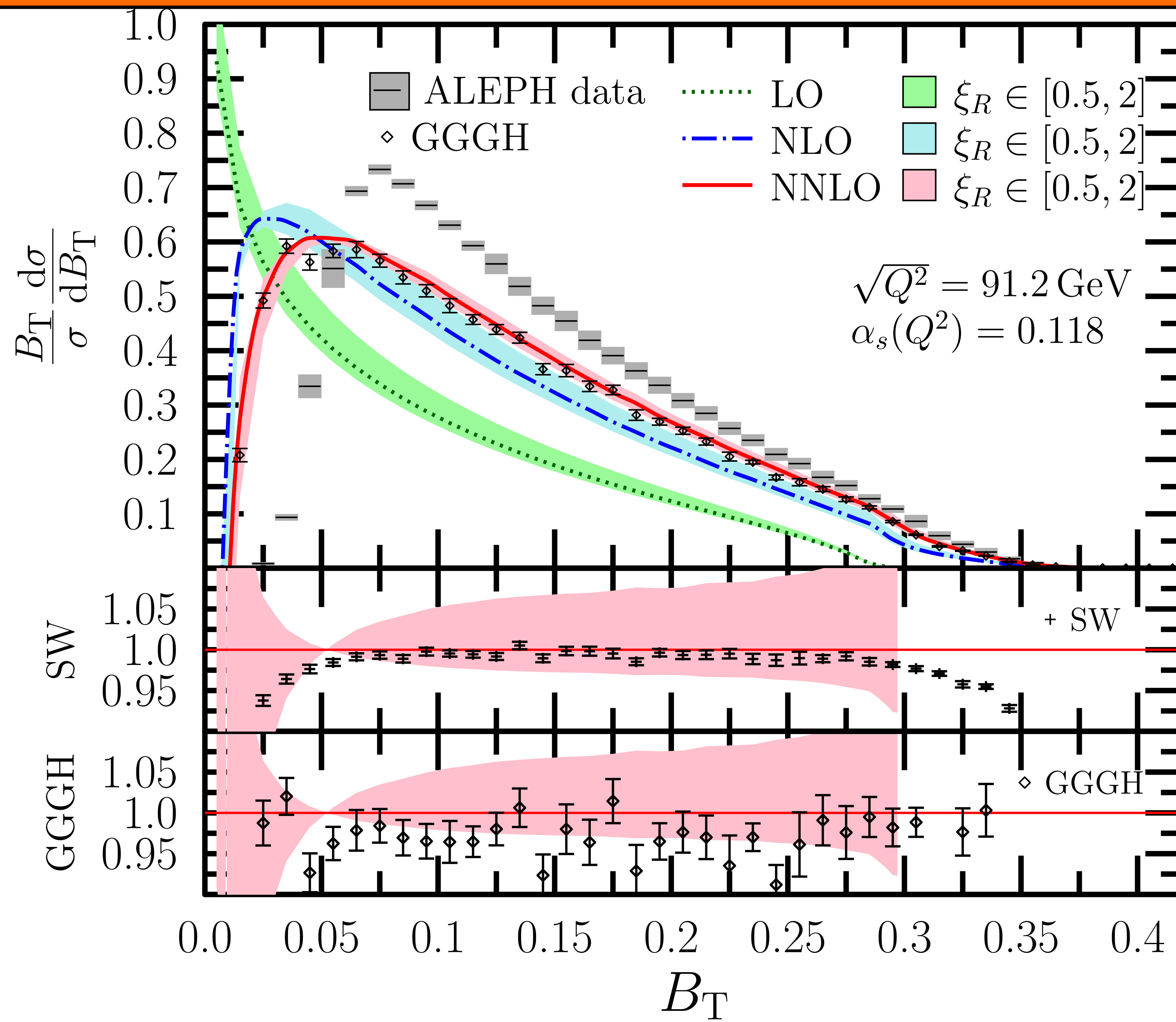
C parameter distribution

$e^+ e^- \longrightarrow 3 jets$



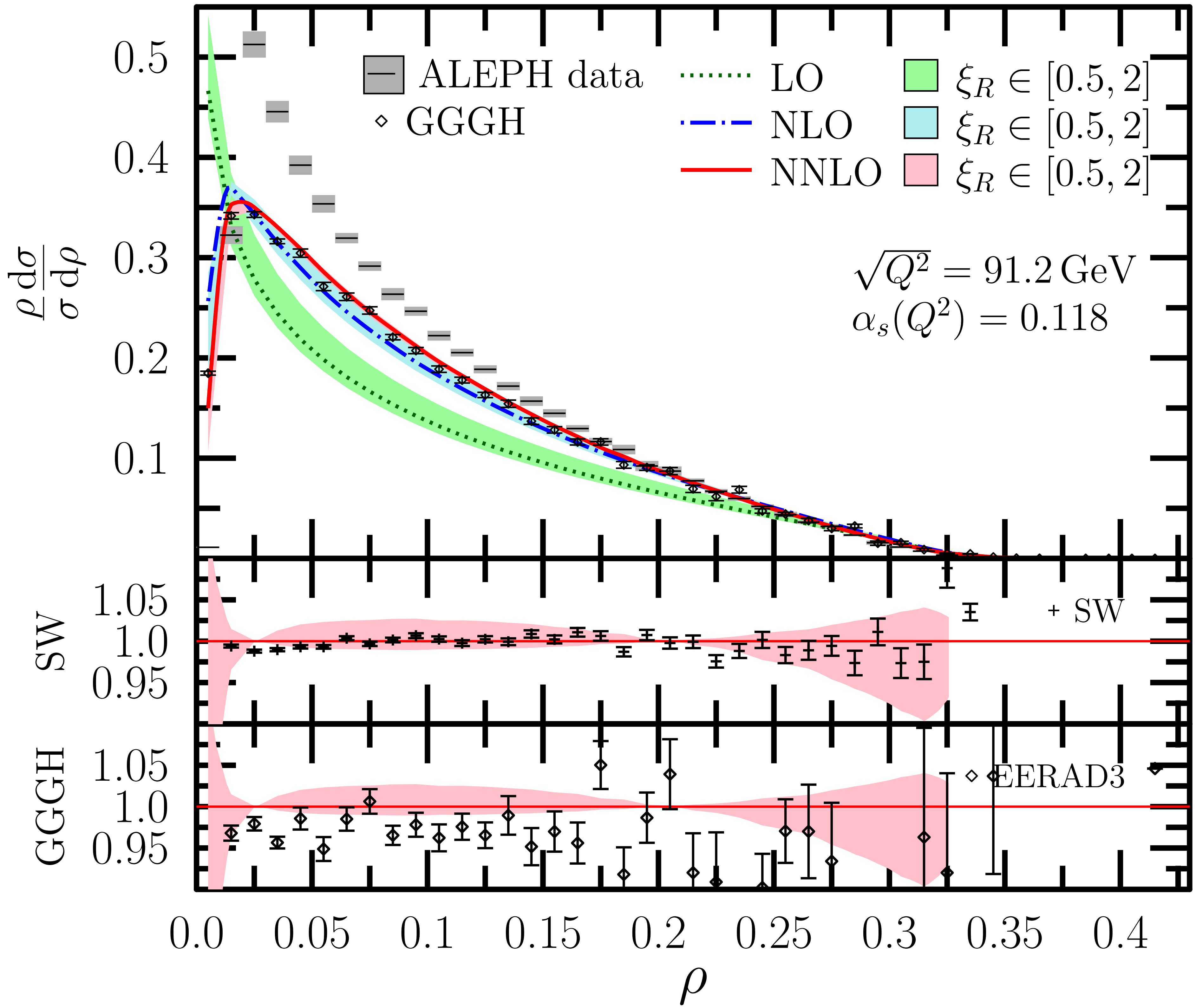
Wide jet broadening distribution

$e^+ e^- \longrightarrow 3$jets



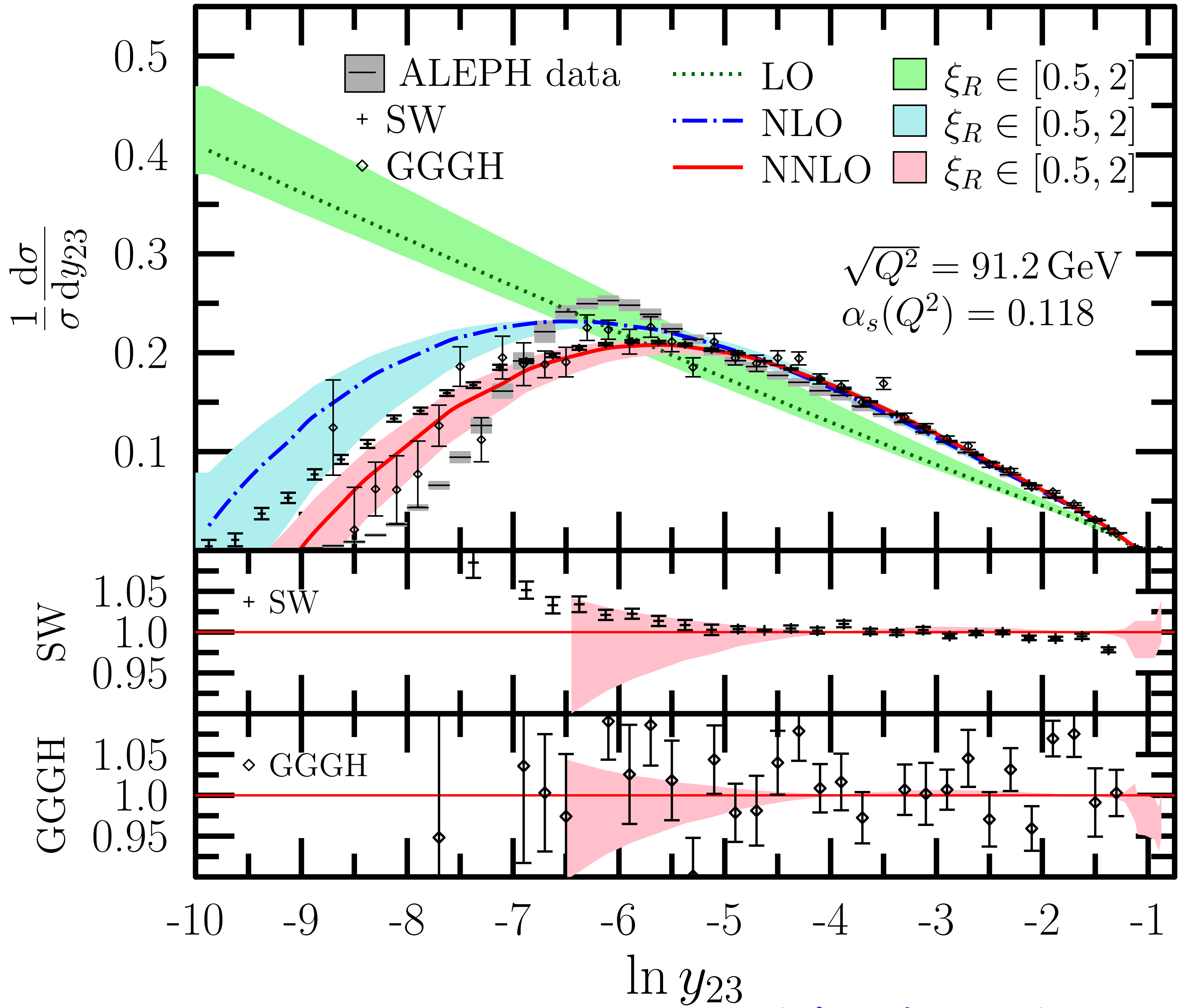
Total jet broadening distribution

$e^+ e^- \longrightarrow 3$jets



Heavy jet mass distribution

$e^+ e^- \longrightarrow 3jets$



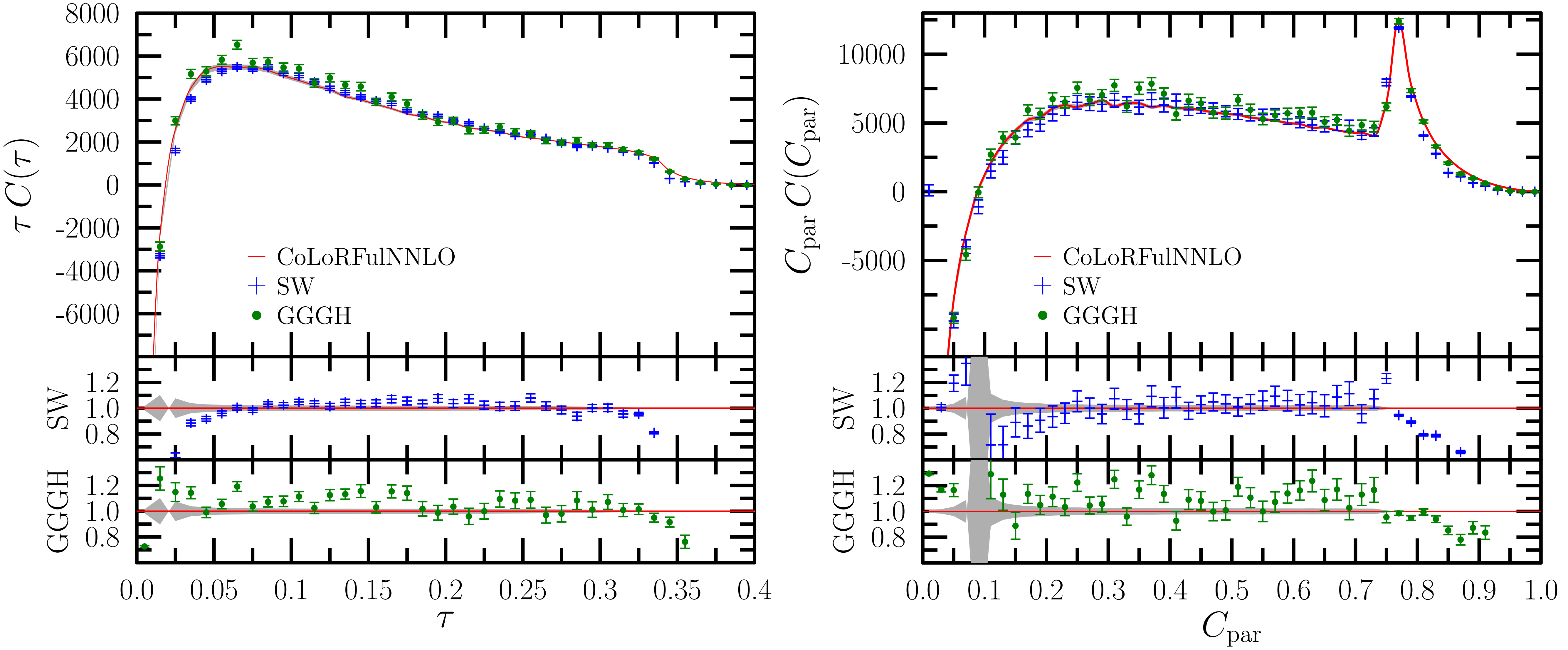
2 to 3 jet transition variable distribution

$e^+ e^- \longrightarrow 3 jets$

To better quantify the size of NNLO corrections, they are plotted separately:

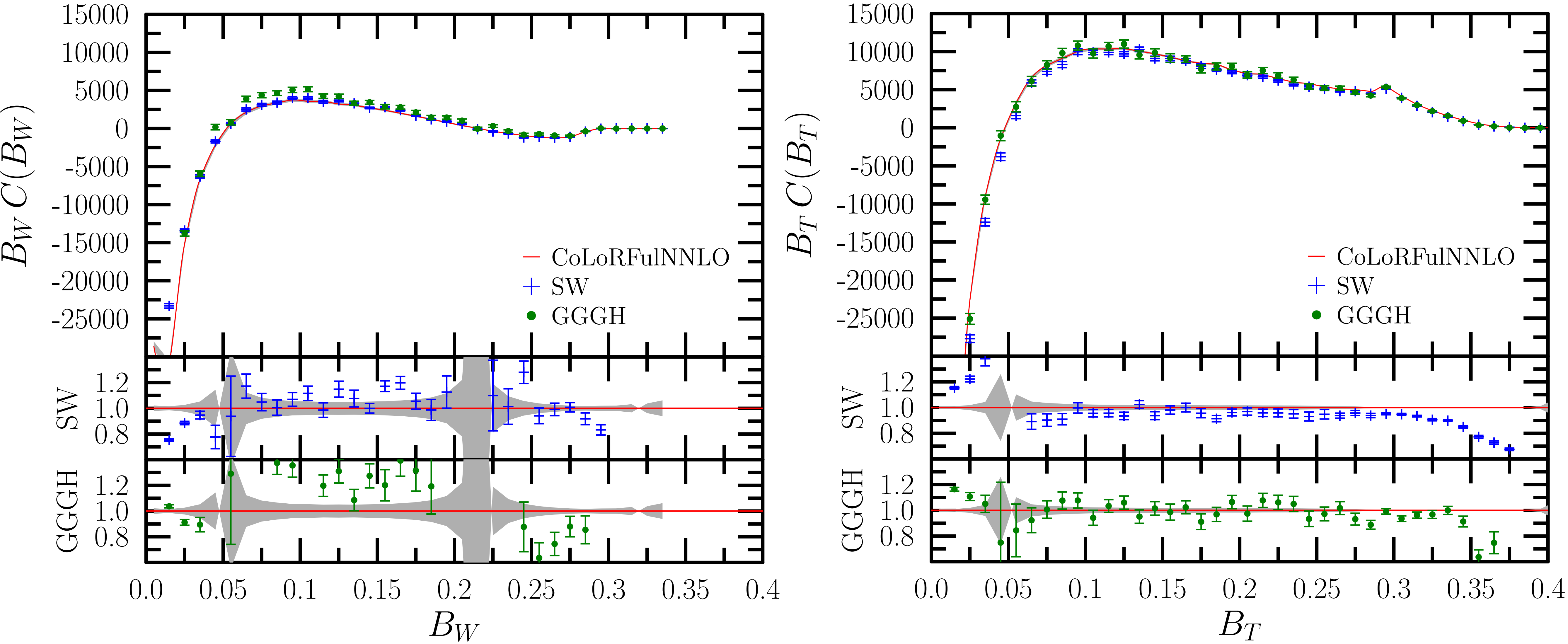
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}O} = \frac{\alpha_S}{2\pi}A(O) + \left(\frac{\alpha_S}{2\pi}\right)^2 B(O) + \left(\frac{\alpha_S}{2\pi}\right)^3 C(O) + \mathcal{O}(\alpha_S^4)$$

$e^+ e^- \longrightarrow 3$jets



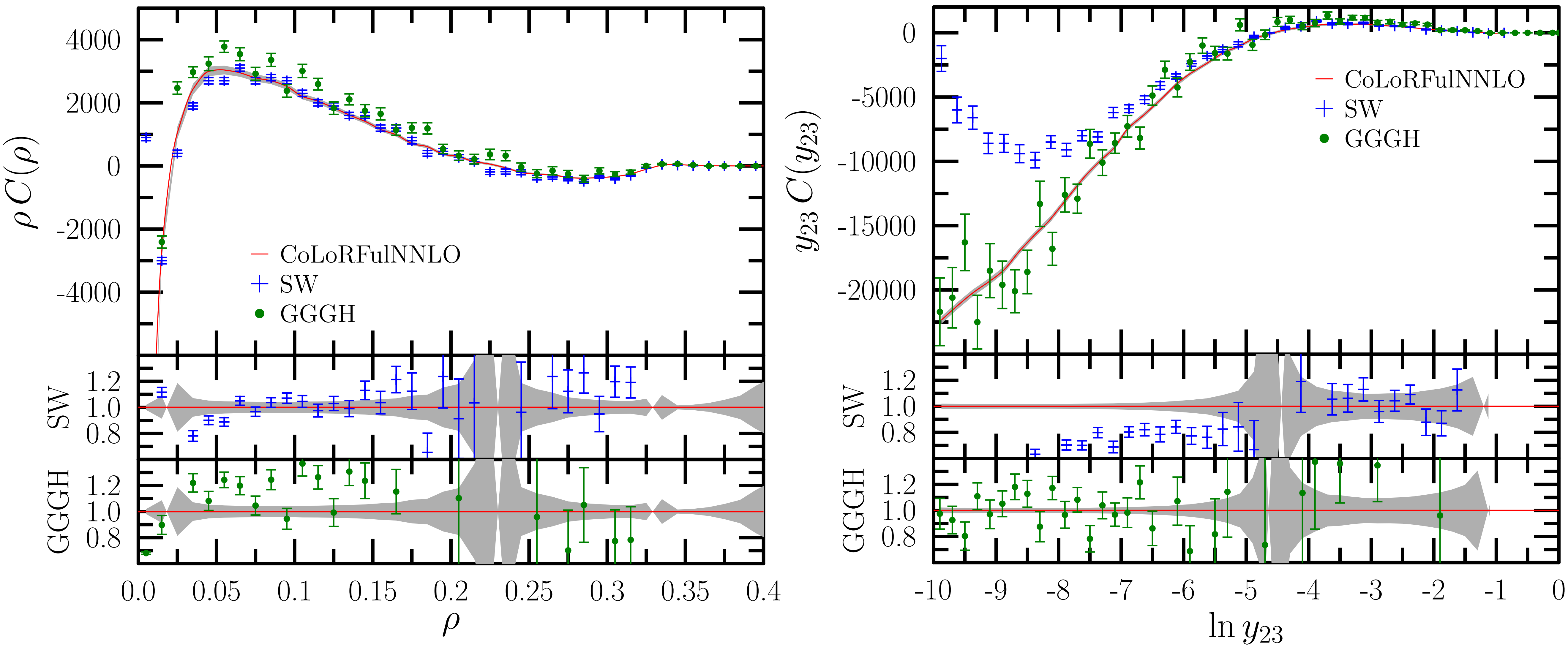
C coefficient for 1-T and C parameter distributions

$e^+ e^- \longrightarrow 3 jets$



C coefficient distribution for wide and total jet broadening

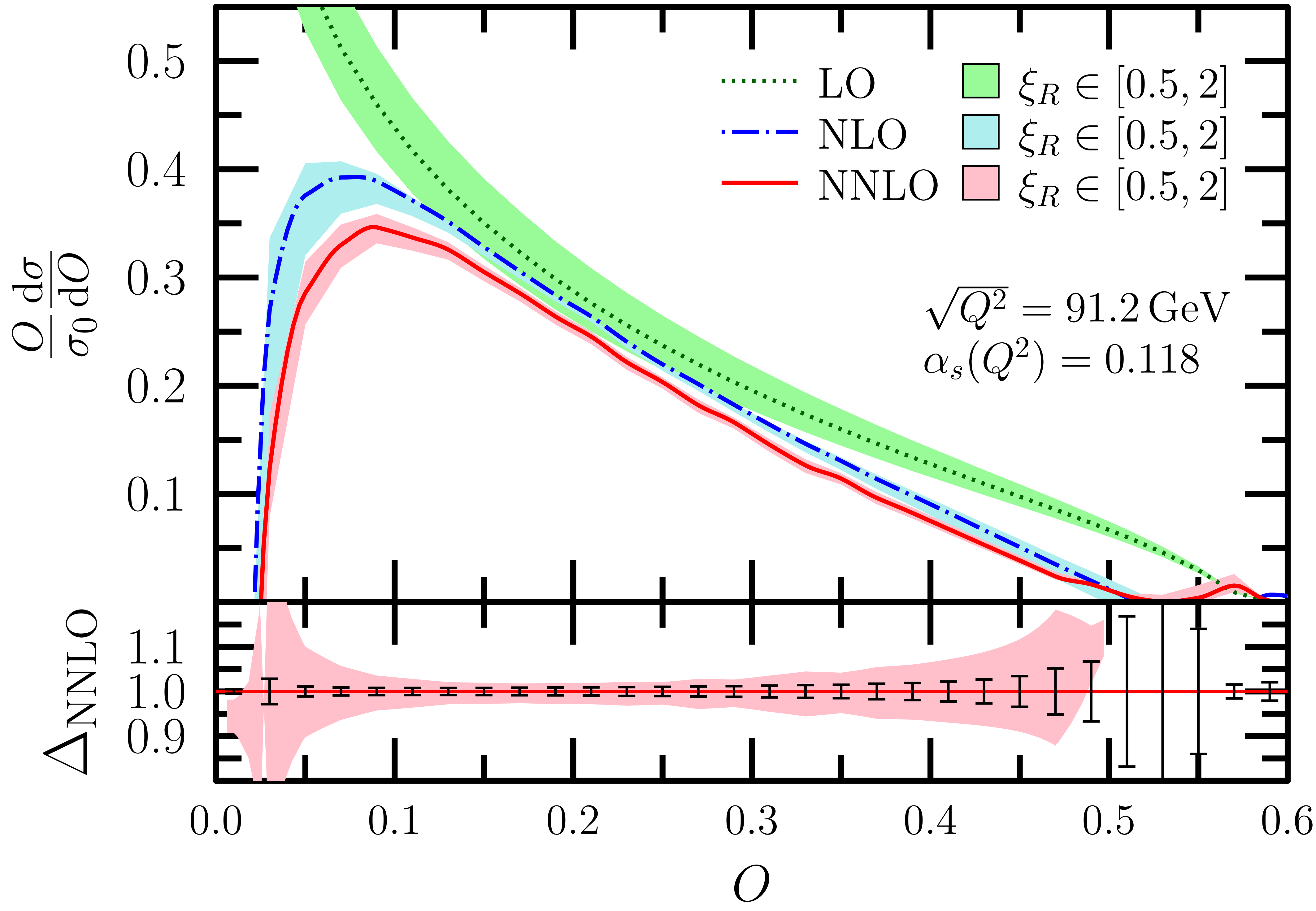
$e^+ e^- \longrightarrow 3 jets$



C coefficient distribution for heavy jet mass and y_{23}

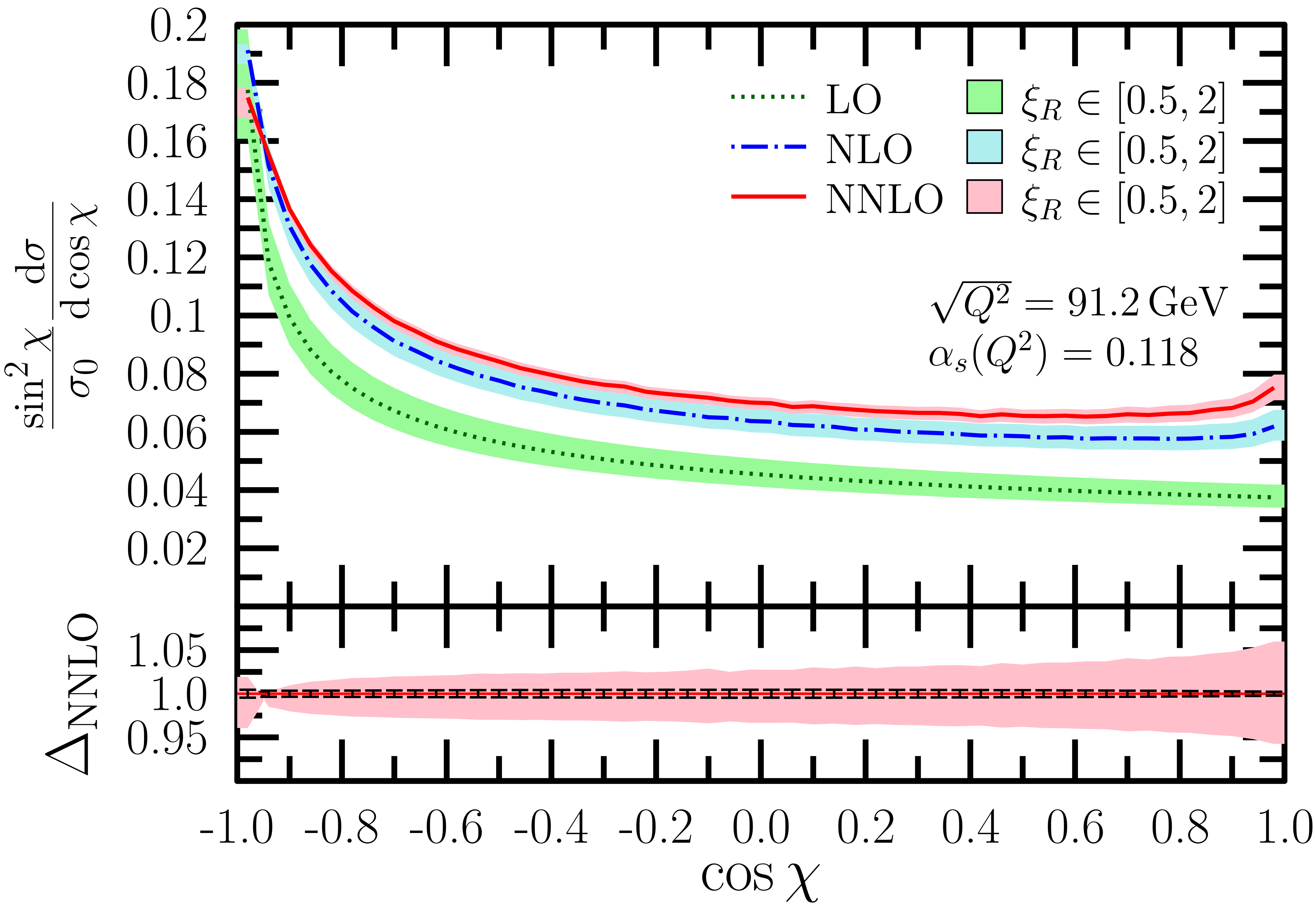
New predictions

$e^+ e^- \longrightarrow 3jets$



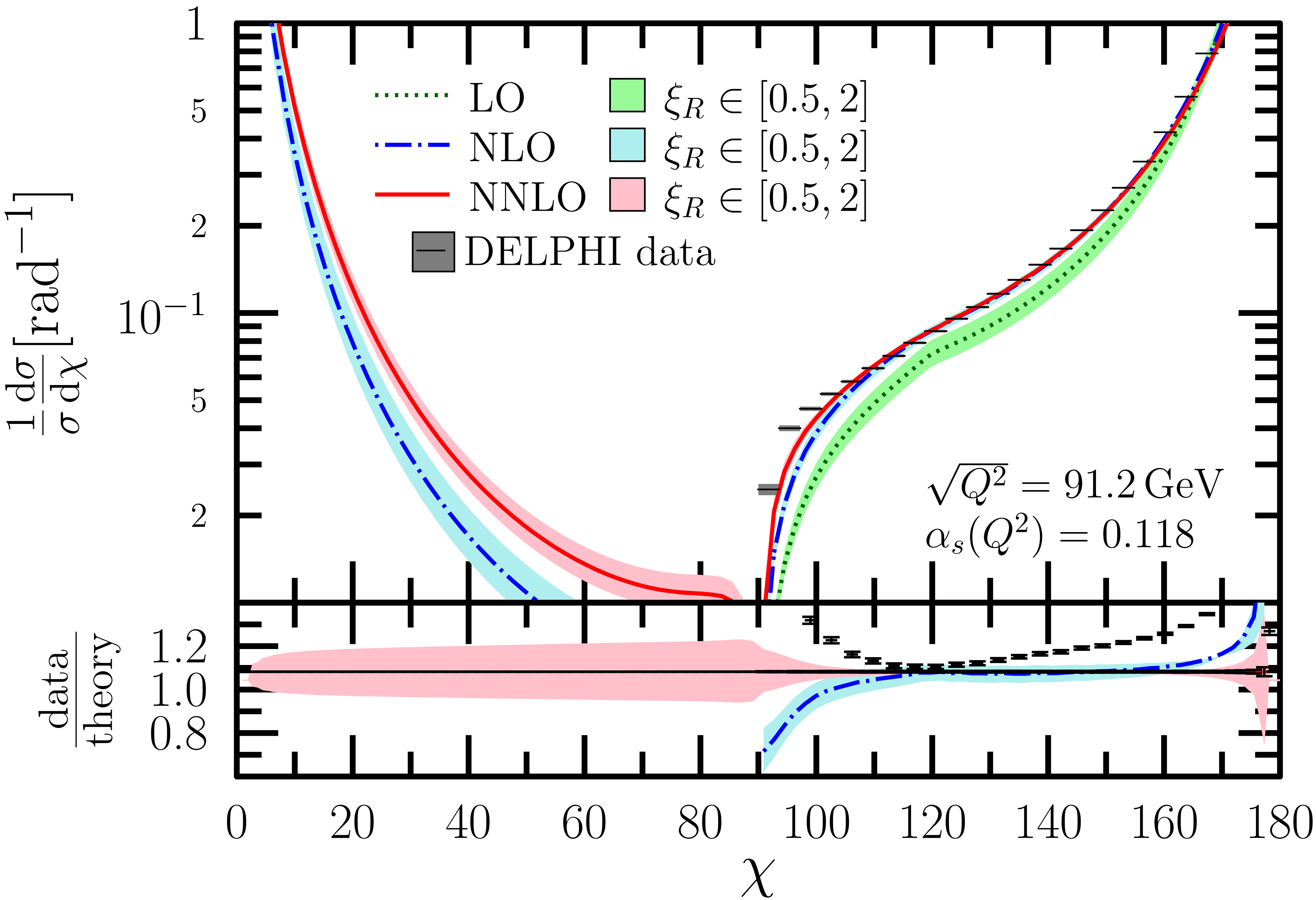
Oblateness

$e^+ e^- \longrightarrow 3jets$



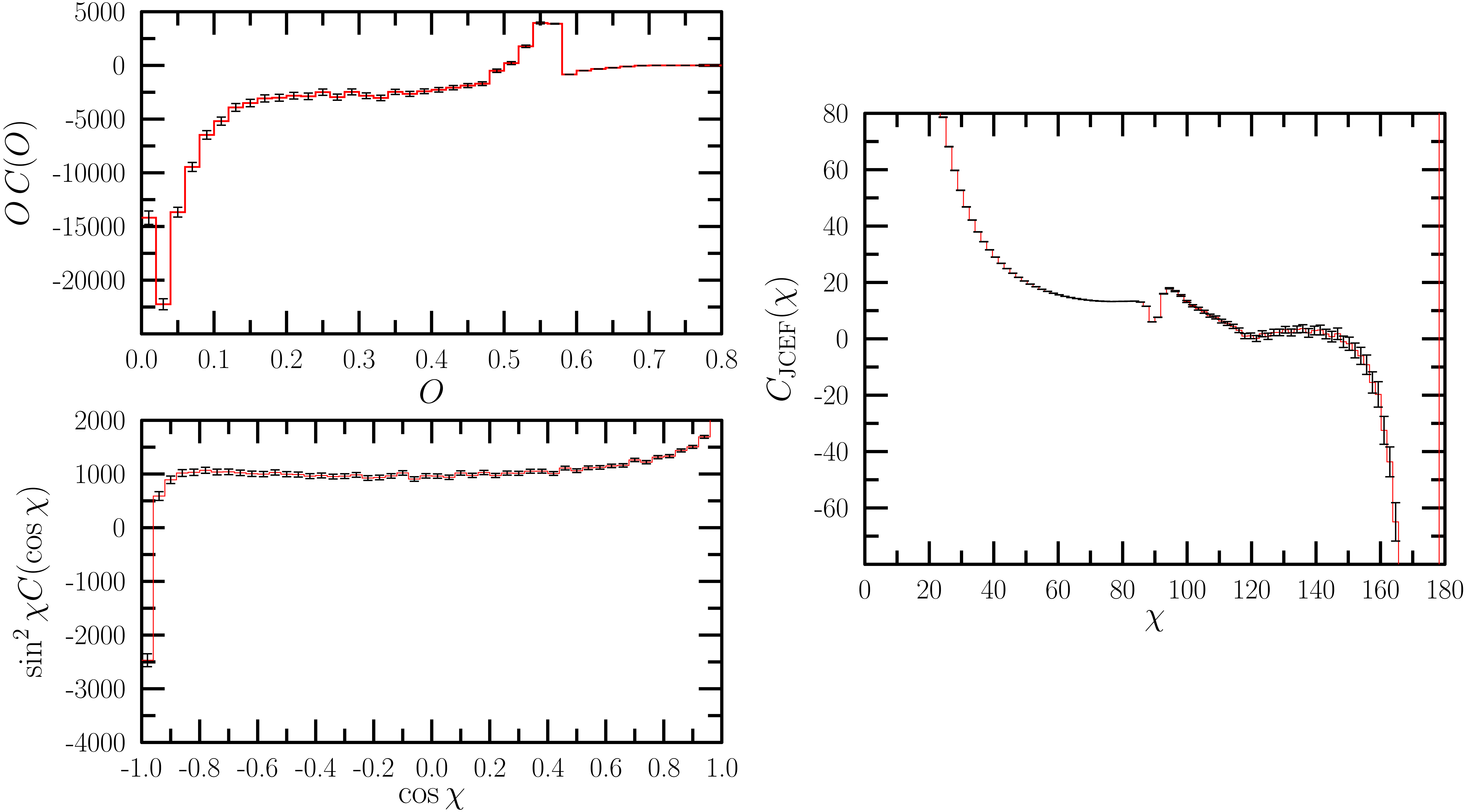
Energy-energy correlation

$e^+ e^- \to 3$jets



Jet cone energy fraction

$e^+ e^- \longrightarrow 3jets$



C coefficients for Oblateness (TL), EEC (BL) and JCEF (CR)

Conclusions

- The MCCSM code is used to obtain NNLO predictions for $e^+ e^- \longrightarrow 3$jets
- Comparisons are made with existing predictions
- New NNLO predictions are made for 3 event shape variables
- The code fully automatic, highly tuneable and completely general (for colorless initial states)
- Incorporating partons in the initial state is already started

Thank you for your attention!

Back-up slides

MCCSM

The generation of all subprocesses is automatic:

```
! We define the process at the Born level:
   proc_LO(1) = 'e+'
   proc_LO(2) = 'e-'
   proc_LO(3) = 'j '
   proc_LO(4) = 'j '
   proc_LO(5) = 'j '
```

```
Number of subprocesses at the NNLO-RR level:   20
-11  11   5  -5   5  -5   0
-11  11   5  -5   4  -4   0
-11  11   3  -3   5  -5   0
-11  11   5  -5   2  -2   0
-11  11   1  -1   5  -5   0
-11  11   5  -5   0   0   0
-11  11   4  -4   4  -4   0
-11  11   3  -3   4  -4   0
-11  11   2  -2   4  -4   0
-11  11   1  -1   4  -4   0
-11  11   4  -4   0   0   0
-11  11   3  -3   3  -3   0
-11  11   3  -3   2  -2   0
-11  11   1  -1   3  -3   0
-11  11   3  -3   0   0   0
-11  11   2  -2   2  -2   0
-11  11   1  -1   2  -2   0
-11  11   2  -2   0   0   0
-11  11   1  -1   1  -1   0
-11  11   1  -1   0   0   0
```

MCCSM

Investigating for possible numerical relations between SMEs:

```
 We found the following relations for the double real:
e+ e- -> b  b~ b  b~ g   : Irreducible
e+ e- -> b  b~ c  c~ g   : Irreducible
e+ e- -> s  s~ b  b~ g   : Irreducible
e+ e- -> b  b~ u  u~ g   ~ 1.0000 e+ e- -> b  b~ c  c~ g
e+ e- -> d  d~ b  b~ g   ~ 1.0000 e+ e- -> s  s~ b  b~ g
e+ e- -> b  b~ g  g  g   : Irreducible
e+ e- -> c  c~ c  c~ g   ~ 4.0000 e+ e- -> b  b~ b  b~ g
e+ e- -> s  s~ c  c~ g   ~ 1.0000 e+ e- -> b  b~ c  c~ g
e+ e- -> u  u~ c  c~ g   ~ 4.0000 e+ e- -> s  s~ b  b~ g
e+ e- -> d  d~ c  c~ g   ~ 1.0000 e+ e- -> b  b~ c  c~ g
e+ e- -> c  c~ g  g  g   ~ 4.0000 e+ e- -> b  b~ g  g  g
e+ e- -> s  s~ s  s~ g   ~ 1.0000 e+ e- -> b  b~ b  b~ g
e+ e- -> s  s~ u  u~ g   ~ 1.0000 e+ e- -> b  b~ c  c~ g
e+ e- -> d  d~ s  s~ g   ~ 1.0000 e+ e- -> s  s~ b  b~ g
e+ e- -> s  s~ g  g  g   ~ 1.0000 e+ e- -> b  b~ g  g  g
e+ e- -> u  u~ u  u~ g   ~ 4.0000 e+ e- -> b  b~ b  b~ g
e+ e- -> d  d~ u  u~ g   ~ 1.0000 e+ e- -> b  b~ c  c~ g
e+ e- -> u  u~ g  g  g   ~ 4.0000 e+ e- -> b  b~ g  g  g
e+ e- -> d  d~ d  d~ g   ~ 1.0000 e+ e- -> b  b~ b  b~ g
e+ e- -> d  d~ g  g  g   ~ 1.0000 e+ e- -> b  b~ g  g  g
```

MCCSM

Automatic detection of all singular regions:

```
~~~~ Cirs ~~~~
iterm:    1 , b (3) -> b (3) || g (5) || g (6)
UBorn: e+ e- -> b  b~ g
                \->  b  g  g
iterm:    2 , b (3) -> b (3) || g (5) || g (7)
UBorn: e+ e- -> b  b~ g
                \->  b  g  g
iterm:    3 , b (3) -> b (3) || g (6) || g (7)
UBorn: e+ e- -> b  b~ g
                \->  b  g  g
iterm:    4 , b~(4) -> b~(4) || g (5) || g (6)
UBorn: e+ e- -> b  b~ g
                   \->  b~ g  g
iterm:    5 , b~(4) -> b~(4) || g (5) || g (7)
UBorn: e+ e- -> b  b~ g
                   \->  b~ g  g
iterm:    6 , b~(4) -> b~(4) || g (6) || g (7)
UBorn: e+ e- -> b  b~ g
                   \->  b~ g  g
iterm:    7 , g (5) -> g (5) || g (6) || g (7)
UBorn: e+ e- -> b  b~ g
                      \->  g  g  g
```

MCCSM

An NNLO calculation is extremely complex. Due to this complexity it is good practice to make as much checks as possible.

In our code the following ones are built in:

- Check upon individual subtraction terms, e.g.:

$$\lim_{p_i||p_r||p_s} \frac{\mathcal{C}_{irs}}{|\mathcal{M}_{\mathrm{RR}}|^2} = 1$$

- Checking bookkeeping and overall consistency by checking complete lines, e.g.:

$$\lim_{p_i||p_r,\, p_s \to 0} \frac{\mathcal{A}_1 + \mathcal{A}_2 - \mathcal{A}_{12}}{|\mathcal{M}_{\mathrm{RR}}|^2} = 1$$

MCCSM

Performance:

- m+2 partonic contribution:
  - 10M PS points per core in 9h
  - Smooth plots with 15B PS points ~45h on 300 cores
- m+1 partonic contribution:
  - 10M PS points per core in 31h
  - Smooth plots with 1.5B PS points ~15h on 300 cores
- m partonic contribution:
  - Never measured, takes virtually no time

MCCSM

Testing the subtraction terms in all limits (even in quad precision):

```
iterm:     5 , g (3) -> g (7) || b (3) || b~(6)
UBorn: e+ e- -> g  b~ b
                \->  g  b  b~
iexp=   1  , Cirs/RR=    1.00803271854102469359760256565251
iexp=   0  , Cirs/RR=    1.00499213240252449541142746655114
iexp=   0  , Cirs/RR=    1.00188210122417253945669893587992
iexp=  -1  , Cirs/RR=    1.00062799209472484026964163812994
iexp=  -2  , Cirs/RR=    1.00020195770962404594799550034461
iexp=  -3  , Cirs/RR=    1.00006420440823578844890246156009
iexp=  -4  , Cirs/RR=    1.00002033728517247971408852818993
iexp=  -5  , Cirs/RR=    1.00000643462401283847535738494549
iexp=  -6  , Cirs/RR=    1.00000203514784404384857605445244
iexp=  -7  , Cirs/RR=    1.00000064360436586785172322902143
iexp=  -8  , Cirs/RR=    1.00000020352898187202854767170299
iexp=  -9  , Cirs/RR=    1.00000006436185636550093621075798
iexp= -10  , Cirs/RR=    1.00000002035304016617958048905596
iexp= -11  , Cirs/RR=    1.00000000643619983435100933779238
iexp= -12  , Cirs/RR=    1.00000000203530543521165651669029
iexp= -13  , Cirs/RR=    1.00000000064362017775031206617771
iexp= -14  , Cirs/RR=    1.00000000020353075587311446332694
iexp= -15  , Cirs/RR=    1.00000000006435764917616243177692
iexp= -16  , Cirs/RR=    1.00000000002031691222958531635813
iexp= -17  , Cirs/RR=    1.00000000000562358122843217515565
```

MCCSM

Testing the whole m+2 parton line:

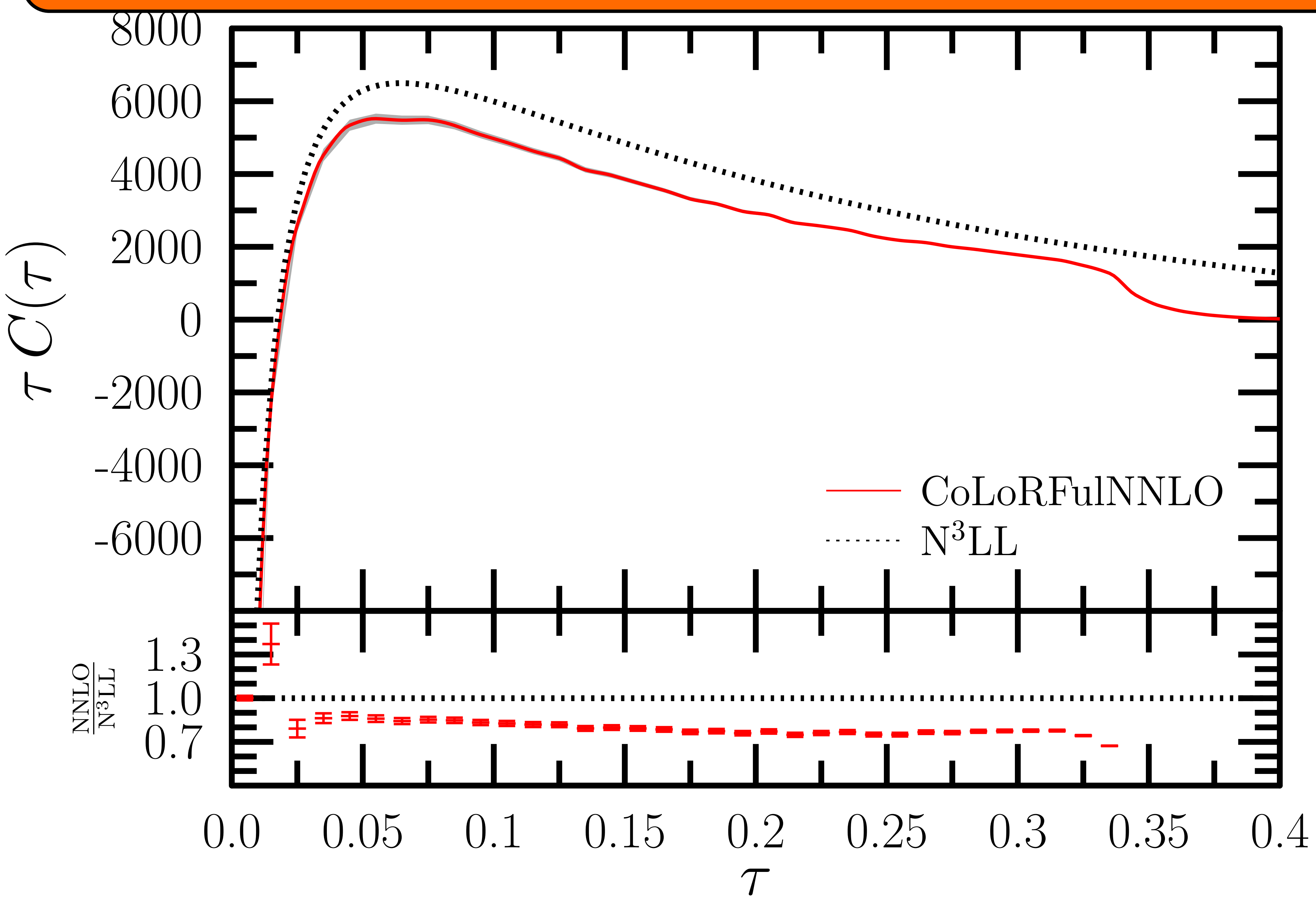
Doubly unresolved

```
CSirs: g (6) -> g (6) || g (7) , g (5) -> 0  VALID
iter no.  1 scale no.  1 1.0626663494874406131036910247582 5 *-WARN-*
iter no.  2 scale no.  1 .99933339118756664131317235085510 9
iter no.  3 scale no.  1 .99993605671620667930196132866217 9
iter no.  4 scale no.  1 .99999321715885735308166967682532 0
iter no.  5 scale no.  1 .99999928952733456236747237157707 3
iter no.  6 scale no.  1 .99999992795555748046415914784189 5
iter no.  7 scale no.  1 .99999999276423133274830626094779 4
iter no.  8 scale no.  1 .99999999927543467248458956328478 1
iter no.  9 scale no.  1 .99999999992751222931850440666947 9
iter no. 10 scale no.  1 .99999999999275023532799673566332 0
iter no. 11 scale no.  1 .99999999999927499230431132728220 4
iter no. 12 scale no.  1 .99999999999992749824289475291072 9
iter no. 13 scale no.  1 .99999999999999274979444747092755 27
iter no. 14 scale no.  1 .99999999999999927500384398391191 8
iter no. 15 scale no.  1 .99999999999999992767541466253552 1
```

Singly unresolved:

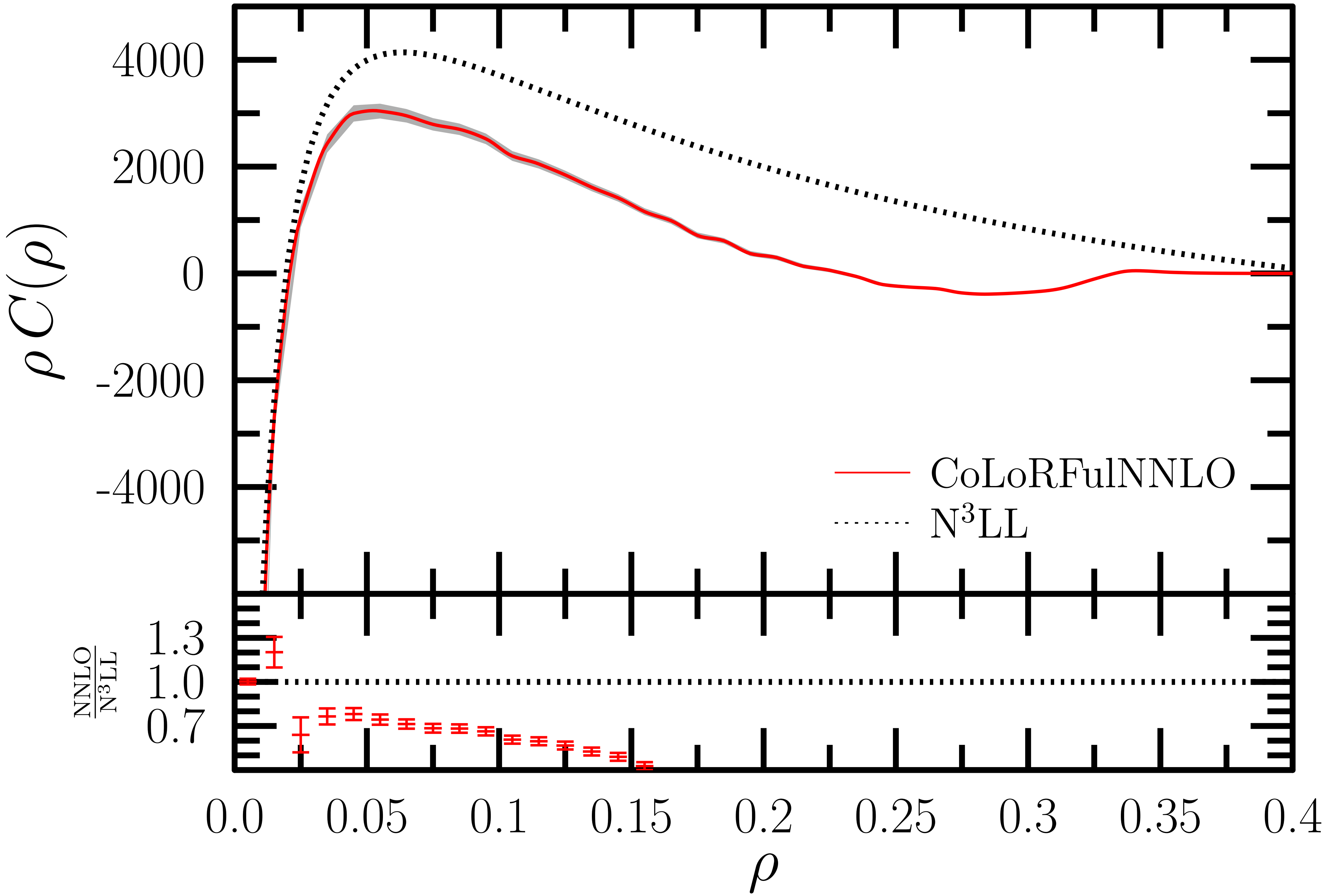
```
Cir: b (3) -> b (3) || g (7)  VALID
iter no.  1 scale no.  1 .96148670801871865442260647152993 8 *-WARN-*
iter no.  2 scale no.  1 1.0060295920978622083723511280477 7
iter no.  3 scale no.  1 1.0006658004717423478286812819735 6
iter no.  4 scale no.  1 1.0000674992486446447146088537433 2
iter no.  5 scale no.  1 1.0000067595112341689215862256272 2
iter no.  6 scale no.  1 1.0000006760473957286239347671044 7
iter no.  7 scale no.  1 1.0000000676057027060685822559986 9
iter no.  8 scale no.  1 1.0000000067605799023491568994038 8
iter no.  9 scale no.  1 1.0000000006760580865527488728314 1
iter no. 10 scale no.  1 1.0000000000676058096184534061560 2
iter no. 11 scale no.  1 1.0000000000067605809714718350712 7
iter no. 12 scale no.  1 1.0000000000006760580972580247363 1
iter no. 13 scale no.  1 1.0000000000000676058092182273659 7
iter no. 14 scale no.  1 1.0000000000000067605779495431716 5
iter no. 15 scale no.  1 1.0000000000000006761539666111960 2
```

Comparison to SCET



1-T distribution compared to SCET, agreement expected for 1-T<<1 (first bin)

Comparison to SCET



Heavy jet mass distribution compared to SCET, agreement expected for rho<<1 (first bin)