

Electroweak corrections to top pair production near threshold

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Loops and Legs in Quantum Field Theory 2016

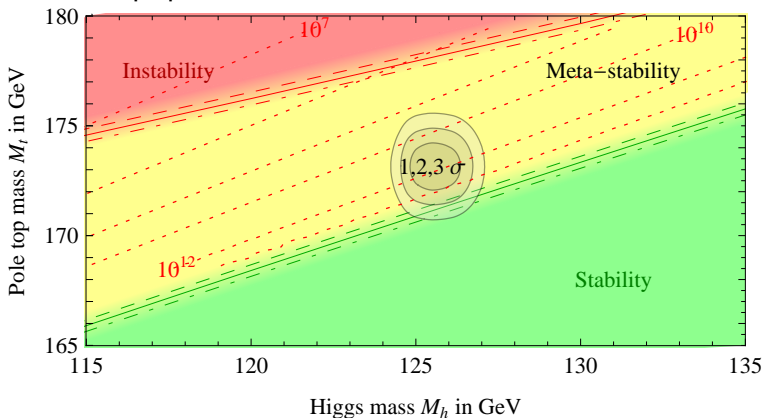
Nucl. Phys. **B899** (2015) 180-193, arXiv:1506.06865



$e^+e^- \rightarrow t\bar{t}$ near threshold

Motivation

- Precise top quark mass



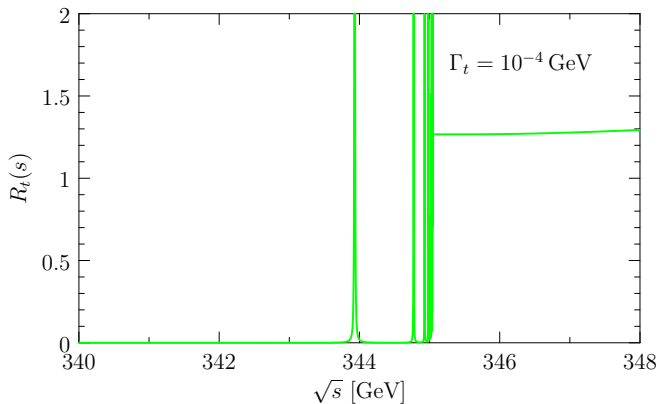
[Degrassi et al. arXiv:1205.6497]

- Width, Yukawa Coupling

$e^+e^- \rightarrow t\bar{t}$ near threshold

- ▶ Nonrelativistic quarks $v \ll 1$
- ▶ Decay during bound state formation

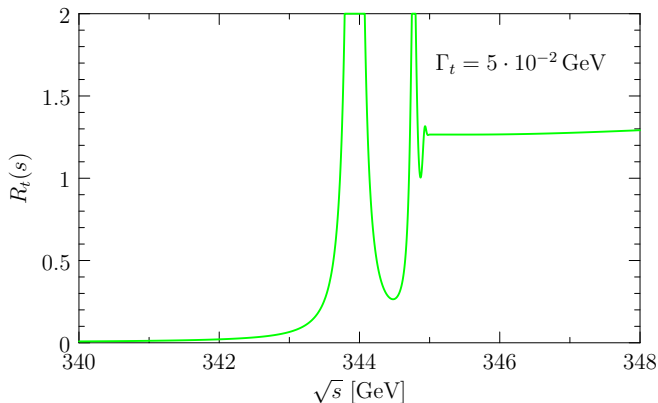
Schrödinger equation with binding potential:



$e^+e^- \rightarrow t\bar{t}$ near threshold

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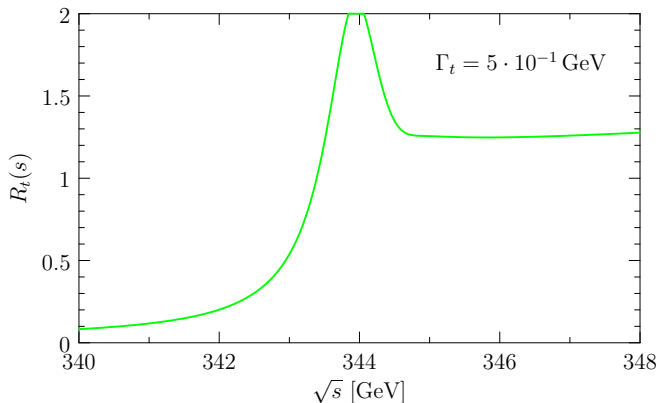
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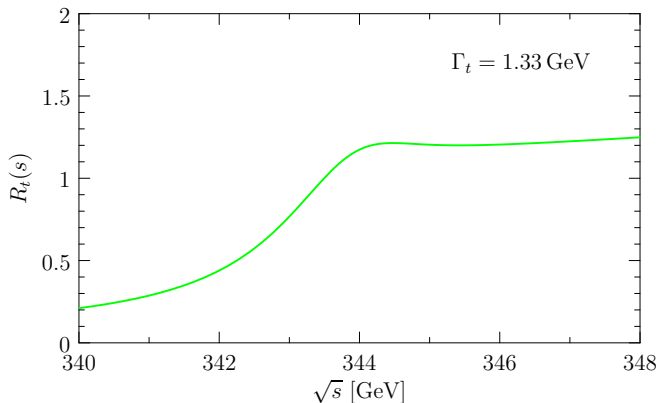
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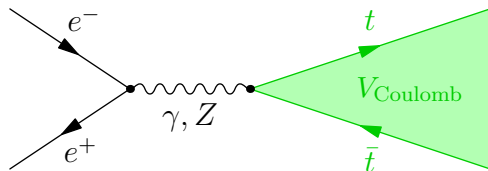
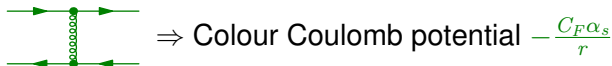
Schrödinger equation with binding potential:



QCD corrections

Need quantum field theory description:

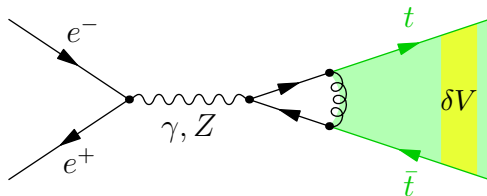
- ▶ Creation of particles
- ▶ Systematic derivation of potential



QCD corrections

Need quantum field theory description:

- ▶ Creation of particles
- ▶ Systematic derivation of potential
- ▶ Precision: perturbative corrections



Effective QFT framework

Scales: $m_t \gg m_t v \gg m_t v^2$

Coulomb potential is LO: $\alpha_s \sim v$

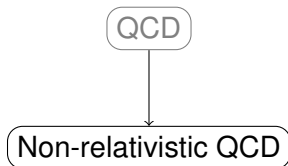
- ▶ hard modes: $k \sim m_t$
- ▶ soft modes: $k \sim m_t v$
- ▶ potential modes: $k_0 \sim m_t v^2, \vec{k} \sim m_t v$
- ▶ ultrasoft modes: $k \sim m_t v^2$

QCD

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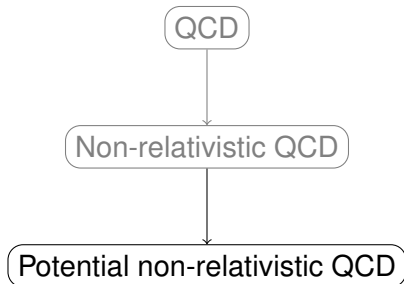
- ▶ hard modes: $k \sim m_t \rightarrow$ (local) effective vertices
- ▶ soft modes: $k \sim m_t v$
- ▶ potential modes: $k_0 \sim m_t v^2, \vec{k} \sim m_t v$
- ▶ ultrasoft modes: $k \sim m_t v^2$



Effective QFT framework

Scales: $m_t \gg m_t v \gg m_t v^2$ Coulomb potential is LO: $\alpha_s \sim v$

- ▶ hard modes: $k \sim m_t \rightarrow$ (local) effective vertices
- ▶ soft modes: $k \sim m_t v \rightarrow$ (non-local) potentials
- ▶ potential light particle modes \rightarrow (non-local) potentials
- ▶ potential top quark modes: $k_0 \sim m_t v^2, \vec{k} \sim m_t v$
- ▶ ultrasoft modes: $k \sim m_t v^2$

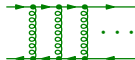


Potential non-relativistic QCD

[Pineda, Soto 97; Beneke, Signer, Smirnov 99; Brambilla et al. 99]

$$\begin{aligned}\mathcal{L}_{\text{PNRQCD}} = & \psi^\dagger \left(i\partial_0 + g_s A_0(t, \mathbf{0}) + \frac{\partial^2}{2m} + \frac{\partial^4}{8m^3} \right) \psi \\ & + \int d^3\mathbf{r} [\psi^\dagger \psi](x + \mathbf{r}) \left(-\frac{C_F \alpha_s}{r} + \delta V(r) \right) [\chi^\dagger \chi](x) \\ & - g_s \psi^\dagger \mathbf{x} \cdot \mathbf{E}(t, \mathbf{0}) \psi \\ & + \mathcal{L}_{\text{anti-quark}} + \{\text{N}^4\text{LO}\}\end{aligned}$$

- ▶ Propagator: Coulomb Green Function



- ▶ Corrections: kinetic energy, potential, ultrasoft gluons

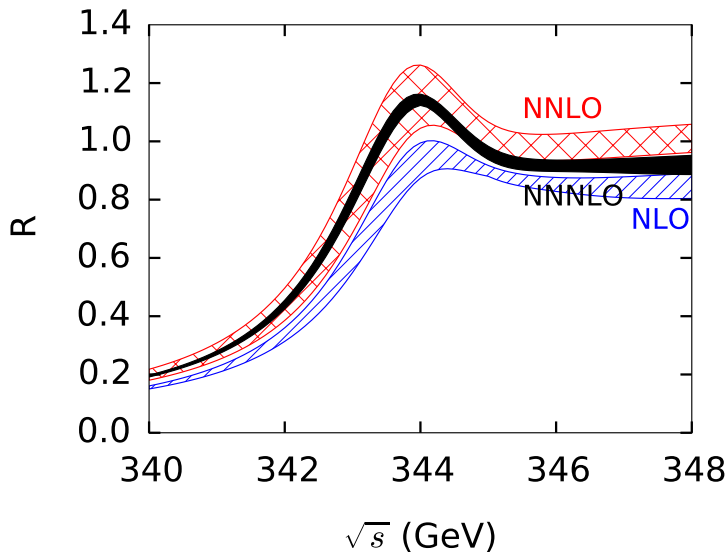
[Anzai, Kiyo, Sumino 09; Smirnov, Smirnov, Steinhauser 09]

- ▶ Production vertex: $c_v \psi^\dagger \sigma^i \chi + \frac{d_v}{6m^2} \psi^\dagger \sigma^i \mathbf{D}^2 \chi$

[Marquard, Piclum, Seidel, Steinhauser 14]

QCD corrections to top production

[Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser 2015]



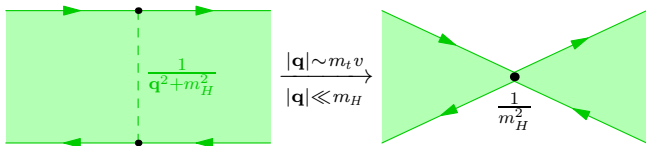
Going beyond QCD

QCD uncertainties under control: $\sim 3\%$

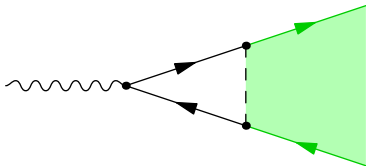
Further corrections ($v^2 \sim \alpha_s^2 \sim y_t^2 \sim \alpha$):

- ▶ Higgs corrections

- ▶ Potential (NNNLO):



- ▶ Production current (NNLO + NNNLO):



[Grzadkowski, Kühn, Krawczyk, Stuart 1986; Guth, Kühn 1991; Hoang, Reibler 2006; Eiras, Steinhauser 2006]

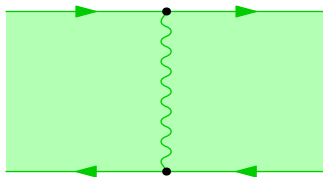
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- ▶ QED Coulomb potential

Starting at NLO:



Going beyond QCD

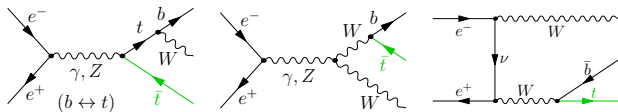
QCD uncertainties under control: $\sim 3\%$

Further corrections ($v^2 \sim \alpha_s^2 \sim y_t^2 \sim \alpha$):

- ▶ Higgs corrections
- ▶ QED Coulomb potential
- ▶ Nonresonant production

Actual final state: $W^+ b W^- \bar{b}$

- ▶ NLO: [Beneke, Jantzen, Ruiz-Femenia 2010]



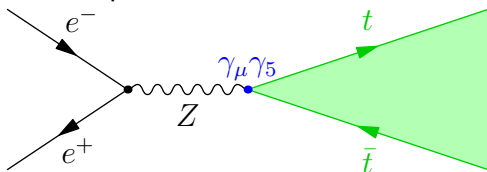
- ▶ NNLO: Partial results [Penin, Piclum 2012; Jantzen, Ruiz-Femenia 2013]
Work in progress

Going beyond QCD

QCD uncertainties under control: $\sim 3\%$

Further corrections ($v^2 \sim \alpha_s^2 \sim y_t^2 \sim \alpha$):

- ▶ Higgs corrections
- ▶ QED Coulomb potential
- ▶ Nonresonant production
- ▶ P-wave production [Penin Pivovarov 1999; Beneke, Piclum, Rauh 2013]



Small contribution at NNLO and NNNLO: $\lesssim 1\%$

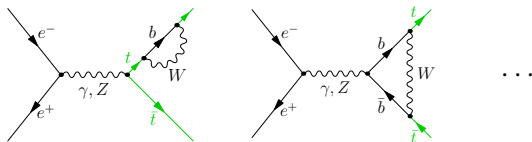
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QCD uncertainties under control: $\sim 3\%$

Further corrections ($v^2 \sim \alpha_s^2 \sim y_t^2 \sim \alpha$):

- ▶ Higgs corrections
- ▶ QED Coulomb potential
- ▶ Nonresonant production
- ▶ P-wave production
- ▶ Electroweak corrections to resonant production

[Grzadkowski, Kühn, Krawczyk, Stuart 1986; Guth, Kühn 1991; Hoang, Reiber 2004 & 2006]



to be combined with NNLO nonresonant

Going beyond QCD

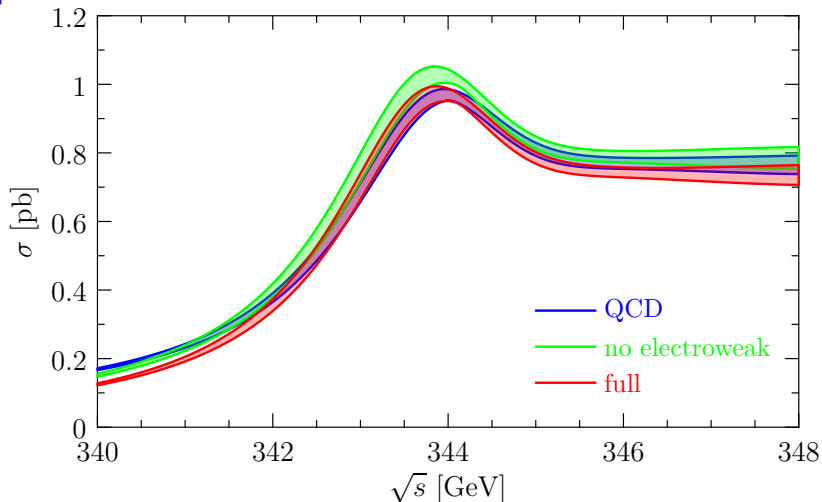
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Further corrections ($v^2 \sim \alpha_s^2 \sim y_t^2 \sim \alpha$):

- ▶ Higgs corrections
- ▶ QED Coulomb potential
- ▶ Nonresonant production
- ▶ P-wave production
- ▶ Electroweak corrections to resonant production
- ▶ Initial state radiation

Results

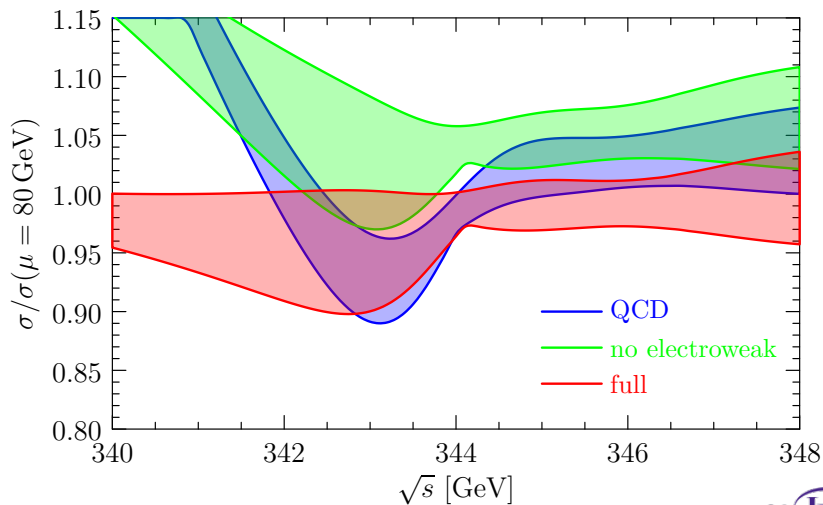
Impact on the cross section



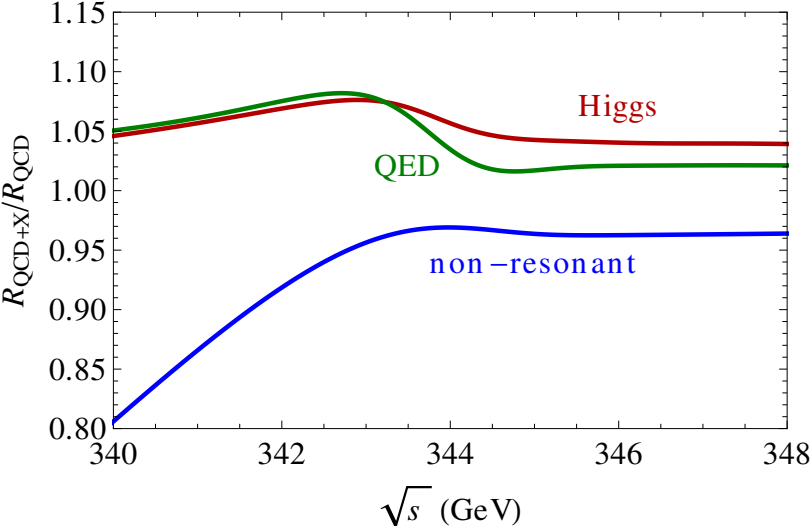
$$m_t^{\text{PS}}(20 \text{ GeV}) = 171.5 \text{ GeV}, \quad \Gamma_t = 1.33 \text{ GeV}, \quad m_H = 125 \text{ GeV},$$
$$\alpha_s(m_Z) = 0.1185, \quad \alpha(m_Z) = 1/128.944 \quad m_W, m_Z$$

Scale uncertainty

$50 \text{ GeV} \leq \mu \leq 350 \text{ GeV}$

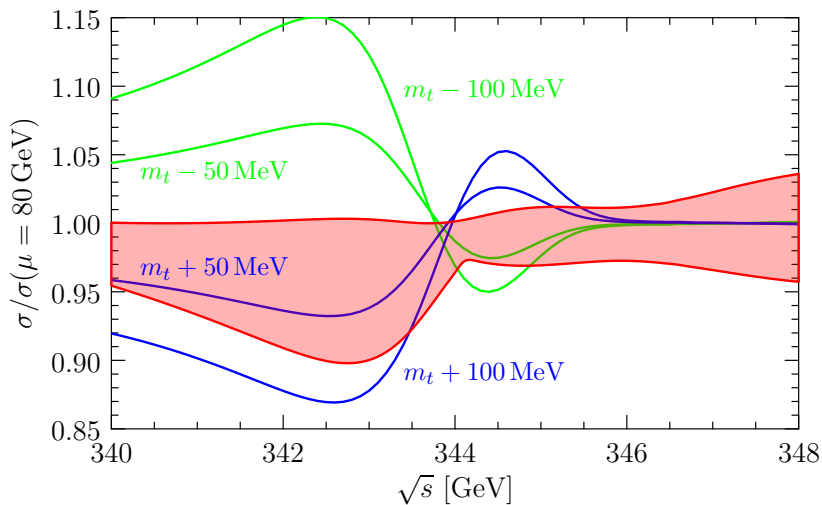


Individual contributions



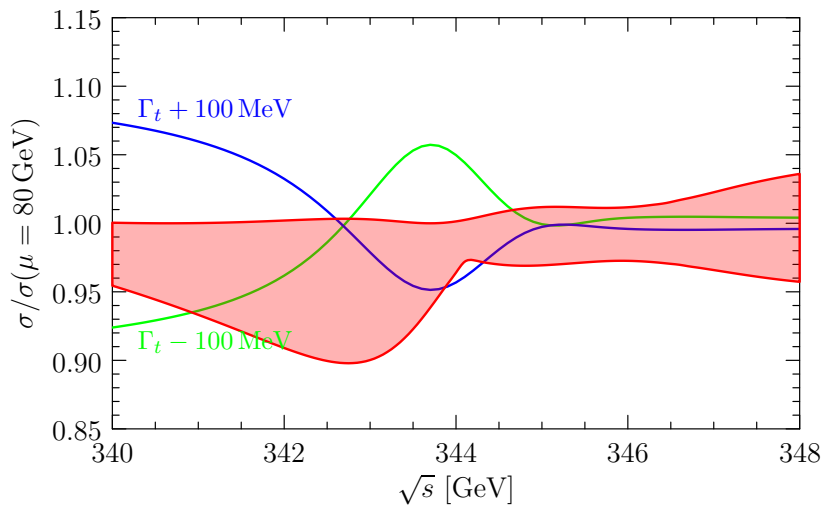
Extracting parameters

Top mass



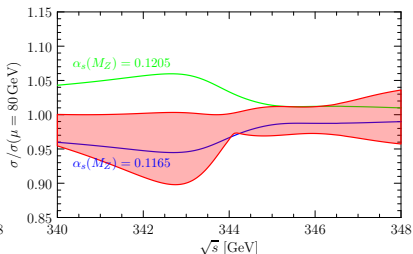
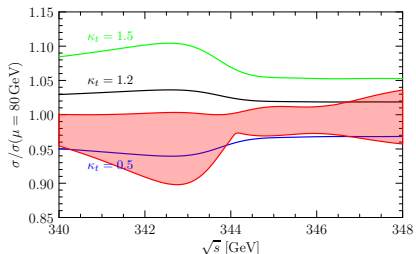
Extracting parameters

Top width



Extracting parameters

Top Yukawa and strong coupling



$$\kappa_t = y_t / y_t^{\text{SM}}, \quad y_t^{\text{SM}} = \frac{\sqrt{2} m_t}{v}$$

from adding operator

$$\Delta\mathcal{L} = -\frac{c_{\text{NP}}}{\Lambda^2} (\phi^\dagger \phi) (\bar{Q}_3 i \sigma^2 \phi^* t_R) + \text{h.c.}, \quad (1)$$

Conclusions

- ▶ $e^+e^- \rightarrow t\bar{t}$ near threshold known at NNNLO QCD + partial electroweak corrections
- ▶ QCD uncertainties under control $\sim 3\%$
- ▶ Corrections beyond QCD significant; up to 10%
- ▶ Precise values for top mass $\Delta m_t < 100 \text{ MeV}$, width
- ▶ Measurement of Yukawa coupling and α_s limited

Outlook

- ▶ Complete electroweak effects up to NNLO
- ▶ Renormalisation group improvement
- ▶ Publish code

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```
Needs["QQbarThreshold"];
LoadGrid[GridDirectory <> "ttbar_grid.tsv"];
Plot[
  TTbarXSection[
    sqrts, {80., 350.}, {171.5, 1.33},
    "N3LO"
  ],
  {sqrts, 340, 348}
]
```

```
#include <iostream>
#include "QQbar_threshold/QQbar_threshold.hpp"
using namespace QQbar_threshold;
int main(){
  load_grid(grid_directory() + "ttbar_grid.tsv");
  std::cout << ttbar_xsection(
    344., {80., 350.}, {171.5, 1.33}, "N3LO"
  ) << '\n';
}
```

