Top quark mass effects in Higgs boson pair production up to NNLO

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Loops and Legs in Quantum Field Theory, Leipzig
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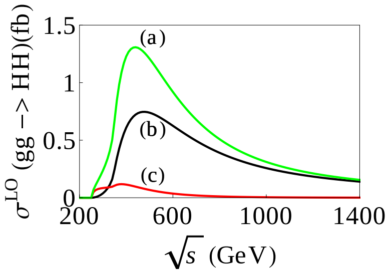
[based on work done in collaboration with J. Grigo and M. Steinhauser; [arXiv:1508.00909]]

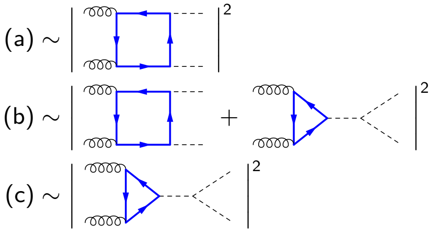
Motivation

Higgs Potential in the Standard Model:

$$V(H) = \frac{1}{2} m_H^2 H^2 + \lambda v H^3 + \frac{1}{4}\lambda H^4, \quad \lambda^{\text{SM}} = \frac{m_H^2}{2v^2} \approx 0.13, \quad v\text{: Higgs vev.}$$

- Verify mechanism of spontaneous symmetry breaking in the SM
- Measure the Higgs self-coupling $\Rightarrow$ sensitive process





Theory status

Prospects for the LHC @ 14 TeV:

- $b\bar{b}\gamma\gamma$-channel, 600 fb^{-1}: $\lambda \neq 0$ [Baur, Plehn, Rainwater; '04]
- $b\bar{b}\gamma\gamma$-, $b\bar{b}\tau^+\tau^-$-channels: "promising";
  $b\bar{b}W^+W^-$-channel: "not promising"
  [Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira; '13]
- 600 fb^{-1}: $\lambda > 0$; 3000 fb^{-1}: $\lambda^{+30\%}_{-20\%}$ (ratio with Higgs cross section)
  [Goertz, Papaefstathiou, Yang, Zurita; '13]
- And many others, e.g.:
  [Dolan, Englert, Spannowsky; '12] [Papaefstathiou, Yang, Zurita; '13]
  [Barr, Dolan, Englert, Spannowsky; '13] [Barger, Everett, Jackson, Shaughnessy; '14]
  [Englert, Krauss, Spannowsky, Thompson; '15] [. . .]
- Until now: Higgs pair production not observed in $b\bar{b}b\bar{b}$- and $b\bar{b}\gamma\gamma$-channels (as expected in the SM) [ATLAS; '15] [CMS; '15]

$\Rightarrow$ **Wait for HL-LHC**

Theory status

Known since long:

- LO result with exact M_t dependence [Glover, van der Bij; '88] [Plehn, Spira, Zerwas; '98]
- NLO result in $M_t \to \infty$ limit [Dawson, Dittmaier, Spira; '98]

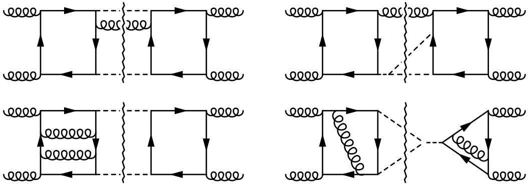
$\boxed{\sigma_H \approx 20^{\text{LO}}\,\text{fb} + 20^{\text{NLO},M_t\to\infty}\,\text{fb}}$ for $\sqrt{s_H} = 14\,\text{TeV}$, $\mu = 2m_H$

More recently:

- NLO + NNLL ($M_t \to \infty$) $\approx$ NLO $+20\%$ [Shao, Li, Li, Wang; '13]
- NNLO w/ or w/o soft-virtual approx. ($M_t \to \infty$) $\approx$ NLO $+20\%$ [de Florian, Mazzitelli; '13]
- $\mathcal{O}\left(1/M_t^8\right)$ corrections at NLO $\approx$ NLO $+10\%$ [Grigo, JH, Melnikov, Steinhauser; '13]
- NLO real exact in M_t, NLO virt. for $M_t \to \infty$ $\approx$ NLO -10% [Maltoni, Vryonidou, Zaro; '14]
- Cross-check of virtual NNLO corrs.; NNLO matching coefficient for $ggHH$-coupling $\approx$ NNLO $+1\%$ [Grigo, Melnikov, Steinhauser; '14]
- Improved $\mathcal{O}\left(1/M_t^{12}\right)$ NLO, $\mathcal{O}\left(1/M_t^{4}\right)$ NNLO soft-virt. corrections (this talk) [Grigo, JH, Steinhauser; 15']
- NLO full M_t result (cf. previous talk by Matthias) [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke; '16]

Generalities

- Operate on full-theory diagrams at NLO and NNLO
- Virt. corrs. in two independent calculations:
  - amplitude (differential; 2-/3-loop)
  - forward scattering (total; 4-/5-loop)
- Real corrs.: only via forward scattering at NLO
- Perform expansion for $M_t \to \infty$; improve upon effective theory results for NLO [Dawson, Dittmaier, Spira; '98], NNLO [de Florian, Mazzitelli; '13] (not expected to be valid for $\sqrt{s} \gtrsim M_t$)
- Laporta reduction to master integrals for the "soft" subdiagrams
- Remaining "hard" massive tadpoles via `MATAD`
- Master integrals as series around $\sqrt{s} = 2m_H$



Differential factorization

Factorization of the LO result exact in M_t for:

Total cross section

$$\sigma^{(i)} = \Delta^{(i)} \sigma^{(0)}_{\text{exact}} = \frac{\sigma^{(0)}_{\text{exact}}}{\sigma^{(0)}_{\text{exp}}} \int_{4m_H^2}^{s} \mathrm{d}Q^2 \, \frac{\mathrm{d}\sigma^{(i)}_{\text{exp}}}{\mathrm{d}Q^2}$$

$$\text{with} \quad \Delta^{(i)} = \frac{\sigma^{(i)}_{\text{exp}}}{\sigma^{(0)}_{\text{exp}}}, \quad \sigma^{(i)}_{\text{exp}} = \sum_{n=0}^{N} c_n^{(i)} \rho^n, \quad \rho = \frac{m_H^2}{M_t^2}$$

Differential cross section

$$\sigma^{(i)} = \int_{4m_H^2}^{s} \mathrm{d}Q^2 \, \frac{\left(\frac{\mathrm{d}\sigma^{(0)}_{\text{exact}}}{\mathrm{d}Q^2}\right)}{\left(\frac{\mathrm{d}\sigma^{(0)}_{\text{exp}}}{\mathrm{d}Q^2}\right)} \frac{\mathrm{d}\sigma^{(i)}_{\text{exp}}}{\mathrm{d}Q^2}$$

"Cure" the invalidity of the $M_t \to \infty$ expansion for the large-Q^2 region

- Virt. corrs. via amplitude: access to Q^2-dependence $\sim \delta\left(s - Q^2\right)$
- Real corrs. via optical theorem (naively): only total cross section

$\Rightarrow$ Use the soft-virtual approximation [de Florian, Mazzitelli; '12]

Soft-virtual approximation

- Split σ into its contributions (works also for $\mathrm{d}\sigma/dQ^2$):

$$\sigma = \sigma^{\text{virt+ren}} + \sigma^{\text{real+split}} = \text{finite}$$
$$= \underbrace{\Sigma_{\text{div}} + \Sigma_{\text{fin}} \quad + \quad \Sigma_{\text{soft}}}_{=\Sigma_{\text{SV}}=\text{finite}} + \underbrace{\Sigma_{\text{hard}}}_{=\Sigma_{\text{H}}=\text{finite}}$$

- Σ_{div} universal for color-less final state [de Florian, Mazzitelli; '12]
- Compute $\sigma^{\text{virt+ren}}$ as ρ-expansion
- Solve $\sigma^{\text{virt+ren}} = \Sigma_{\text{div}} + \Sigma_{\text{fin}}$ for Σ_{fin}
- Σ_{div} and Σ_{soft} (soft coll. counterterms + soft real corrs.) $\sim$ exact σ^{LO} (include M_t effects)

$$Q^2 \frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \sigma^{\text{LO}} z G(z) \quad \text{with} \quad z = \frac{Q^2}{s}, \quad G(z) = G_{\text{SV}}(z) + G_{\text{H}}(z)$$
$$\sigma_{(\text{SV})} = \int_{1-\delta}^{1} \mathrm{d}z \sigma^{\text{LO}}(zs)\, G_{(\text{SV})}(z) \quad \text{with} \quad \delta = 1 - \frac{4m_H^2}{s}$$

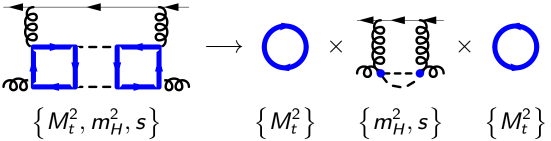
- $G_{\text{SV}}^{(i)}(z)$ constructed from $\sigma_{\text{fin}}^{(i)}$ and σ^{LO} only
  [de Florian, Mazzitelli; '12] [Grigo, JH, Steinhauser; '15]

Asymptotic expansion

- Expand at integrand level for all contributing regions (scalings of loop momenta)
  $\equiv$ series expansion in analytic result
- Hierarchy: $M_t^2 \gg s, m_H^2 \;\Rightarrow$ series in $\rho = m_H^2/M_t^2$
- Effectively <u>reduce</u> number of loops and scales

- Here: regions correspond to subgraphs (in general more than one)
- Hard mass expansion: subgraphs must contain all heavy lines

Example: NLO real with one region



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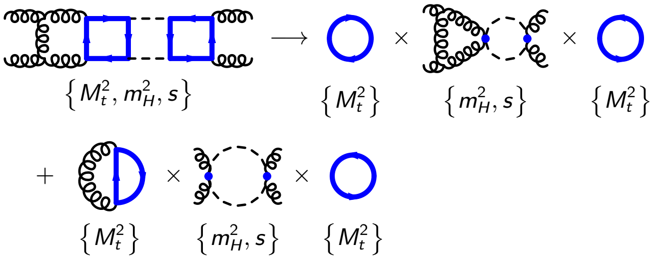
Example: NLO virt. with two regions



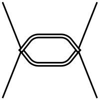
Calculation (toolchain)

Reduction

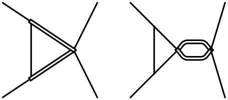
1. Generate Feynman diagrams
   QGRAF [Nogueira; '93]
2. Select diagrams with specific cuts for $gg \to gg$ (17 millions)
   filter [JH, Pak; (unpublished)]
3. Map diagrams to topologies ($\leftarrow$ graph information)
   exp [Harlander, Seidensticker, Steinhauser; '98]
4. Reduction to scalar integrals ($\leftarrow$ generic topologies)
   FORM [Kuipers, Ueda, Vermaseren, Vollinga; '13]
5. Reduction to master integrals ($\leftarrow$ basic topologies)
   $gg \to gg$: rows [JH, Pak; (unpublished)]
   $gg \to hh$: FIRE [Smirnov]
6. Minimal basis of master integrals
   TopoID [JH, Pak]

Calculation (via optical theorem)

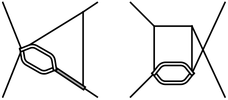
LO topology:



Virt. NLO topologies:



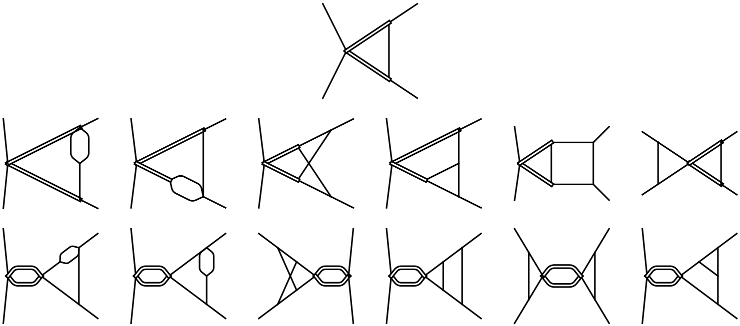
Real NLO topologies:



(shaded Higgs line cannot be cut)

Calculation (via optical theorem)

Virt. NNLO topologies:

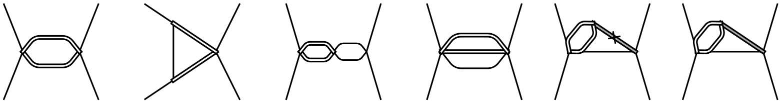


Note:

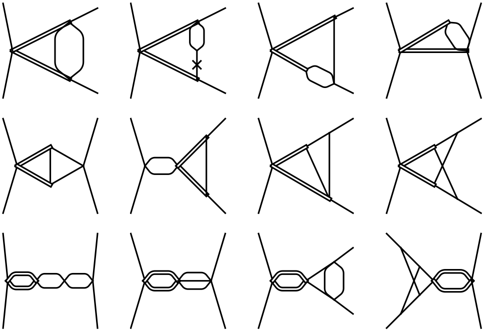
- Different regions in asymptotic expansion $\Rightarrow$ different loop-orders
- Here: multiplied with 1- to 3-loop massive tadpoles

Calculation (via optical theorem)

Virt. and real LO-NLO master integrals:



Virt. NNLO master integrals (in addition):



Calculation (via optical theorem)

- NLO: 3 real and 1 virtual (+ 2-loop tadpoles) master integrals
- NNLO master integrals:
  - 3-loop: 1-loop phase space $\times$ 2-loop form factors
  - 2-loop: massless sub-loops; same approach as one NLO
- Phase space integrals depend on $s = (q_1 + q_2)^2$ and m_H
- Derive 1-dimensional integral representation: e.g. for the simplest NLO case (cf. fourth diagram on previous slide)

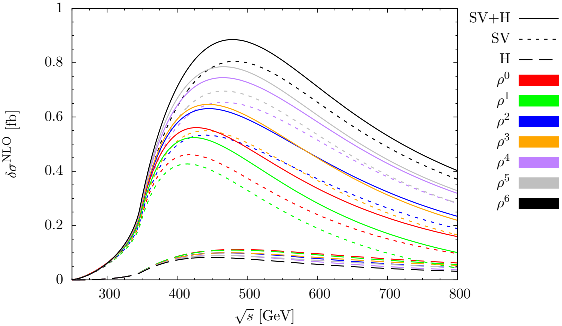
$$I_1 = \mathcal{N} s^{1-2\epsilon} \delta^{5/2-3\epsilon} \int_0^1 \frac{d\mu}{\sqrt{1-\mu\delta}} (1-\mu)^{1/2-\epsilon} \mu^{1-2\epsilon}, \quad \delta = 1 - \frac{4m_H^2}{s}$$

- Expand up to $\mathcal{O}\left(\delta^{100}\right)$ on NLO, $\mathcal{O}\left(\delta^{10}\right)$ on NNLO
  $\Rightarrow$ very good convergence, small impact on numerics
  $\Rightarrow$ analytic results for partonic cross sections

- Note: All master integrals for the amplitude available in the literature

Splitting in soft-virtual and real contributions at NLO

- Partonic NLO correction; total factorization



Using $\mu = 2m_H$ (also in the following)

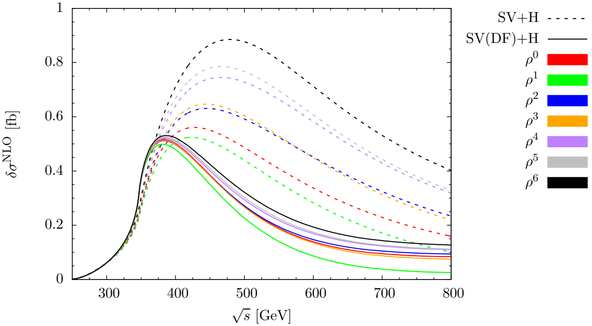
- Different behavior for higher orders in ρ expansion:

SV increasing
H descreasing (flat for $\sqrt{s} \gtrsim 400$ GeV)

$\Rightarrow$ SV numerically dominant

Total vs. differential factorization (DF) at NLO

- DF applied only to SV part;
  H treated via total factorization (i.e. identical)



- Maxima of DF curves at lower $\sqrt{s}$; smaller cross sections

$\Rightarrow$ Improvement of convergence:
difference of ρ^0 and corrections to ρ^6 (for $\sqrt{s} = 400\,\mathrm{GeV}$):
0.25 fb vs. 0.05 fb

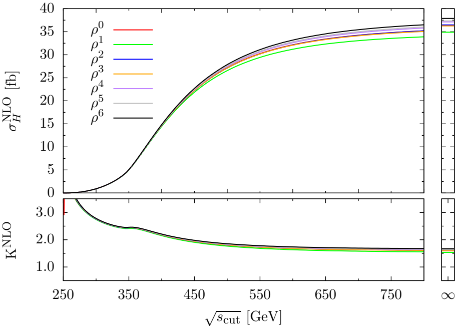
- (Partonic K-factor: behavior at top quark pair threshold not washed out)

Hadronic cross section and K-factor

- Technical upper cut on $\sqrt{s}$ (good proxy to Q^2):

$$\sigma_H(s_H, s_{\text{cut}}) = \int_{4m_H^2/s_H}^{1} \mathrm{d}\tau \left(\frac{\mathrm{d}\mathcal{L}_{gg}}{\mathrm{d}\tau}\right)(\tau)\, \sigma(\tau s_H)\, \theta(s_{\text{cut}} - \tau S_H)$$

- $\sqrt{s_{\text{cut}}} \to \infty$: total cross section for 14 TeV



- Spread of ρ-orders $\Rightarrow$ $\boxed{\pm 10\% \text{ uncertainty of EFT at NLO due to } M_t}$

Revisiting NLO

Lessons from NLO for NNLO:

- SV approximation contructed for $z \to 1$;
  $G_{\mathrm{SV}}(z)$ can be replaced by $f(z)\, G_{\mathrm{SV}}(z)$ with $f(1) = 1$
  Splitting into SV and H not unique

- Tune $f(z)$ at NLO such that $\sigma \approx \Sigma_{\mathrm{SV}}$

$\Rightarrow$ $f(z) = z$ accurate to 2%

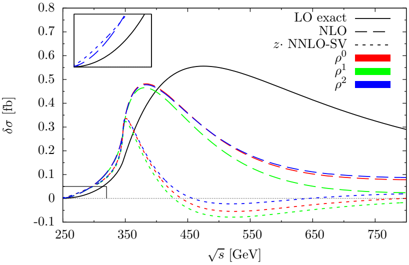
- Replace RGE logarithms ($\sqrt{s} \approx Q^2$ in the soft limit):

$\Rightarrow$ $\log\left(\mu^2/s\right) \to \log\left(\mu^2/Q^2\right)$

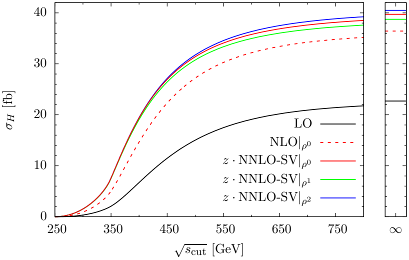
Discrepancy to [Maltoni, Vryonidou, Zaro; '14]:

- Real corrs.: treated exactly; Virt. corrs.: EFT result
- Claim: -10% correction at NLO
- Difference due to treating M_t dependence for real and virt. corrs. as consistent $1/M_t$ expansion or not

NNLO SV corrections



- EFT result plus ρ- and ρ^2-terms
- Peaks at smaller $\sqrt{s}$
- Same pattern of ρ-corrections

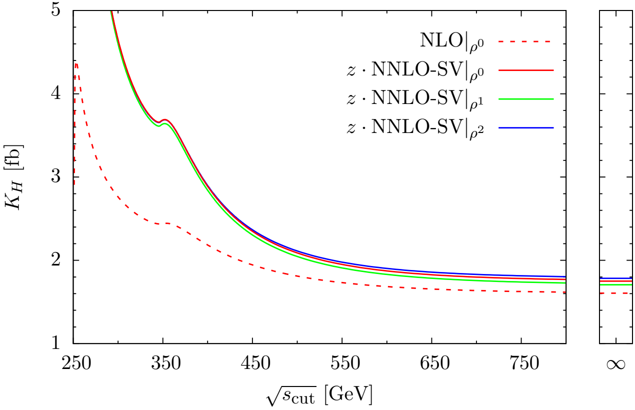


- Conv. up to $\sqrt{s_{\rm cut}} \approx 400$ GeV
- ρ- and ρ^2-corrections: $\pm 2.5\%$

$\Rightarrow$ **M_t-uncertainty at NNLO: 5%**

(NNLO corrs. $\approx$ 20%)

NNLO SV K-factor



- Behavior at top threshold not washed out
- Strong raise close to threshold $\Leftarrow$ steeper NNLO correction
- For total cross section: $\boxed{K_H^{\text{NNLO}} \approx 1.7 - 1.8}$

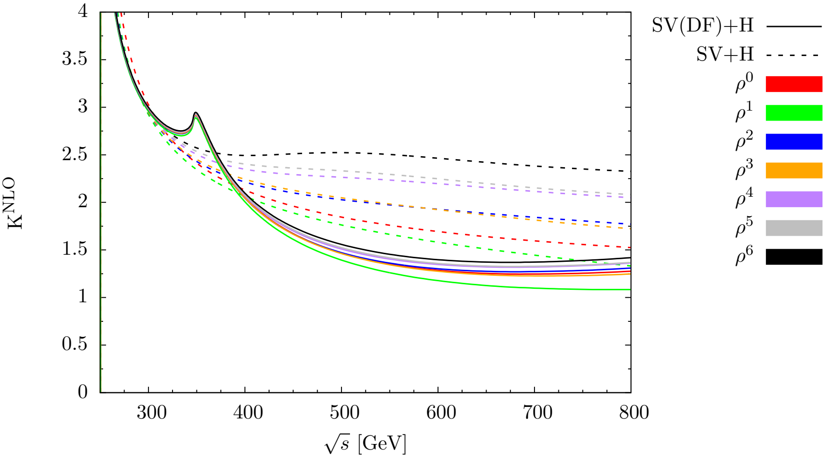
Conclusion and Outlook

- Real corrections at NLO via opt. theorem; employ SV approx. at NNLO
- Computed expansion to $\mathcal{O}\left(1/M_t^{12}\right)$ at NLO, $\mathcal{O}\left(1/M_t^{4}\right)$ at NNLO
- Improvement of convergence by differential factorization
- Uncertainty of EFT results due to M_t:
  10% at NLO, 5% at NNLO
- Small-$\sqrt{s}$ behavior as benchmark for exact calculations

- $1/M_t$ corrections for real contributions at NNLO?
- Adapt approximation procedure to match exact NLO calculation
  $\Rightarrow$ Improved NNLO prediction

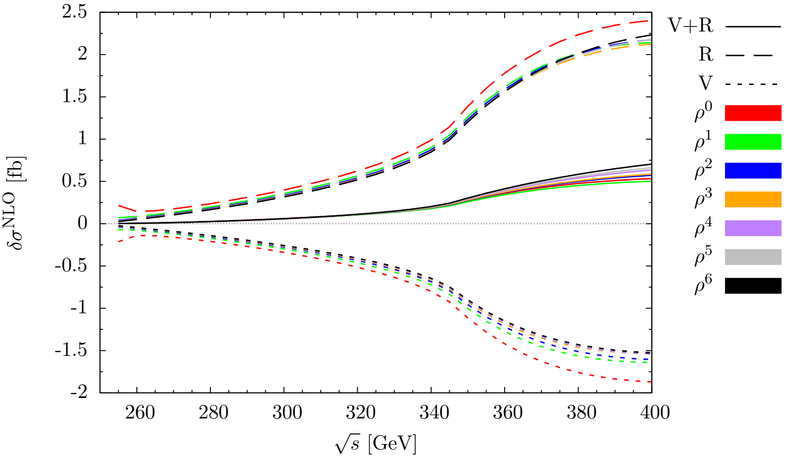
Thank you for your attention!

Partonic NLO K-factor



Note: Behavior around top quark pair threshold not washed out

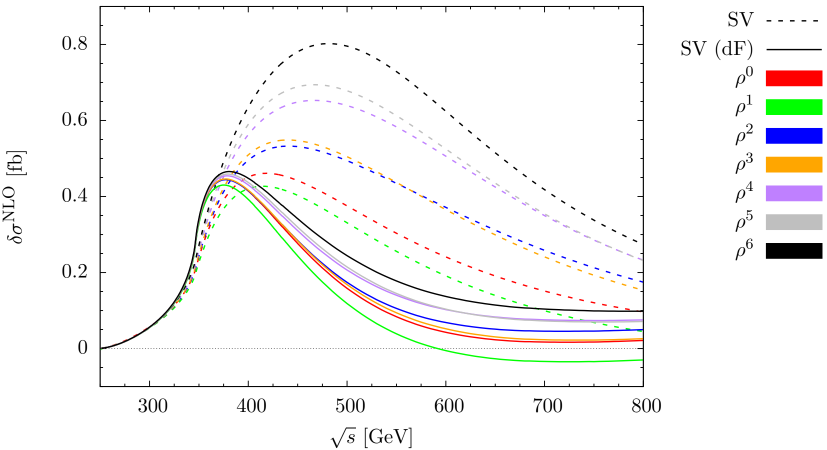
Splitting into real and virtual corrections at NLO



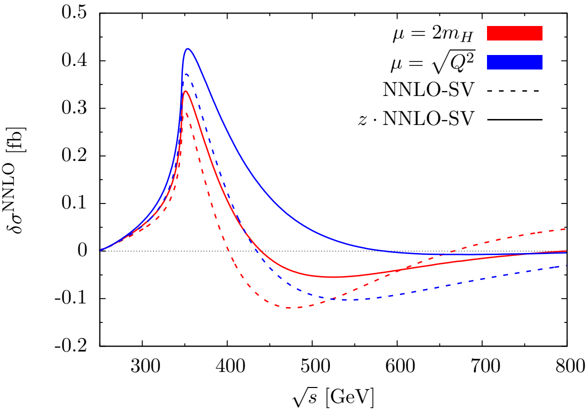
Note:

- R and V separately divergent; only finite contributions shown;
- Dominant positive shift from virtual corrections
- Different correction patterns (R negative, V positive)

Total vs. differential factorization without hard contributions at NLO



Partonic NNLO cross section for different scales choices and $f(z)$



Note: $f(z) = z$ (better proxy) leads to higher values