Two-loop conformal anomaly in QCD: Nonsinglet operators

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based on

V. Braun, A. Manashov, Eur. Phys. J. C **73** *(2013) 2544*
V. Braun, A. Manashov, Phys. Lett. B **734** *(2014) 137*
V. Braun, A. Manashov, S. Moch, M. Strohmaier, JHEP **1603** *(2016) 142*

Loops and Legs, Leipzig, 28.04.2016

Motivation

- **DIS**: Structure functions : scale dependence is governed by anomalous dimensions of twist-2 operators (known at three loops), Larin, Vermaseren, Moch, Vogt, et al)
- **Deeply Virtual Compton Scattering**: (Müller, Ji, Radyushkin) Generalized Parton Distributions: scale dependence $\longleftrightarrow$ anomalous dimension matrix (nonforward kernel) (two loops) Belitsky, Müller, (2000)

Explore the road to NNLO (three-loop) evolution equations for GPDs

One loop: anomalous dimensions+conformal symmetry→ full anomalous dimension matrix.
(Makeenko, 1980)

$$O_N \sim (\partial_{z_1} + \partial_{z_2})^N C_N^{3/2} \left(\frac{\partial_{z_1} - \partial_{z_2}}{\partial_{z_1} + \partial_{z_2}} \right) \bar{q}(z_1 n) \gamma_+ q(z_2 n)$$

In any realistic $d = 4$ QFT the conformal symmetry is broken, $\beta(g) \neq 0$.

D. Müller, Constraints for anomalous dimensions of local light cone operators in ϕ^3 in six-dimensions theory, Z. Phys. C 49 (1991) 293.
(Conformal Ward Identities, Conformal anomaly, Conformal scheme, etc)

Belitsky, Müller, (2000) two loop kernels in QCD.

Difference to D.Müller:

Instead of considering consequences of broken conformal symmetry in QCD we make use of exact conformal symmetry of a modified theory: Large N_f QCD in $4-2\epsilon$ dimensions at critical coupling:

$$\beta^{QCD}(a) = 2a\,[-\epsilon - \beta_0 a + \ldots] \qquad a_* = -4\pi\epsilon/\beta_0 + \ldots \qquad \beta^{QCD}(a_*) = 0$$

$$a_* = a_*(\epsilon) \qquad \longleftrightarrow \qquad \epsilon = \epsilon(a_*)$$

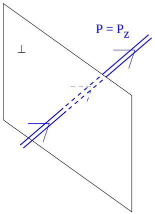
$1/N_f$ **expansion, Gracey, Cuccini, Derkachov, A.M.**

Premium:

- **Exact symmetry $\Rightarrow$ algebraic group-theory methods**
- **Answer obtained directly in $\overline{\mathrm{MS}}$ scheme**
- **Light-ray operator basis, do not need to restore evolution eqs. from local operators.**

$$\mathcal{O}(x;z_1,z_2) \equiv [\bar{q}(x+z_1 n)\not{n}\, q(x+z_2 n)] \equiv \sum_{m,k} \frac{z_1^m z_2^k}{m!k!}[(D_+^m \bar{q})(x)\not{n}(D_+^k q)(x)]$$

$$= \sum_{N,k} \Psi_{Nk}(z_1,z_2)\,\partial_+^k \mathcal{O}_N(x)$$



$$p_+ = \frac{1}{\sqrt{2}}(p_0 + p_z) \to \infty$$
$$p_- = \frac{1}{\sqrt{2}}(p_0 - p_z) \to 0$$
$$px \to p_+ x_-$$

- Special conformal transformation
$$x_- \to x'_- = \frac{x_-}{1 + 2ax_-}$$
- translations $x_- \to x'_- = x_- + c$
- dilatations $x_- \to x'_- = \lambda x_-$

form the so-called **collinear subgroup** $SL(2, R)$

$$\alpha \to \alpha' = \frac{a\,\alpha + b}{c\,\alpha + d}, \quad ad - bc = 1$$
$$\Phi(\alpha) \to \Phi'(\alpha) = (c\,\alpha + d)^{-2j}\Phi\left(\frac{a\alpha + b}{c\alpha + d}\right)$$

where $\Phi(x) \to \Phi(x_-) = \Phi(\alpha n_-)$ is the quantum field with scaling dimension ℓ and spin projection s "living" on the light-ray

Conformal spin:
$$j = (l + s)/2$$

Light-ray operators satisfy the RG equation Balitsky, Braun '89

$$\Big(M\partial_M + \beta(a)\partial_a + \mathbb{H}(a)\Big)\mathcal{O}(z_1, z_2) = 0$$

where $\mathbb{H}$ is an integral operator acting on the light-cone coordinates of the fields:

$$\mathbb{H}(a)\mathcal{O}(z_1, z_2) = \int_0^1 d\alpha \int_0^1 d\beta\, h(\alpha, \beta)\, \mathcal{O}(z_{12}^{\alpha}, z_{21}^{\beta})$$

$$z_{12}^{\alpha} \equiv z_1\bar{\alpha} + z_2\alpha \qquad \bar{\alpha} = 1 - \alpha$$

$$h(\alpha, \beta) = a\, h^{(1)}(\alpha, \beta) + a^2 h^{(2)}(\alpha, \beta) + \ldots$$ **[does not depend on ϵ]**

One can show that the powers $\mathcal{O}(z_1, z_2) \mapsto (z_1 - z_2)^N$ are eigenfunctions of $\mathbb{H}$, and the corresponding eigenvalues are the anomalous dimensions of local operators of spin N (with $N - 1$ derivatives)

$$\gamma_N = \int d\alpha d\beta\, h(\alpha, \beta)(1 - \alpha - \beta)^{N-1}$$ **(NNLO)**

Is it possible to restore $h(\alpha, \beta)$ from γ_N?

It is expected that the evolution kernel $\mathbb{H}$ commutes with the generators of $SL(2,R)$ collinear subgroup, $[S_\alpha, \mathbb{H}] = 0, \quad \alpha = \pm, 0$.

- **Leading order generators**

$$S_+^{(0)} = z_1^2\partial_{z_1} + z_2^2\partial_{z_2} + 2(z_1+z_2), \quad S_0^{(0)} = z_1\partial_{z_1} + z_2\partial_{z_2} + 2, \quad S_-^{(0)} = -\partial_{z_1} - \partial_{z_2}$$

if $[\mathbb{H}, S_\alpha] = 0$ **then** $h(\alpha,\beta) = h(\tau)$**,** $\tau = \alpha\beta/\bar{\alpha}\bar{\beta}$**.**

$$\gamma_N \Leftrightarrow h(\tau)$$

- **Exact conformal symmetry, but the generators are modified by quantum corrections**

$$a = \frac{\alpha_s}{4\pi}$$

$$\begin{aligned} S_- &= S_-^{(0)}\,, \\ S_0 &= S_0^{(0)} - \epsilon(a_*) + \frac{1}{2}\mathbb{H}(a_*)\,, \qquad \mathbb{H}(a_*) = a_*\,\mathbb{H}^{(1)} + \dots \\ S_+ &= S_+^{(0)} + (z_1+z_2)\Big(-\epsilon(a_*) + \frac{1}{2}\mathbb{H}(a_*)\Big) + (z_1-z_2)\Delta_+(a_*)\,, \end{aligned}$$

Expansion over conformal operators:

$$\mathcal{O}(x;z_1,z_2) = \sum_{N,k} \Psi_{Nk}(z_1,z_2)\,\partial_+^k \mathcal{O}_N(x),$$

$$\mathbb{H}(a_*)\Psi_{Nk} = \gamma_N\,\Psi_{Nk} \qquad\qquad \Psi_{Nk}(z_1,z_2) = S_+^k (z_1-z_2)^N$$

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$\mathbb{H}(a_*)$ **commutes with all generators** $\Longrightarrow \mathbb{H}(a_*) = f(J)$

$$J(J-1) = S_+S_- + S_0(S_0-1)$$

The lowest eigenfunctions are known: $\Psi_{N0}(z_1, z_2) = z_{12}^N$ **(for any coupling).**

$$J(a_*)z_{12}^N = (N+2-\epsilon+\gamma_N/2)\, z_{12}^N, \qquad \mathbb{H}(a_*)z_{12}^N = \gamma_N z_{12}^N$$

$$\gamma_N = f(N+2-\epsilon+\gamma_N/2)\,.$$

One can restore the function f **from anomalous dimensions:** $f(x) = a\, f_1(x) + a^2\, f_2(x) + \ldots$.

$$\mathbb{H}(a) = a\, f_1(J(a)) + a^2\, f_2(J(a)) + a^3\, f_3(J(a)) + \ldots$$

- **Expansion over conformal operators**

$$\mathcal{O}^{(n)}(x; z_1, z_2) = \sum_{N,k} \Psi_{Nk}(z_1, z_2)\, \partial_+^k \mathcal{O}_N^{(n)}(x),$$

- **Conformal symmetry at the critical coupling implies**

$$\left(S_+^{(z)} - \frac{1}{2}x^2(\bar{n}\partial_x)\right)\Big\langle \mathcal{O}_n(0, z_1, z_2), \mathcal{O}_{\bar{n}}(x, w_1, w_2)\Big\rangle = 0 \qquad \boxed{(nx) = (\bar{n}x) = 0}$$

- **To find explicit expression for S_+, consider Ward identity ($\delta_+ = \mathbf{K}\bar{n}$)**

$$\Big\langle \delta_+ \mathcal{O}^{(n)}(z)\, \mathcal{O}^{(\bar{n})}(x, w)\Big\rangle + \Big\langle \mathcal{O}^{(n)}(z)\, \delta_+ \mathcal{O}^{(\bar{n})}(x, w)\Big\rangle = \Big\langle \delta_+ S_R\, \mathcal{O}^{(n)}(z)\, \mathcal{O}^{(\bar{n})}(x, w)\Big\rangle$$

Then

$$\begin{aligned}
\delta_+ \mathcal{O}^{(\bar{n})}(x; w_1, w_2) &= -x^2(\bar{n}\partial_x)\mathcal{O}^{(\bar{n})}(x; w_1, w_2) \\
\delta_+ \mathcal{O}^{(n)}(0; z_1, z_2) &= 2(n\bar{n})\Big(S_+^{(0)} - \epsilon(z_1 + z_2) - \frac{a}{2}[\mathbb{H}^{(1)}, z_1 + z_2)] + \cdots\Big)\mathcal{O}^{(n)}(0; z_1, z_2)
\end{aligned}$$

and for the last term

$$\delta_+ S^{QCD} = 4\epsilon \int d^d x (x\bar{n}) L^{QCD} + 2(d-2)\bar{n}^\mu \int d^d x\, \delta_{BRST}(\bar{c}^a A_\mu^a).$$

$\Big\langle \mathcal{O}^{(n)}(0; z_1, z_2)\, q(x)\bar{q}(y)\Big\rangle$ **– bad object for an analysis**

- **Reexpand ϵL in terms of renormalized operators**

A.N. Vasil'ev, The field theoretic renormalization group in critical behaviour theory and stochastic dynamics

$$2\epsilon L = -\beta(a)/a\,[L^{YM+gf}] + EOM + BRST$$

$[\beta(a_*) = 0]$

$$\Delta S_+ \langle \mathcal{O}_n(0,z), \mathcal{O}_{\bar n}(x,w)\rangle = KR'\left(\langle \int d^D y (\bar n y) L^{YM+gf}(y)\mathcal{O}_n(0,z), \mathcal{O}_{\bar n}(x,w)\rangle\right)_{\text{SIMPLE RESIDUE IN }\epsilon}$$

One loop: **A. Belitsky, D. Müller, 2001, (V. Braun, A.M., (2014))**

$$\Delta_+^{(1)}\mathcal{O}(z_1,z_2) = -2C_F\int_0^1 du\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}\Big[\mathcal{O}(z_{12}^{u\alpha},z_2) - \mathcal{O}(z_1,z_{21}^{u\alpha})\Big]$$

$$[\Delta_+^{(2)}\mathcal{O}](z_1,z_2) = \int_0^1 d\alpha \int_0^{\bar\alpha} d\beta \Big[\omega(\alpha,\beta) + \omega^{\mathbb{P}}(\alpha,\beta)\mathbb{P}_{12}\Big]\Big[\mathcal{O}(z_{12}^\alpha, z_{21}^\beta) - \mathcal{O}(z_{12}^\beta, z_{21}^\alpha)\Big]$$
$$+ \int_0^1 du \int_0^1 d\alpha\, \varkappa(\alpha)\,\Big[\mathcal{O}(z_{12}^{u\alpha}, z_2) - \mathcal{O}(z_1, z_{21}^{u\alpha})\Big].$$

Color structures; $\beta_0 C_F$, C_F^2 **and** C_F/N_C: **(**$\tau = \alpha\beta/\bar\alpha\bar\beta$**)**

$$\omega_{\mathsf{P}}(\alpha,\beta) = \frac{4}{\alpha}\Big[\mathrm{Li}_2(\bar\alpha) - \mathrm{Li}_2(1)\Big] + \frac{1}{\alpha}\ln^2\bar\alpha - (\alpha-\beta)\ln^2\left(\frac{\alpha}{\bar\alpha}\right) - \beta\ln^2\alpha$$
$$+ 2\alpha\left(\frac{\pi^2}{3} - \frac{15}{2}\right) - 2\left(\alpha+\beta+\frac{1}{\bar\alpha}\right)\ln\alpha + \Big(\beta - 2\bar\alpha\Big)\left(1+\frac{2}{\alpha}\right)\ln\bar\alpha\,.$$

$$\omega_{\mathsf{NP}}(\alpha,\beta) = 2\left[\left(\frac{1}{\alpha} - \alpha\right)\left[\mathrm{Li}_2\left(\frac{\beta}{\bar\alpha}\right) - \mathrm{Li}_2(\beta) - 2\,\mathrm{Li}_2(\alpha) - \ln\alpha\ln\bar\alpha\right] + \frac{\alpha}{\tau}\Big[\tau\ln\tau + \bar\tau\ln\bar\tau\Big]\right.$$
$$\left. - \bar\beta\ln\alpha - \frac{\bar\alpha}{\alpha}\ln\bar\alpha\right],$$

$$\omega_{\mathsf{NP}}^{\mathbb{P}}(\alpha,\beta) = 2\left[\left(\bar\alpha - \frac{1}{\bar\alpha}\right)\left[\mathrm{Li}_2\left(\frac{\alpha}{\bar\beta}\right) - \mathrm{Li}_2(\alpha) - \ln\bar\alpha\ln\bar\beta\right] + \alpha\bar\tau\ln\bar\tau + \frac{\beta^2}{\bar\beta}\ln\bar\alpha\right]$$

- **one loop:**

$$\gamma_N = 4C_F\Big(S_1(N+1)+S_1(N-1)-3/2\Big) \qquad \Rightarrow \qquad \mathbb{H}^{(1)} = 4C_F\Big(\widehat{H} - H^+ + \frac{1}{2}\Big)$$

$$\widehat{H}f(z_1,z_2) = \int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}\Big[2f(z_1,z_2) - f(z_{12}^\alpha, z_2) - f(z_1, z_{21}^\alpha)\Big]$$

$$H^+ f(z_1,z_2) = \int_0^1 d\alpha \int_0^{\bar\alpha} d\beta\, f(z_{12}^\alpha, z_{21}^\beta)$$

- **two loop:** determine nonivariant part

$$[S_+^{(0)}, \mathbb{H}^{(2)}] = \Big[\mathbb{H}^{(1)}, z_1+z_2\Big]\left(\beta_0 + \frac{1}{2}\mathbb{H}^{(1)}\right) + [\mathbb{H}^{(1)}, (z_1-z_2)\Delta_+^{(1)}]$$

$$\mathbb{H}^{(2)} = \mathbb{H}^{(2)}_{inv} + \mathbb{T}^{(1)}\left(\beta_0 + \frac{1}{2}\mathbb{H}^{(1)}\right) + \Big[\mathbb{H}^{(1)}, \mathbb{X}^{(1)}\Big]$$

$$\Big[S_+^{(0)}, \mathbb{X}^{(1)}\Big] = (z_1-z_2)\Delta^{(1)} \qquad\qquad \Big[S_+^{(0)}, \mathbb{T}^{(1)}\Big] = \Big[\mathbb{H}^{(1)}, z_1+z_2\Big]$$

e.g.

$$\mathbb{X}^{(1)} f(z_1,z_2) = 2C_F \int_0^1 d\alpha\,\frac{\ln\alpha}{\alpha}\Big[2f(z_1,z_2) - f(z_{12}^\alpha, z_2) - f(z_1, z_{21}^\alpha)\Big]$$

- **Restoration of $\mathbb{H}_{inv}$:**

$$\mathbb{H}_{inv}(N) = \mathbb{H}_2(N) - T_1(N)\left(\beta_0 + \frac{1}{2}\mathbb{H}_1(N)\right).$$

$$\mathbb{H}_1(N) = 4C_F\Big(S_1(N+1) + S_1(N-1) - 3/2\Big) \qquad T_1(N) = 4C_F\Big(2 - S_2(N+1) - S_2(N-1)\Big)$$

$$\mathbb{H}^{(2)}_{inv} = 4C_F\Bigg\{\beta_0\left(\frac{13}{12} + \frac{5}{3}\widehat{\mathcal{H}} - \frac{11}{3}\mathcal{H}^+\right) + 4C_F\left(\frac{19}{48} - \frac{1}{3}\widehat{\mathcal{H}} - \frac{2}{3}\mathcal{H}^+ - \frac{1}{4}(\mathcal{H}^+)^2\right)$$
$$+ \frac{2}{N_c}\Bigg[\left(3\zeta(3) - \frac{\pi^2}{3} + \frac{52}{48}\right) - \frac{\pi^2-4}{6}\left(\widehat{\mathcal{H}} - \mathcal{H}^+\right) - \mathcal{H}^+ - (\mathcal{H}^+)^2 - (\mathcal{H}^+)^3$$
$$+ \mathcal{H}^{1/\tau\ln\bar\tau} - \frac{1}{2}P_{12}\Big(\mathcal{H}^{\ln^2\bar\tau} - 2\mathcal{H}^{\tau\ln\bar\tau}\Big)\Bigg]\Bigg\}$$

Conclusions

- QCD evolution equations possess a "hidden" conformal symmetry
- n-loop evolution kernels for twist-two operators in MS scheme can be restored from the $(n-1)$-loop calculation of the special conformal anomaly and n-loop anomalous dimensions
- $SL(2)$ symmetry properties manifest in the light-ray operator representation
- S_+ is known with two loop accuracy