

# Power corrections to flavour non-singlet structure functions and sum rules at NNLO

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# Outline

## Introduction

DIS structure functions and the contribution of heavy quarks

## Two-loop power corrections

### Structure functions

- Neutral current DIS:  $g_1(x, Q^2)$ ,  $F_1(x, Q^2)$  and  $F_2(x, Q^2)$
- Charged current DIS:  $F_1(x, Q^2)$ ,  $F_2(x, Q^2)$  and  $F_3(x, Q^2)$

### DIS sum rules

- polarized Bjorken sum rule
- unpolarized Bjorken sum rule

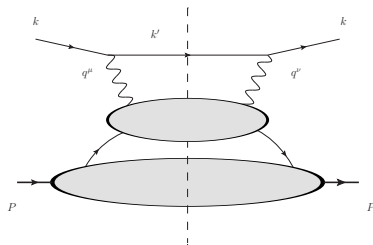
## Conclusion

Summary and future developments

# Introduction to DIS

- Scattering experiments of leptons off nucleon targets provide an ideal environment to study QCD interactions
- To lowest order in the electroweak interaction, the cross section is factorized in leptonic and hadronic tensor

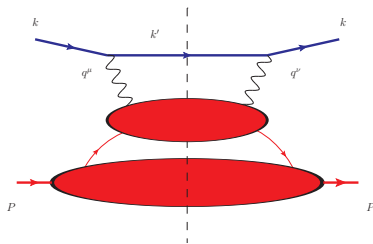
$$\frac{d\sigma}{dx dq^2} \propto L_{\mu\nu}(k, q) W^{\mu\nu}(P, q) \quad (1)$$



# Introduction to DIS

- Scattering experiments of leptons off nucleon targets provide an ideal environment to study QCD interactions
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$$\frac{d\sigma}{dxdq^2} \propto L_{\mu\nu}(k, q) W^{\mu\nu}(P, q) \quad (1)$$



The hadronic tensor is parameterized by **structure functions**: e.g. for unpolarized e.m.  $e p$  scattering

$$W^{\mu\nu}(P, q) = \left[ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right] F_L(x, Q^2) + \left[ P^\mu P^\nu + \frac{q^\mu P^\nu + P^\mu q^\nu}{2x} - \frac{Q^2}{4x^2} g^{\mu\nu} \right] \frac{2x}{Q^2} F_2(x, Q^2) \quad (2)$$

$$Q^2 = -q^2 \quad x = \frac{Q^2}{2P \cdot q}$$

# Heavy flavour

Focus on the contribution of the heavy flavour quark with mass  $m_Q$  to the structure functions

$$F_i(x, Q^2, m_Q^2) = F_i^{\text{light}}(x, Q^2) + F_i^{\text{heavy}}(x, Q^2, m_Q^2). \quad (3)$$

$F_i^{\text{light}}(x, Q^2)$  and  $F_i^{\text{heavy}}(x, Q^2, m_Q^2)$  are convolutions of PDFs and Wilson coefficients. The latter encode the short-distance (perturbative) dynamics of the scattering process.

## Current status

- The massless Wilson coefficients  $C_{i,j}$  are currently known up to three loops.
- The calculation of massive Wilson coefficients in the asymptotic limit  $Q^2 \gg m_Q^2$  at three-loop order is ongoing  $\rightarrow$  talk by A. Behring

## This work

- Two-loop power corrections in the ratio  $\frac{m_Q^2}{Q^2}$  to the structure functions in neutral current and charged current DIS.
- Two-loop power corrections to the associated sum rules.



## Neutral current

Lepton-proton scattering via photon exchange has  $W_{\mu\nu} = W_{\mu\nu}^{(S)} + i W_{\mu\nu}^{(A)}$

$$\begin{aligned}
 W_{\mu\nu}^{(S)} &= \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \frac{F_L(x, Q^2)}{2x} + \left[ P_\mu P_\nu + \frac{q^\mu P^\nu + P^\mu q^\nu}{2x} - \frac{Q^2}{4x^2} g^{\mu\nu} \right] \frac{2x}{Q^2} F_2(x, Q^2), \\
 W_{\mu\nu}^{(A)} &= -\frac{M}{P \cdot q} \epsilon_{\mu\nu\rho\sigma} q^\rho \left[ S^\sigma \hat{g}_1(x, Q^2) + \left( S^\sigma - \frac{S \cdot q}{P \cdot q} P^\sigma \right) \hat{g}_2(x, Q^2) \right],
 \end{aligned} \tag{4}$$

$M$ ,  $P^\mu$  and  $S^\mu$  are respectively the nucleon mass, momentum and spin.

- Flavour non-singlet (NS) contributions are determined in the difference of measures e.g.

$$\left( \frac{d\sigma}{dx dQ^2} \right)^{\text{ep}} - \left( \frac{d\sigma}{dx dQ^2} \right)^{\text{en}}$$



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NS structure functions are determined using the factorization formula

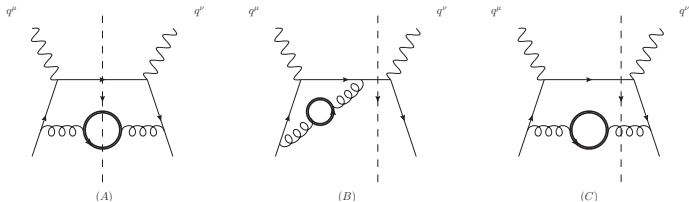
$$\hat{g}_1(x, Q^2) = \frac{1}{2} \int_x^1 \frac{dz}{z} \left[ C_{g_1, q}^{\text{NS}} \left( z, \frac{Q^2}{\mu^2} \right) + L_{g_1, q}^{\text{NS}} \left( z, \frac{Q^2}{m_Q^2}, \mu^2 \right) \right] \cdot \tilde{\Delta} \left( \frac{x}{z}, \mu^2 \right),$$
(5)

- PDF combination:  $\tilde{\Delta}(x, \mu^2) = \sum_i e_i^2 [\Delta f_i(x, \mu^2) + \Delta \bar{f}_i(x, \mu^2)]$ ,
- $C_{g_1, q}^{\text{NS}}$ ,  $L_{g_1, q}^{\text{NS}}$ : light quark-photon scattering subprocess.



# $L_{g1,q}^{NS}$ : two-loop power corrections

Inclusive scattering process  $q + \gamma^* \rightarrow q + X$ , heavy quarks in the final state  $X$  or in loops



$$\begin{aligned}
 L_{g1,q}^{NS}(z, Q^2, m_Q^2) &= \underbrace{\Theta\left(\frac{\xi}{\xi+4} - z\right) L_{g1,q}^{NS,(R)}(z, \xi)}_{(A)} + \underbrace{\delta(1-z) L_{g1,q}^{NS,(V)}(z, \xi)}_{(B)} \\
 &\quad - \underbrace{\left(\frac{\alpha_s}{4\pi}\right)^2 \beta_{0,Q} \log\left(\frac{m_Q^2}{\mu^2}\right) \left[ \frac{1}{2} P_{qq}^{(0)} \log\left(\frac{Q^2}{\mu^2}\right) + c_{g1,q}^{(1)} \right]}_{(C)}, \quad (6)
 \end{aligned}$$

$$\text{where } z = \frac{Q^2}{2p \cdot q}, \quad \xi = \frac{Q^2}{m_Q^2}.$$



## Real radiation

- Re-calculation of the Compton process

$$q + \gamma^* \rightarrow q + Q + \bar{Q}$$

The following variables are used

$$sq_1 = \sqrt{1 - \frac{4}{\xi} \frac{z}{1-z}}, \quad sq_2 = \sqrt{1 - \frac{4}{\xi} z}, \quad L_i = \log \left( \frac{1 + sq_i}{1 - sq_i} \right) \quad (i=1,2), \quad L_3 = \log \left( \frac{sq_2 + sq_1}{sq_2 - sq_1} \right),$$

$$di_1 = \text{Li}_2 \left[ (1-z) \frac{1 + sq_1}{1 + sq_2} \right], \quad di_2 = \text{Li}_2 \left( \frac{1 - sq_2}{1 + sq_1} \right), \quad di_3 = \text{Li}_2 \left( \frac{1 - sq_1}{1 + sq_2} \right), \quad di_4 = \text{Li}_2 \left( \frac{1 + sq_1}{1 + sq_2} \right).$$



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- Agreement with the literature (Buza, Matiounine, Smith, Mignerone, van Neerven '96)

$$L_{g_1, q}^{\text{NS}, (R)}(z, Q^2) = \left(\frac{\alpha_s}{4\pi}\right)^2 C_F T_F \left\{ -\frac{8L_3 sq_2}{9(z-1)\xi} \left(50z^3 - 11\xi + z(6\xi + 20) - 2z^2(7\xi + 12)\right) \right.$$

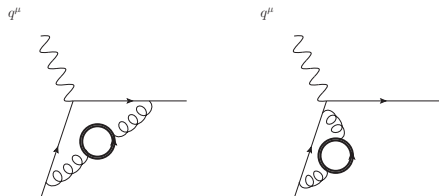
$$+ \frac{2sq_1}{27(z-1)^2\xi} \left(1200z^4 + 265\xi - 4z^3(109\xi + 490) + 2z^2(389\xi + 618) - z(607\xi + 466)\right)$$

$$+ \frac{4L_1}{3(z-1)^3\xi^2} \left(24z^4 - \xi^2 + 3z^2(\xi^2 + 6) - 2z^3(\xi^2 + 18)\right) + \frac{12z^3 - \xi^2 - z^2\xi^2}{3(z-1)\xi^2}$$

$$\left. \times \left[4L_1L_2 + 8(-di_1 + di_2 + di_3 - di_4) - 4L_1 \log\left(\frac{z^2}{1-z}\right)\right]\right\}. \quad (7)$$



## Virtual corrections



- The virtual corrections are given by

$$L_{g_1, q}^{\text{NS}, (V)}(\xi) = 2\mathcal{F}_1^{(2)}\left(-\frac{Q^2}{m_Q^2}\right),$$

where  $\mathcal{F}_1^{(2)}$  come from the form factor diagrams.

Introducing the variable  $\tilde{\lambda} = \sqrt{1 - \frac{4}{\xi}}$  the result is

$$\begin{aligned} L_{g_1, q}^{\text{NS}, (V)}(\xi) = & 2\left(\frac{\alpha_s}{4\pi}\right)^2 C_F T_F \left\{ \frac{3355}{81} - \frac{952}{9\xi} + \left(\frac{32}{\xi^2} - \frac{16}{3}\right) \zeta(3) \right. \\ & + \left(\frac{440}{9\xi} - \frac{530}{27}\right) \log(\xi) + \tilde{\lambda} \left[ \frac{184}{9\xi} - \frac{76}{9} \right] \left[ \text{Li}_2\left(\frac{\tilde{\lambda}+1}{\tilde{\lambda}-1}\right) - \text{Li}_2\left(\frac{\tilde{\lambda}-1}{\tilde{\lambda}+1}\right) \right] \\ & \left. + \left[ \frac{8}{3} - \frac{16}{\xi^2} \right] \left[ \text{Li}_3\left(\frac{\tilde{\lambda}-1}{\tilde{\lambda}+1}\right) + \text{Li}_3\left(\frac{\tilde{\lambda}+1}{\tilde{\lambda}-1}\right) \right] \right\}. \end{aligned} \quad (8)$$

# $g_1(x, Q^2)$ structure function

Both the  $c$  quark and the  $b$  quark contributions are taken into account by summing

$$L_{g_1,q}^{\text{NS,tot}} = L_{g_1,q}^{\text{NS}}(z, Q^2, m_c^2) + L_{g_1,q}^{\text{NS}}(z, Q^2, m_b^2). \quad (9)$$

The effect of power correction can be seen both on the isolated charm term and on the combined heavy flavour contribution<sup>1</sup>

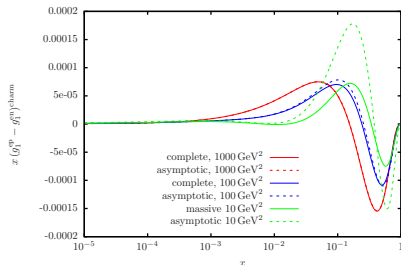


Figure: Comparison of the asymptotic and the exact charm quark contribution to  $g_1^{\text{NS}}(x, Q^2)$ .

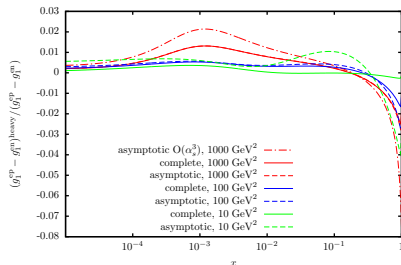


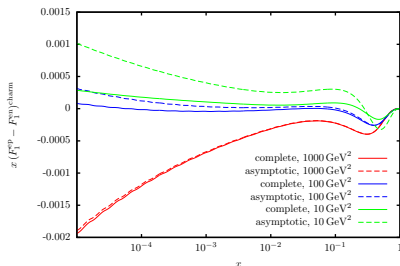
Figure: Relative impact of heavy flavours on  $g_1^{\text{NS}}(x, Q^2)$ .

<sup>1</sup>These plots are done with BB09 POLPDF (polarized) and abm12.3.nnlo (unpolarized),  $m_c = 1.59$  GeV and  $m_b = 4.78$  GeV

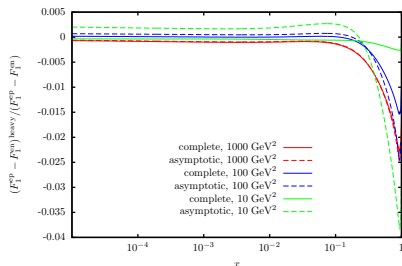
# $F_1(x, Q^2)$ structure function

Similarly, the symmetric part of the hadronic tensor,  $W_{\mu\nu}^{(S)}$ , is obtained from the unpolarized  $q \gamma^*$  scattering process. Illustrations for the combination

$$F_1(x, Q^2) = \frac{1}{2x} [F_2(x, Q^2) - F_L(x, Q^2)] \quad (10)$$



**Figure:** Comparison of the asymptotic and the exact charm quark contribution to  $F_1^{NS}(x, Q^2)$ .



**Figure:** Relative impact of heavy flavours on  $F_1^{NS}(x, Q^2)$ .

## $F_2(x, Q^2)$ structure function

- The asymptotic approximation works better for the structure function  $F_2(x, Q^2)$  alone. Indeed  $F_1(x, Q^2)$  includes also  $F_L(x, Q^2)$ , which approaches the asymptotic limit at higher values of  $Q^2$ .

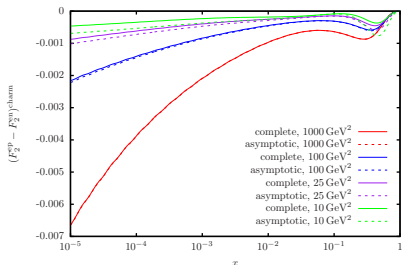


Figure: Comparison of the asymptotic and the exact charm quark contribution to  $F_2^{NS}(x, Q^2)$ .

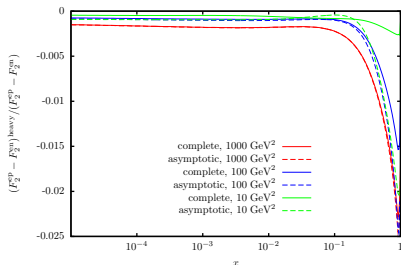


Figure: Relative impact of heavy flavours on  $F_2^{NS}(x, Q^2)$ .

## Charged current

Charged current DIS  $q + W^\pm \rightarrow X$  has

$$W_{\mu\nu}^{(A)} = i \epsilon_{\mu\nu\rho\sigma} \frac{P^\rho q^\sigma}{2P \cdot q} F_3^{W^\pm}(x, Q^2). \quad (11)$$

It is useful to consider the antisymmetric combinations under crossing:

$$F_i^{(-)}(x, Q^2) = [F_i^{W^+}(x, Q^2) - F_i^{W^-}(x, Q^2)], \quad F_3^{(-)}(x, Q^2) = [F_3^{W^+}(x, Q^2) + F_3^{W^-}(x, Q^2)],$$

$i = 2, L$ . Factorization involves a new term, depending on the variable  $\tilde{x} = x \frac{Q^2 + m_Q^2}{Q^2}$

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$i = 2, L$ . Factorization involves a new term, depending on the variable  $\tilde{x} = x \frac{Q^2 + m_Q^2}{Q^2}$

$$F_2^{(-)}(x, Q^2) = 2 \left\{ x \int_x^1 \frac{dz}{z} \left[ |V_{du}|^2 d_v \left( \frac{x}{z} \right) - (|V_{du}|^2 + |V_{su}|^2) u_v \left( \frac{x}{z} \right) \right] (C_{2,q}^{\text{NS}}(z) + L_{2,q}^{\text{NS}}(z)) \right. \\ \left. + \tilde{x} \int_{\tilde{x}}^1 \frac{dz}{z} |V_{dc}|^2 d_v \left( \frac{\tilde{x}}{z} \right) H_{2,q}^{\text{NS}}(z) \right\} \quad (12)$$

$$F_3^{(-)}(x, Q^2) = 2 \left\{ \int_x^1 \frac{dz}{z} \left[ |V_{du}|^2 d_v \left( \frac{x}{z} \right) + (|V_{du}|^2 + |V_{su}|^2) u_v \left( \frac{x}{z} \right) \right] (C_{3,q}^{\text{NS}}(z) + L_{3,q}^{\text{NS}}(z)) \right. \\ \left. + \int_{\tilde{x}}^1 \frac{dz}{z} |V_{dc}|^2 d_v \left( \frac{\tilde{x}}{z} \right) H_{3,q}^{\text{NS}}(z) \right\},$$

$q_v(x, \mu^2) = q(x, \mu^2) - \bar{q}(x, \mu^2)$  are the valence quark  $q$  PDFs.





# Charged current

## New feature

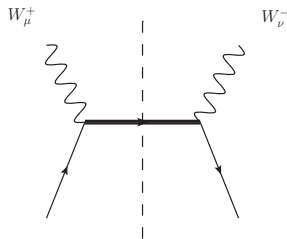
The Wilson coefficient  $H_{i,q}^{\text{NS}}$  describes the coupling of the gauge boson to a heavy flavour quark.

- $H_i^{\text{NS}}$  is already present at tree level.
- The Bjorken variable  $x$  is bounded by

$$\tilde{x} = x \frac{Q^2 + m_Q^2}{Q^2} \leq 1, \quad (13)$$

because part of the incoming energy goes in the production of the massive final state (*slow rescaling*).

- The contribution of  $H^{\text{NS}}$  is suppressed by the CKM matrix element  $|V_{dc}|^2 \simeq 0.05$  with respect to  $L^{\text{NS}}$  and  $C^{\text{NS}}$ .



The coefficients  $H_{i,q}^{\text{NS}}$  are known exactly at one loop (Gottschalk '81; Glück, Kretzer, Reya '96; Blümlein, Hasselhuhn, Kovacikova, Moch '11). CKM suppression allows to use the asymptotic two-loop formulae (Blümlein, Pfoh, Hasselhuhn '14).

# Charged current plots: $F_1(x, Q^2)$

- Relying on this approximation  $H_{i,q}^{\text{NS}}$  are implemented up to two-loops.
- Massless Wilson coefficients at the same order are also available (Zijlstra, van Neerven '92; Moch, Vermaseren '99; Moch, Rogal, Vogt '07).
- Two-loop power corrections to  $L_{i,q}^{\text{NS}}$  for  $i = 2, L$  are known from the neutral current calculation.
- Regarding  $F_3(x, Q^2)$ ,  $L_{3,q}^{\text{NS}} = L_{g1,q}^{\text{NS}}$  at two loops.

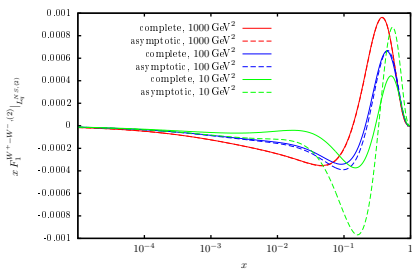


Figure: Exact and asymptotic  $L_{1,q}^{\text{NS}}$  contribution to the structure function.

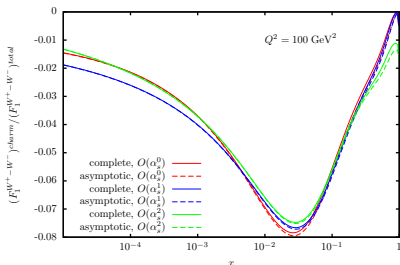
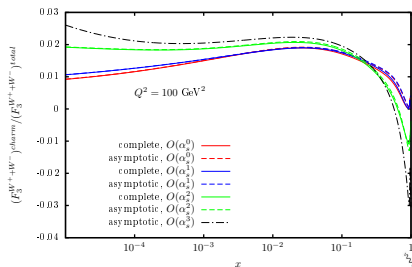
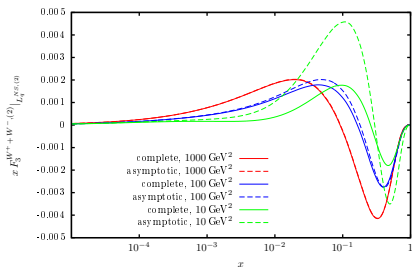
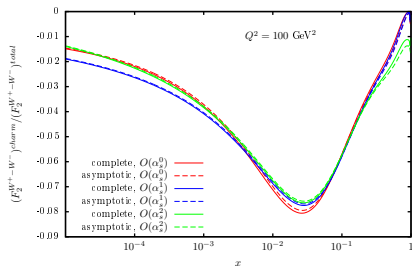
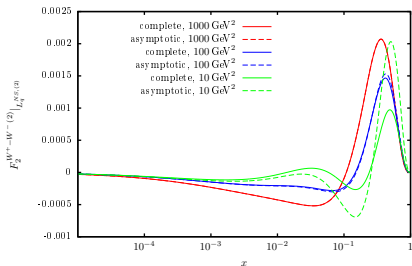


Figure: Impact of the heavy flavours (both  $H_{1,q}^{\text{NS}}$  and  $L_{1,q}^{\text{NS}}$ ) on the structure function.

Charged current plots:  $F_2(x, Q^2)$  and  $F_3(x, Q^2)$ 

## QCD sum rules

The integrals of non-singlet structure functions give a set of sum rules

$$\begin{aligned} \Delta g_1(Q^2) &= \int_0^1 dx \left[ g_1^{e p}(x, Q^2) - g_1^{e n}(x, Q^2) \right] = K_{g_1}(n_f) A^{g_1}(\alpha_s, Q^2) \quad \text{Pol. Bjorken,} \\ \Delta F_1(Q^2) &= \int_0^1 dx \left[ F_1^{\bar{\nu} p}(x, Q^2) - F_1^{\nu p}(x, Q^2) \right] = K_1(n_f) A^{F_1}(\alpha_s, Q^2) \quad \text{Unpol. Bjorken,} \\ \Delta F_2(Q^2) &= \int_0^1 \frac{dx}{x} \left[ F_2^{\bar{\nu} p}(x, Q^2) - F_2^{\nu p}(x, Q^2) \right] = K_2(n_f) \quad \text{Adler sum rule,} \quad (14) \\ \Delta F_3(Q^2) &= \int_0^1 dx \left[ F_3^{\bar{\nu} p}(x, Q^2) + F_3^{\nu p}(x, Q^2) \right] = K_3(n_f) A^{F_3}(\alpha_s, Q^2) \quad \text{Gross Llewellyn Smith.} \end{aligned}$$

- $K_i(n_f)$  is the parton model result, QCD radiative corrections are encoded in  $A^i(\alpha_s, Q^2)$ .  
Massless contributions known at 4 loops ([Baikov, Chetyrkin, Kühn '10](#); [Chetyrkin '14](#)).



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### Heavy flavour contribution

Having the analytic expression of  $F_i(x, Q^2)$ , we compute the power corrections to  $A^i$ . We verified the Adler sum rule as non-trivial check on the structure function  $F_2$ .

## $A^{g_1}$ : two-loop power corrections

The contribution of massive flavours to  $A^{g_1}$  is

$$\begin{aligned} \Delta g_1^{\text{massive}}(\xi) &= \int_0^1 dx \int_x^1 \frac{dz}{z} L_{g_1, q}^{\text{NS}}(z) \cdot \left\{ \frac{1}{2} \left[ \tilde{\Delta}_p \left( \frac{x}{z} \right) - \tilde{\Delta}_n \left( \frac{x}{z} \right) \right] \right\} \\ &= \underbrace{\int_0^1 dy \frac{\tilde{\Delta}_p(y) - \tilde{\Delta}_n(y)}{2}}_{K_{g_1}(n_f) = \frac{1}{6} \left| \frac{G_A}{G_V} \right|} \times \underbrace{\int_0^1 dz L_{g_1, q}^{\text{NS}}(z)}_{C_{\text{pBj}}^{\text{Q}}(\xi)} \end{aligned} \quad (15)$$

By introducing  $\lambda = \sqrt{1 + \frac{4}{\xi}}$  and computing the integral of the Wilson coefficient<sup>2</sup> we get

$$\begin{aligned} C_{\text{pBj}}^{\text{Q}}(\xi) &= - \left( \frac{\alpha_s}{4\pi} \right)^2 \frac{C_F T_F}{315 \xi^2} \left\{ 2100 \log \left( \frac{\lambda + 1}{\lambda - 1} \right)^2 - \xi (6\xi^2 + 2735\xi + 11724) + \lambda \log \left( \frac{\lambda + 1}{\lambda - 1} \right) \right. \\ &\quad \left. \times \xi (3\xi^3 + 106\xi^2 + 1054\xi + 4812) - \xi^2 (3\xi^2 + 112\xi + 1260) \log(\xi) \right\}. \end{aligned} \quad (16)$$

$$C_{\text{pBj}}^{\text{Q}}(\xi) \xrightarrow{\xi \gg 1} \left( \frac{\alpha_s}{4\pi} \right)^2 C_F T_F \left[ 8 + \frac{1}{\xi} \left( \frac{272}{9} - \frac{64}{3} \log(\xi) \right) + \mathcal{O} \left( \frac{\log^2(\xi)}{\xi^2} \right) \right].$$

Differently to the non-inclusive case (Blümlein, van Neerven '98), no large logarithm survives in the massless limit  $Q^2 \gg m_Q^2$ .

<sup>2</sup>Two-loop power corrections to the Gross Llewellyn Smith sum rule are immediately derived, as  $L_{g_1, q}^{\text{NS}}(z, \xi) = L_{3, q}^{\text{NS}}(z, \xi)$ .

# Polarized Bjorken sum rule

## Features

The contribution of heavy flavours

- implements the transition to one more massless flavour  $n_f \rightarrow n_f + 1$  in the limit  $Q^2 \gg m_Q^2$

$$A^{\text{g1}}(\xi) = 1 - \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[ -4.58333 + 0.33333 n_f + C_{\text{pBj}}^{\text{Q},(2)}(\xi) \right] + \mathcal{O}(\alpha_s^3), \quad (17)$$

- becomes even negative at small scales  $Q^2 = m_Q^2$ , because of the virtual corrections.

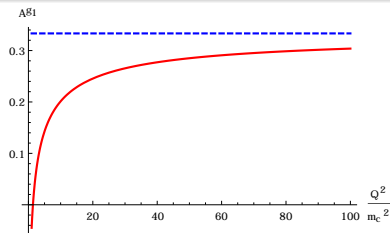


Figure:  $C_{\text{pBj}}^{\text{c},(2)}(\xi)$  (solid) compared to the transition to one more massless flavour  $n_f \rightarrow n_f + 1$  (dashed).

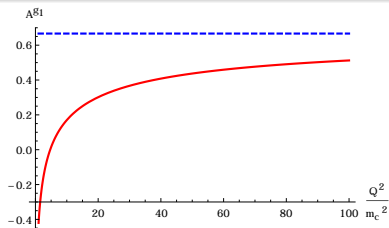


Figure: Contribution of bottom and charm quarks  $C_{\text{pBj}}^{\text{c},(2)}(\xi) + C_{\text{pBj}}^{\text{b},(2)}(\xi)$  (solid) vs  $n_f \rightarrow n_f + 2$ .



## Unpolarized Bjorken sum rule

$$A^{F_1} = 1 - \left(\frac{\alpha_s}{\pi}\right) \left[0.66667 + C_{uBj}^{Q,(1)}(\xi)\right] + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-3.83333 + 0.29630 n_f + C_{uBj}^{Q,(2)}(\xi)\right] + \mathcal{O}(\alpha_s^3), \quad (18)$$

$C_{uBj}^{Q,(1)}(\xi)$ : first moment of the one-loop Wilson coefficient  $H_1$  (Blümlein, van Neerven '98).



## Unpolarized Bjorken sum rule

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$C_{\text{uBj}}^{\text{Q},(1)}(\xi)$ : first moment of the one-loop Wilson coefficient  $H_1$  (Blümlein, van Neerven '98).

$$C_{\text{uBj}}^{\text{Q},(2)}(\xi) = C_F T_F \left\{ \frac{1129}{2520} - \frac{1}{2\xi^2} \log\left(\frac{\lambda+1}{\lambda-1}\right)^2 + \frac{107}{42\xi} - \frac{\xi}{420} + \lambda \log\left(\frac{\lambda+1}{\lambda-1}\right) \right. \\ \left. \times \left(-\frac{67}{420} - \frac{43}{42\xi} - \frac{\xi}{420} + \frac{\xi^2}{840}\right) + \left(\frac{1}{6} - \frac{\xi^2}{840}\right) \log(\xi) \right\}, \quad (19)$$

$$C_{\text{uBj}}^{\text{Q},(2)}(\xi) \xrightarrow{\xi \gg 1} C_F T_F \left[ \frac{4}{9} + \frac{1}{\xi} \left( \frac{20}{9} - \frac{4}{3} \log(\xi) \right) \right] + \mathcal{O}\left(\frac{\log^2(\xi)}{\xi^2}\right).$$

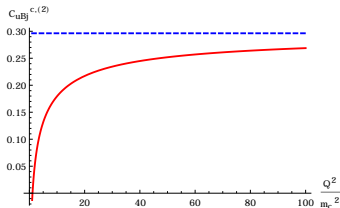
## Unpolarized Bjorken sum rule

$$A^{F_1} = 1 - \left(\frac{\alpha_s}{\pi}\right) \left[0.66667 + C_{uBj}^{Q,(1)}(\xi)\right] + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-3.83333 + 0.29630 n_f + C_{uBj}^{Q,(2)}(\xi)\right] + \mathcal{O}(\alpha_s^3), \quad (18)$$

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$$C_{uBj}^{Q,(2)}(\xi) \xrightarrow{\xi \gg 1} C_F T_F \left[ \frac{4}{9} + \frac{1}{\xi} \left( \frac{20}{9} - \frac{4}{3} \log(\xi) \right) \right] + \mathcal{O}\left(\frac{\log^2(\xi)}{\xi^2}\right).$$



- No logarithmic corrections in the mass scales involved.
- Smooth interpolation to the regime  $n_f \rightarrow n_f + 1$ .



# Summary and outlook

## Power corrections to structure functions

- We calculate the power corrections to the Wilson coefficients  $L_{g1,q}^{\text{NS}}$ ,  $L_{2,q}^{\text{NS}}$ ,  $L_{L,q}^{\text{NS}}$  at two loops.
- These Wilson coefficients are used to compute all the non singlet structure functions in neutral and charged current DIS. In the latter case it is necessary to include also single-excitation processes, already known in the literature.
- A natural development of this project is an improvement of the analysis of DIS data with the complete heavy flavour effects at two loops.

## Sum rules

- We compute the power corrections to the polarized and unpolarized Bjorken sum rules and to the Gross Llewellyn Smith sum rule. We verified the Adler sum rule as a check on  $L_{2,q}^{\text{NS}}$ .
- Massive quarks provide a smooth interpolation of the sum rules from  $n_f$  to  $n_f + 1$  in the limit  $\frac{m_Q^2}{Q^2} \rightarrow 0$ .
- The transition to  $n_f + 1$  takes place at large scales; at  $Q^2 \simeq m_Q^2$  the effect can be negative.



Thank you for your attention!

