Precision Observables for 2-Doublet Higgs Sectors with/out SUSY

Loops and Legs in Quantum Field Theory

Leipzig

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- Electroweak precision observables have been a powerful tool since the days of LEP
- test compatibility of models with experimental precision data
- constrain range of model parameters
- since LHC (2012): existence of a Higgs boson confirmed, mass M_H precisely measured
  - Standard Model precision observables uniquely determined
  - constraints on BSM model parameters more severe

Outline

- Electroweak precision observables – the Standard Model
- Electroweak precision observables in SUSY models
- Electroweak precision observables in 2-Higgs-doublet models

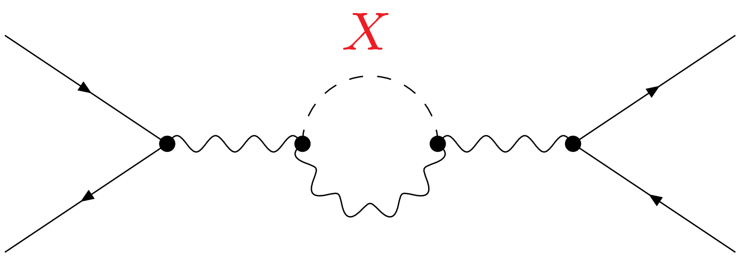
electroweak precision observables

- μ lifetime: $G_{\rm F}$
- Z observables: M_Z, Γ_Z, $\sin^2\theta_{\rm eff}$, $\ldots$
- LEP 2, Tevatron, LHC: M_W, m_t $+$ M_H
- low energy: $(g-2)_\mu$

electroweak precision observables

- μ lifetime: G_F
- Z observables: $M_Z, \Gamma_Z, \sin^2\theta_\mathrm{eff}, \ldots$
- LEP 2, Tevatron, LHC: $M_W, m_t \quad + \; M_H$
- low energy: $(g-2)_\mu$

quantum structure of the Standard Model



sensitivity to heavy internal particles (X)

Standard Model: X = Higgs, top

experimental input

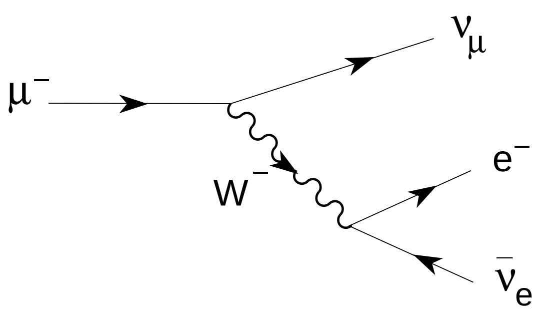
G_F	=	$1.1663787(6) \cdot 10^{-5}\,\mathrm{GeV}^{-2}$
M_Z	=	$91.1875 \pm 0.0021\,\mathrm{GeV}$
M_W	=	$80.385 \pm 0.015\,\mathrm{GeV}$
m_t	=	$173.2 \pm 0.9\,\mathrm{GeV}$
M_H	=	$125.09 \pm 0.24\,\mathrm{GeV}$
$\alpha_s(M_Z)$	=	0.1185 ± 0.0006

$M_W - M_Z$ correlation

Definition of Fermi constant G_F via muon lifetime:

$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)(1 + \Delta q)$$

Δq: QED corrections in Fermi Model,



$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2\left(1 - M_W^2/M_Z^2\right)} \quad + \ [\text{higher orders}]$$

$M_W - M_Z$ correlation

Definition of Fermi constant G_F via muon lifetime:

$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)(1 + \Delta q)$$

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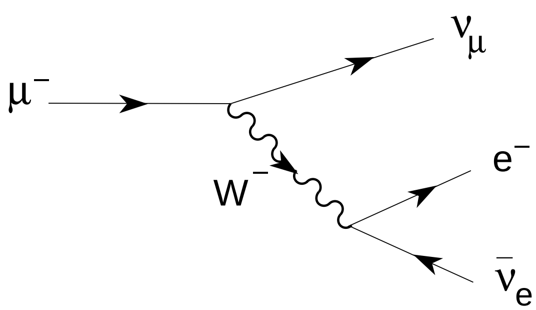
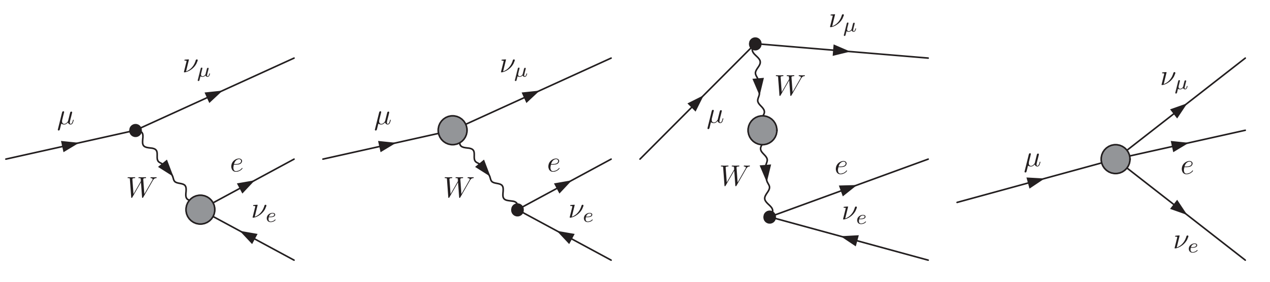




with loop contributions

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2\,(1 - M_W^2/M_Z^2)} \cdot (1 + \Delta r)$$

Δr : quantum correction

$$\Delta r = \Delta r(M_Z, M_W, m_t, M_H)$$

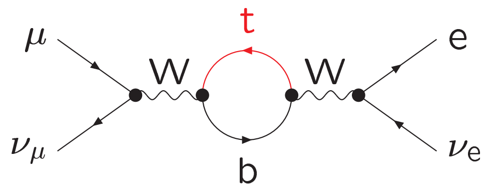
determines W mass

$$M_W = M_W(\alpha, G_F, M_Z, m_t, M_H)$$

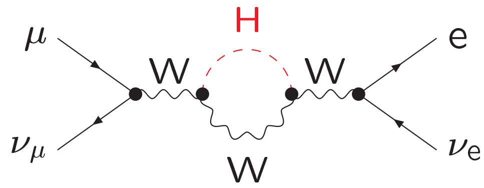
complete at 2-loop order α^2 and $\alpha\alpha_s$

1-loop examples

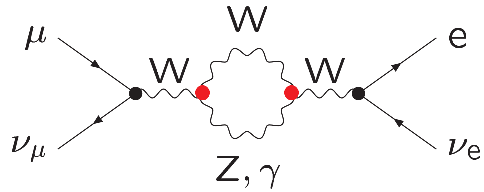
- top quark



- Higgs boson



- gauge-boson self-couplings



$O(\alpha\alpha_s)$ 2-loop calculations for Δr

Halzen, Kniehl

Djoaudi, Gambino

EW 2-loop calculations for Δr

Freitas, Hollik, Walter, Weiglein

Awramik, Czakon

Onishchenko, Veretin

Degrassi, Gambino, Giardino

universal terms at 3- and 4-loops (EW and QCD)

van der Bij, Chetyrkin, Faisst, Jikia, Seidensticker

Faisst, Kühn Seidensticker, Veretin

Boughezal, Tausk, van der Bij

Schröder, Steinhauser

Chetyrkin, Faisst, Kühn *Chetyrkin, Faisst, Kühn, Maierhofer, Sturm*

Boughezal, Czakon

dominant contributions to Δr

$$\Delta r = \Delta\alpha - \frac{c_{\mathrm{w}}^2}{s_{\mathrm{w}}^2}\Delta\rho + \cdots$$

Large universal terms:

$$\Delta\alpha = \Pi^{\gamma}_{\mathrm{ferm}}(M_Z^2) - \Pi^{\gamma}_{\mathrm{ferm}}(0) = 0.05907 \pm 0.0001$$

$$\Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2} = 3\frac{G_F m_t^2}{8\pi^2\sqrt{2}} = 0.0094 \quad \textit{[one-loop]} \quad \sim \frac{m_t^2}{v^2} \sim \alpha_t$$

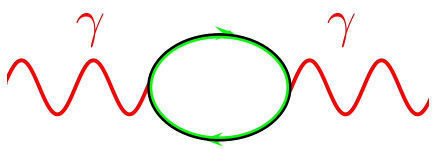
beyond 2-loop order: $\Delta\rho^{(3)} + \Delta\rho^{(4)} \quad \sim \quad \alpha_s^2\alpha_t,\ \alpha_s\alpha_t^2,\ \alpha_t^3,\ \alpha_s^3\alpha_t$

reducible higher order terms from $\Delta\alpha$ and $\Delta\rho$ via

$$1 + \Delta r \quad \rightarrow \quad \frac{1}{\left(1-\Delta\alpha\right)\left(1+\frac{c_{\mathrm{w}}^2}{s_{\mathrm{w}}^2}\Delta\rho\right)+\cdots}$$

Consoli, WH, Jegerlehner

photon vacuum polarization



$$\Pi^{\gamma}_{\rm ferm}(M_Z^2) - \Pi^{\gamma}_{\rm ferm}(0) \equiv \Delta\alpha \quad \rightarrow \quad \alpha(M_Z) = \frac{\alpha}{1-\Delta\alpha} \simeq \frac{1}{129}$$

$$\Delta\alpha = \Delta\alpha_{\rm lept} + \Delta\alpha_{\rm had},$$

$$\Delta\alpha_{\rm lept} = 0.031498 \; (4-{\rm loop}) \qquad \text{Steinhauser 1998; Sturm 2013}$$

$$\Delta\alpha_{\rm had} = 0.02757 \quad \pm 0.00010 \qquad \text{Davier et al. 2010}$$

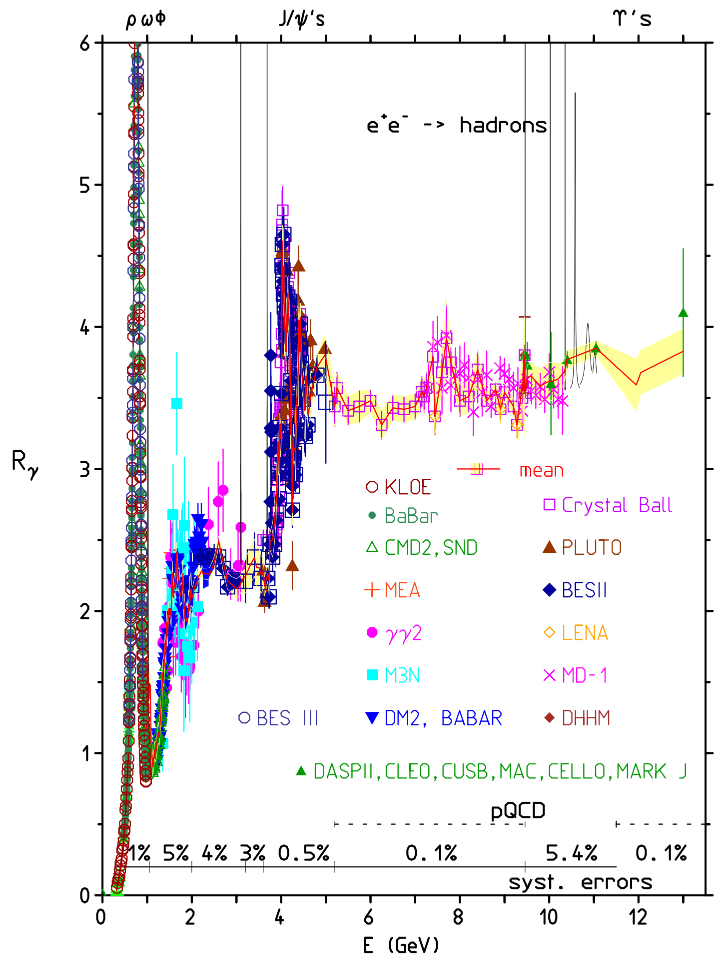
$$= 0.027626 \pm 0.000103 \qquad \text{Hagiwara et al. 2011}$$

$$= 0.027504 \pm 0.000118 \qquad \text{Jegerlehner 2015}$$

significant parametric uncertainty (universal)

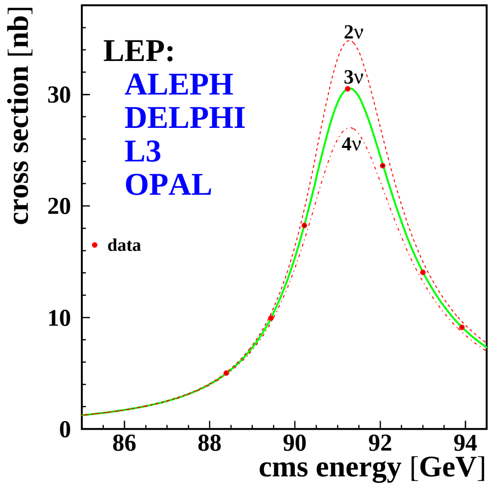
$$\Delta\alpha_{\rm had} = -\frac{\alpha}{3\pi} M_Z^2 \, {\rm Re} \int_{4m_\pi^2}^{\infty} ds' \, \frac{R_{\rm had}(s')}{s'(s' - M_Z^2 - i\epsilon)}$$

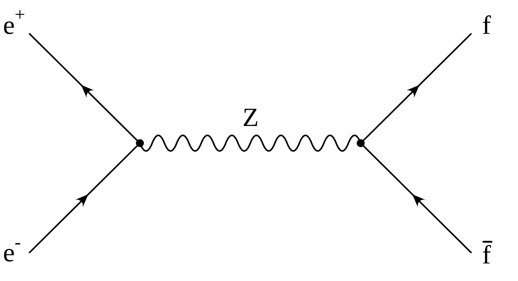
$$R_{\rm had} = \frac{\sigma(e^+e^- \to \gamma^* \to {\rm hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$



Jegerlehner 2015

Z resonance





- effective Z boson couplings with higher-order $\Delta g_{V,A}$

$$g_V^f \to g_V^f + \Delta g_V^f, \qquad g_A^f \to g_A^f + \Delta g_A^f$$

- effective ew mixing angle (for $f = e$):

$$\sin^2\theta_{\text{eff}} = \frac{1}{4}\left(1 - \text{Re}\,\frac{g_V^e}{g_A^e}\right) = 1 - \frac{M_W^2}{M_Z^2} + \frac{M_W^2}{M_Z^2}\,\Delta\rho + \cdots$$

EW 2-loop calculations for $\sin^2\theta_{\text{eff}}$

Awramik, Czakon, Freitas, Weiglein

Awramik, Czakon, Freitas

Hollik, Meier, Uccirati

$O(\alpha\alpha_s)$ 2-loop calculations (universal)

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universal terms at 3- and 4-loops (EW and QCD)

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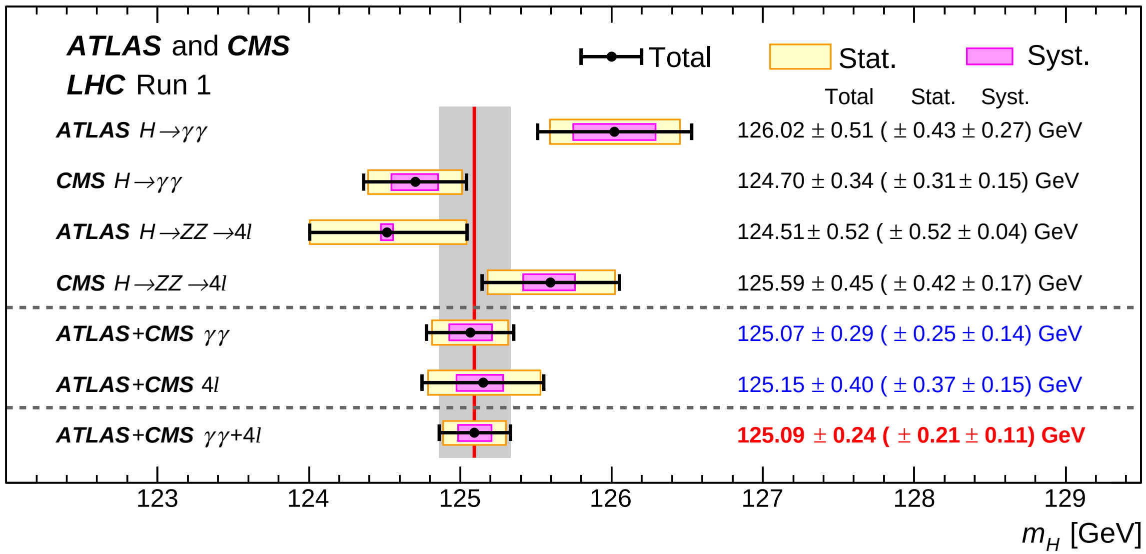
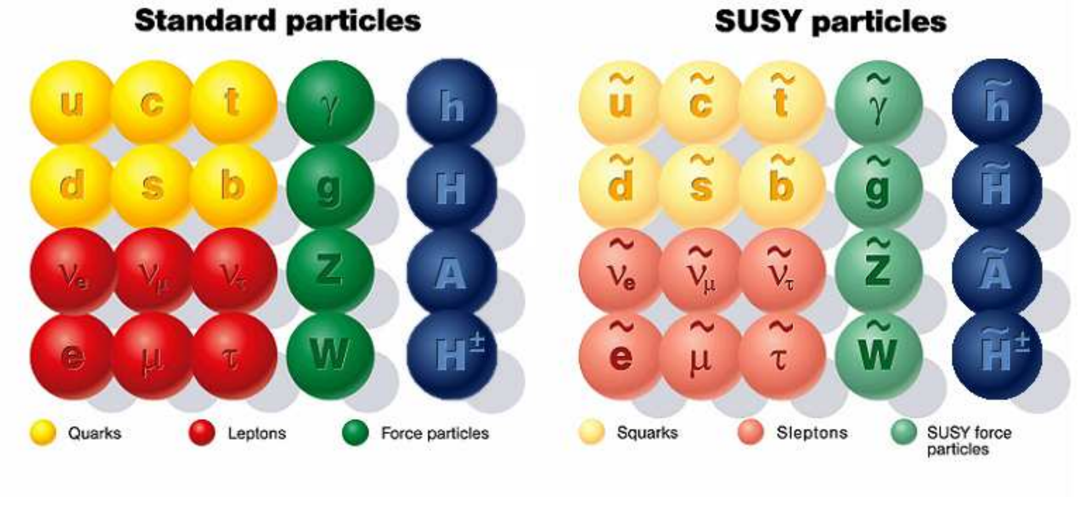


SM input completely determined $\Rightarrow$ precision observables uniquely predicted

	theo	exp
$\sin^2\theta_{\text{eff}}$	$0.23152 \pm 0.00005 \pm 0.00005$	0.23153 ± 0.00016
M_W (GeV)	$80.361 \pm 0.006 \pm 0.004$	80.385 ± 0.015

Supersymmetry



- gauge bosons $W^{\pm}, Z$
- fermions f_L, f_R
- charginos and neutralinos $\chi^{\pm}_{1,2}$, $\chi^{0}_{1,2,3,4}$
- sfermions $\tilde{f}_L,\ \tilde{f}_R \rightarrow\ \tilde{f}_1,\ \tilde{f}_2$
- Higgs bosons: $h^0,\ H^0,\ \ A^0,\ \ H^{\pm}$

Higgs fields – MSSM

two scalar doublets from $\mathbf{H}_1, \mathbf{H}_2$ superfields:

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} H_1^0 \\ \phi_1^- \end{pmatrix}, \qquad < H_1 >_0 = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ H_2^0 \end{pmatrix}, \qquad < H_2 >_0 = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

$$V_H^{\text{susy}} = \mu^2 \, H_1^\dagger H_1 + \mu^2 \, H_2^\dagger H_2 + \frac{g_1^2 + g_2^2}{8} \left(H_1^\dagger H_1 - H_2^\dagger H_2 \right)^2 + \frac{g_2^2}{2} \, |H_1^\dagger H_2|^2 \,,$$

$$V_H^{\text{soft}} = m_1^2 \, H_1^\dagger H_1 + m_1^2 \, H_2^\dagger H_2 - m_3^2 \, \varepsilon_{ij} \left(H_1^i \, H_2^j + h.c. \right)$$

Higgs potential: $V_H = V_H^{\rm susy} + V_H^{\rm soft}$

$$= (\mu^2 + m_1^2)\, H_1^\dagger H_1 + (\mu^2 + m_2^2)\, H_2^\dagger H_2 \;-\; m_3^2\, \varepsilon_{ij} \left(H_1^i\, H_2^j + h.c. \right)$$

$$+ \frac{g_1^2+g_2^2}{8} \left(H_1^\dagger H_1 - H_2^\dagger H_2 \right)^2 + \frac{g_2^2}{2} \left| H_1^\dagger H_2 \right|^2$$

EW symmetry breaking: minimum of V_H at

$$H_1^0 = v_1 \neq 0, \;\; H_2^0 = v_2 \neq 0, \quad \phi_1^- = 0, \;\; \phi_2^+ = 0$$

SM particle masses:

$$M_{W,Z}^2 \sim v_1^2 + v_2^2 \equiv v^2, \quad m_d, m_e \sim v_1, \quad m_u \sim v_2$$

free parameters: $\boxed{m_3^2, \; \tan\beta = \frac{v_2}{v_1}}$

mass spectrum: 3 unphysical + 5 physical degrees of freedom

- 3 Goldstone bosons $G^0,\ G^\pm$
- 2 neutral CP-even Higgs bosons $h^0,\ H^0$
- 1 neutral CP-odd Higgs boson A^0 *"pseudoscalar"'*

$$M_A^2 = m_3^2\,(\cot\beta + \tan\beta)$$

conventional input parameters: $M_A,\ \tan\beta = \frac{v_2}{v_1}$

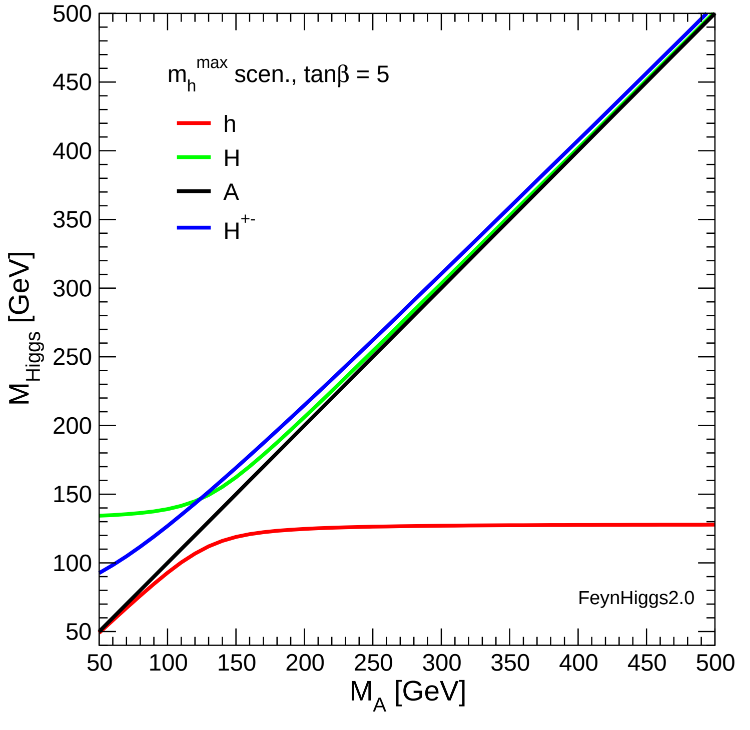
- other masses predicted (tree-level):

$$m_{H^\pm}^2 = M_A^2 + M_W^2$$

$$m_{H,h}^2 = \frac{1}{2}\left(M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2\,\cos^2 2\beta}\right)$$

- substantial higher-order corrections to masses and couplings

Higgs boson spectrum: $h^0,\ H^0,\ A^0,\ H^\pm$



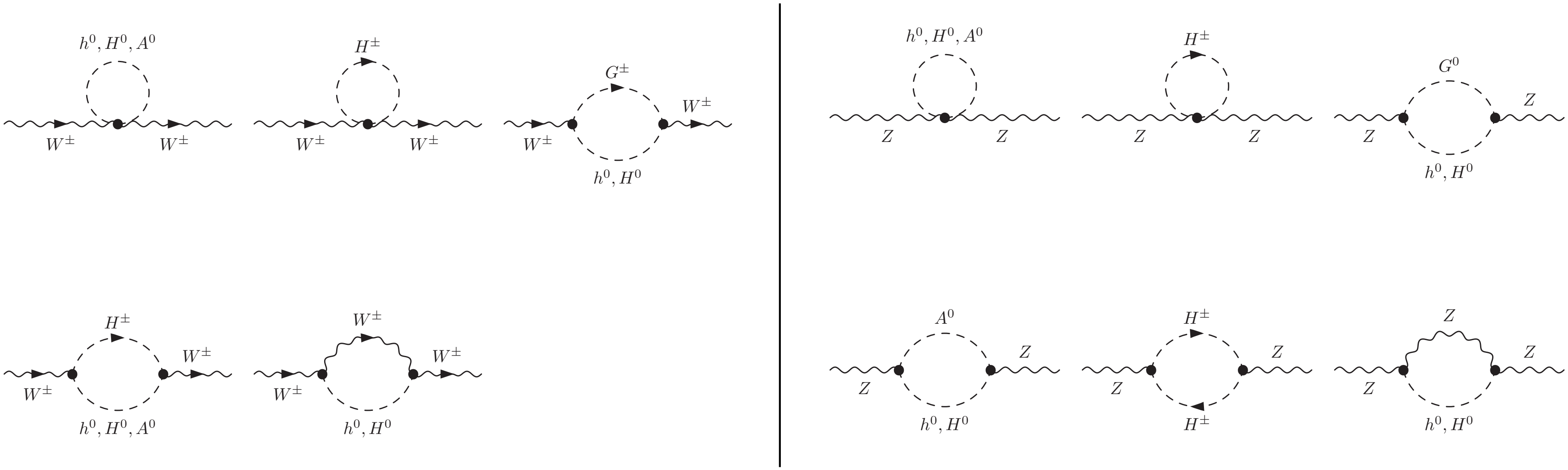
- *light Higgs boson* h^0

  $$m_h \leq m_Z |\cos(2\beta)| + \Delta m_{h^0}$$

- *for heavy* $A^0,\ H^0,\ H^\pm$*:*

  h^0 *like Standard Model Higgs boson*

higher-order contributions for masses and couplings taken into account for precision observables *vector-boson self energies: Higgs contributions*



- Higgs masses from FEYNHIGGS
- Higgs couplings as effective couplings
- \+ loops with charginos, neutralinos, sleptons, squarks in self-energies, vertex corrections, box diagrams

long-standing activities in precision observables in SUSY

Grifols, Solá 1985

Chankowski, Dabelstein, WH, Moesle, Pokorski, Rosiek 1994

Garcia, Solá 1994

Pierce, Bagger, Matchev, Zhang 1997

Djouadi, Gambino, Heinemeyer, WH, Weiglein 1997, 1998

two-loop $\Delta\rho^{\rm susy} \sim \alpha_s \alpha_t$

Hastier, Heinemeyer, Stöckinger, Weiglein 2005

two-loop $\Delta\rho^{\rm susy} \sim \alpha_t^2, \alpha_t\alpha_b, \alpha_b^2$

Heinemeyer, WH, Stöckinger, Weber, Weiglein 2006

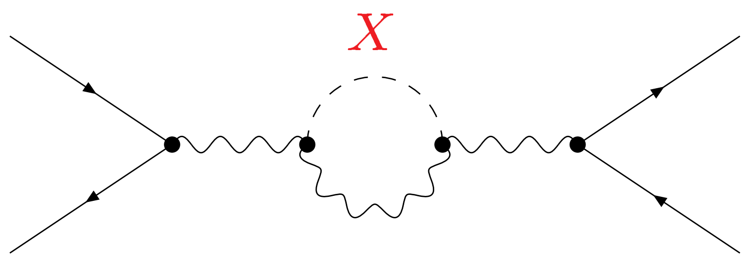
Heinemeyer, WH, Weber, Weiglein 2008

Heinemeyer, WH, Weiglein, Zeune 2013

M_W *and* Δr *in view of LHC results*

precision observables with SUSY quantum loops

muon decay $\mu^- \to e^- \, \nu_\mu \, \bar{\nu}_e$



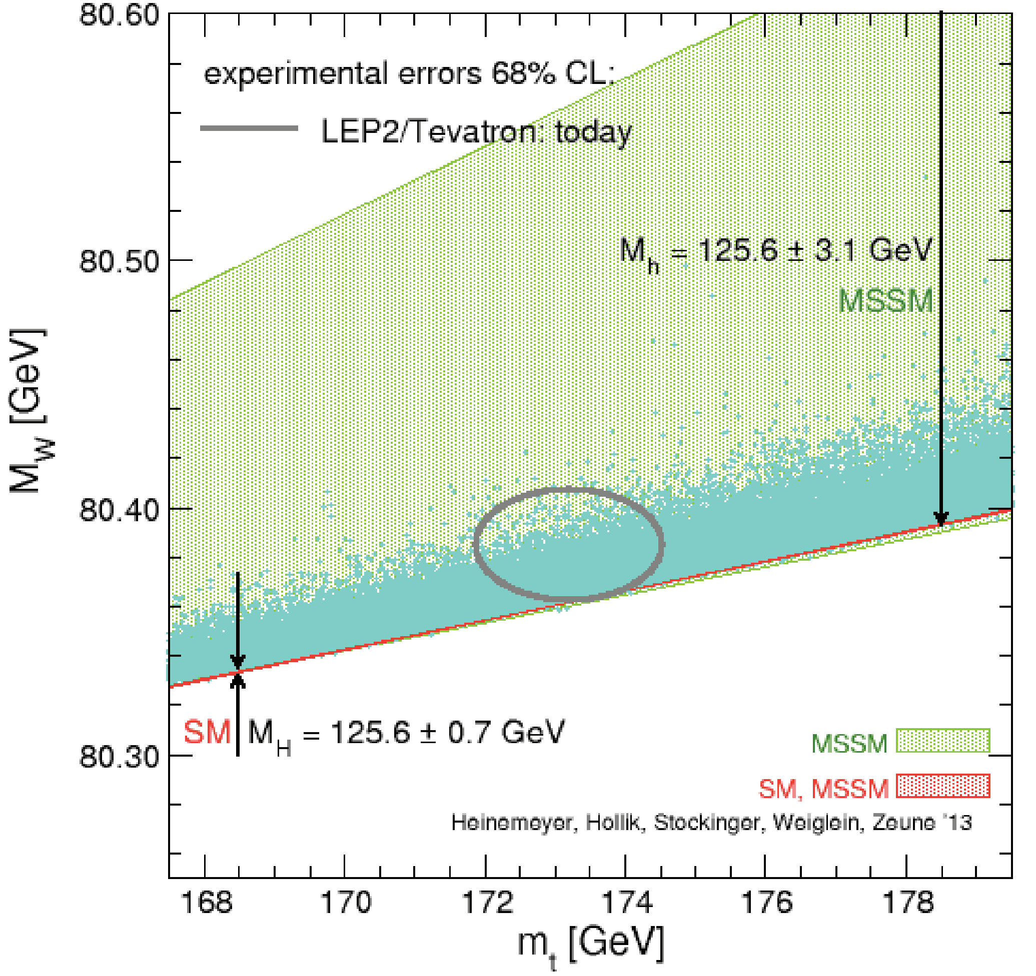
X = Higgs bosons, SUSY particles

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2 \left(1 - M_W^2/M_Z^2\right)} \cdot [1 + \Delta r(m_t, X)]$$

determines W mass $M_W = M_W(\alpha, G_F, M_Z, m_t, X)$

- ⋆ ⇒ MSSM prediction of M_W
- ⋆ ⇒ each amplitude $\sim G_F$ requires Δr at higher order

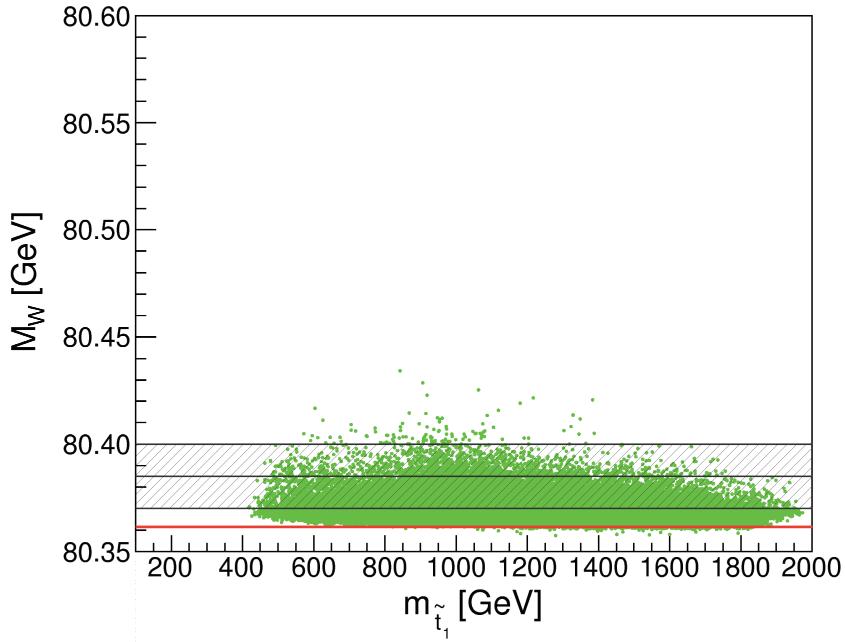
W mass with SUSY quantum loops

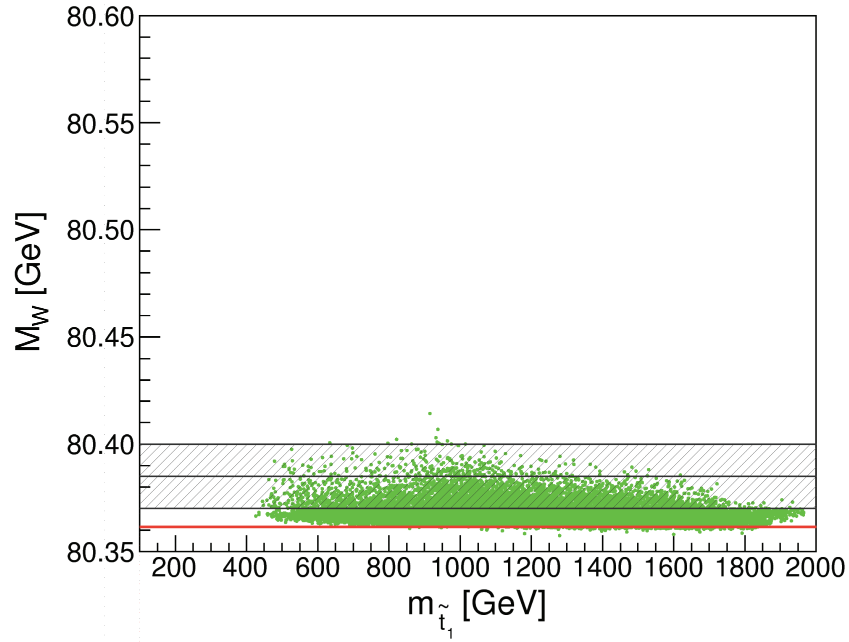


dark: $m_{\tilde{t}},\, m_{\tilde{b}} > 500\,\mathrm{GeV}$

$m_{\tilde{q}},\, m_{\tilde{g}} > 1200\,\mathrm{GeV}$

$$m_{\tilde{t}_1} < m_{\tilde{t}_2} < 2.5\, m_{\tilde{t}_1}$$



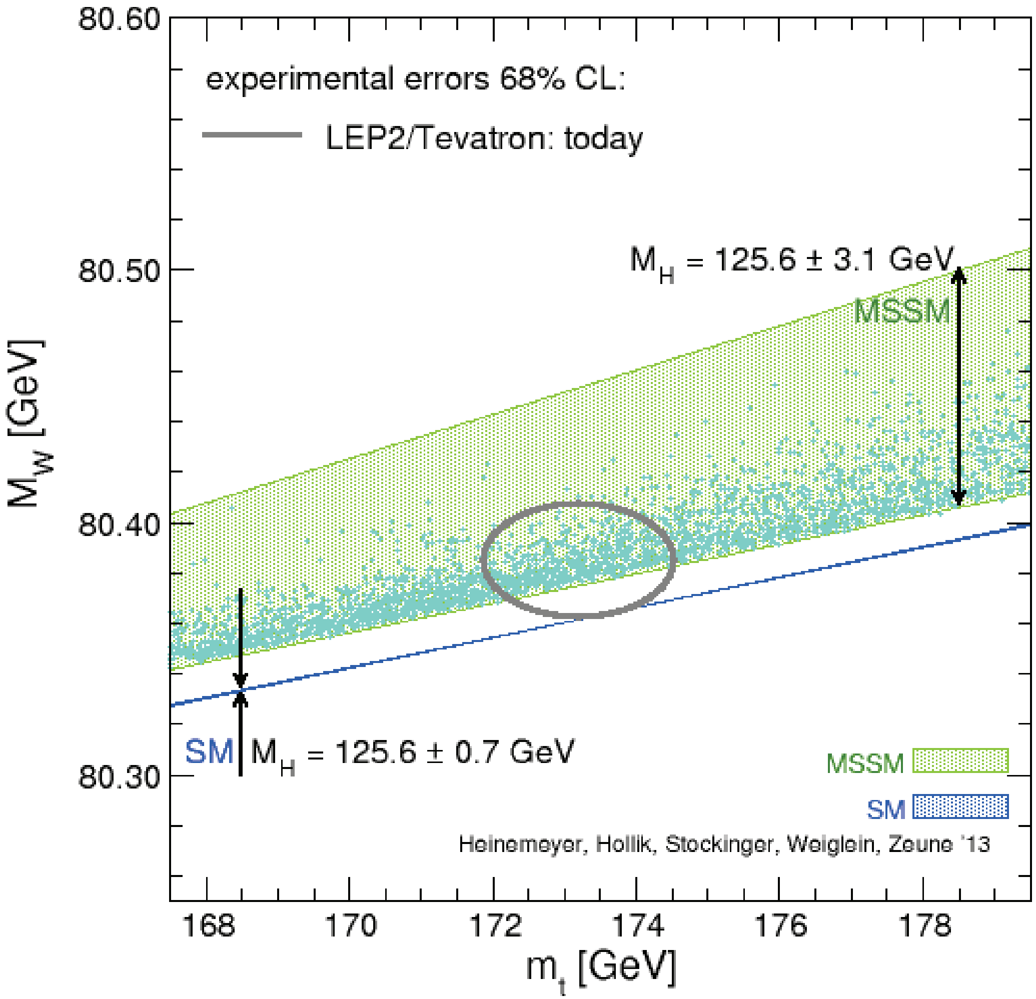


$$m_{\tilde{b}} > 1000\,\mathrm{GeV}$$

$$(m_{\tilde{q}},\, m_{\tilde{g}} > 1200\,\mathrm{GeV})$$

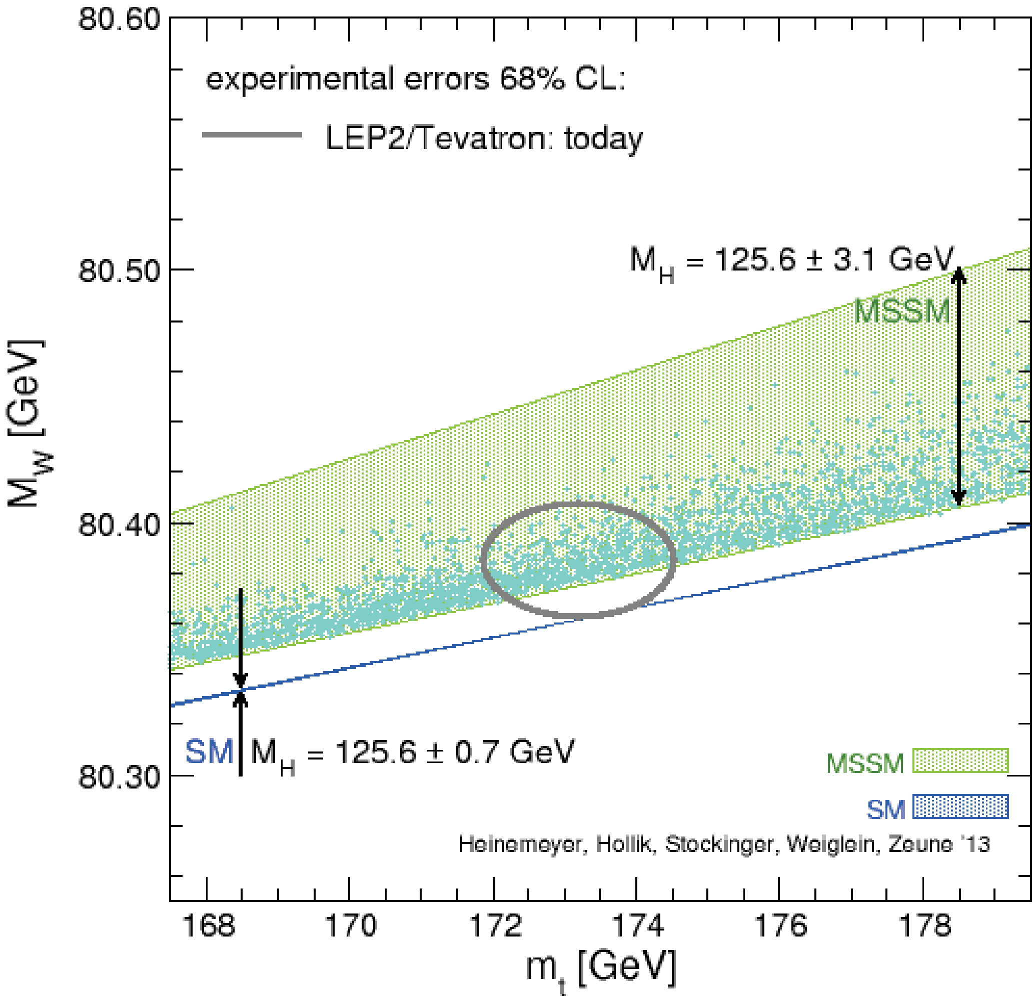
+ *charginos and sleptons above 500 GeV*

other interpretation: heavier H^0 = observed signal



dark: $m_{\tilde{t}},\, m_{\tilde{b}} > 500\,\mathrm{GeV}, \quad m_{\tilde{q}},\, m_{\tilde{g}} > 1200\,\mathrm{GeV}$

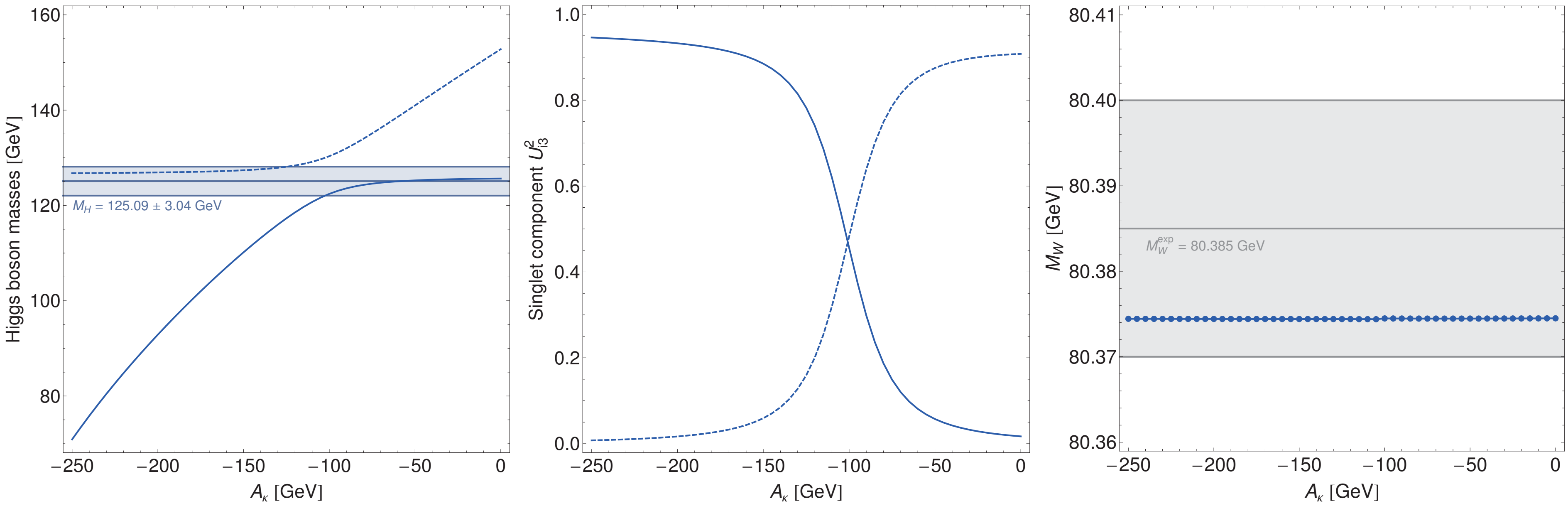
other interpretation: **heavier H^0 = observed signal**



dark: $m_{\tilde{t}}, m_{\tilde{b}} > 500\,\mathrm{GeV}, \quad m_{\tilde{q}}, m_{\tilde{g}} > 1200\,\mathrm{GeV}$

light $H^{\pm}$ needed – ruled out soon?

inverted hierarchy – NMSSM



Stal, Weiglein, Zeune, arxiv:1506.07465

Two-Doublet-Model Higgs Bosons

$$V(\Phi_1,\Phi_2) = \lambda_1(\Phi_1^+\Phi_1 - {v_1}^2)^2 + \lambda_2(\Phi_2^+\Phi_2 - {v_2}^2)^2 + \lambda_3[(\Phi_1^+\Phi_1 - {v_1}^2) + (\Phi_2^+\Phi_2 - {v_2}^2)]^2$$

$$+\lambda_4[(\Phi_1^+\Phi_1)(\Phi_2^+\Phi_2) - (\Phi_1^+\Phi_2)(\Phi_2^+\Phi_1)] + \lambda_5[Re(\Phi_1^+\Phi_2) - v_1 v_2]^2 + \lambda_6[Im(\Phi_1^+\Phi_2)]^2$$

mass eigenstates: $h^0, H^0, A^0, H^\pm$

free parameters: $m_h, m_H, m_A, m_{H^\pm},\quad \tan\beta = \frac{v_2}{v_1},\ \alpha \quad \lambda_5$

$$\lambda_1 = \frac{g^2}{16\cos^2\beta m_W^2}[m_H^2 + m_h^2 + (m_H^2 - m_h^2)\frac{\cos(2\alpha+\beta)}{\cos\beta}] + \lambda_3(-1+\tan^2\beta)$$

$$\lambda_2 = \frac{g^2}{16\sin^2\beta m_W^2}[m_H^2 + m_h^2 + (m_h^2 - m_H^2)\frac{\sin(2\alpha+\beta)}{\sin\beta}] + \lambda_3(-1+\cot^2\beta)$$

$$\lambda_4 = \frac{g^2 m_{H^\pm}^2}{2m_W^2} \quad , \quad \lambda_5 = \frac{g^2}{2m_W^2}\frac{\sin 2\alpha}{\sin 2\beta}(m_H^2 - m_h^2) - 4\lambda_3 \quad , \quad \lambda_6 = \frac{g^2 m_A^2}{2m_W^2}$$

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \mathrm{Re}\,\phi_1^0 \\ \mathrm{Re}\,\phi_2^0 \end{pmatrix}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \mathrm{Im}\,\phi_1^0 \\ \mathrm{Im}\,\phi_2^0 \end{pmatrix}$$

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$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

vector-boson masses: $M_{W,Z} \sim v, \quad v = \sqrt{v_1^2 + v_2^2}$

coupling to vector bosons $V = W, Z$:

$$[h, H]VV = [\sin(\alpha - \beta), \cos(\alpha - \beta)] \cdot [\text{SM}]$$

coupling to fermions, type I: $[h, H]ff = [\frac{\cos\alpha}{\sin\beta}, \frac{\sin\alpha}{\sin\beta}]\,[\text{SM}]$

coupling to fermions, type II $(u \to \Phi_2,\ d \to \Phi_1)$:

$$[h, H]bb = [\frac{\sin\alpha}{\cos\beta}, \frac{\cos\alpha}{\cos\beta}][\text{SM}], \quad [h, H]tt = [\frac{\cos\alpha}{\sin\beta}, \frac{\sin\alpha}{\sin\beta}][\text{SM}]$$

$$A\,bb = \tan\beta\,[\text{SM}] \qquad\qquad A\,tt = \cot\beta\,[\text{SM}]$$

$\alpha = \beta - \frac{\pi}{2}$ (or $\alpha = \beta$) : $h^0(H^0)$ is SM-like

$H^0(h^0), A^0$ with non-standard couplings

$m_H \sim m_A \sim m_{H^\pm} \gg m_Z,\ \alpha \to \beta - \frac{\pi}{2}$: 'decoupling regime'

bounds from unitarity and vacuum stability

Kanemura, Kubota, Takasugi 1993

Akeroyd, Arhrib, Naimi 2000

Horeijsi, Kladiva 2006

vacuum stability conditions

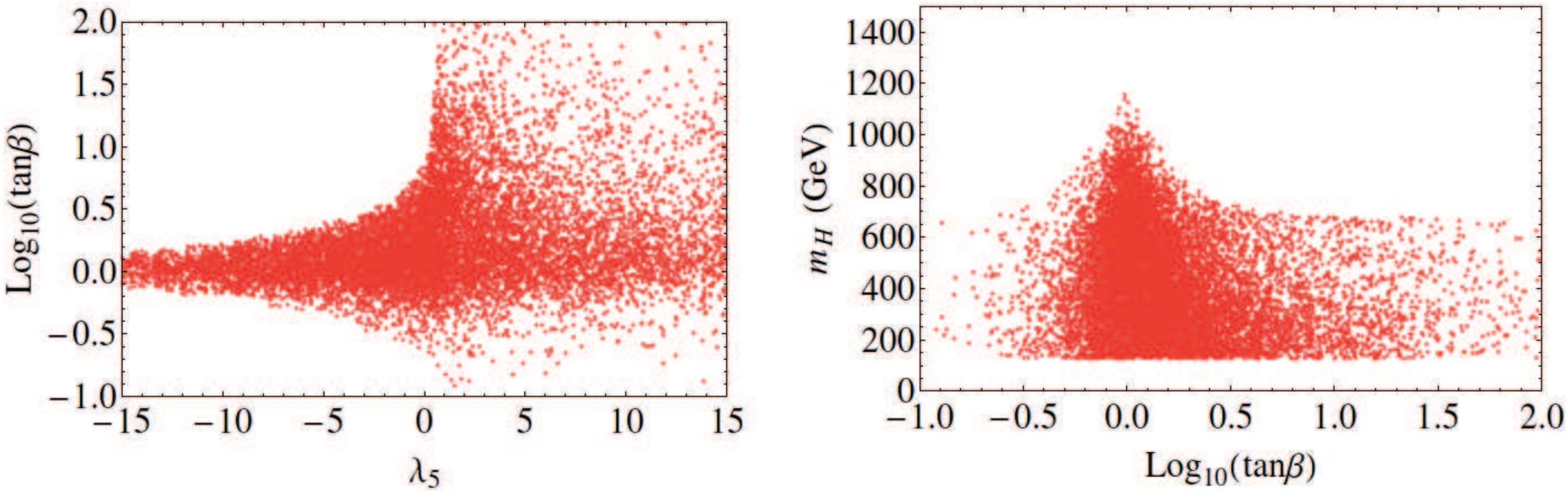
$$\lambda_1 + \lambda_3 > 0\,,$$
$$\lambda_2 + \lambda_3 > 0\,,$$
$$2\lambda_3 + \lambda_4 + 2\sqrt{(\lambda_1 + \lambda_3)(\lambda_2 + \lambda_3)} > 0\,,$$
$$2\lambda_3 + \frac{\lambda_5 + \lambda_6}{2} - \frac{|\lambda_5 - \lambda_6|}{2} + 2\sqrt{(\lambda_1 + \lambda_3)(\lambda_2 + \lambda_3)} > 0$$

unitarity conditions

$$
\begin{aligned}
a_1^{\pm} &= 3(\lambda_1+\lambda_2+2\lambda_3) \pm \sqrt{9(\lambda_1-\lambda_2)^2+\left(4\lambda_3+\lambda_4+\frac{\lambda_5+\lambda_6}{2}\right)^2}\,, \\
a_2^{\pm} &= (\lambda_1+\lambda_2+2\lambda_3) \pm \sqrt{(\lambda_1-\lambda_2)^2+\frac{1}{4}\left(2\lambda_4-\lambda_5-\lambda_6\right)^2}\,, \\
a_3^{\pm} &= (\lambda_1+\lambda_2+2\lambda_3) \pm \sqrt{(\lambda_1-\lambda_2)^2+\frac{1}{4}\left(\lambda_5-\lambda_6\right)^2}\,, \\
b_1 &= 2\lambda_3-\lambda_4-\frac{1}{2}\lambda_5+\frac{5}{2}\lambda_6\,, \\
b_2 &= 2\lambda_3+\lambda_4-\frac{1}{2}\lambda_5+\frac{1}{2}\lambda_6\,,
\end{aligned}
$$

$$
\begin{aligned}
b_3 &= 2\lambda_3-\lambda_4+\frac{5}{2}\lambda_5-\frac{1}{2}\lambda_6\,, \\
b_4 &= 2\lambda_3+\lambda_4+\frac{1}{2}\lambda_5-\frac{1}{2}\lambda_6\,, \\
b_5 &= 2\lambda_3+\frac{1}{2}\lambda_5+\frac{1}{2}\lambda_6\,, \\
b_6 &= 2(\lambda_3+\lambda_4)-\frac{1}{2}\lambda_5-\frac{1}{2}\lambda_6\,.
\end{aligned}
$$

$l = 0$ *partial wave amplitudes* $|a_i^{\pm}|,\ |b_i| < 16\pi$



D. Das, arXiv:150102610

precision observables and two-Higgs doublet models – history

Bertolini 1986

WH 1986, 1988

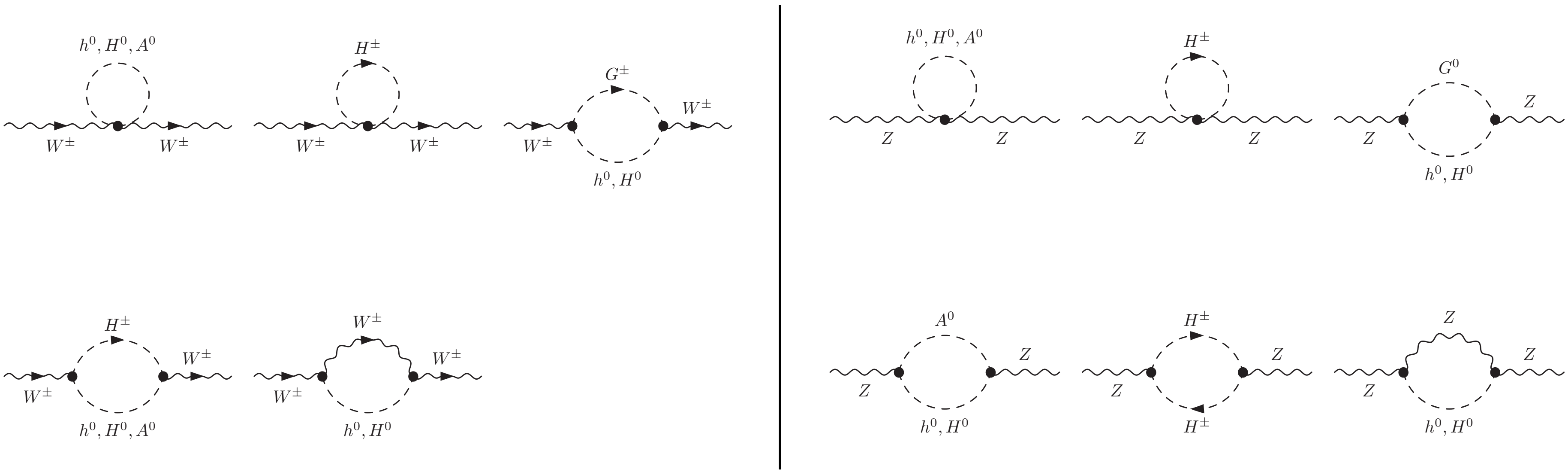
Denner, Guth, WH, Kühn 1991

Chankowski, Krawczyk, Zochowski 1999

Solá, Lopez Val 2013

Stefan Hessenberger, Diploma Thesis, MPI and TUM,
ongonig PhD thesis

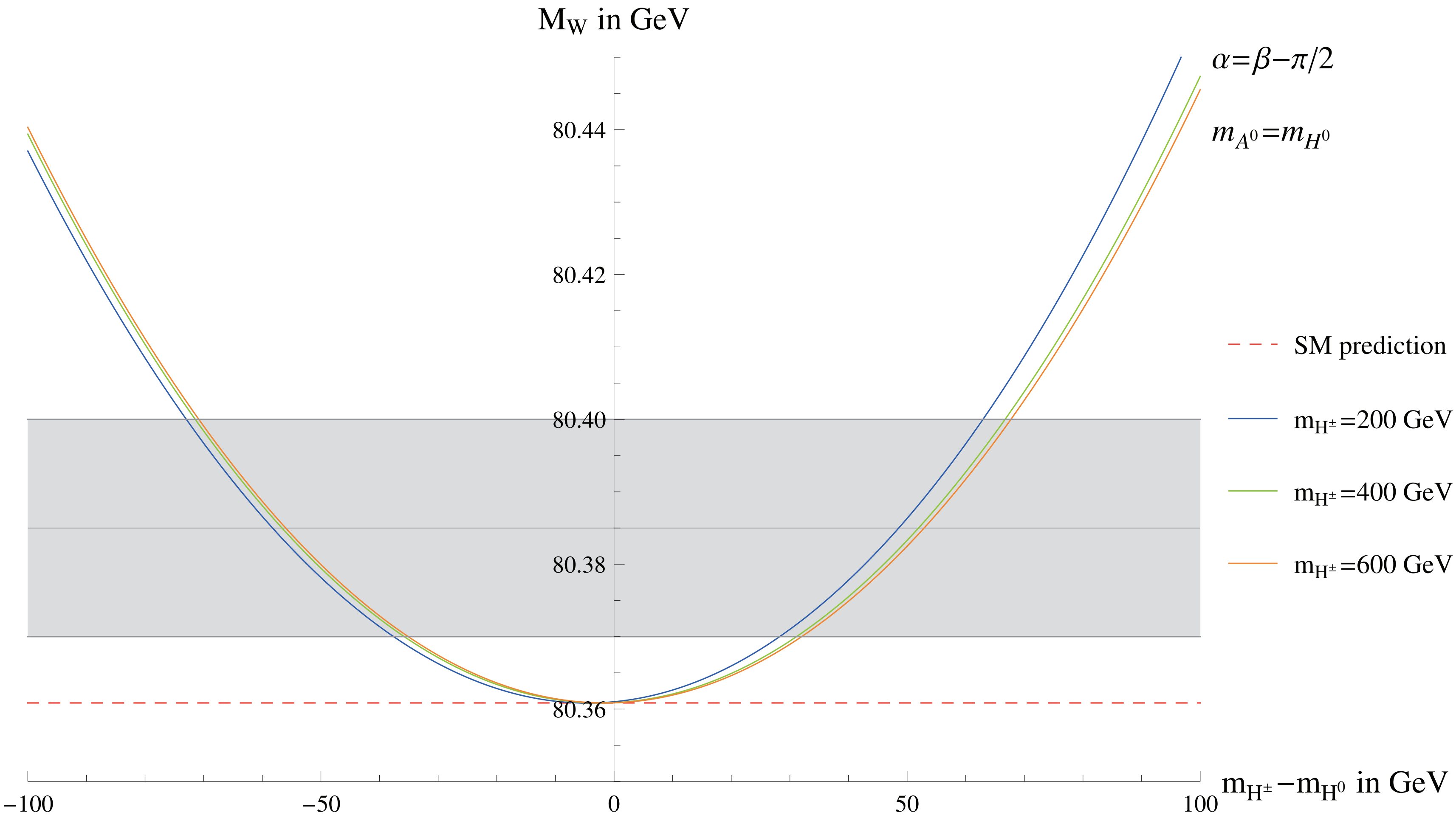
Higgs contributions to vector-boson self energies



alignment limit: $\alpha = \beta - \frac{\pi}{2}$

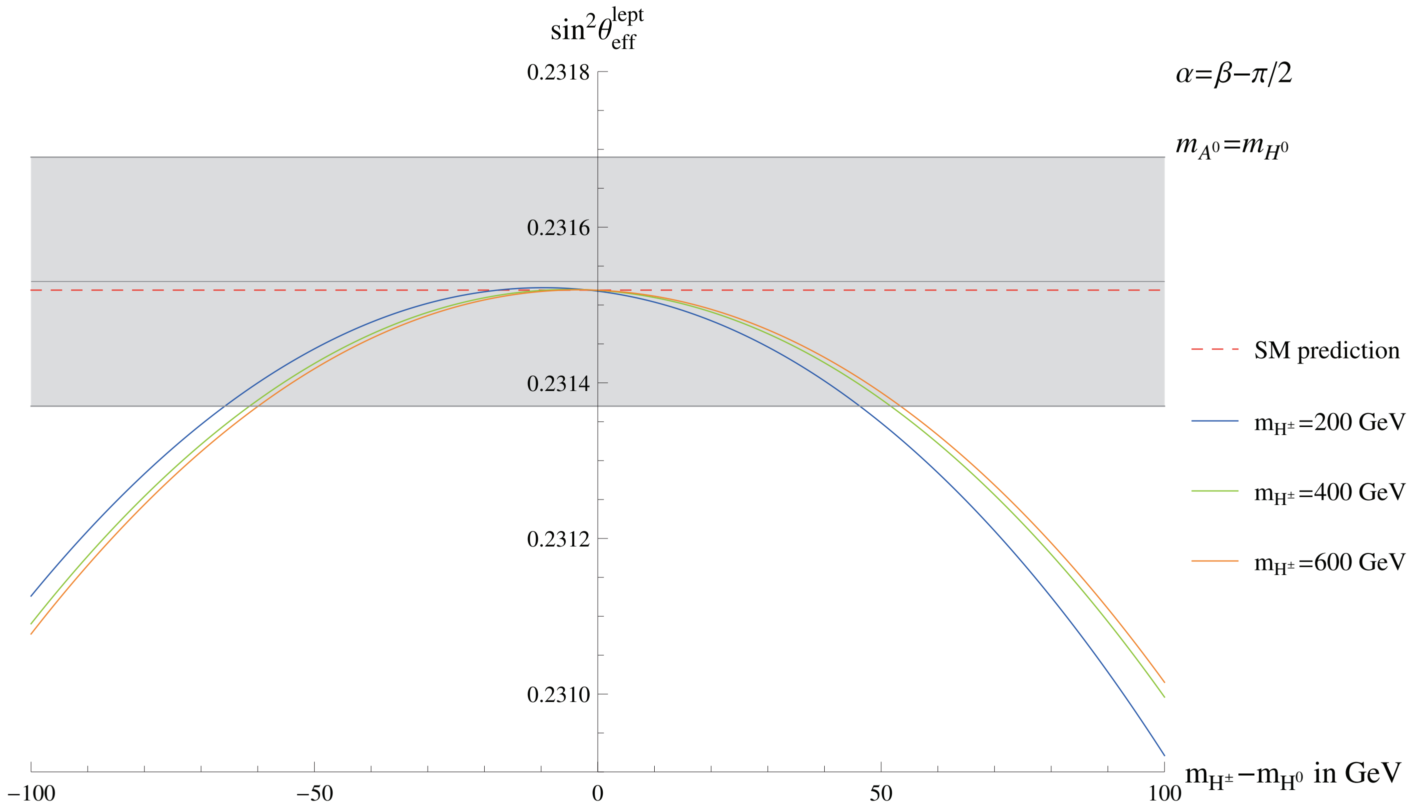
$$h^0 = H_{SM}, \quad m_{h^0} = 125\,\mathrm{GeV}$$

$H^0, A^0, H^\pm$ separated from SM part

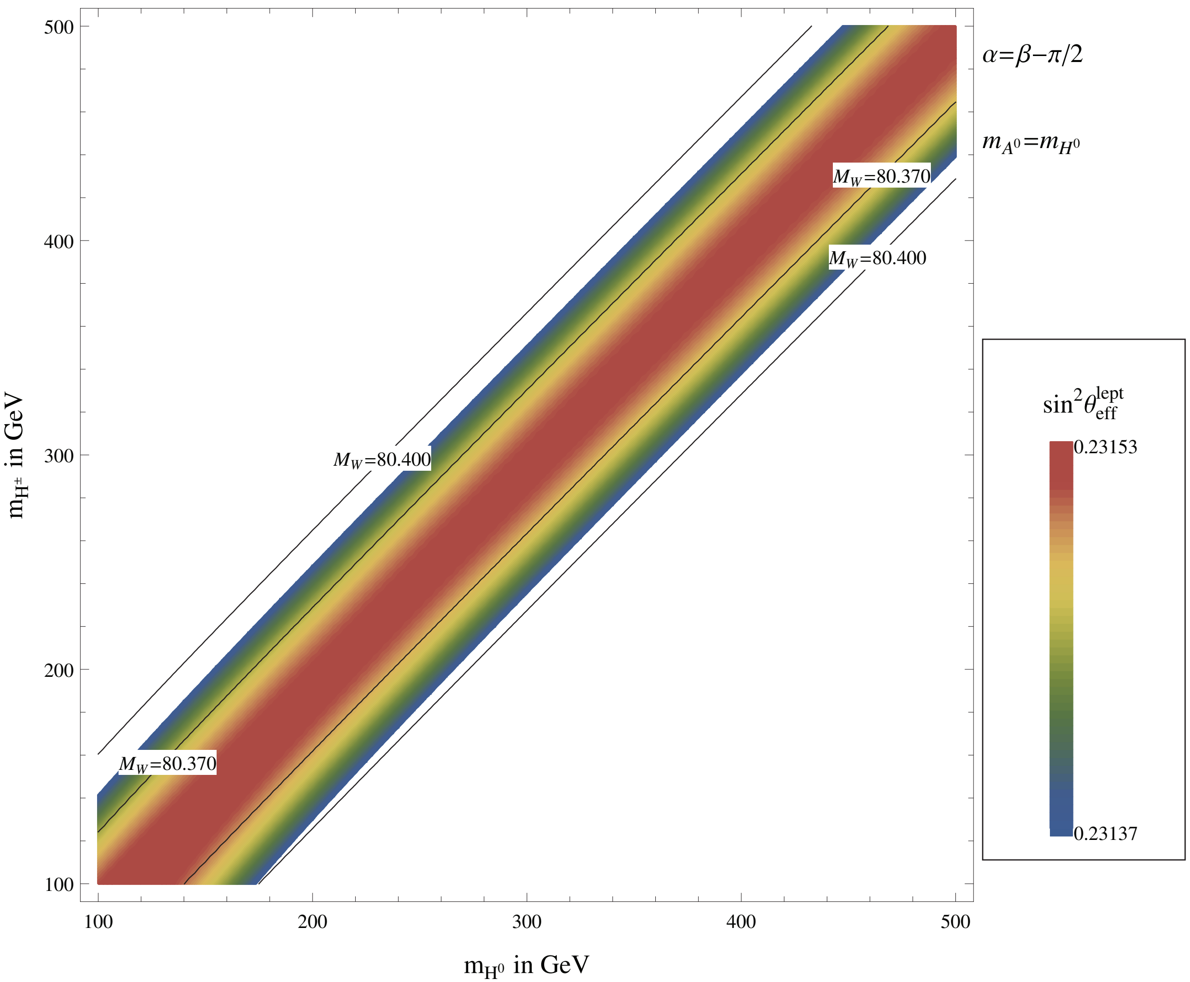


dominating term $\sim \Delta\rho$

$$\Delta r = \Delta\alpha - \frac{c_{\mathrm{w}}^2}{s_{\mathrm{w}}^2}\Delta\rho + \cdots$$



$\sin^2\theta_{\rm eff}^{\rm lept}$
0.2318
0.2316
0.2314
0.2312
0.2310
$\alpha=\beta-\pi/2$
$m_{A^0}=m_{H^0}$
SM prediction
$m_{H^\pm}$=200 GeV
$m_{H^\pm}$=400 GeV
$m_{H^\pm}$=600 GeV
−100
−50
0
50
100
$m_{H^\pm}-m_{H^0}$ in GeV



$\alpha=\beta-\pi/2$
$m_{A^0}=m_{H^0}$
M_W=80.370
M_W=80.400
M_W=80.400
M_W=80.370
$\sin^2\theta_{\rm eff}^{\rm lept}$
0.23153
0.23137
$m_{H^\pm}$ in GeV
m_{H^0} in GeV
100
200
300
400
500

two-loop calculations for two-doublet models

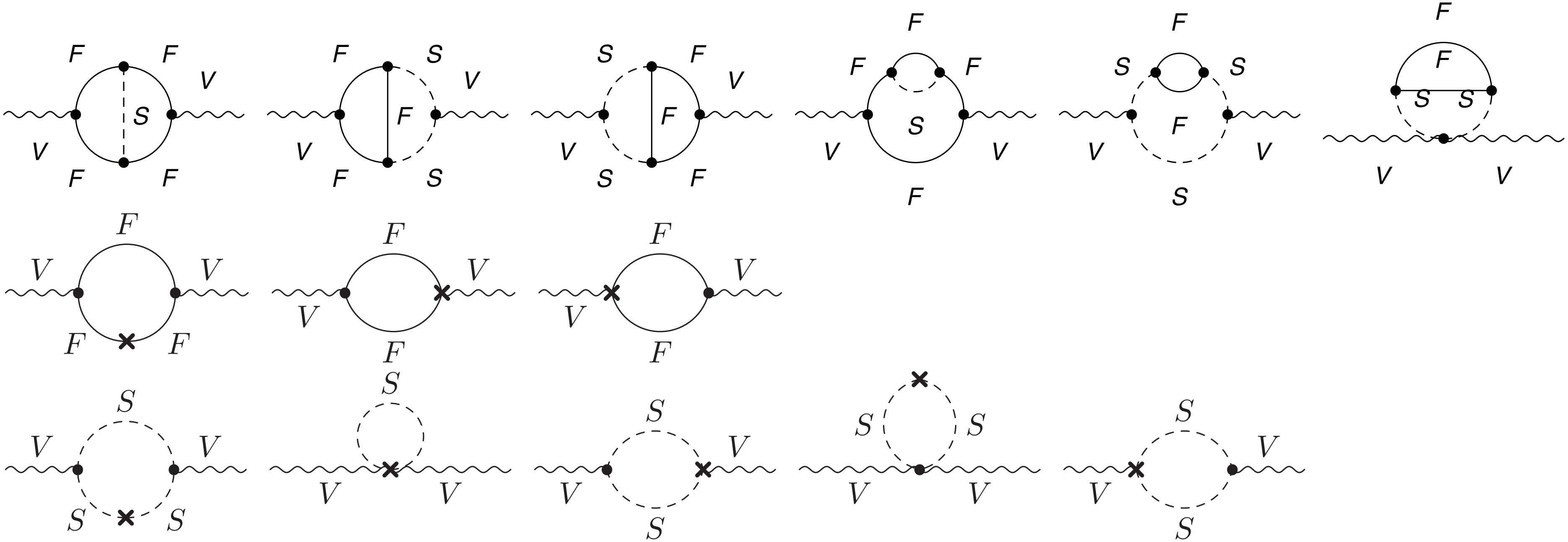
- dominating one-loop contributions from the ρ-parameter via $\Delta\rho$
  - ⋆ top-Yukawa contributions $\sim \alpha_t \sim \frac{m_t^2}{v^2}$
  - ⋆ Higgs-self-coupling contributions $\sim \frac{M_\phi^2}{v^2}\,, \quad \phi = H^0, A^0, H^\pm$
- most significant two-loop terms from the top-Yukawa and from the Higgs sector
- obtained in the "gaugeless limit"

$$g_1,\, g_2 \;\to\; 0$$
$$M_W \to 0, \quad M_Z \to 0, \quad \text{but} \quad c_\text{w} = \frac{M_W}{M_Z} = \text{const}$$

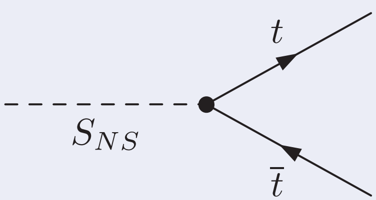
- new parameters $\tan\beta$ and λ_5 enter at the two-loop level
- NOTE: different from SUSY $m_{h^0} \neq 0$

  no gauge term, but independent Higgs-potential parameter

Top Yukawa contribution



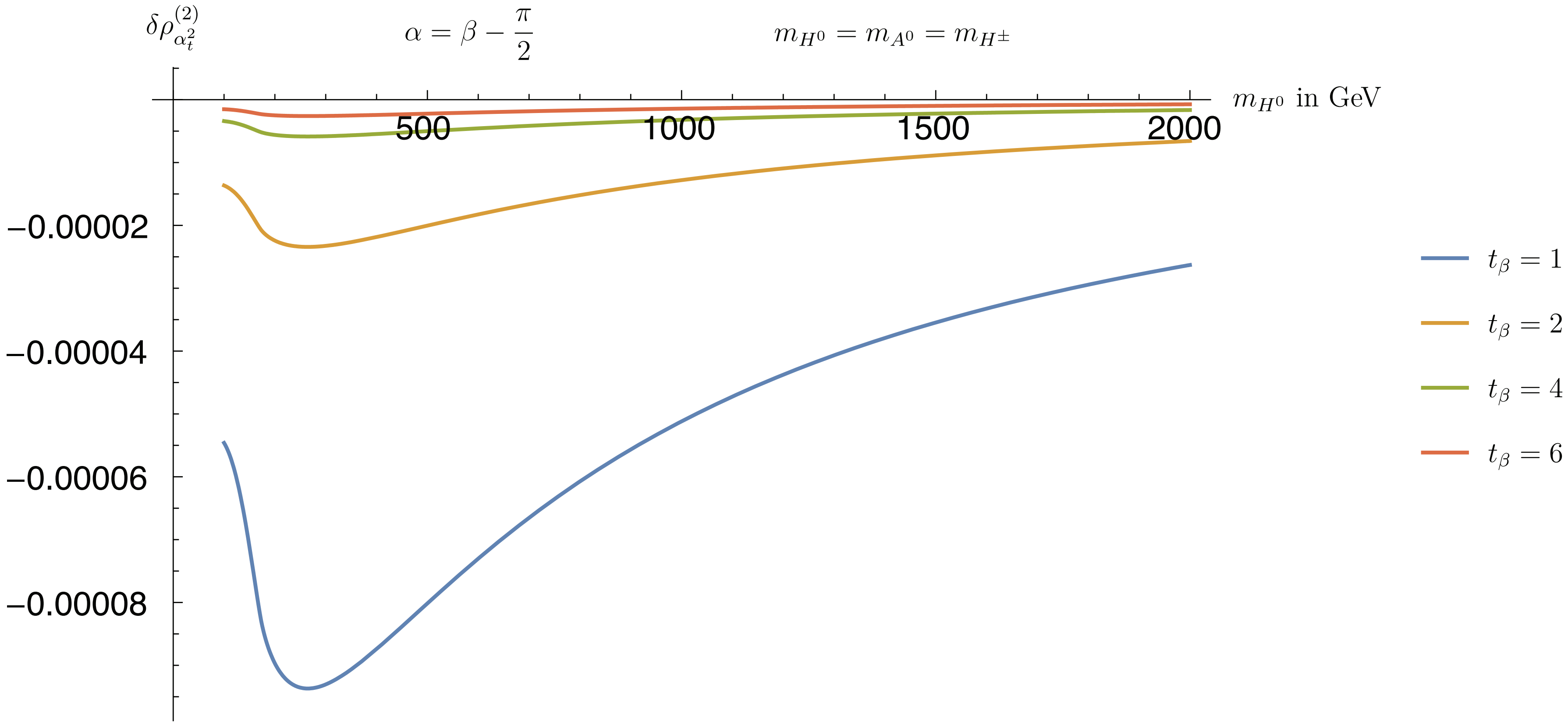
Top Yukawa coupling



$$\propto \frac{m_t}{v t_\beta}$$

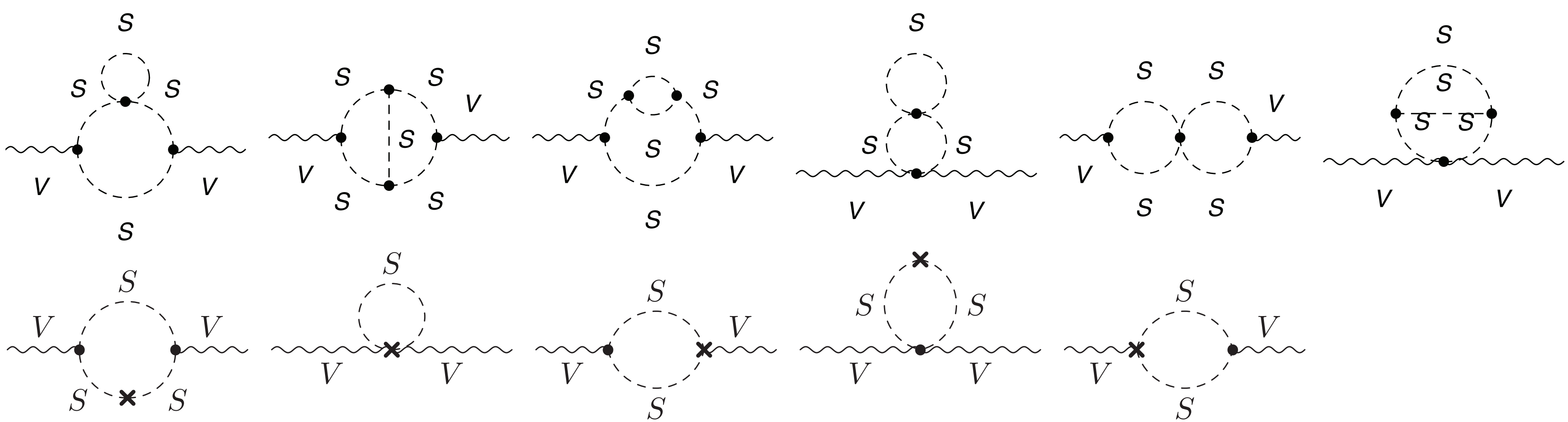
- neglect bottom quark mass: $m_b = 0$
- coupling proportional to large top mass
- coupling suppressed for large t_β
- $S_{NS} = H^0, A^0, H^\pm$

$\Rightarrow$ over 60 diagrams to calculate

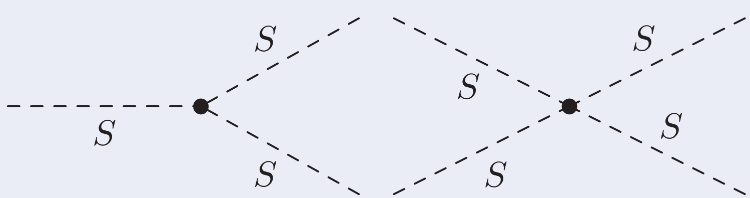


$$\delta(\Delta\rho) = 10^{-4} \quad \Rightarrow \quad \delta M_W \simeq 6\,\mathrm{MeV}$$

Contribution from scalar self interaction

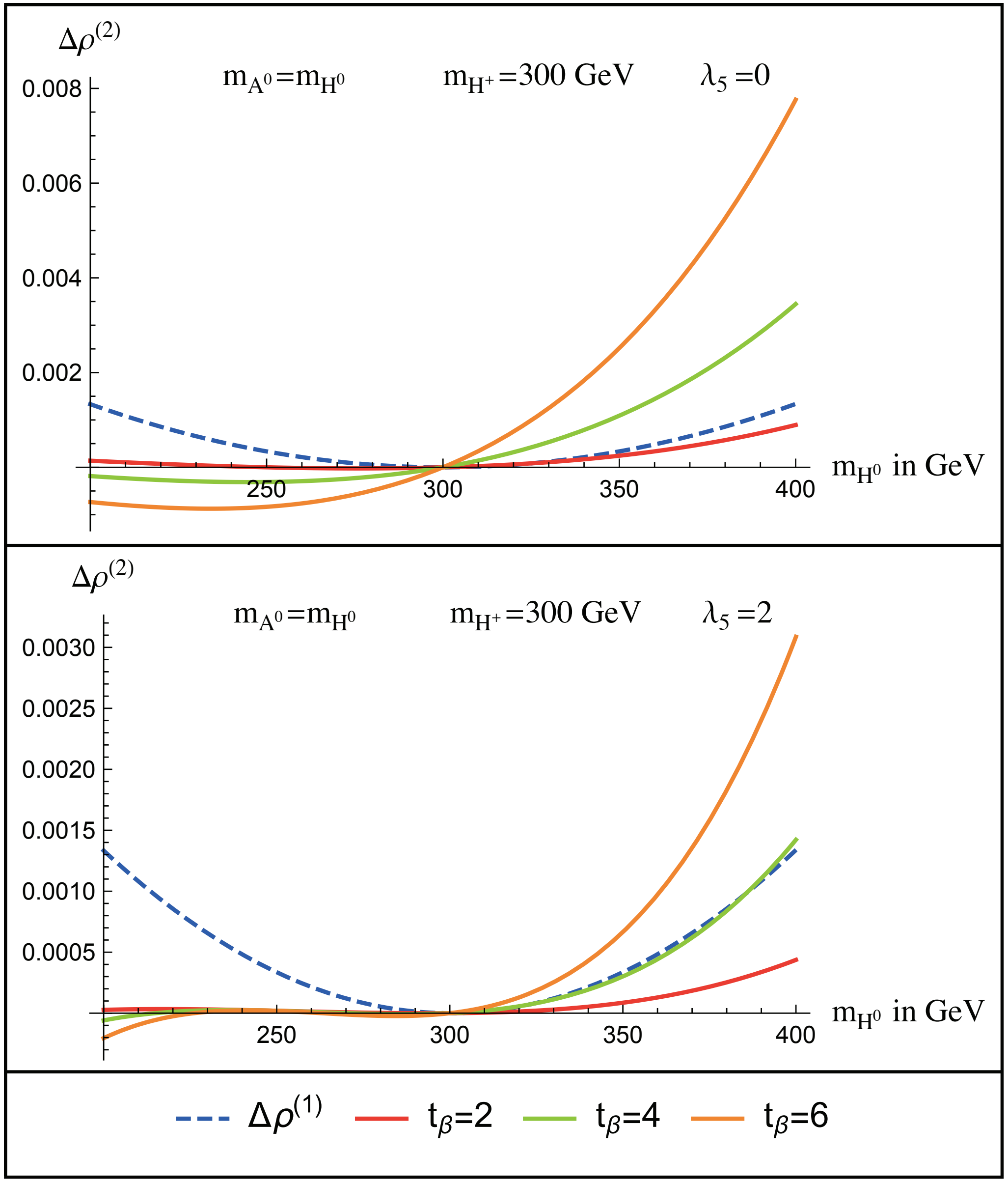


scalar self couplings



- contribution from the scalar self couplings ⇒ contain t_β and λ_5
- $S = h^0, G^0, G^\pm, H^0, A^0, H^\pm$

⇒ over 200 diagrams to calculate



$\Delta\rho^{(2)}$
$m_{A^0}=m_{H^0}$
$m_{H^+}=300$ GeV
$\lambda_5=0$
0.008
0.006
0.004
0.002
250
300
350
400
m_{H^0} in GeV
$\Delta\rho^{(2)}$
$m_{A^0}=m_{H^0}$
$m_{H^+}=300$ GeV
$\lambda_5=2$
0.0030
0.0025
0.0020
0.0015
0.0010
0.0005
250
300
350
400
m_{H^0} in GeV
$\Delta\rho^{(1)}$
$t_\beta=2$
$t_\beta=4$
$t_\beta=6$

- **counterterms for 1-loop subrenormalization**

Vff / VSS couplings $\qquad \delta s_{\rm w}^2 \;=\; c_{\rm w}^2\,\Delta\rho^{(1)}$

top mass renormalization $\qquad \delta m_t \;=\; \Sigma_t^{(1)}\big(\not{p}=m_t\big)$

scalar mass renormalization $\qquad \delta m_S^2 \;=\; \Sigma_S^{(1)}\big(m_S^2\big), \quad S=h,H,A,H^\pm$

$$\delta m_G^2 \;=\; \frac{1}{v}\,\delta t_h^{(1)}\,, \quad G=G^0,G^\pm$$

all with standard (t, h, G) and non-standard (H,A,$H^\pm$) contributions

- **counterterms for 1-loop subrenormalization**

Vff / VSS couplings $$\delta s_{\rm w}^2 \;=\; c_{\rm w}^2\, \Delta\rho^{(1)}$$

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$$\delta m_G^2 \;=\; \frac{1}{v}\,\delta t_h^{(1)}\,, \qquad G = G^0, G^{\pm}$$

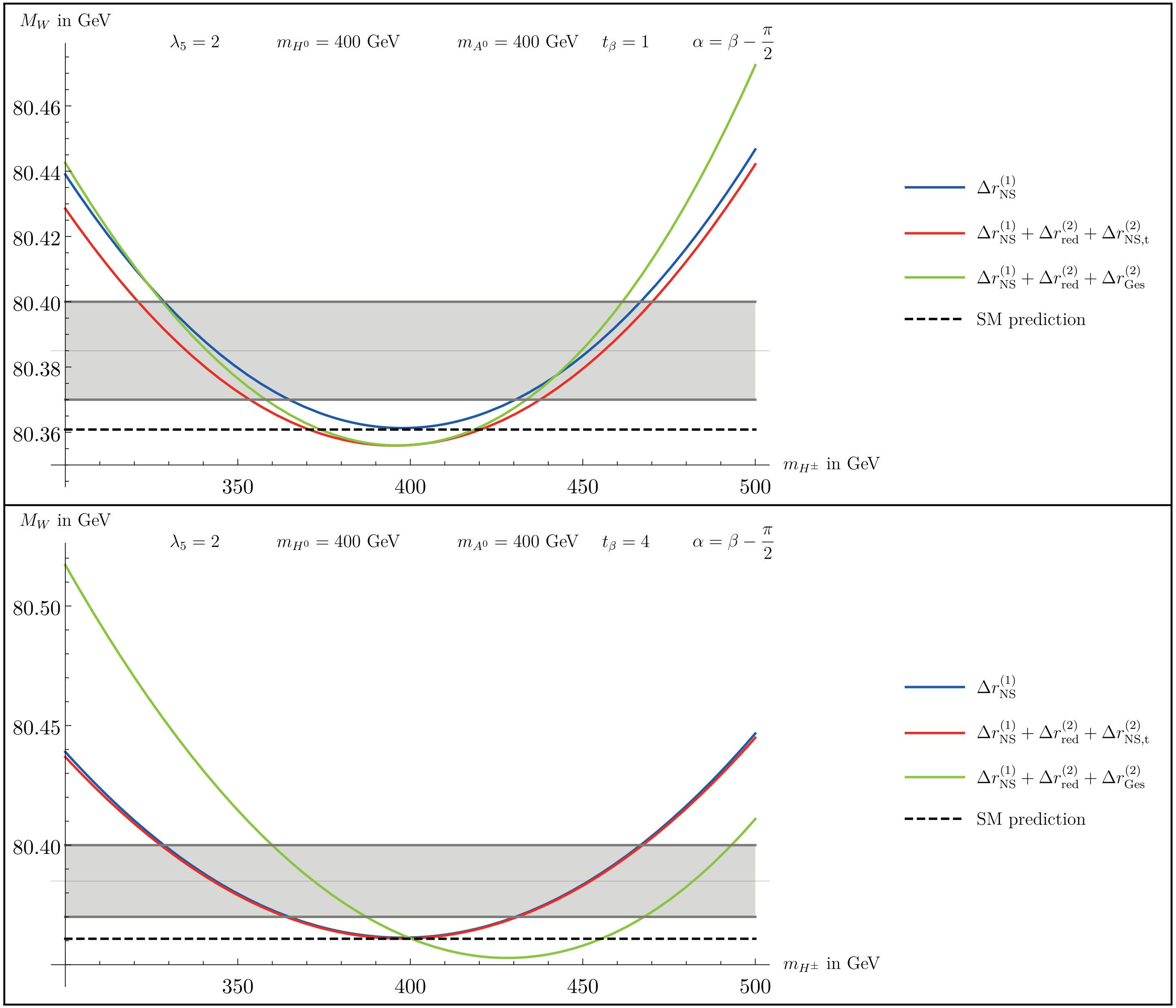
all with standard (t, h, G) and non-standard (H,A,$H^{\pm}$) contributions

- **Ward identity**

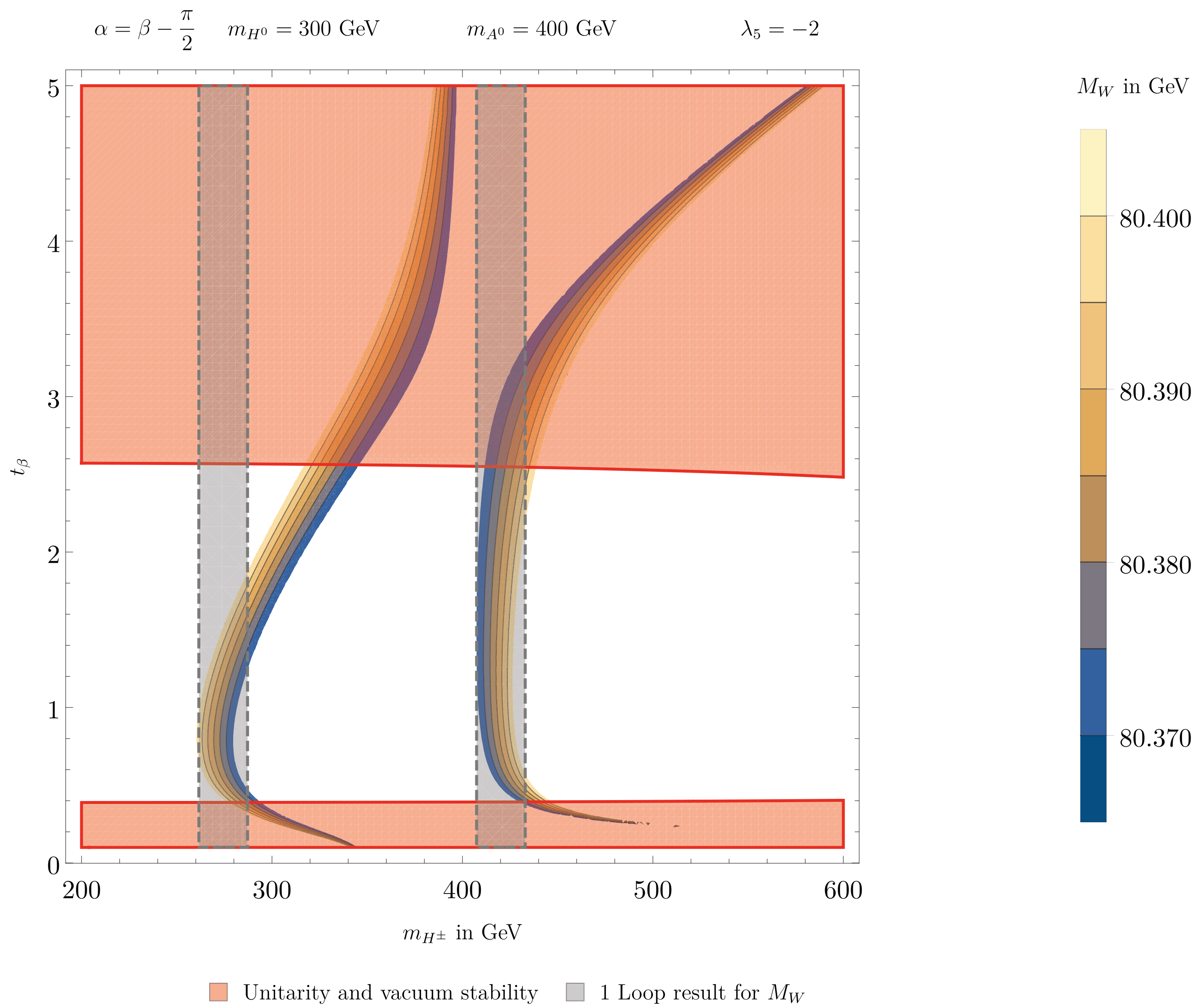
$$\Delta\rho \;=\; \Sigma'_{G^{\pm}}(0) - \Sigma'_{G^0}(0)$$

Barbieri et al. ’93 / Fleischer, Tarasov, Jegerlehner ’95

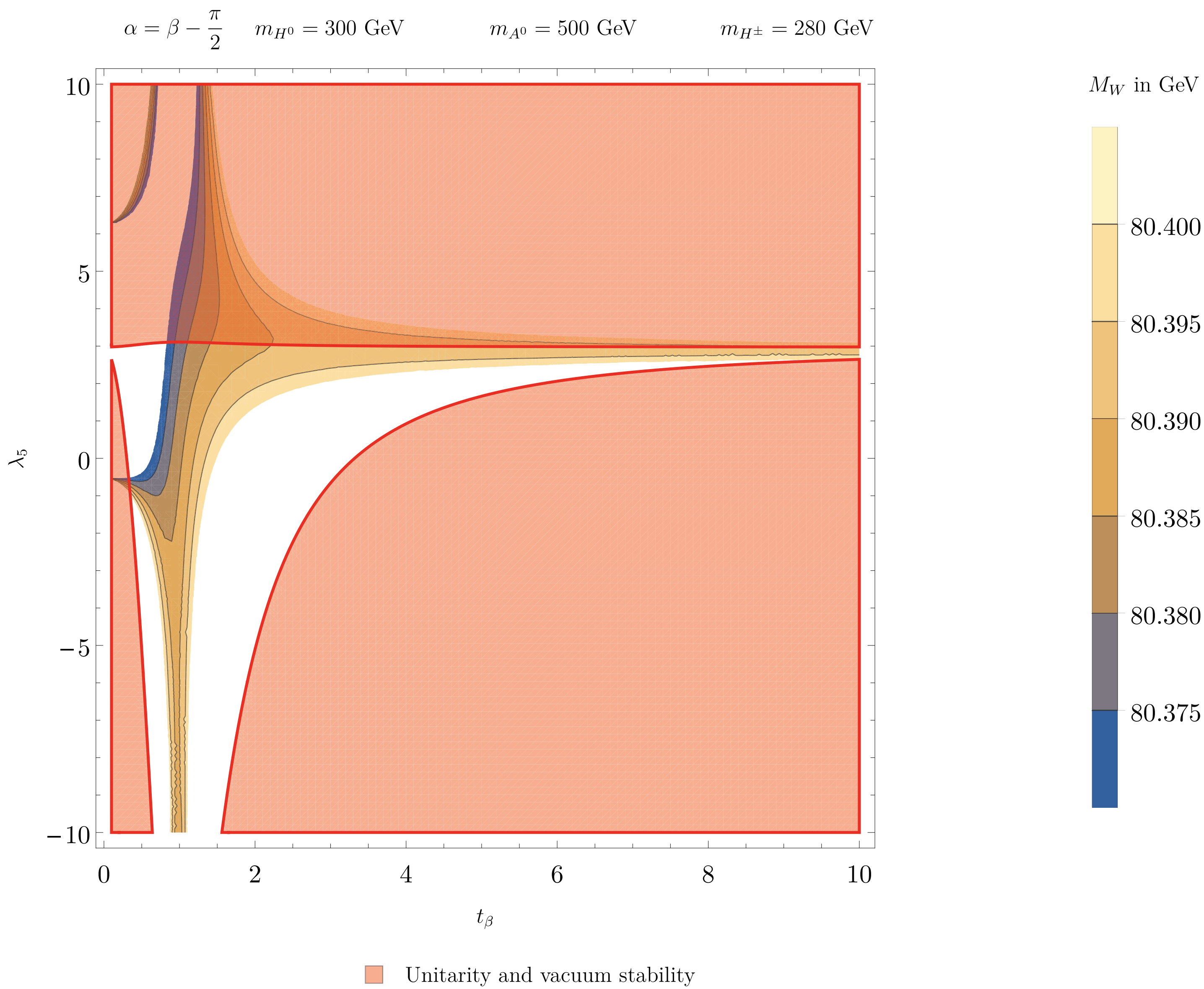
used for checking the results



M_W in GeV
$\lambda_5 = 2$
$m_{H^0} = 400$ GeV
$m_{A^0} = 400$ GeV
$t_\beta = 1$
$\alpha = \beta - \frac{\pi}{2}$
80.46
80.44
80.42
80.40
80.38
80.36
350
400
450
500
$m_{H^\pm}$ in GeV
$\Delta r^{(1)}_{\text{NS}}$
$\Delta r^{(1)}_{\text{NS}} + \Delta r^{(2)}_{\text{red}} + \Delta r^{(2)}_{\text{NS,t}}$
$\Delta r^{(1)}_{\text{NS}} + \Delta r^{(2)}_{\text{red}} + \Delta r^{(2)}_{\text{Ges}}$
SM prediction
M_W in GeV
$\lambda_5 = 2$
$m_{H^0} = 400$ GeV
$m_{A^0} = 400$ GeV
$t_\beta = 4$
$\alpha = \beta - \frac{\pi}{2}$
80.50
80.45
80.40
350
400
450
500
$m_{H^\pm}$ in GeV
$\Delta r^{(1)}_{\text{NS}}$
$\Delta r^{(1)}_{\text{NS}} + \Delta r^{(2)}_{\text{red}} + \Delta r^{(2)}_{\text{NS,t}}$
$\Delta r^{(1)}_{\text{NS}} + \Delta r^{(2)}_{\text{red}} + \Delta r^{(2)}_{\text{Ges}}$
SM prediction



$\alpha = \beta - \frac{\pi}{2}$
$m_{H^0} = 300$ GeV
$m_{A^0} = 400$ GeV
$\lambda_5 = -2$
t_β
0
1
2
3
4
5
200
300
400
500
600
$m_{H^\pm}$ in GeV
M_W in GeV
80.400
80.390
80.380
80.370
Unitarity and vacuum stability
1 Loop result for M_W



$\alpha = \beta - \frac{\pi}{2}$ $m_{H^0} = 300$ GeV $m_{A^0} = 500$ GeV $m_{H^\pm} = 280$ GeV
M_W in GeV
80.400
80.395
80.390
80.385
80.380
80.375
10
5
0
−5
−10
λ_5
0
2
4
6
8
10
t_β
Unitarity and vacuum stability

Conclusions

- interplay of precison measurements and precision calculations has been a convincing concept for the Standard Model
- could be repeated also for BSM,
  decisive input: $M_H = 125\,\mathrm{GeV}$
- yields important information on supersymmetric models, complementary to direct searches
- for general two-doublet models:
  – important constraints besides unitarity and vacuum stability
  – large one-loop effects
  – two-loop calculations required
  – specific parameters ($\tan\beta$, λ_5) enter at two-loop level


RESEARCH INSTITUTE
UNANSWERED QUESTIONS
UNQUESTIONED ANSWERS
S. Harris