Threshold corrections and vacuum stability

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Loops and Legs in Quantum Field Theory
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In collaboration w/ A. Bednyakov, F. Bezrukov, M. Yu. Kalmykov, A. Pikelner, M. Shaposhnikov, and O. Veretin.

Outline

Introduction

The Nobel Prize in Physics 2013 was awarded jointly to François Englert and Peter W. Higgs "for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider."

Higgs potential

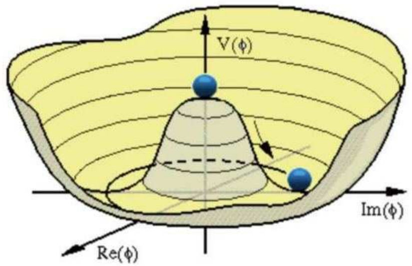
SM Higgs sector: complex scalar doublet Φ

$$\mathscr{L} = \partial_\mu \Phi^\dagger \partial^\mu \Phi - V(|\Phi|^2), \qquad V = \mu^2 |\Phi|^2 + \lambda |\Phi|^4$$

Unbroken phase: $\mu^2 > 0$

Broken phase: $\mu^2 < 0$

After SSB and Higgs mechanism:



$$\Phi = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}$$

$$\mathscr{L} = \frac{1}{2}\partial_\mu H \partial^\mu H - V(H), \qquad V = -\frac{\lambda v^4}{4} + \lambda v^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4$$

- $\left.\frac{\partial V}{\partial H}\right|_{H=v} = 0 \quad \rightsquigarrow \quad -\mu^2 = \lambda v^2 \equiv \frac{m_H^2}{2}$
- $\frac{gv}{2} \equiv m_W \quad \rightsquigarrow \quad v = 2^{-1/4} G_F^{-1/2} = 246.220\ \mathrm{GeV}$
- m_H is free parameter.

So far, bare fields and parameters.

Properties of the Higgs boson

$$\mathcal{L}_H = \left(m_W^2 W_\mu^+ W^{-\mu} + \frac{m_Z^2}{2} Z_\mu Z^\mu \right) \left(1 + \frac{H}{v} \right)^2 - \sum_f m_f \bar{f} f \left(1 + \frac{H}{v} \right) - \frac{m_H^2}{2} H^2 \left(1 + \frac{H}{2v} \right)^2$$

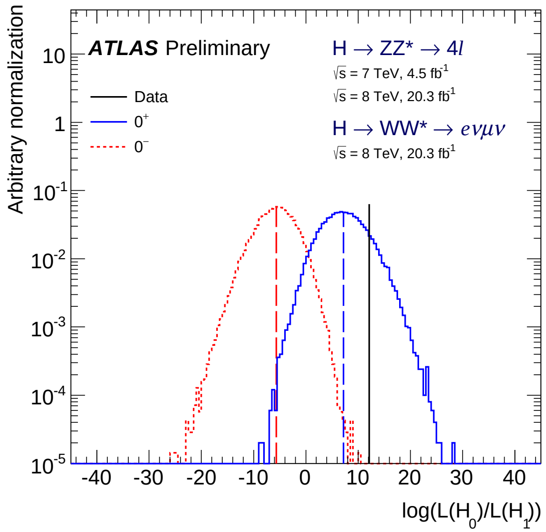
Quantum numbers	$Q = 0$ $J^{PC} = 0^{++}$
VEV	$v = 2^{-1/4} G_F^{-1/2} \approx 246.22$ GeV
Couplings	$g_{VVH} = 2^{5/4} G_F^{1/2} m_V^2 \qquad V = W, Z$ $g_{VVHH} = 2^{3/2} G_F m_V^2$ $g_{ffH} = 2^{3/4} G_F^{1/2} m_f$ $\lambda = 2^{-1/2} G_F m_H^2$ $g_{HHH} = 6 v \lambda$ $g_{HHHH} = 6\lambda$
Mass	M_H is free parameter

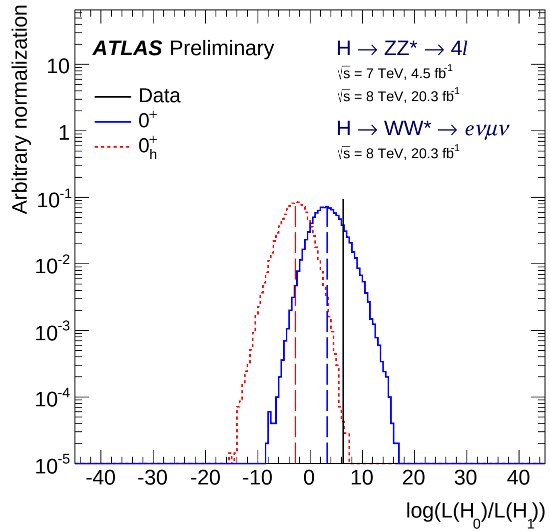
J^P & Lagrangian tensor structure

0^+: $HV_\mu V^\mu$

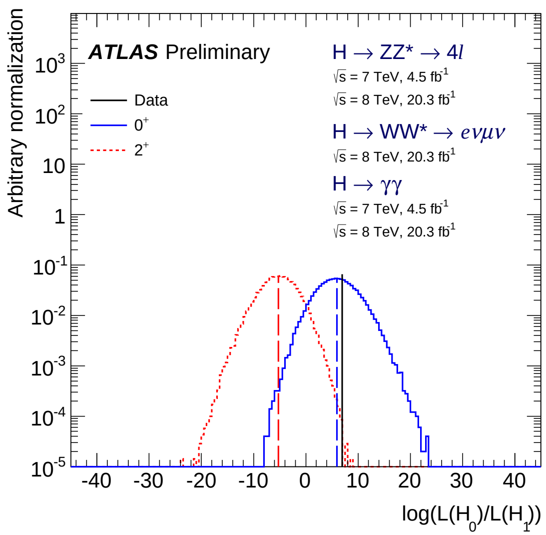
0^-: $\frac{1}{\Lambda} HV_{\mu\nu}\widetilde{V}^{\mu\nu}$

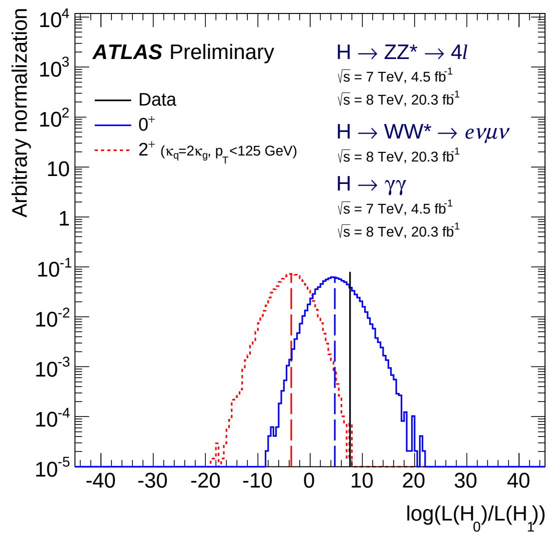
0_h^+: $\frac{1}{\Lambda} HV_{\mu\nu}V^{\mu\nu}$





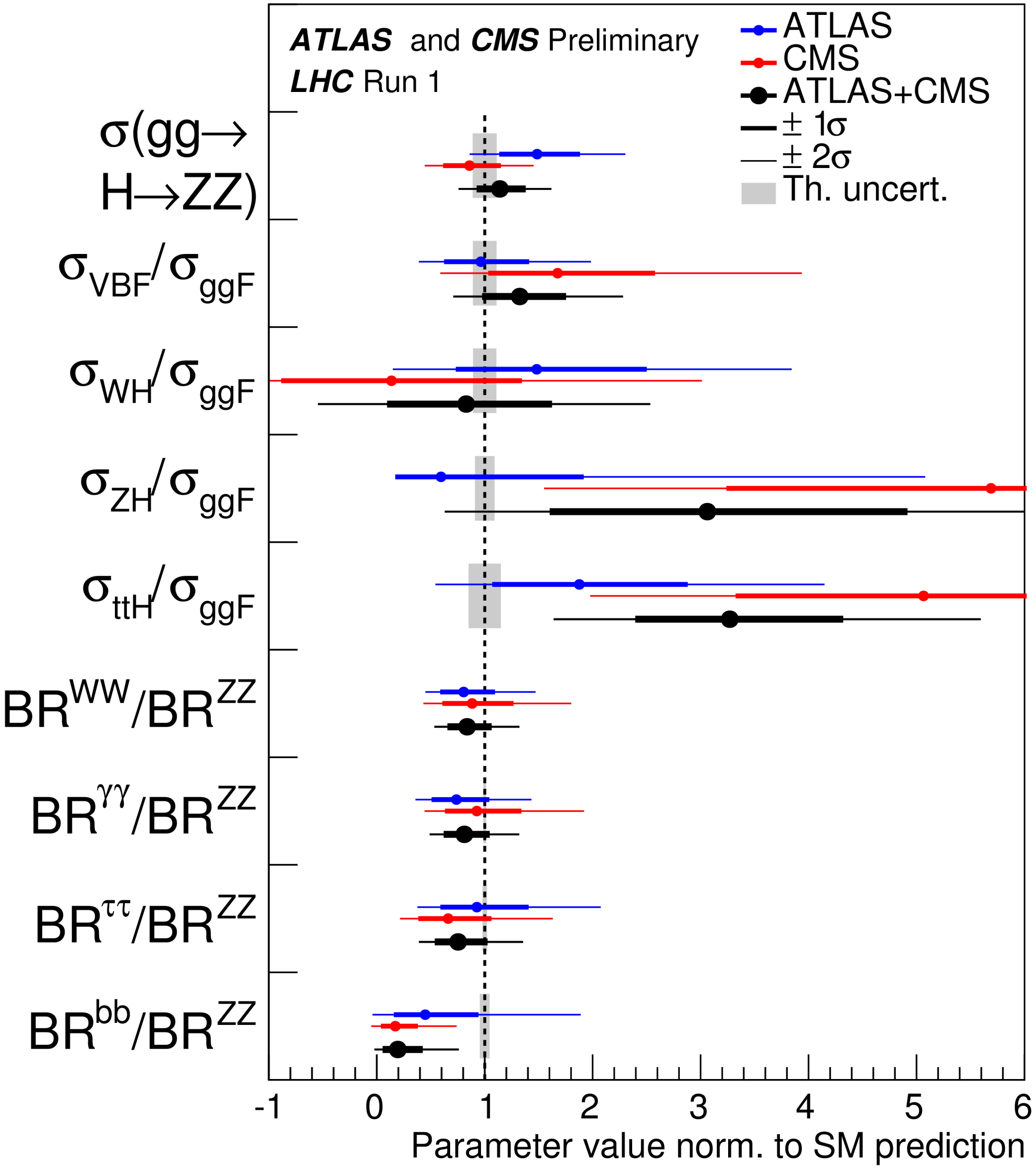
2^-: $\frac{1}{\Lambda} H_{\mu\nu} \times [\kappa_g \mathscr{T}_g^{\mu\nu} + \kappa_q \mathscr{T}_q^{\mu\nu}]$





ATLAS-CONF-2015-008 (13 Mar 2015) 4.5 fb^{-1} @ $\sqrt{s} = 7$ TeV, 20.3 fb^{-1} @ $\sqrt{s} = 8$ TeV

Cross sections & branching ratios



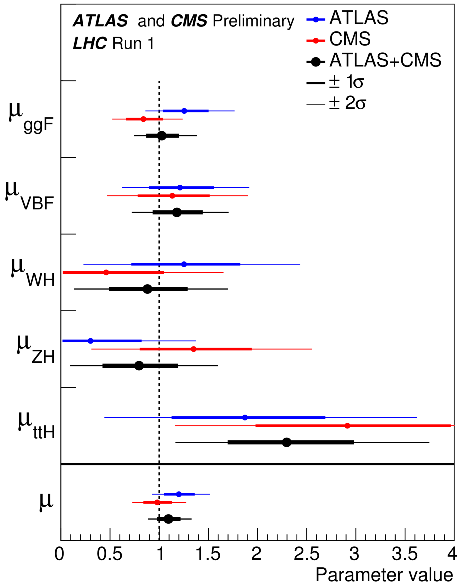
ATLAS-CONF-2015-044
CMS-PAS-HIG-15-002
(15 September 2015)
5 fb^{-1} @ $\sqrt{s} = 7$ TeV
20 fb^{-1} @ $\sqrt{s} = 8$ TeV
per experiment

Signal strengths

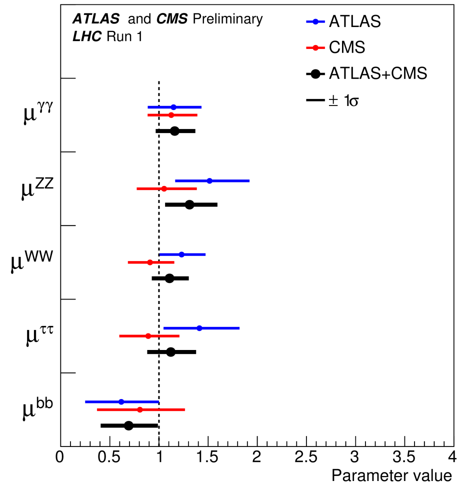
Production and decay channel $i \to H \to f$

Narrow-width approximation: $\sigma(i \to H \to f) = \sigma_i \times \mathrm{BR}^f = \sigma_i \times \frac{\Gamma^f}{\Gamma_H}$

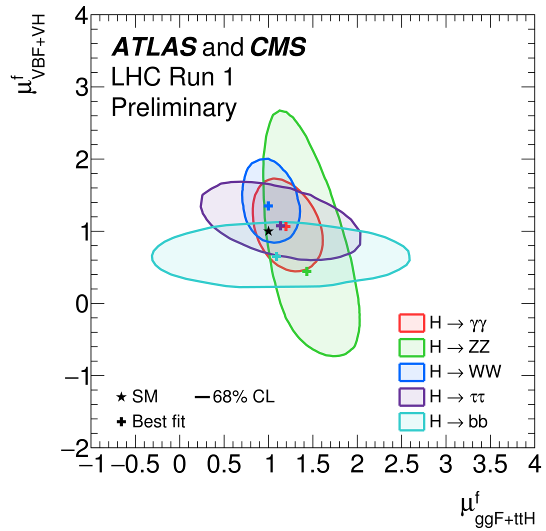
$\mu_i = \frac{\sigma_i}{(\sigma_i)_{\mathrm{SM}}}$



$\mu_f = \frac{\mathrm{BR}^f}{(\mathrm{BR}^f)_{\mathrm{SM}}}$



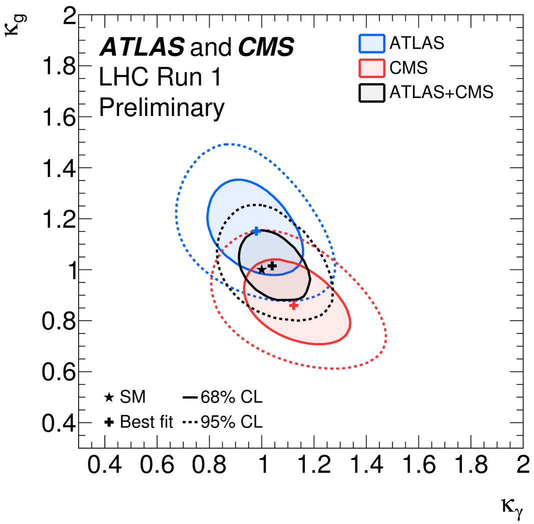
$\mu_i^f = \mu_i \times \mu^f$

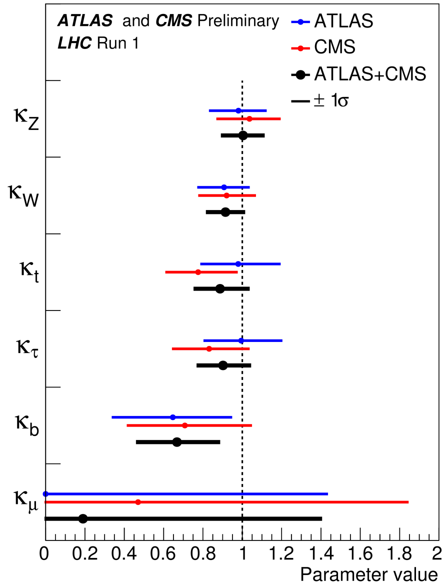


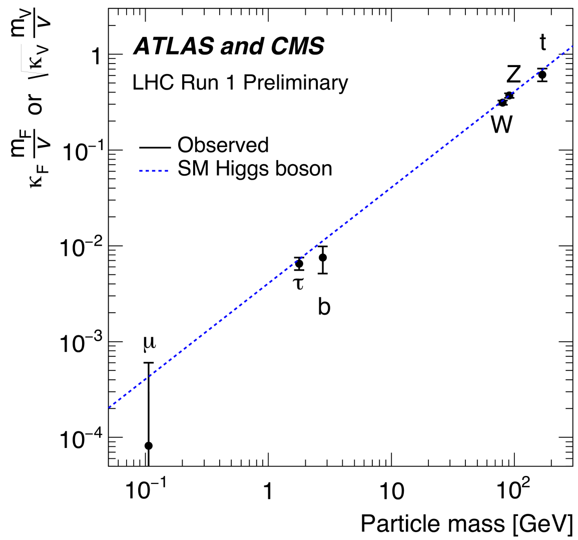
Coupling modifiers

$$\kappa_i^2 = \frac{\sigma_i}{(\sigma_i)_{\rm SM}} \text{ except for } \kappa_g$$

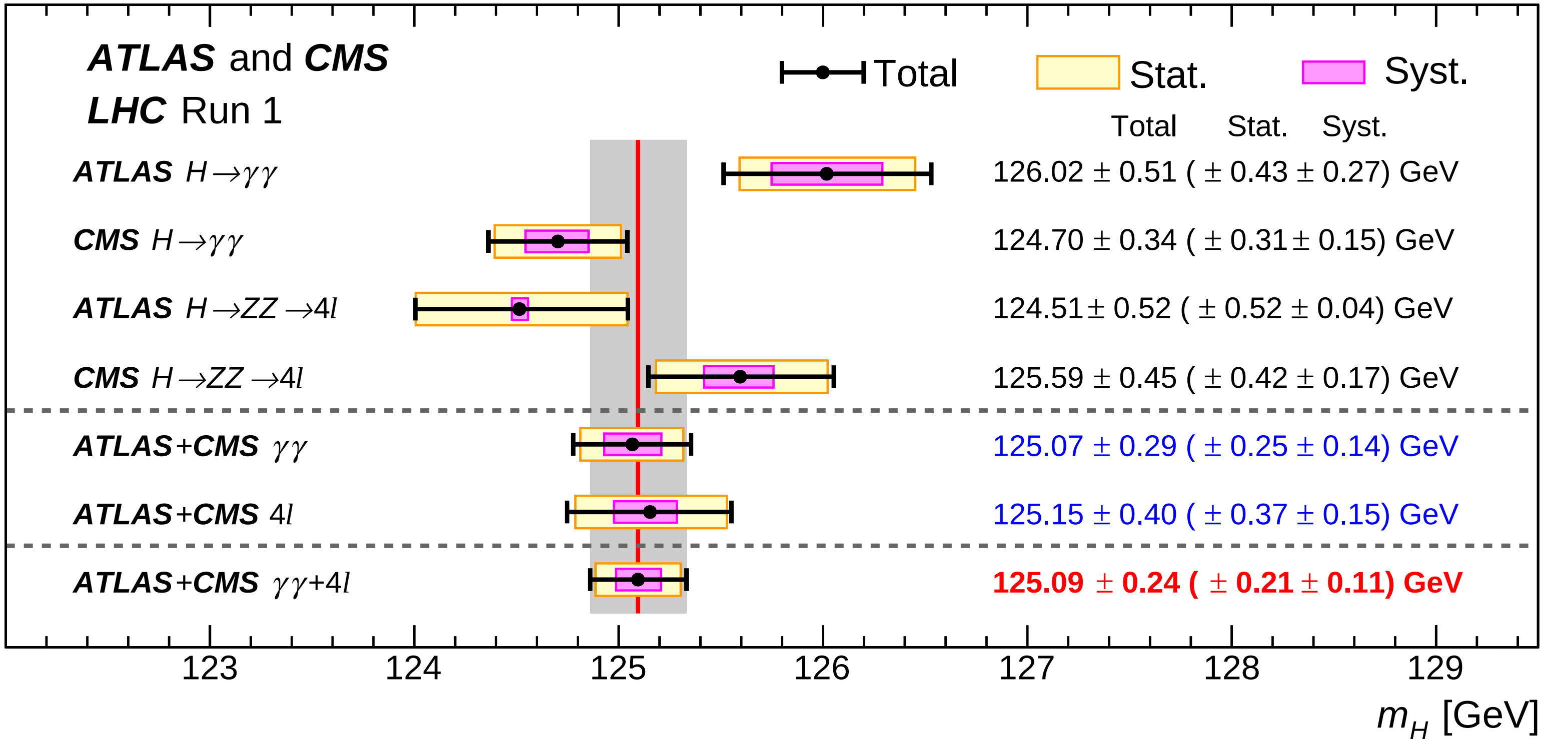
$$\kappa_f^2 = \frac{\Gamma^f}{(\Gamma^f)_{\rm SM}} \text{ except for } \kappa_\gamma$$







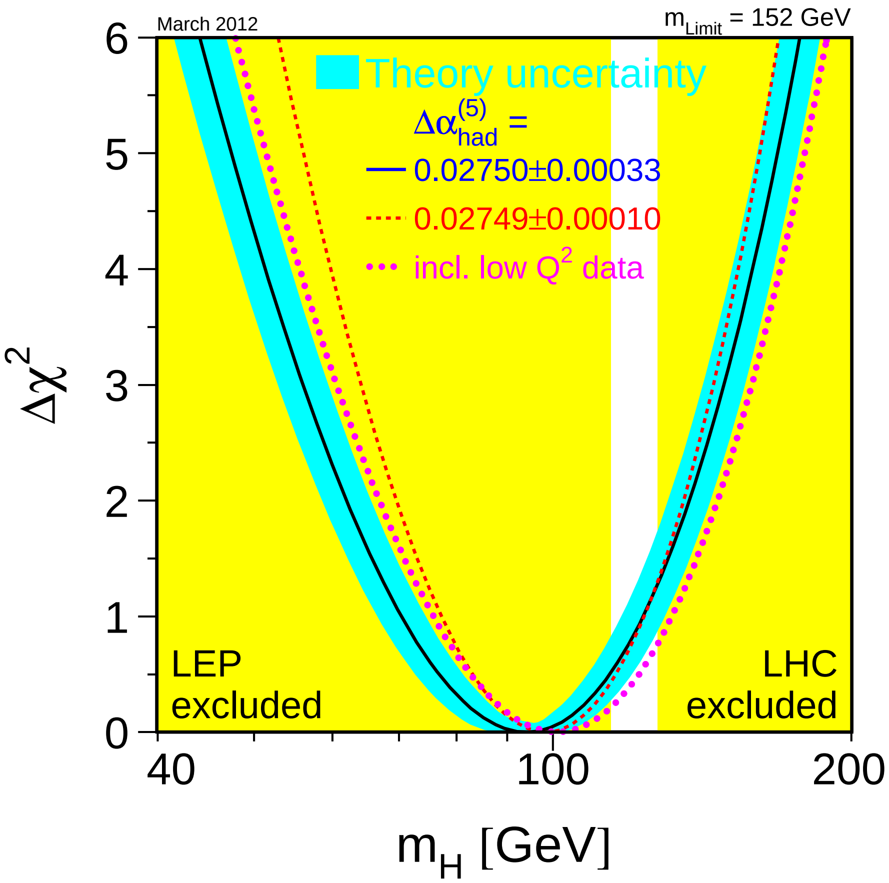
Mass measurements

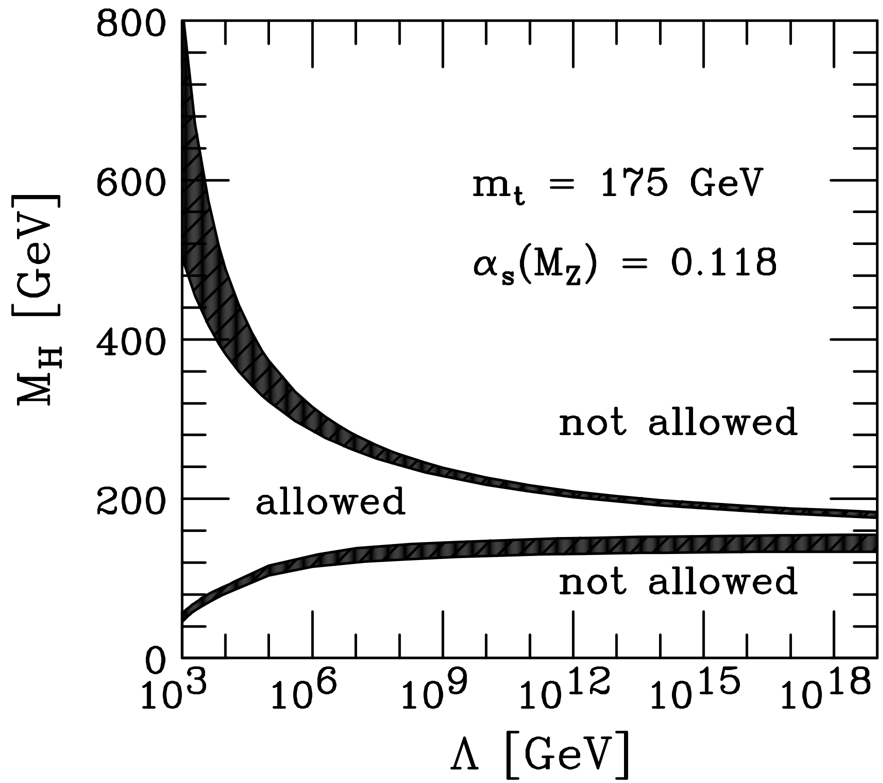


$M_H = (125.09 \pm 0.24)$ GeV

ATLAS & CMS, PRL114(2015)191803

EW precision tests, triviality & vacuum stability





- $m_H = (125.09 \pm 0.24)$ GeV agrees w/ EW precision data.
- Triviality bound satisfied.
- How about vacuum stability bound?

Renormalizaton: RG evolution

Cosmological applications require reliable predictions over very large range of scales: $v \lesssim \mu \lesssim M_P$

Use $\overline{\mathrm{MS}}$ renromalization scheme: running couplings $\lambda(\mu), y_t(\mu), g_s(\mu), \ldots$

Two-step procedure: 1. RG evolution:

$$\mu^2 \frac{d\lambda(\mu)}{d\mu^2} = \beta_\lambda = \frac{1}{16\pi^2}(12\lambda^2 + 6\lambda y_t^2 - 3y_t^4) + \cdots$$

$$\mu^2 \frac{dy_t(\mu)}{d\mu^2} = \beta_{y_t} = \frac{1}{16\pi^2} y_t \left(\frac{9}{4} y_t^2 - 4g_s^2\right) + \cdots$$

$$\mu^2 \frac{dg_s(\mu)}{d\mu^2} = \beta_{g_s} = \frac{1}{16\pi^2} g_s^3 \left(-\frac{11}{2} + \frac{n_f}{3}\right) + \cdots$$

$\beta_\lambda^{(3)}, \beta_{y_t}^{(3)}$	Chetyrkin, Zoller, JHEP06(2012)033; 04(2013)091
	Bednyakov *et al.*, JHEP01(2013)017; PLB722(2013)336; NPB875(2013)552
$\beta_{\ldots}^{(3)}, \beta_{g_s,y_t}^{(3)}$	Mihaila *et al.*, PRL108(2012)151602; PRD86(2012)096008
$\beta_{g_s}^{(3)}$	Tarasov *et al.*, PLB93(1980)429

Threshold corrections

2. Matching at $\mu = \mathcal{O}(v)$:

$$\lambda(\mu) = 2^{-1/2} G_F m_H^2 [1 + \delta_H^{(1)}(\mu) + \cdots]$$

$$\delta_H^{(1)}(\mu) = \frac{G_F m_H^2}{8\pi^2\sqrt{2}} \left[6 \ln \frac{\mu^2}{m_H^2} + \frac{25}{2} - \frac{3}{2}\pi\sqrt{3} + \mathcal{O}\left(\frac{m_Z^2}{m_H^2} \ln \frac{m_H^2}{m_Z^2} \right) \right]$$

Sirlin, Zucchini, NPB266(1986)389

$$y_t(\mu) = 2^{3/4} G_F^{1/2} m_t [1 + \delta_t^{(1)}(\mu) + \cdots]$$

$$\delta_t^{(1)}(\mu) = \frac{Q_t^2 \alpha + C_F \alpha_s(\mu)}{4\pi} \left(-3 \ln \frac{\mu^2}{m_t^2} - 4 \right) + \frac{G_F m_t^2}{8\pi^2\sqrt{2}} \left[\frac{9}{2} \ln \frac{\mu^2}{m_t^2} + \frac{11}{2} - 2\pi \frac{m_H}{m_t} + \mathcal{O}\left(\frac{m_H^2}{m_t^2} \ln \frac{m_t^2}{m_H^2} \right) \right]$$

Hempfling, BK, PRD51(1995)1386

$\delta_H^{(\alpha\alpha_s)}, \delta_t^{(\alpha\alpha_s)}$ Bezrukov, Kalmykov, BK, Shaposhnikov, JHEP10(2012)140

$\delta_H^{(y_t^4)}, \delta_t^{(y_t^4)}$ Degrassi *et al.*, JHEP08(2012)098; BK, Veretin, NPB885(2014)459

$\delta_H^{(\alpha^2)}, \delta_t^{(\alpha^2)}$ Buttazzo *et al.*, JHEP12(2013)089

$\delta_x^{(\alpha^2)}$ for all x BK, Veretin, Pikelner, NPB896(2015)19

$\overline{\mathrm{MS}}$ renormalization scheme

Parameters of the symmetric phase: $g, g', \lambda, m_\phi, y_f$
Parameters of the broken phase: e, m_W, m_Z, m_H, m_f
Tree-level relationships:

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}$$

$$\frac{4m_W^2}{v^2} = g^2 \qquad \frac{4m_Z^2}{v^2} = g^2 + g'^2 \qquad \frac{m_H^2}{2v^2} = \lambda \qquad \frac{2m_f^2}{v^2} = y_f^2$$

$$\frac{1}{v^2} = \frac{\lambda}{-m_\phi^2} = \frac{e^2}{4m_W^2(1 - m_W^2/m_Z^2)}$$

Treat as exact in the $\overline{\mathrm{MS}}$ renormalization scheme.

On-shell renormalization scheme

- Pole masses:

$$p^2 = M_B^2 : \; 0 = p^2 - m_{B,0}^2 - \Pi_{BB}(p^2) \qquad (B = H, W)$$

$$p^2 = M_Z^2 : \; 0 = p^2 - m_{Z,0}^2 - \Pi_{ZZ,T}(p^2) - \frac{\Pi^2_{\gamma Z,T}(p^2)}{p^2 - \Pi_{\gamma\gamma,T}(p^2)}$$

$$\not{p} = m_f : \; 0 = \not{p} - m_{f,0} - \Sigma_f(\not{p})$$

- Fine-structure constant: α_{Th} absorbs radiative corrections to Thomson scattering.
  Induces large corrections $\propto \alpha \ln(q^2/m_\ell^2)$ and hadronic uncertainties! $\rightsquigarrow$ Use instead Sirlin, PRD22(1980)971
  $$G_F = \frac{\pi \alpha_{\mathrm{Th}}}{\sqrt{2} M_W^2 (1 - M_W^2/M_Z^2)} (1 + \Delta r)$$

Matching

- Masses:

$$m_0^2 = M^2 - \Pi(M^2) = m^2(\mu)\left(1 + \frac{Z^{(1)}}{\varepsilon} + \frac{Z^{(2)}}{\varepsilon^2} + \cdots\right)$$

$$Z^{(j)} = \frac{g^2}{16\pi^2} Z_{\alpha}^{(j)} + \frac{g^2}{16\pi^2}\frac{g_s^2}{16\pi^2} Z_{\alpha\alpha_s}^{(j)} + \left(\frac{g^2}{16\pi^2}\right)^2 Z_{\alpha^2}^{(j)} + \cdots$$

- Couplings:

$$2^{1/2} G_F = \frac{1 + \Delta\bar{r}(\mu)}{v^2(\mu)}$$

$$\frac{e^2}{8 m_W^2 (1 - m_W^2/m_Z^2)}(1 + \Delta\bar{r}) = \left[\sqrt{Z_{2,e} Z_{2,\nu_e} Z_{2,\mu} Z_{2,\nu_\mu}}\, A(e + \nu_e \to \mu + \nu_\mu)\right]_{\rm hard}$$

hard: Nullify external four-momenta and light-fermion masses before loop integration. Awramik *et al.*, PRD68(2003)053004

Threshold corrections

- Couplings:

$$g^2(\mu) = 2^{5/2} G_F M_W^2 [1 + \delta_W(\mu)]$$
$$g^2(\mu) + g'^2(\mu) = 2^{5/2} G_F M_Z^2 [1 + \delta_Z(\mu)]$$
$$e^2(\mu) = 2^{5/2} G_F M_W^2 [1 + \delta_W(\mu)] \left[1 - \frac{M_W^2}{M_Z^2} \frac{1 + \delta_W(\mu)}{1 + \delta_Z(\mu)}\right]$$
$$\lambda(\mu) = 2^{-1/2} G_F M_H^2 [1 + \delta_H(\mu)]$$
$$y_f(\mu) = 2^{3/4} G_F^{1/2} M_f [1 + \delta_f(\mu)]$$
$$g_s^2(\mu) = 4\pi \alpha_s^{(5)}(\mu) [1 + \delta_{\alpha_s}(\mu)]$$

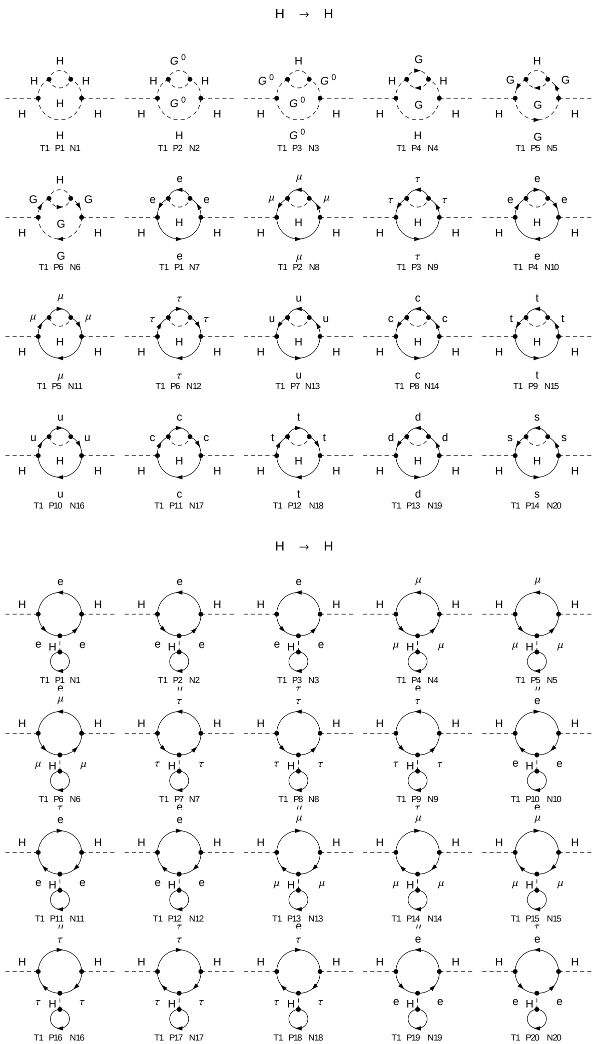
- Masses:

$$m_B^2(\mu) = M_B^2 [1 + \Delta\bar{r}(\mu)][1 + \delta_B(\mu)] \qquad B = W, Z, H$$
$$m_f(\mu) = M_f [1 + \Delta\bar{r}(\mu)]^{1/2} [1 + \delta_f(\mu)] \qquad f = t, b$$

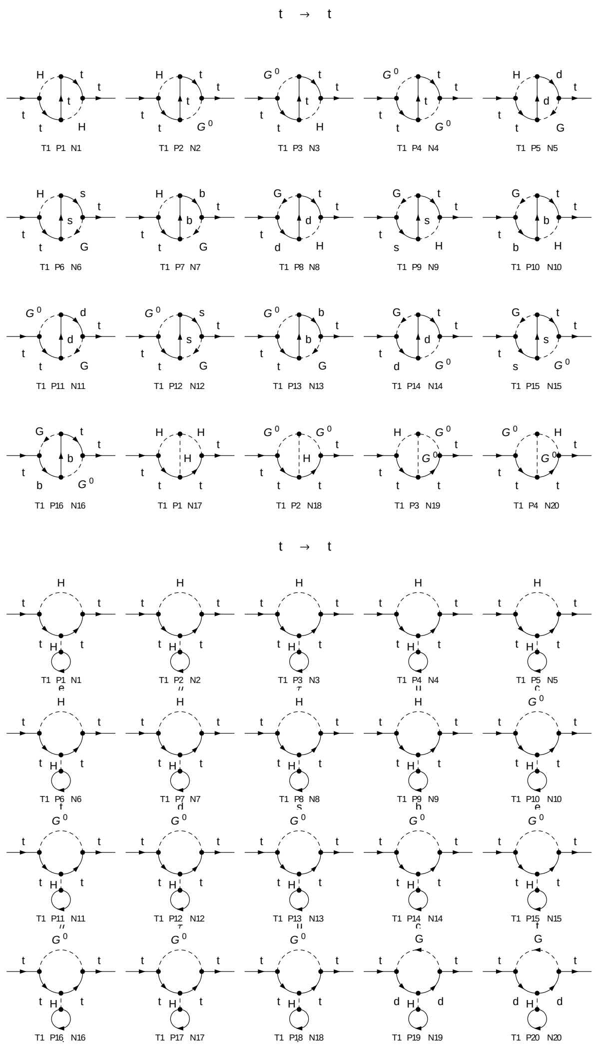
Exact two-loop results. BK, Veretin, Pikelner, NPB896(2015)19

Typical Feynman diagrams

$\delta_H(\mu)$



$\delta_t(\mu)$



Tools

Packages used:

- Generation of diagrams: QGRAF, DIANA — Nogueira, Tentyukov
- Reduction: TARCER (*Mathematica*) — Mertig
  $\rightsquigarrow$ Gauge invariance upon inclusion of all tadpoles $\surd$
- Numerical evaluation of master integrals: TSIL (*C++*) — Martin

Program library created: mr for matchng and running (*C++*)

BK, Pikelner, Veretin, 1601.08143 [hep-ph]

- Matching @ 2-loop EW & 4-loop QCD level
- RG evolution @ 3-loop EW & 4-loop QCD level

Available for download from URL: http://apik.github.io/mr/

Numerical results

- Corrections to $\delta_H(M_t)$ in 10^{-4}

M_H [GeV]	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha\alpha_s)$	$\mathcal{O}(\alpha^2)$	total
124	-114.8	-107.5	-26.6 (-29.1)	-248.7
125	-114.5	-105.2	-26.4 (-29.2)	-246.1
126	-114.1	-103.1	-26.3 (-29.3)	-243.5

- Corrections to $\delta_t(M_t)$ in 10^{-4}

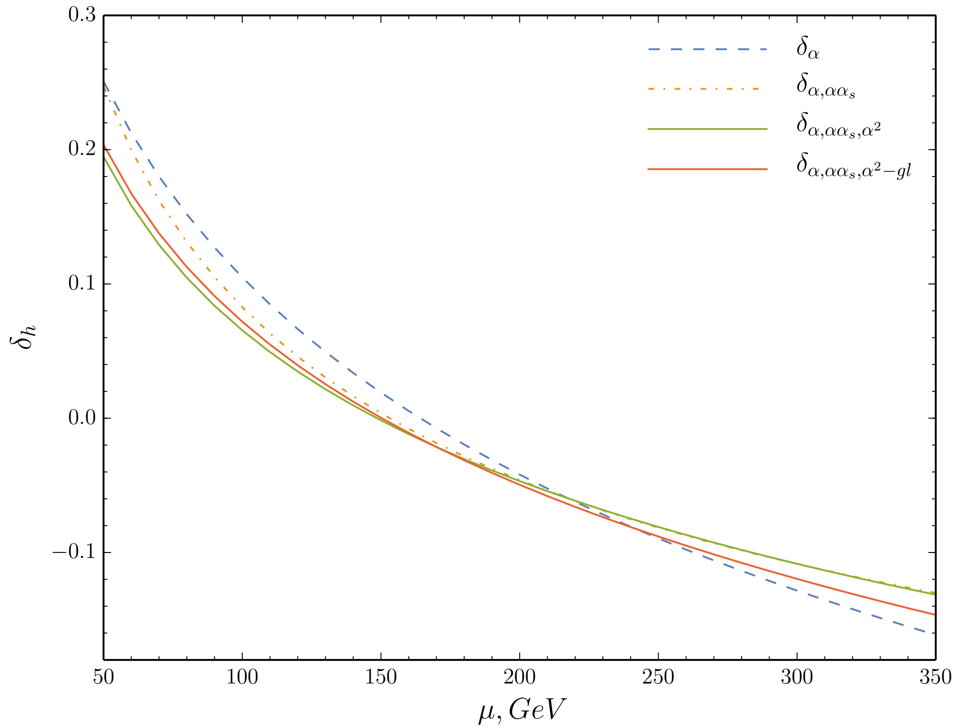
M_H [GeV]	QCD	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha\alpha_s)$	$\mathcal{O}(\alpha^2)$	total
124	-599.3	13.5	-4.4	2.7 (3.1)	-587.4
125	-599.3	13.2	-4.3	2.7 (3.1)	-587.7
126	-599.3	12.9	-4.2	2.7 (3.1)	-587.9

- Corrections to $\delta_b(M_b)$
  $\{1+\delta_b(M_b)\}_{\mathrm{QCD},O(\alpha),O(\alpha\alpha_s),O(\alpha^2)} =$
  $1-0.1728-0.0190-0.0112+0.0032(0.0033)$

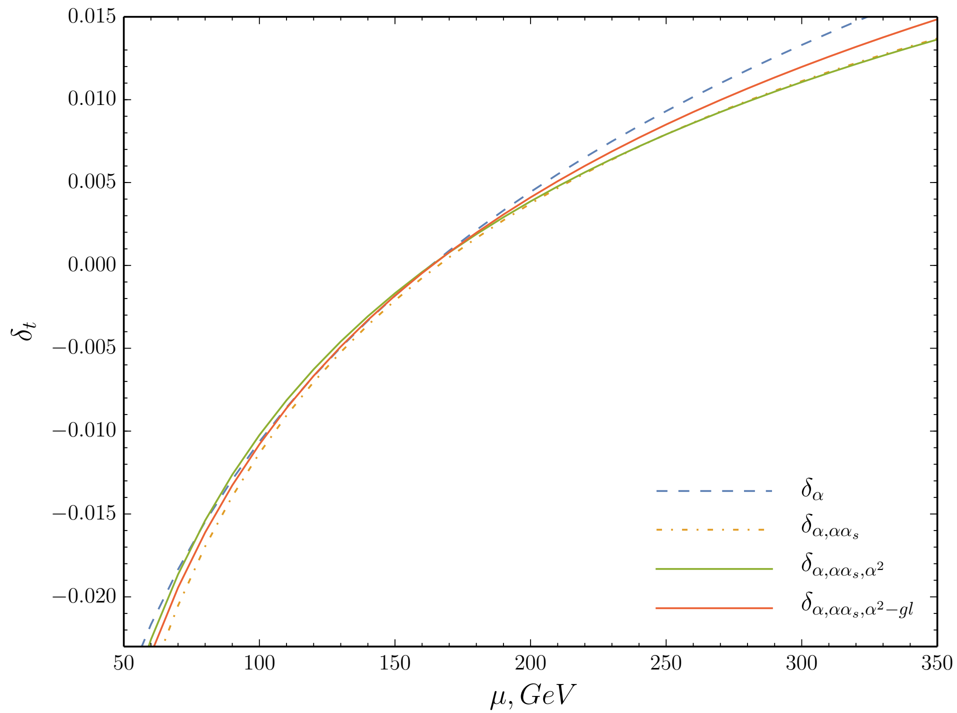
BK, Pikelner, Veretin, NPB896(2015)19

Numerical results

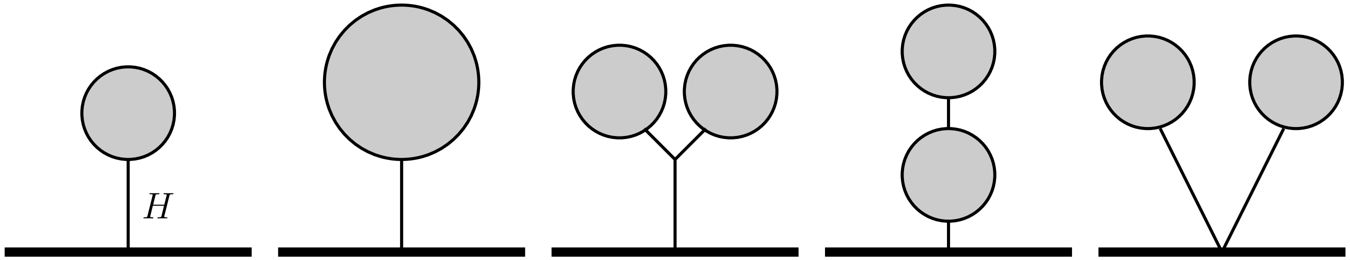
$\delta_H(\mu)$



$\delta_t(\mu)$



Role of tadpoles



- Tadpole is gauge dependent and $\propto 1/M_H^{2n}$ for $M_H \to 0$.
- Adjust vev $v^0 = \sqrt{-(m_\Phi^0)^2/\lambda^0}$ to eliminate term $\propto H$ in bare $\mathscr{L}$.
  Hempfling, BK, PRD51(1995)1386
- No tadpole counterterm.
- Include tadpoles order by order to ensure finiteness and gauge independence.
- $\Delta\bar{r}(\mu)$ and $\delta_x(\mu)$ are gauge independent through $\mathscr{O}(\alpha^2)$.
- At $\mathscr{O}(\alpha^2)$, $\delta_x(\mu) \propto M_H^0$ for $x = W, Z, f$; $\delta_H(\mu) \propto M_H^{-2}$; $\Delta\bar{r}(\mu) \propto M_H^{-4}$ for $M_H \to 0$.
- $m_f(\mu)$ gauge independent, but receive large EW corrections. $\rightsquigarrow$
  Use instead Jegerlehner, Kalmykov, BK, PLB722(2013)123
  $m_f^Y(\mu) = 2^{-3/4} G_F^{-1/2} y_f(\mu) = M_f[1 + \delta_f(\mu)] = m_f(\mu)[1 + \Delta\bar{r}(\mu)]^{-1/2}$.

Tadpole cancellation

- Consider $m_f(\mu)$ and $y_f(\mu)$ at $\mathcal{O}(\alpha)$ Hempfling, BK, PRD51(1995)1386

$$m_f(\mu) = M_f(1 + \delta M_f/M_f)_{\overline{\mathrm{MS}}}$$

$$y_f(\mu) = 2^{3/4} G_F^{1/2} M_f(1 + \delta M_f/M_f - \delta v/v)_{\overline{\mathrm{MS}}}$$

$$\delta M_f/M_f = \mathrm{Re}[\Sigma_V^f(M_f^2) + \Sigma_S^f(M_f^2)] - 2^{1/4} G_F^{1/2} T/M_H^2$$

$$\delta v/v = [\Pi_W(0)/M_W^2 + E]/2 - 2^{1/4} G_F^{1/2} T/M_H^2$$

- Exact tadpole cancellation also in $\mathcal{O}(\alpha\alpha_s)$. Jegerlehner, Kalmykov, NPB676(2004)365; BK, Piclum, Steinhauser, NPB695(2004)199
- Incomplete tadpole cancellation in $\mathcal{O}(\alpha^2)$ BK, Veretin, NPB885(2014)459; BK, Pikelner, Veretin, NPB896(2015)19
- Similar for $\lambda(\mu)$. Sirlin, Zucchini, NPB266(1986)389; Bezrukov *et al.*, JHEP01(2012)140

Running top and bottom masses

- Corrections to $m_t(M_t) - M_t$ in GeV

M_H [GeV]	QCD	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha\alpha_s)$	$\mathcal{O}(\alpha^2)$	total
124	-10.38	12.08	-0.39	-0.99 (-0.47)	0.32
125	-10.38	11.88	-0.39	-0.96 (-0.45)	0.14
126	-10.38	11.67	-0.38	-0.94 (-0.44)	-0.03

- Corrections to $m_t^Y(M_t) - M_t$ in GeV

M_H [GeV]	QCD	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha\alpha_s)$	$\mathcal{O}(\alpha^2)$	total
124	-10.38	0.234	-0.076	0.047 (0.054)	-10.17
125	-10.38	0.229	-0.075	0.047 (0.054)	-10.18
126	-10.38	0.223	-0.073	0.047 (0.054)	-10.18

- Corrections to $m_b(M_b) - M_b$
  $\{m_b(M_b) - M_b\}_{\mathrm{QCD},O(\alpha),O(\alpha\alpha_s),O(\alpha^2)} =$
  $-0.85 - 1.90 - 1.53 + 1.75\ (1.80)$ GeV
- Corrections to $m_b^Y(M_b) - M_b$
  $\{m_b^Y(M_b) - M_b\}_{\mathrm{QCD},O(\alpha),O(\alpha\alpha_s),O(\alpha^2)} =$
  $-0.847 - 0.093 - 0.055 + 0.016(0.016)$ GeV

$\rightsquigarrow$ m_q^Y is much more perturbatively stable than $m_q(M_q)$.

Vaccum stability in a nutshell

Recall

$$\mu^2 \frac{d\lambda(\mu)}{d\mu^2} = \frac{1}{16\pi^2}(12\lambda^2 + 6\lambda y_t^2 - 3y_t^4) + \cdots$$

$$\mu^2 \frac{dy_t(\mu)}{d\mu^2} = \frac{1}{16\pi^2} y_t \left(\frac{9}{4} y_t^2 - 4g_s^2\right) + \cdots$$

$$\lambda(m_H) = 0.130 \times \left(\frac{m_H}{125.7\ \mathrm{GeV}}\right)^2, \quad y_t(m_t) = 0.995 \times \frac{m_t}{173.21\ \mathrm{GeV}}, \quad g_s(m_Z) = 1.220$$

For $m_t \gg m_H$, $\lambda(\mu) = \lambda(v) - \dfrac{(3/8\pi^2) y_t^4(v) \ln(\mu/v)}{1 - (9/16\pi^2) y_t^2(v) \ln(\mu/v)}$

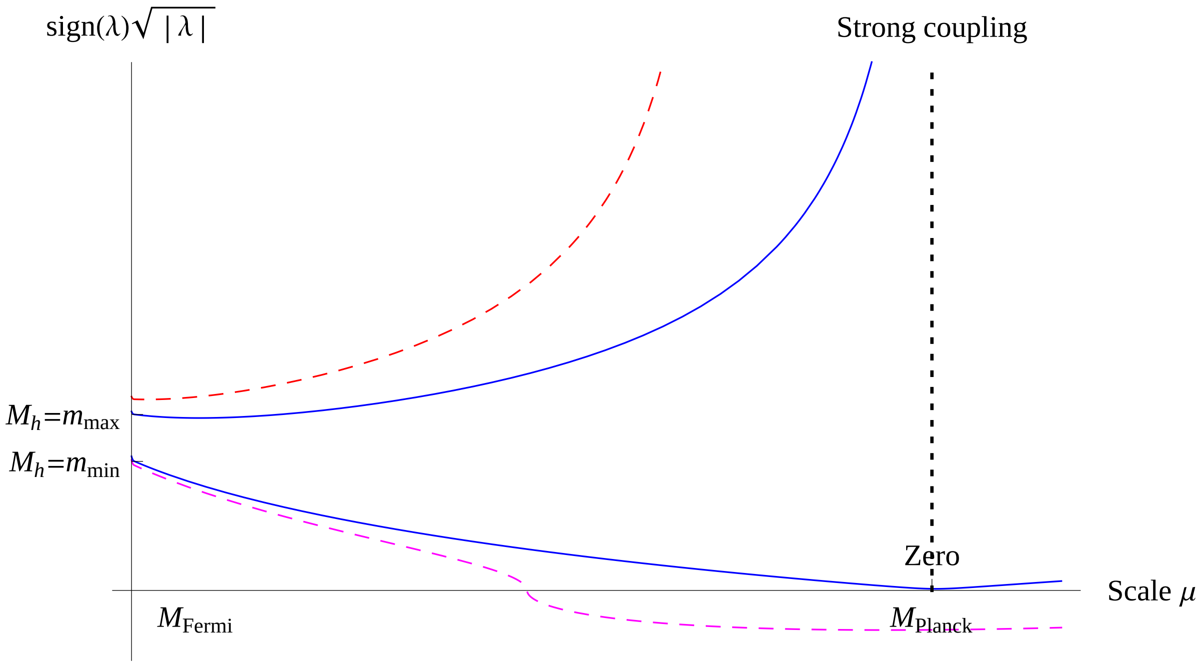
$\rightsquigarrow$ $\mu^{\mathrm{cri}} \approx v \exp(4\pi^2 v^2 m_H^2 / 3m_t^4) = 8.0 \times 10^3\ \mathrm{GeV}$

Vacuum stability bound: Lindner, ZPC31(1986)295

If $m_H < m_H^{\mathrm{cri}}$, then $\lambda(\mu) < 0$ for $\mu > \mu^{\mathrm{cri}}$.

$\rightsquigarrow$ *Decay of universe*

Vaccum stability condition



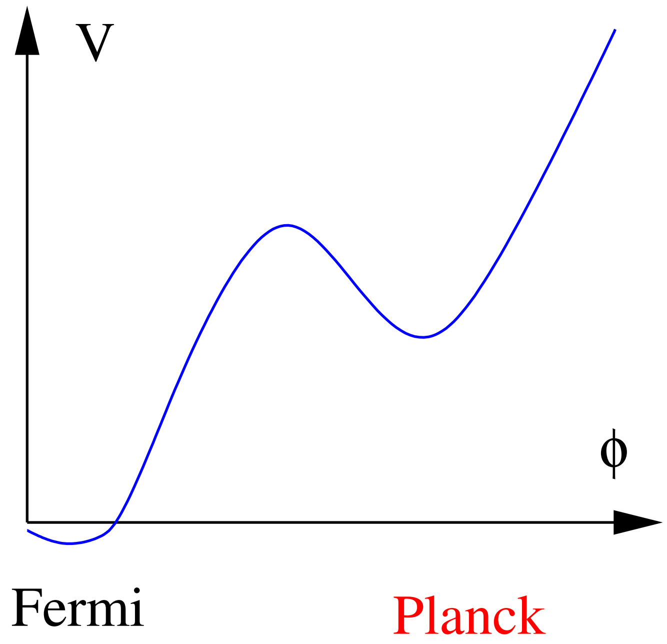
Determine μ^{cri} and M_H^{cri} for given M_t (or M_t^{cri} for given M_H) so that

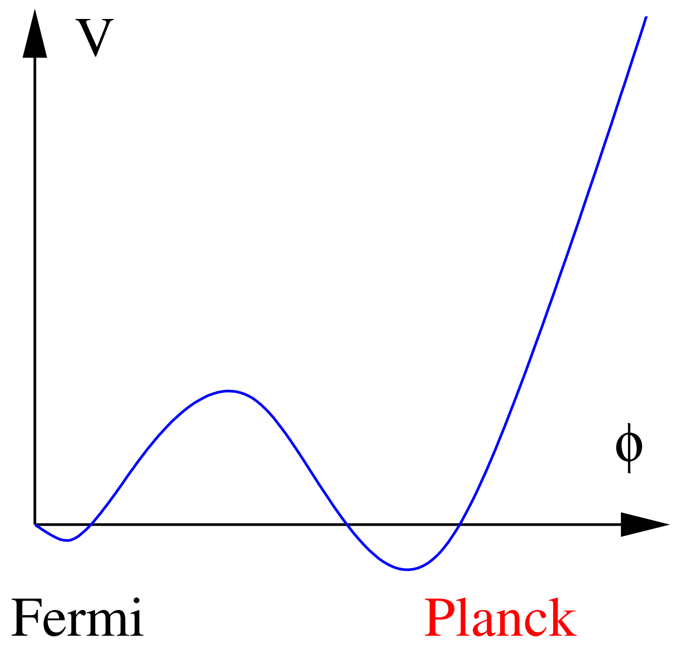
$$\lambda(\mu^{\mathrm{cri}}) = \beta_\lambda(\lambda(\mu^{\mathrm{cri}})) = 0$$

$\rightsquigarrow$ Vacuum is stable for $M_H \geq M_H^{\mathrm{cri}}$ (or $M_t \leq M_t^{\mathrm{cri}}$).

Caveat: $\mu^{\mathrm{cri}}, M_H^{\mathrm{cri}}, M_t^{\mathrm{cri}}$ are gauge independent, but (slightly) scheme dependent. $\rightsquigarrow$ theoretical uncertainty

Effective potential





Determine $\tilde{\mu}^{\rm cri}$ and $\tilde{M}_H^{\rm cri}$ for given M_t (or $\tilde{M}_t^{\rm cri}$ for given M_H) so that

$$V_{\rm eff}(\tilde{\mu}^{\rm cri}) = V_{\rm eff}(v) \approx 0, \qquad V'_{\rm eff}(\tilde{\mu}^{\rm cri}) = 0$$

$\rightsquigarrow$ Vacuum is stable for $M_H \geq \widetilde{M}_H^{\rm cri}$ (or $M_t \leq \widetilde{M}_t^{\rm cri}$).

Caveat: $\tilde{\mu}^{\rm cri}, \widetilde{M}_H^{\rm cri}, \widetilde{M}_t^{\rm cri}$ are gauge dependent!

Degassi *et al.*, JHEP08(2012)098; Buttazzo *et al.*, JHEP12(2013)089

Consistent approach to effective potential

- Reorganize $V_{\text{eff}}(H)$ in powers of $\hbar$ so that expansion coefficients are gauge independent at its extrema Andreassen *et al.*, PRL113(2014)241801
- Solve $V'_{\text{eff}}(H) = 0$ for $H = \tilde{\mu}^{\text{cri}}$:

$$\lambda = \frac{1}{256\pi^2}\left[(g^2+g'^2)^2\left(1-3\ln\frac{g^2+g'^2}{4}\right)\right.$$
$$\left.+\,2g'^4\left(1-3\ln\frac{g'^2}{4}\right)-48y_t^4\left(1-\ln\frac{y_t^2}{4}\right)\right]$$

- Require that $V_{\min}^{\text{NLO}} = V_{\text{eff}}^{\text{NLO}}(\tilde{\mu}^{\text{cri}}) \geq 0$ for $M_H \geq \widetilde{M}_H^{\text{cri}}$ (or $M_t \leq \widetilde{M}_t^{\text{cri}}$) e.g. in the Landau gauge
- Caveat: $\tilde{\mu}^{\text{cri}} > M_P$!

Critical parameters

$$X = X_0 + \Delta X_{\alpha_s} \frac{\alpha_s^{(5)}(M_Z) - \alpha_s^{(5),\mathrm{exp}}(M_Z)}{\Delta\alpha_s^{(5),\mathrm{exp}}(M_Z)} + \Delta X_M \frac{M - M^{\mathrm{exp}}}{\Delta M^{\mathrm{exp}}} \pm \delta X_{\mathrm{par}} + \delta X_\mu^\pm \pm \delta X_{\mathrm{tru}}$$

X	X_0	ΔX_{α_s}	ΔX_M	δX_{par}	δX_μ^+	δX_μ^-	δX_{tru}
M_t^{cri}	171.44	0.23	0.20	0.001	-0.36	0.17	-0.02
$\log_{10}\mu_t^{\mathrm{cri}}$	17.752	-0.051	0.083	0.007	0.007	-0.006	-0.002
M_H^{cri}	129.30	-0.49	1.79	0.002	0.72	-0.33	0.04
$\log_{10}\mu_H^{\mathrm{cri}}$	18.512	-0.158	0.381	0.008	0.173	-0.082	0.008
$\widetilde{M}_t^{\mathrm{cri}}$	171.64	0.23	0.20	0.001	-0.36	0.17	-0.02
$\log_{10}\tilde{\mu}_t^{\mathrm{cri}}$	21.442	-0.059	0.094	0.005	-0.083	0.022	0.002
$\widetilde{M}_H^{\mathrm{cri}}$	128.90	-0.49	1.79	0.003	0.73	-0.34	0.04
$\log_{10}\tilde{\mu}_H^{\mathrm{cri}}$	22.209	-0.181	0.436	0.007	0.092	-0.062	0.013

Importance of higher orders

- $\mathcal{O}(\alpha^2)$ corrections to all $\delta_i(\mu)$ BK, Pikelner, Veretin, NPB896(2015)19
- $\mathcal{O}(\alpha_s\alpha)$ and $\mathcal{O}(\alpha_s^4)$ corrections to $\delta_{\alpha_s}(\mu)$ Bednyakov, PLB741(2015)262; Schröder, Steinhauser, JHEP01(2006)051; Chetyrkin, Kühn, Sturm, NPB744(2006)121; BK *et al.*, PRL97(2006)042001
- $\mathcal{O}(\alpha_s^4)$ corrections to $\delta_q(\mu)$ Marquard *et al.*, PRL114(2015)142002

X	$X_0+\delta X_\mu^\pm$	w/o $\delta_i^{O(\alpha^2)}$	w/o $\delta_{\alpha_s}^{O(\alpha\alpha_s,\alpha_s^4)}$	w/o $\delta_q^{O(\alpha_s^4)}$
$M_t^{\rm cri}$	$171.44_{+0.17}^{-0.36}$	$171.55_{+1.04}^{-0.47}$	$171.43_{+0.17}^{-0.36}$	$171.24_{+0.19}^{-0.38}$
$\log_{10}\mu_t^{\rm cri}$	$17.752_{-0.006}^{+0.007}$	$17.783_{-0.008}^{+0.062}$	$17.754_{-0.006}^{+0.007}$	$17.751_{-0.007}^{+0.007}$
$M_H^{\rm cri}$	$129.30_{-0.33}^{+0.72}$	$129.06_{-2.14}^{+0.95}$	$129.32_{-0.33}^{+0.73}$	$129.72_{-0.38}^{+0.76}$
$\log_{10}\mu_H^{\rm cri}$	$18.512_{-0.082}^{+0.173}$	$18.495_{-0.531}^{+0.226}$	$18.518_{-0.082}^{+0.174}$	$18.602_{-0.094}^{+0.184}$
$\widetilde{M}_t^{\rm cri}$	$171.64_{+0.17}^{-0.36}$	$171.74_{+1.04}^{-0.46}$	$171.63_{+0.17}^{-0.36}$	$171.43_{+0.19}^{-0.37}$
$\log_{10}\tilde{\mu}_t^{\rm cri}$	$21.442_{+0.022}^{-0.083}$	$21.485_{+0.343}^{-0.085}$	$21.445_{+0.022}^{-0.083}$	$21.441_{+0.014}^{-0.072}$
$\widetilde{M}_H^{\rm cri}$	$128.90_{-0.34}^{+0.73}$	$128.67_{-2.15}^{+0.95}$	$128.92_{-0.34}^{+0.73}$	$129.32_{-0.38}^{+0.76}$
$\log_{10}\tilde{\mu}_H^{\rm cri}$	$22.209_{-0.062}^{+0.092}$	$22.201_{-0.171}^{+0.146}$	$22.217_{-0.062}^{+0.094}$	$22.312_{-0.082}^{+0.113}$

Combined results

Selected for a Viewpoint in *Physics*
PRL **115,** 201802 (2015) PHYSICAL REVIEW LETTERS week ending 13 NOVEMBER 2015

Stability of the Electroweak Vacuum: Gauge Independence and Advanced Precision

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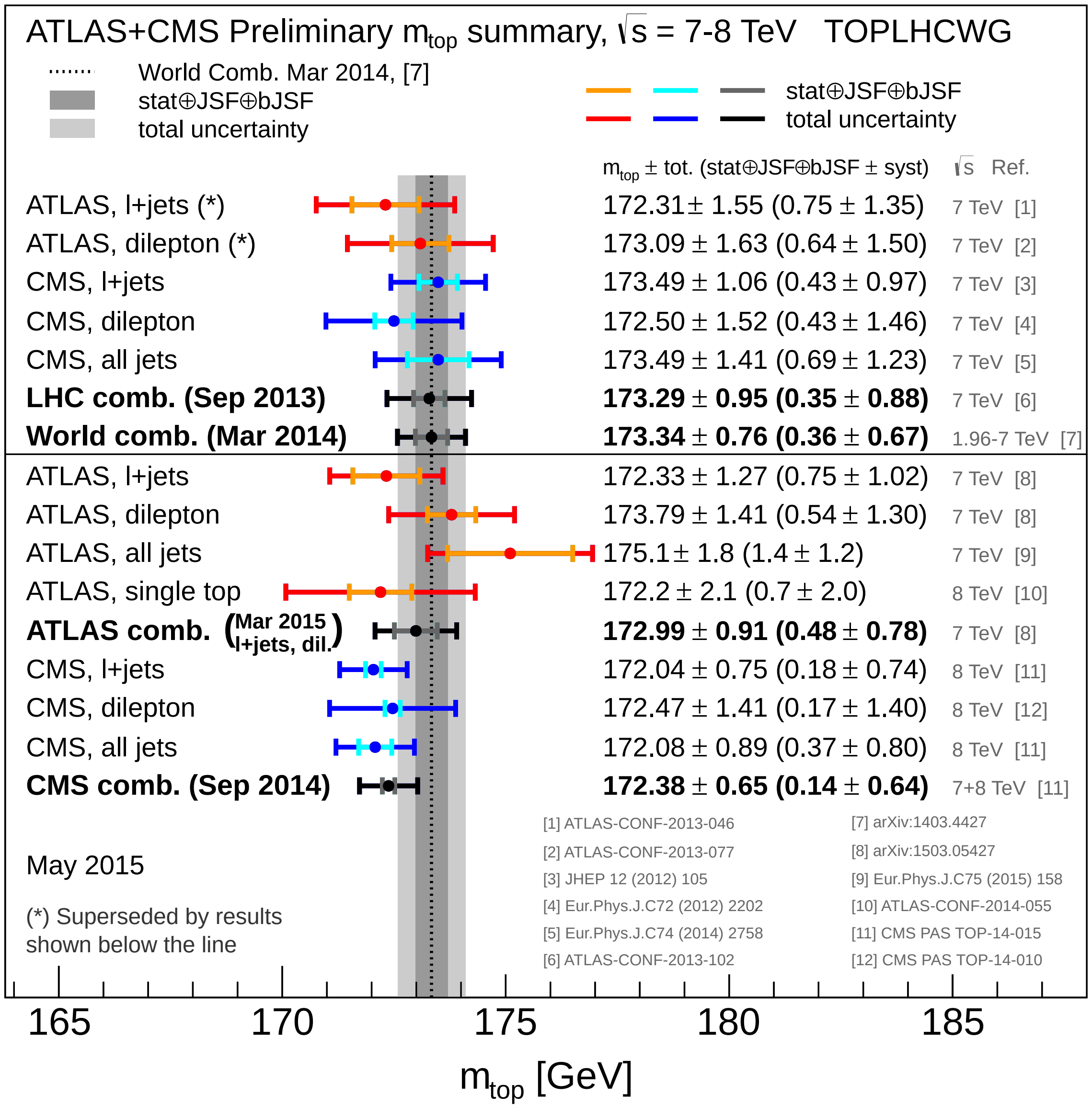
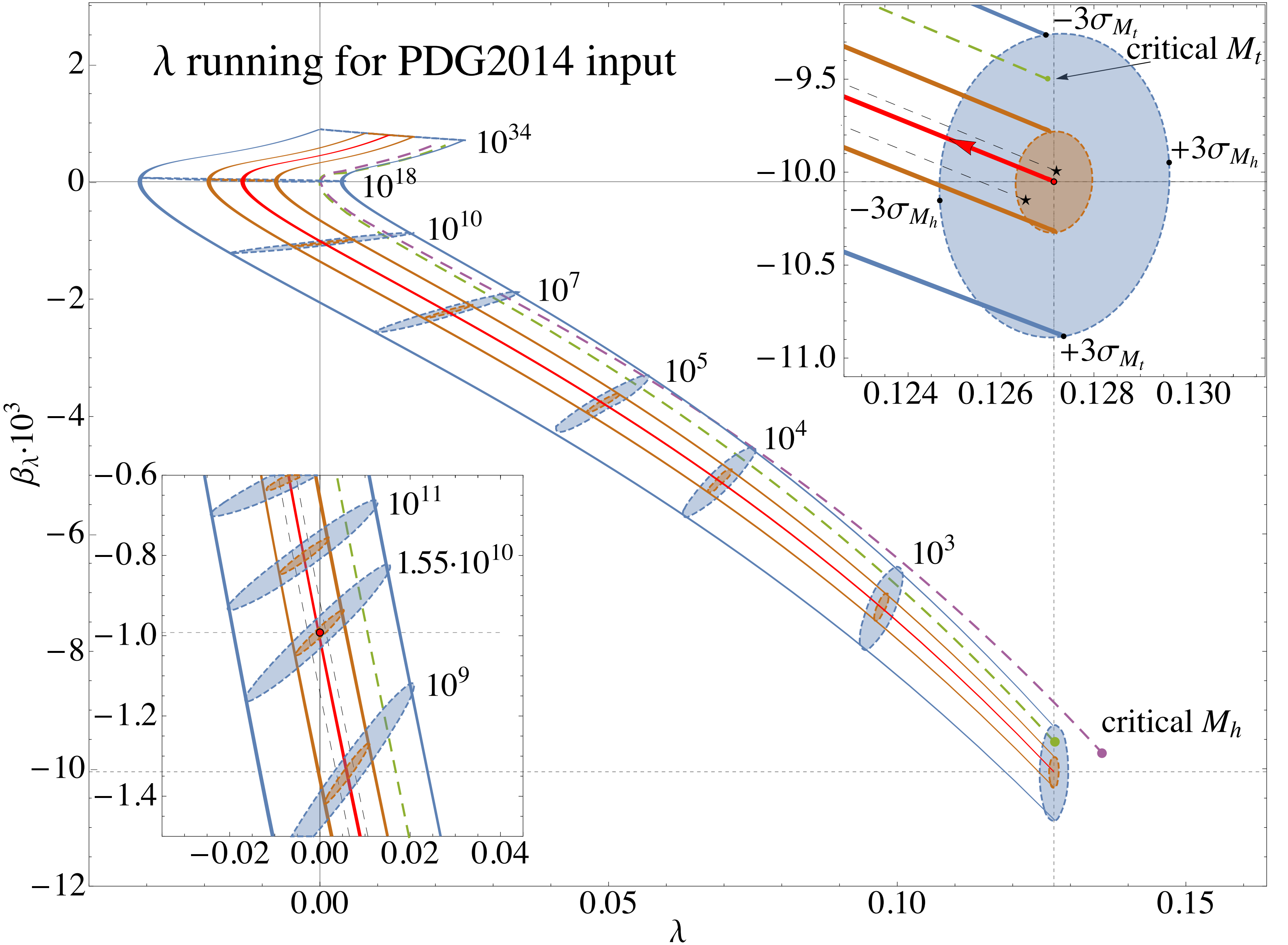
- From $\lambda(\mu)$: $M_t^{\rm cri} = (171.44 \pm 0.30\,^{+0.17}_{-0.36})$ GeV
- From $V_{\rm eff}(H)$: $\widetilde{M}_t^{\rm cri} = (171.64 \pm 0.30\,^{+0.17}_{-0.36})$ GeV
- Combination: $\widehat{M}_t^{\rm cri} = (171.54 \pm 0.30\,^{+0.26}_{-0.41})$ GeV
- Experiment: $M_t^{\rm MC} = (172.38 \pm 0.66)$ GeV ATLAS & CMS, arXiv:1512.02244 [hep-ex]

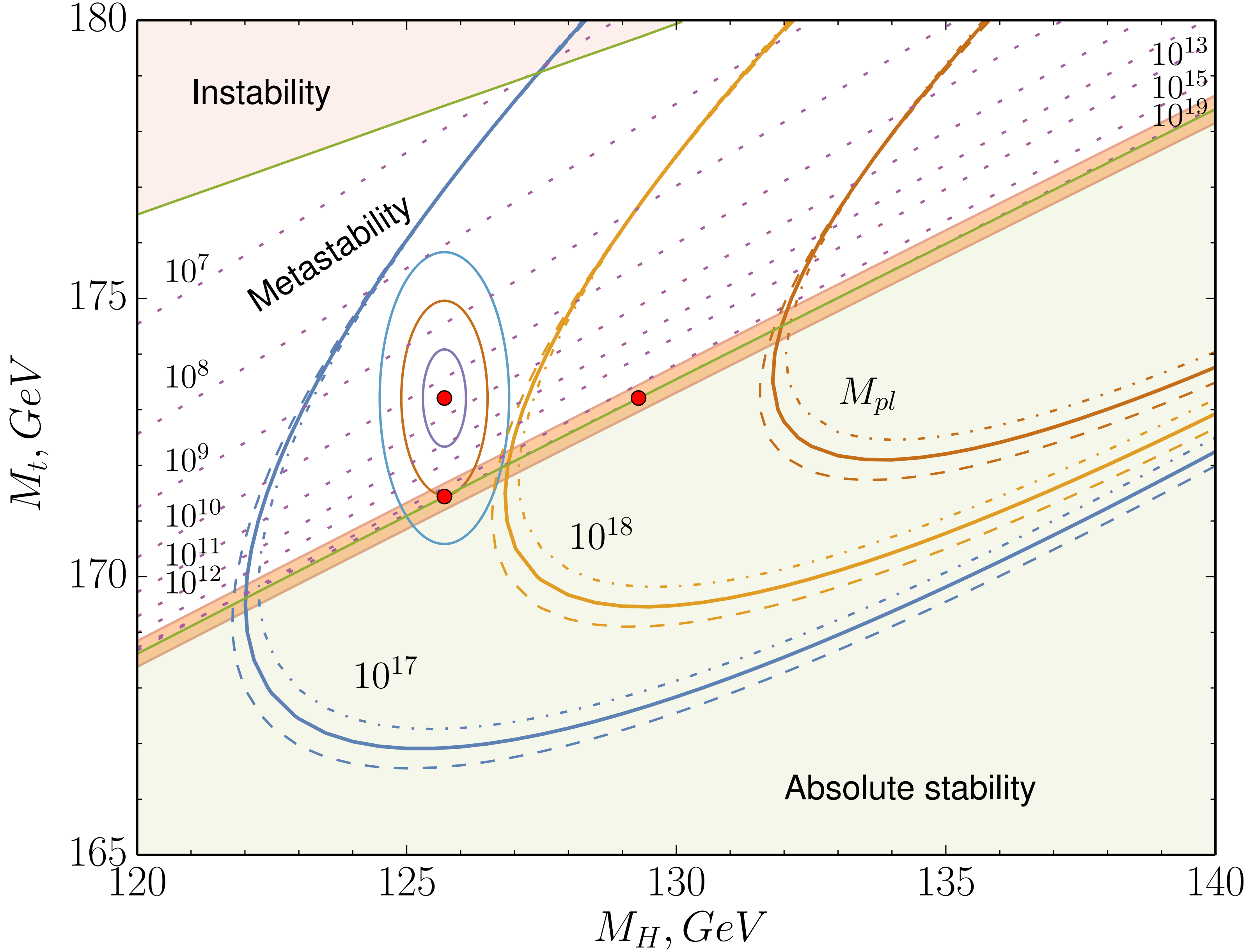
M_t Measurements



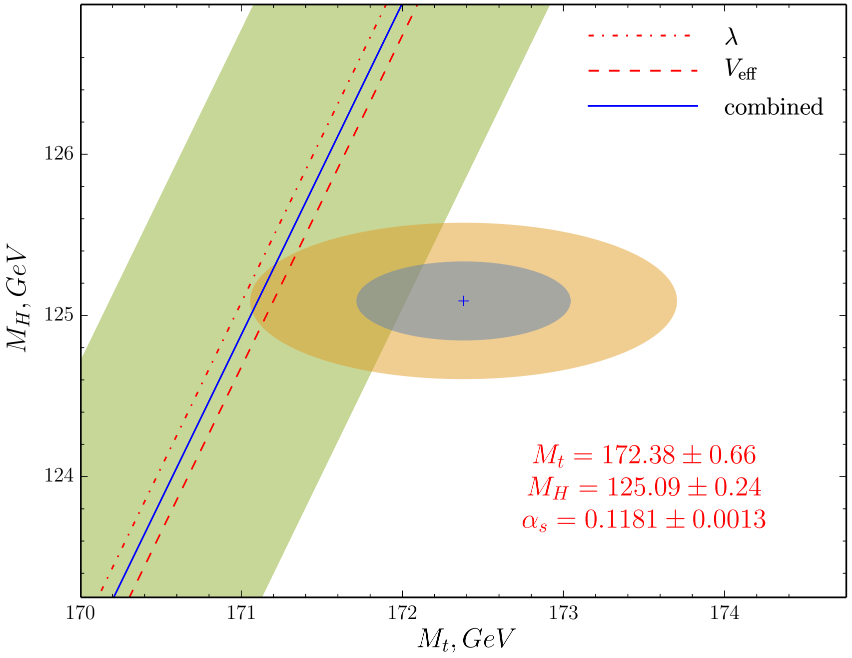
RG flow


λ running for PDG2014 input
10^{34}
10^{18}
10^{10}
10^{7}
10^{5}
10^{4}
10^{3}
critical M_h
$\beta_\lambda \cdot 10^3$
λ
$-3\sigma_{M_t}$
critical M_t
$+3\sigma_{M_h}$
$-3\sigma_{M_h}$
$+3\sigma_{M_t}$
10^{11}
$1.55 \cdot 10^{10}$
10^{9}

Phase diagram

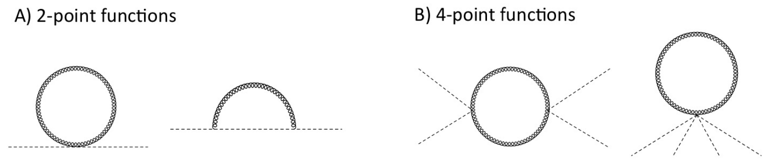


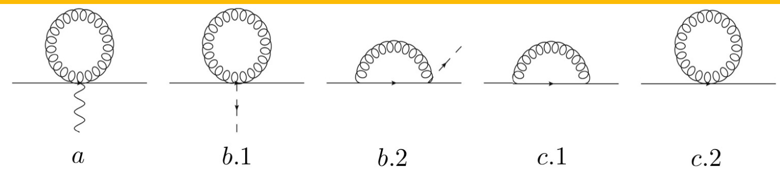
SM stable all the way up to M_P?

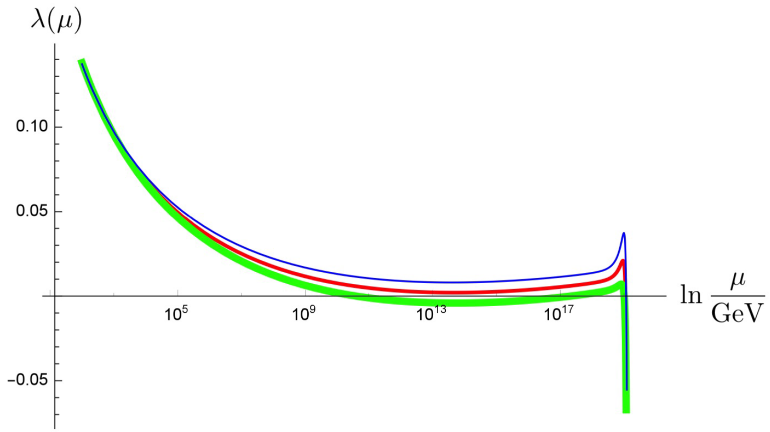


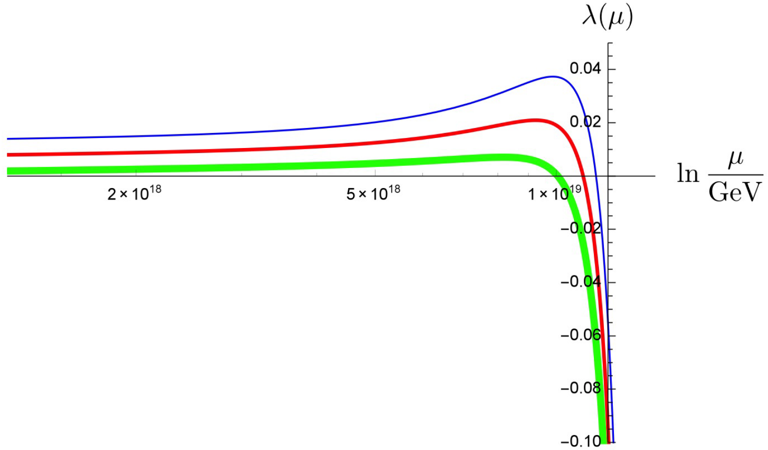
- Intriguing conspiracy of SM particle masses $\rightsquigarrow$ $\mu^{cri} \approx M_P = 1.22 \times 10^{18}$ GeV
- μ^{cri} stable w.r.t. parametric and higher-order uncertainties due to asymptotic safety
- Relationship between M_P and SM parameters?
- Electroweak scale determined by Planck scale physics?
- Implicit reduction of fundamental couplings?

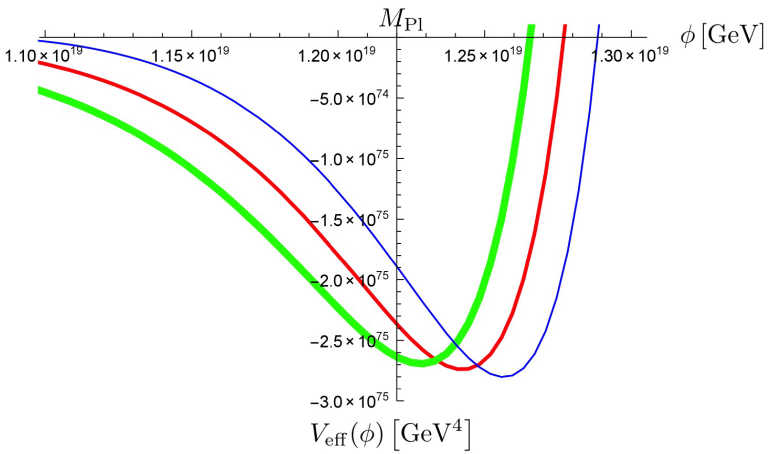
Einstein gravity loop corrections

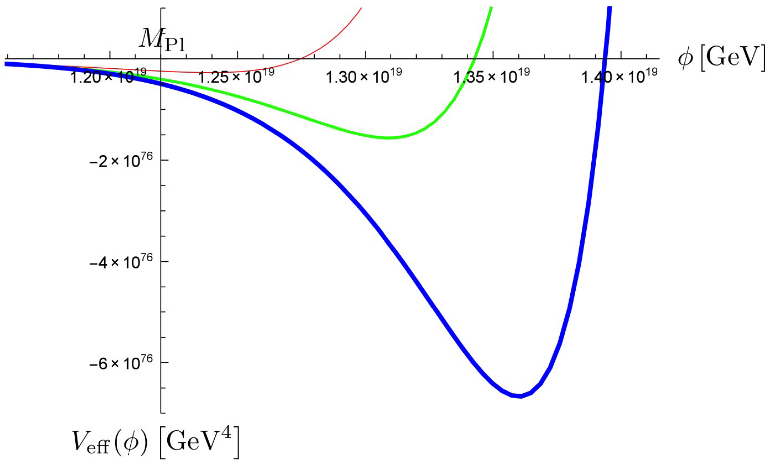








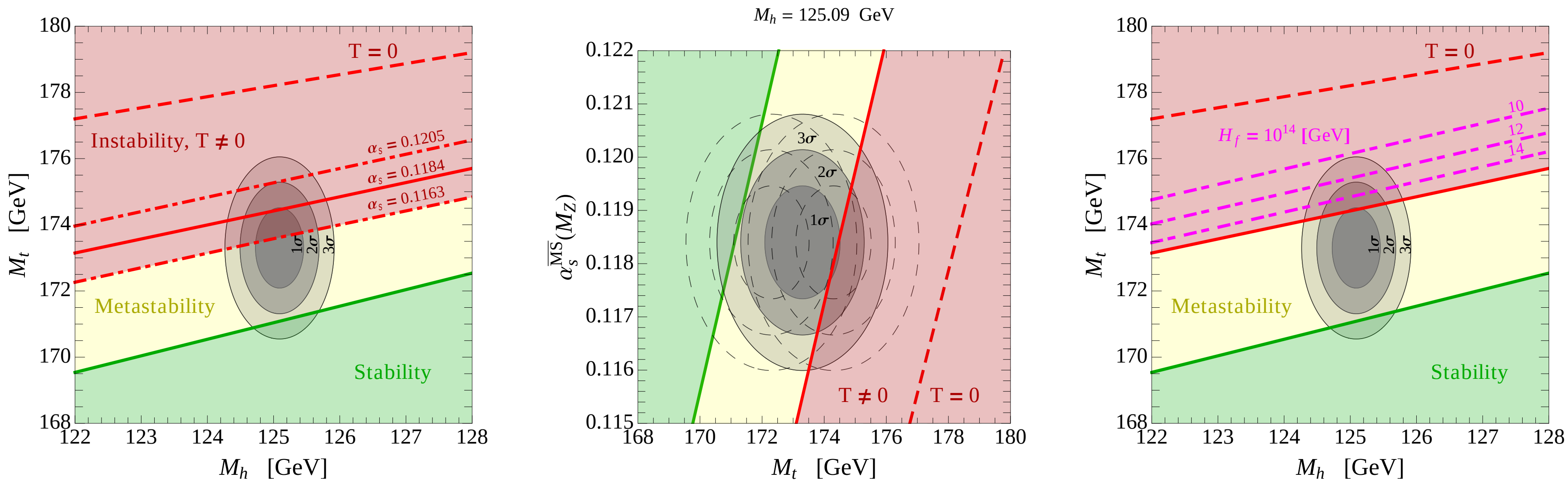


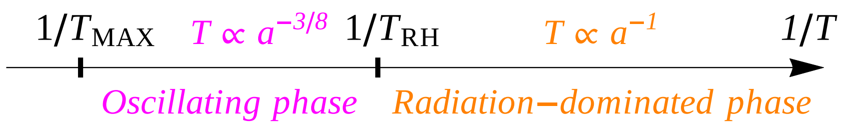


Quantum gravity corrections stabilize V_{eff}! Abe *et al.*, 1602.03792 [hep-ph]

Thermal corrections

Cut-off scale $\Lambda = 10^{19}$ GeV, *i.e.* $T_{cut} \approx 10^{18}$ GeV $\qquad$ T_{RH} dependence



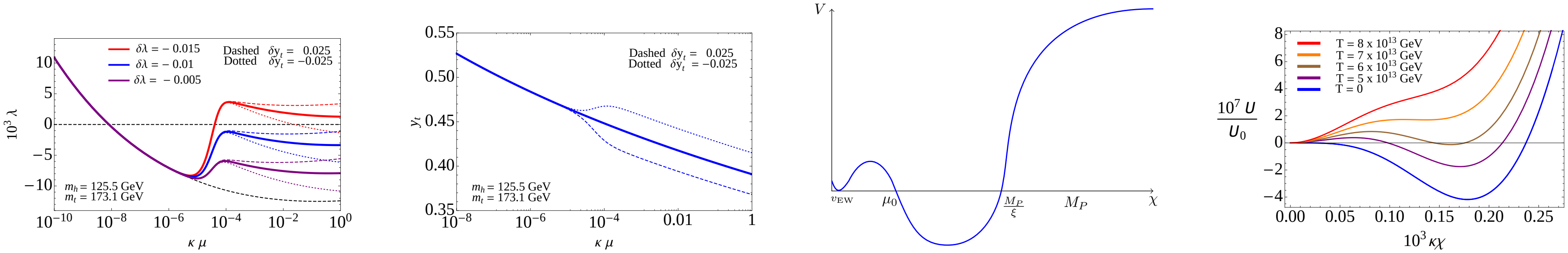


Finite-T corrections increase tunneling probability and expand instable phase!

Delle Rose *et al.*, 1507.06912 [hep-ph]

Higgs inflaton

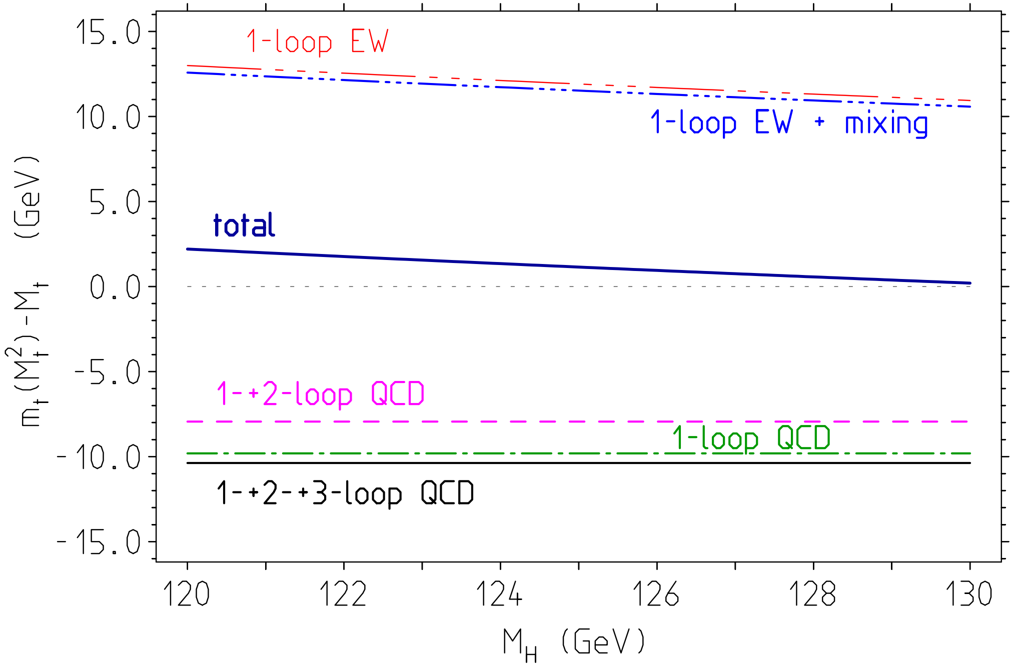
- Higgs field, nonminimally coupled to gravity with strength ξ, can be responsible for inflation
- Successful scenario possible even if EW vacuum is metastable
- Effective renormalization of SM couplings at scale M_P/ξ
- Symmetry restoration after inflation due to high-T effects temporarily eliminating vacuum at $H \approx M_P$



Bezrukov *et al.*, PRD92(2015)083512

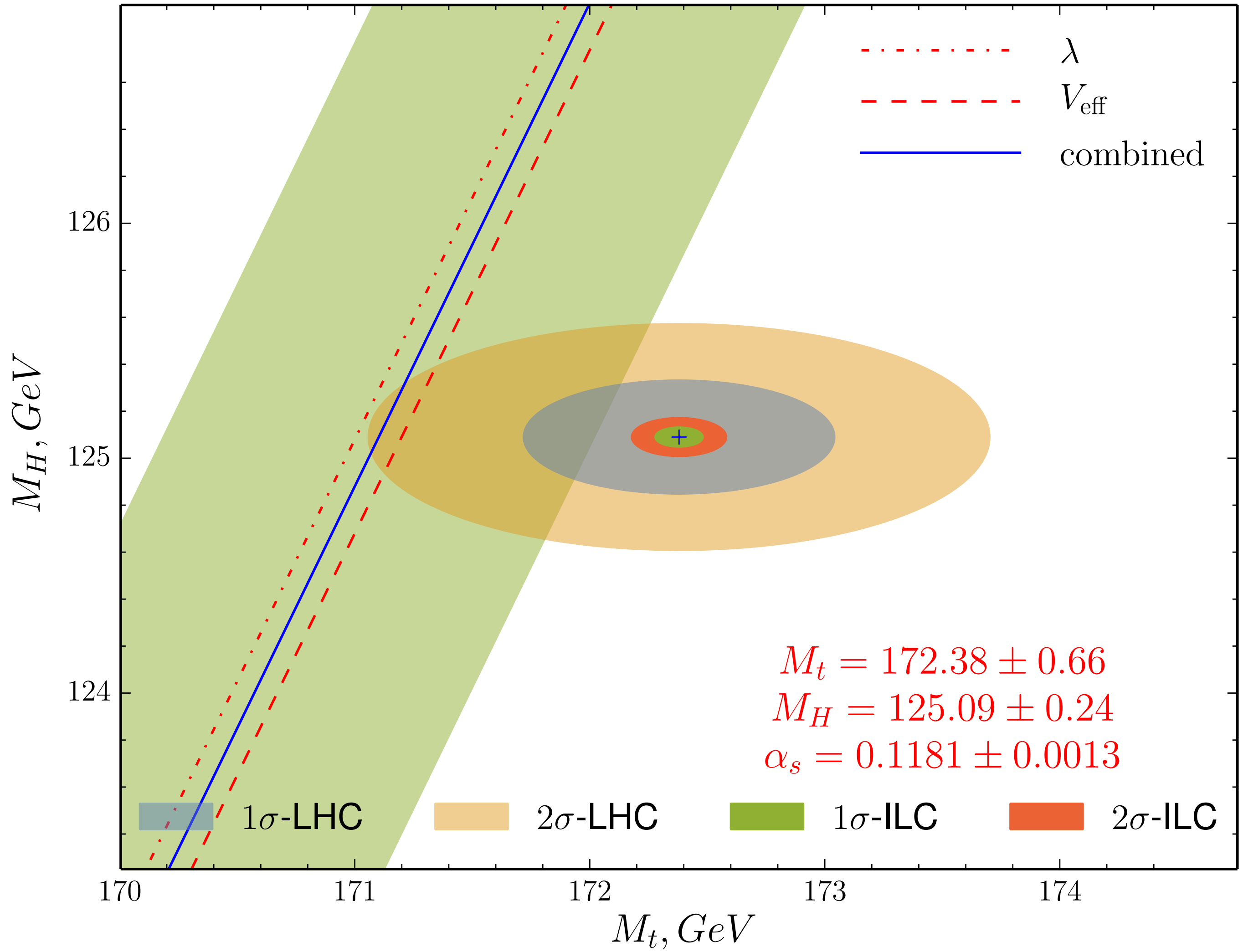
Outlook: pole mass M_t

- PDG value $M_X(t \to X) = (173.21 \pm 0.87)$ GeV is not pole mass M_t, but just parameter in MC programs w/o RC to partonic cross sections.
- Rigorous determination of $\overline{\text{MS}}$ mass $m_t(\mu)$ from $\sigma_{\text{tot}}(p\bar{p}, pp \to t\bar{t} + X)$
- $m_t(\mu) - M_t$ receives large EW RC from tadpole contributions.



Jegerlehner, Kalmykov, BK, PLB722(2013)123

ILC as top and Higgs factory



Anticipated errors $\delta M_t = 100$ MeV, $\delta M_H = 40$ MeV Moortgat-Pick *et al.*, EPJC75(2015)371

BSM physics

- Depending on future precision measurements of M_H, M_t, α_s and higher-loop RC calculations, SM may be stable all the way up to M_P.
- Reduction of $M_t \approx M_t^{\mathrm{MC}} = 172.38\ \mathrm{GeV}$ by $0.84\ \mathrm{GeV}$ $\rightsquigarrow$ $M_H^{\mathrm{cri}} = M_H = 125.09\ \mathrm{GeV}$
- *Cf.* $m_t - m_{\bar{t}} = (-0.6 \pm 0.6)\ \mathrm{GeV}$, $\Gamma_t = (2.0 \pm 0.5)\ \mathrm{GeV}$
- BSM physics still necessary to solve open problems, *e.g.*
  - smallness of neutrino masses
  - strong CP problem
  - dark matter
  - baryon asymmetry of universe
  - unification with gravity
- Higgs portals?