Loops and Legs 2016 (25 minutes)

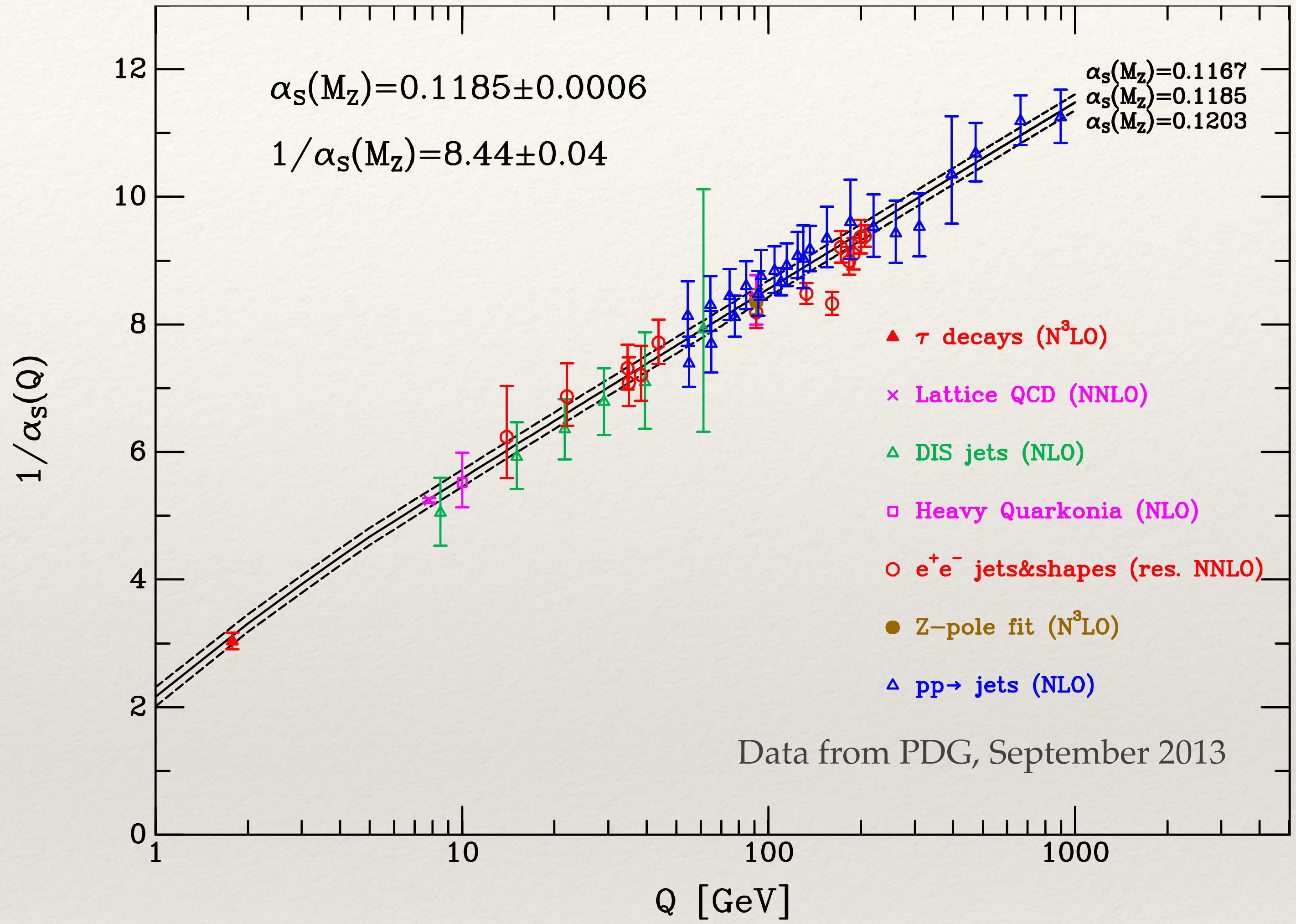
Results from slicing methods for NNLO calculations.

Keith Ellis, IPPP, Durham

Campbell, Ellis, Li, Williams,1602.0663
Campbell, Ellis, Li, Williams,1601.00658
Boughezal, Campbell, Ellis, Focke,Giele, Liu,Petriello, Williams (to appear)

The promise of perturbative QCD

- Incontrovertible fact that α_s is small at high energies.
- $\alpha_s(M_Z) \sim 0.1185+0.0006$
- NLO ~ 11%?
- NNLO~1%?



Also some outliers from thrust and C-parameter, Parton fits
Abate et al, 1060.3080, $\alpha s(M_Z)=0.1135+0.0010$
Hoang et al, 1501.04111,1501.04753, $\alpha s(M_Z)=0.1123\pm0.0015$
Alekhin et al, 0908.2766, $\alpha s(M_Z)=0.1135\pm0.0014$

The reality

- The reality for the Higgs cross section (known to N^{3}LO) is less optimistic.
- Consequence of the special nature of renormalisation group improved perturbation theory.
- At least NNLO is needed.

Anastasiou et al, 1602.00695

$$\boxed{\sigma = 48.58\,\mathrm{pb}^{+2.22\,\mathrm{pb}\,(+4.56\%)}_{-3.27\,\mathrm{pb}\,(-6.72\%)}\ (\mathrm{theory}) \pm 1.56\,\mathrm{pb}\,(3.20\%)\ (\mathrm{PDF}+\alpha_s)\,.}$$

$$\begin{aligned}
48.58\,\mathrm{pb} = \quad & 16.00\,\mathrm{pb} && (+32.9\%) && (\mathrm{LO,\ rEFT}) \\
& +20.84\,\mathrm{pb} && (+42.9\%) && (\mathrm{NLO,\ rEFT}) \\
& -\ \ 2.05\,\mathrm{pb} && (-4.2\%) && ((t,b,c),\ \mathrm{exact\ NLO}) \\
& +\ \ 9.56\,\mathrm{pb} && (+19.7\%) && (\mathrm{NNLO,\ rEFT}) \\
& +\ \ 0.34\,\mathrm{pb} && (+0.2\%) && (\mathrm{NNLO},\ 1/m_t) \\
& +\ \ 2.40\,\mathrm{pb} && (+4.9\%) && (\mathrm{EW,\ QCD\text{-}EW}) \\
& +\ \ 1.49\,\mathrm{pb} && (+3.1\%) && (\mathrm{N^3LO,\ rEFT})
\end{aligned}$$

σ=16pb x(1+1.30+0.59+0.09)

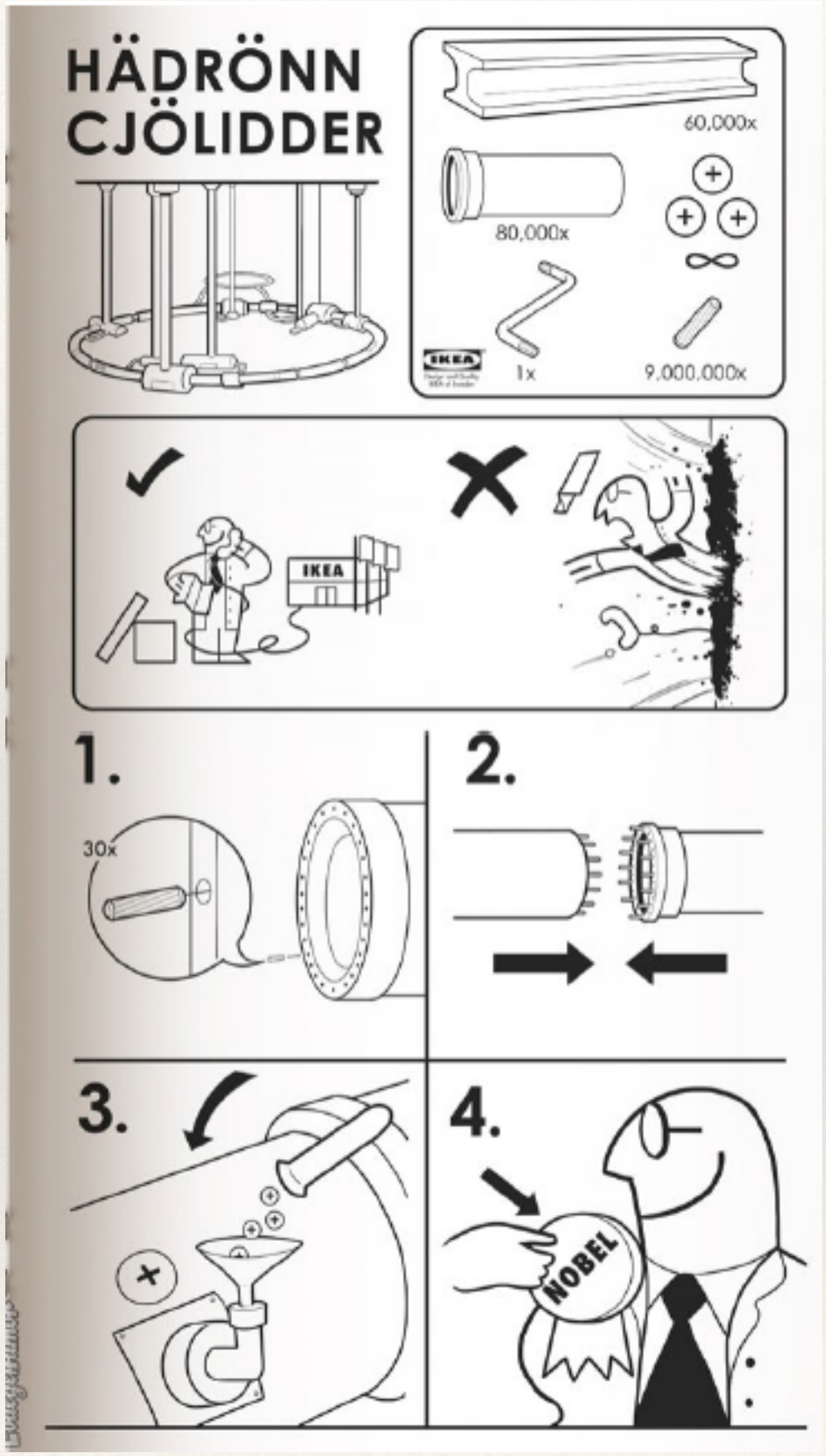
Processes currently known through NNLO

H+0jet	fully inclusive N^3LO	Higgs couplings	1503.06056
H+1jet	exclusive	Higgs couplings	1604.04085,1408.5325,1504.07922, 1505.03893
WBF	exclusive VBF cuts	Higgs couplings	1506.02660
H->bb	exclusive, massless	Higgs couplings boosted	1110.2368,1501.07226
W+0jet	fully exclusive, decays	PDFs	0903.2120,1208.5967
Z/gamma+0jet	fully exclusive, decays	PDFs	0903.2120,1208.5967
W+j	fully exclusive, decays	PDFs	1504.02131
Z+j	decay, off-shell effects	PDFs	1601.04569,1507.20850, 1507.02850
ZH	decays to bb at NLO	Higgs couplings	1407.4747,1601.00658
WH	fully exclusive	Higgs couplings	1312.1669, 1601.00658
ZZ	fully exclusive, off-shell	trilinear gauge couplings,BSM	1405.2219, 1507.06257,1509.06734
WW	fully inclusive	trilinear gauge couplings,BSM	1408.5243,1511.08617
$W\gamma$,$Z\gamma$	fully exclusive	trilinear gauge couplings,BSM	1601.06751
$\gamma\gamma$	fully differential	Background studies	1110.2375,1603.02663
tt pair	fully exclusive, stable tops	top cross section ,mass pt, FB asymmetry,PDFs BSM	1601.05375, 1506.04037
single top	fully exclusive, stable tops, t-channel	Vtb,width, PDfs	1404.7116
top decay	exclusive	Top couplings	1210.2808, 1301.7133
dijets	gluon-gluon	PDFs,strong couplings,BSM	1407.5558

Adapted from K. Melnikov, Aspen Winter Conference 2016

Some assembly required

- Several techniques for assembly of fully differential NNLO cross sections have reached maturity in the last few years.
- Antenna Subtraction
- Sector decomposition
- Phase space mapping
- Non-local QT/SCET-based slicing methods, separating phase space into single and double unresolved regions and make use of a factorisation theorem for the latter.



Slicing methods

- Slicing methods fell out of favour at NLO because of lack of local cancellation.
- We take a second look because they:-
  - Mesh well with existing NLO calculations
  - It could be that with the increase in computing power, large numerical cancellations can be handled.

For colour neutral final states the transverse momentum of the recoiling EW particles can be used to separate the double and singly unresolved regions of phase space. (Catani Grazzini 07)

$$\sigma_{NNLO} = \int dq_T \frac{d\sigma}{dq_T} \theta(q_T^{cut} - q_T) + \int dq_T \frac{d\sigma}{dq_T} \theta(q_T - q_T^{cut})$$

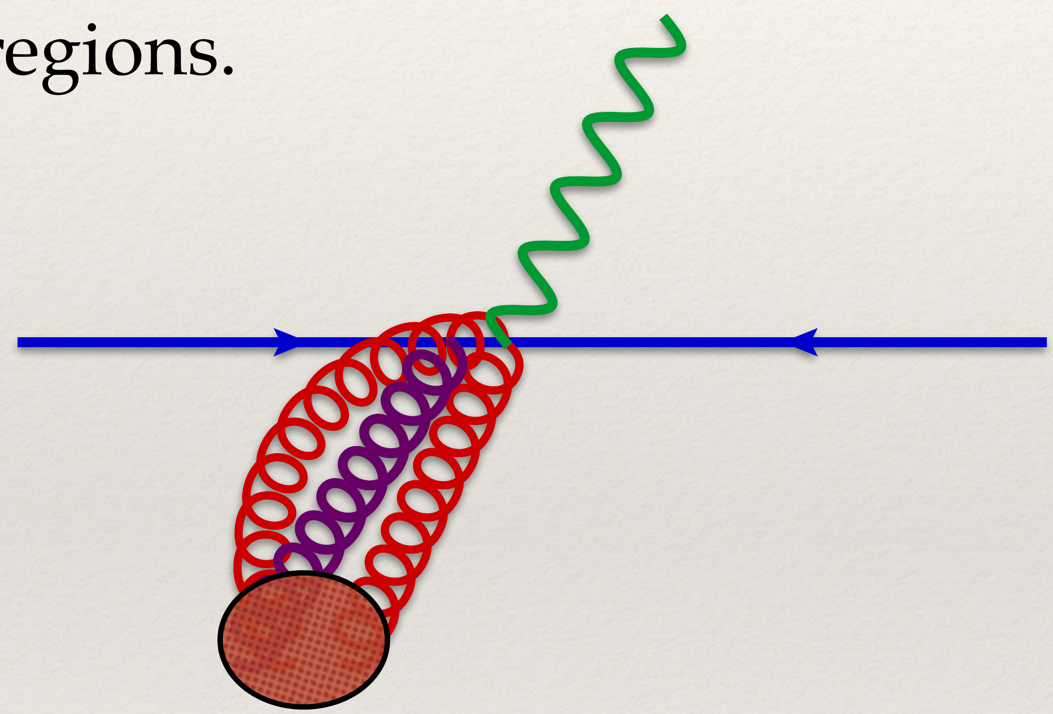
Obtained from the Collins-Soper-Sterman factorization theorem for small q_T

This is an NLO cross section for one additional parton

MCFM contains an extensive library of NLO processes, which in this approach are a component of the NNLO process with one fewer parton.

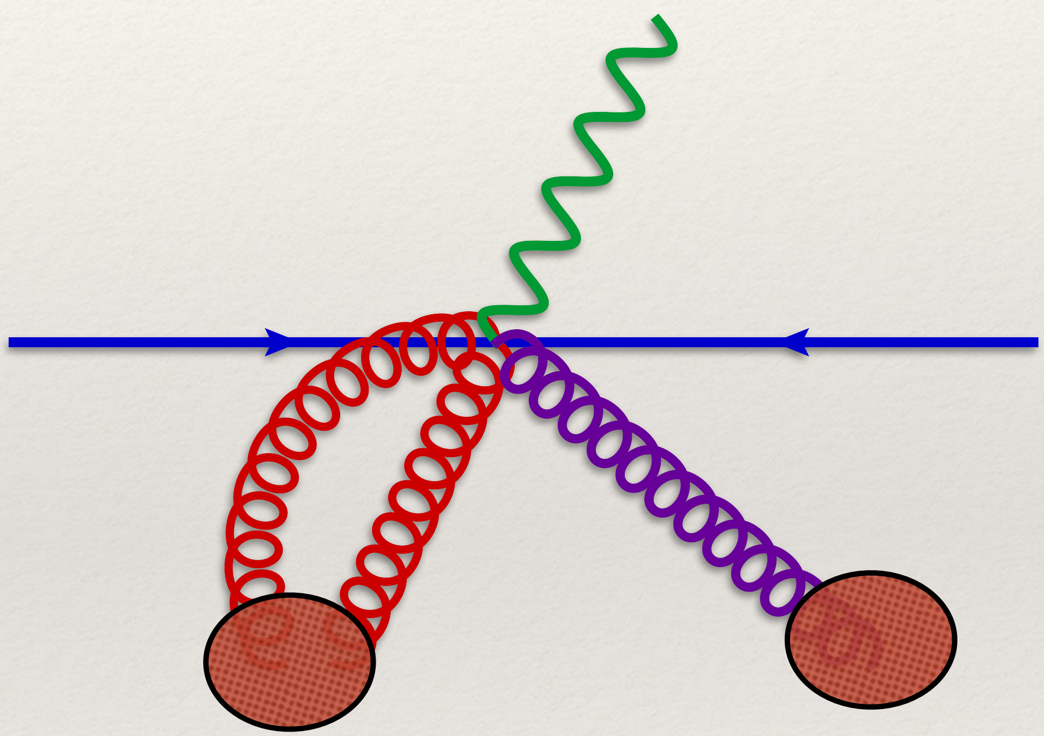
Extension to coloured final states

The idea is to use the event shape variable N-jettiness (Stewart, Tackmann, Waalewijn 2009) to separate the phase space into two regions (Boughezal, Liu, Petriello 2015, Gaunt, Stahlhofen, Tackmann Walsh 2015) which separates the doubly-from singly unresolved regions.



Doubly unresolved

Small N-jettiness, use factorization theorem.



Singly unresolved

“Large” N-jettiness, what is required is an NLO calculation. Can use existing tools, like MCFM

N-jettiness

N-jettiness is an event shape variable, designed to classify final state jets (Stewart, Tackmann, Waalewijn 09)

$$\mathcal{T}_N(\Phi_M) = \sum_{k=1}^{M} \min_i \left\{ \frac{2q_i \cdot p_k}{Q_i} \right\}$$

Number of final state jets

Number of final state partons

All final state partons

A hard scale (e.g. Energy of jets)

Momentum of final state jets and beams

We need to understand the below cut region for the method to be applied. We use the factorization theorem (Stewart, Tackmann, Waalewijn 09), based upon SCET.

Beam functions, describes radiation collinear to initial state

Jet functions, describes radiation collinear to final state jets

$$\sigma(\tau_N < \tau_N^{cut}) = \int H \otimes B \otimes B \otimes S \otimes \left[\prod_n^N J_n\right] + \mathcal{O}(\tau_N^{cut})$$

Process dependent hard function, includes 2-loop virtual

Soft function, describes soft radiation

- B@NNLO : Gaunt, Stahlhofen, Tackmann (14) …
- S@NNLO : Boughezal, Liu, Petriello (14) ….
- J@NNLO:Becher, Neubert (06), Becher, Bell (11) …
- H@NNLO: Derived from two loop virtual corrections

In order to test our implementation of the SCET N-jettiness routine we establish cross checks using the following public codes for the total cross sections for color singlet production.

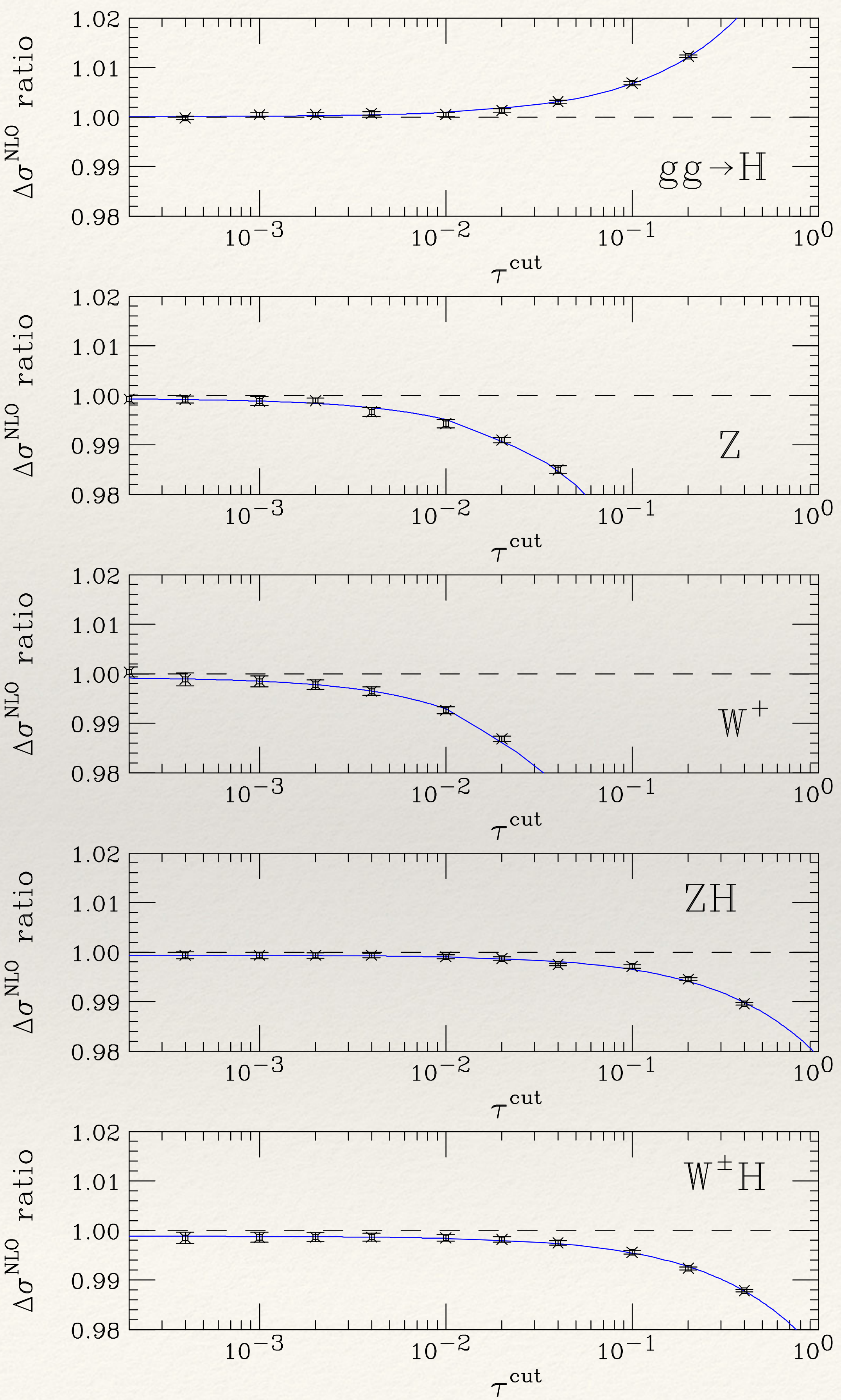
Process	μ_R	μ_F	Cross-section to NNLO	Reference
$gg \to H$	M_H	M_H	$12.937 \times (1 + 1.28 + 0.77)$ pb	ggh@nnlo [65]
Z	$2M_Z$	$M_Z/2$	$44.303 \times (1 + 0.22 + 0.05)$ nb	ZWMS [66]
W^+	$2M_W$	$M_W/2$	$81.561 \times (1 + 0.23 + 0.06)$ nb	ZWMS [66]
ZH	$\sqrt{q^2}$	$\sqrt{q^2}$	$0.68255 \times (1 + 0.16 + 0.10)$ pb	vh@nnlo [67, 68]
$W^+H + W^-H$	$\sqrt{q^2}$	$\sqrt{q^2}$	$1.2593 \times (1 + 0.16 + 0.02)$ pb	vh@nnlo [67, 68]

[65] R. V. Harlander and W. B. Kilgore, *Higgs boson production in bottom quark fusion at next-to-next-to leading order*, *Phys. Rev.* **D68** (2003) 013001, [hep-ph/0304035].

[66] R. Hamberg, W. L. van Neerven, and T. Matsuura, *A Complete calculation of the order α_s^2 correction to the Drell-Yan K factor*, *Nucl. Phys.* **B359** (1991) 343–405. [Erratum: Nucl. Phys.B644,403(2002)].

[67] O. Brein, A. Djouadi, and R. Harlander, *NNLO QCD corrections to the Higgs-strahlung processes at hadron colliders*, *Phys. Lett.* **B579** (2004) 149–156, [hep-ph/0307206].

[68] O. Brein, R. V. Harlander, and T. J. E. Zirke, *vh@nnlo - Higgs Strahlung at hadron colliders*, *Comput. Phys. Commun.* **184** (2013) 998–1003, [arXiv:1210.5347].

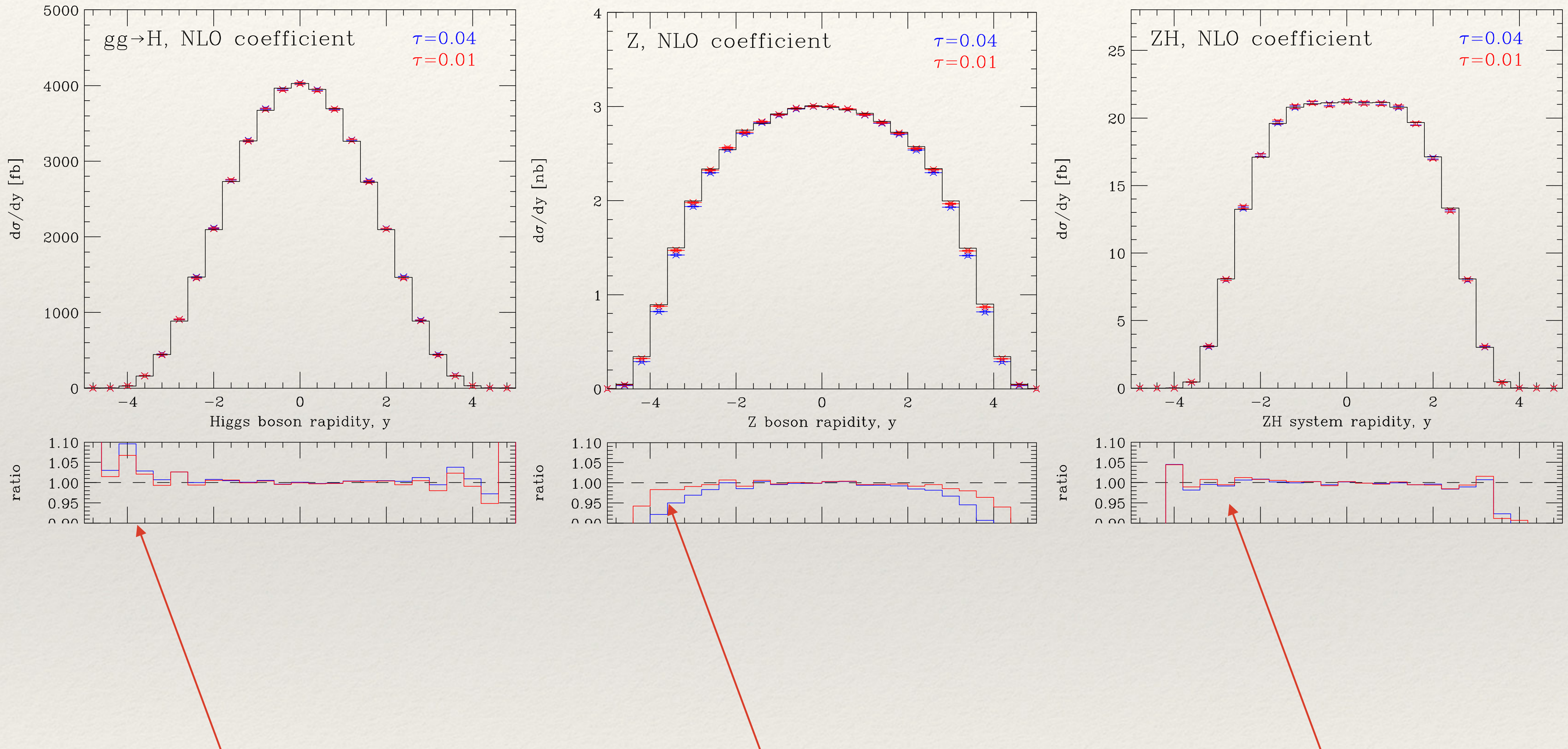


We first establish the validity of our comparison by using standard MCFM to reproduce NLO results from other codes.

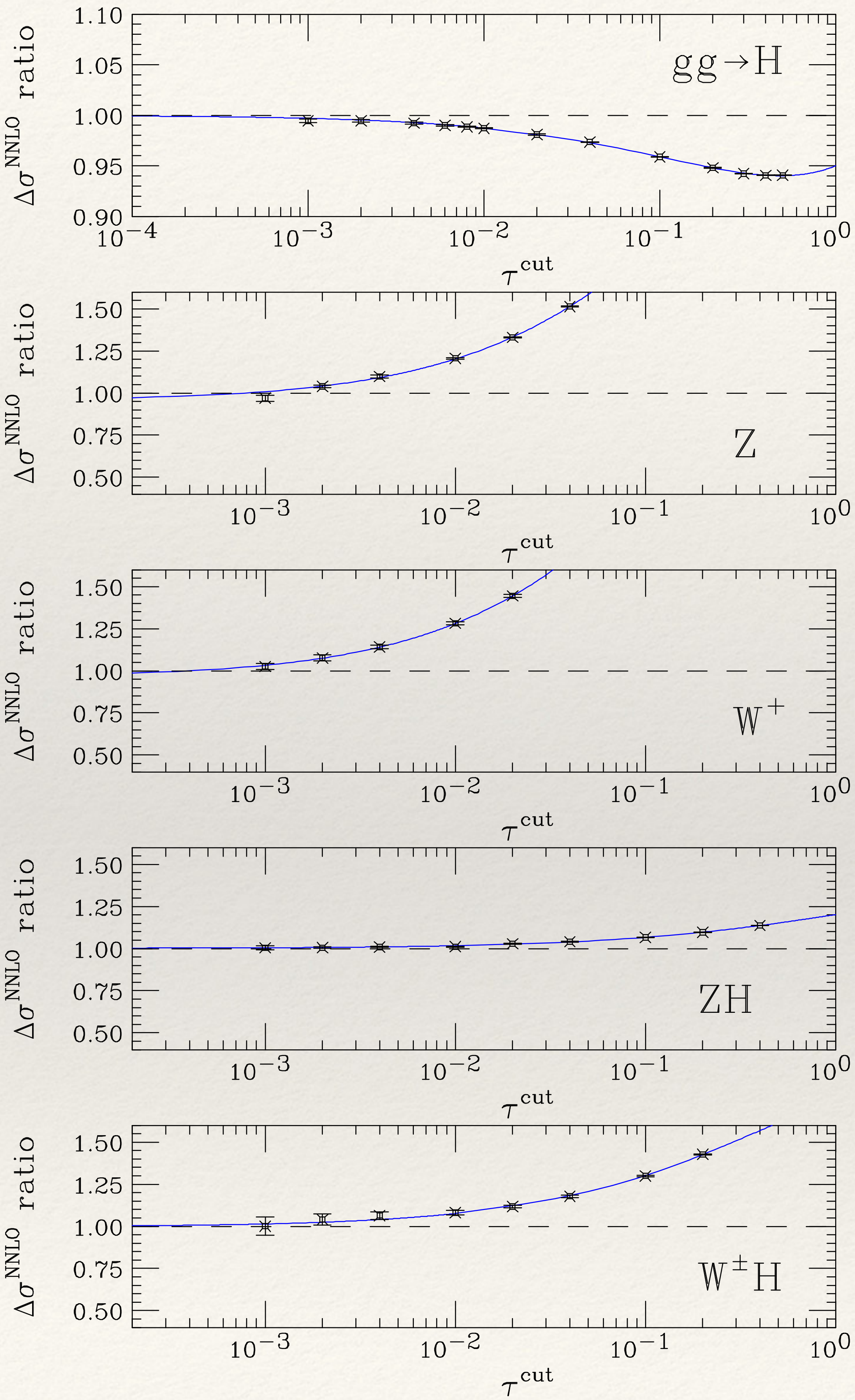
Process	Order	MCFM cross-section	Cross-check
H production	LO	12.937 ± 0.001 pb	12.937 pb
	NLO	29.520 ± 0.001 pb	29.521 pb
Z	LO	44.303 ± 0.001 nb	44.303 nb
	NLO	53.958 ± 0.002 nb	53.957 nb
W^+	LO	81.559 ± 0.002 nb	81.561 nb
	NLO	100.298 ± 0.003 nb	100.299 nb
ZH	LO	0.68254 ± 0.00001 pb	0.68255 pb
	NLO	0.79073 ± 0.00003 pb	0.79079 pb
$W^+H + W^-H$	LO	1.2592 ± 0.02 pb	1.2593 pb
	NLO	1.4629 ± 0.04 pb	1.4630 pb

We calculate the cross sections at NLO with Catani-Seymour Dipoles as validation.

We can then turn our attention to differential predictions (still at NLO) and see the impact of the tau dependence differentially.



Dependence on the tau cutoff becomes more acute for larger values of the rapidity.



Next we study the dependence at NNLO for the inclusive cross sections.

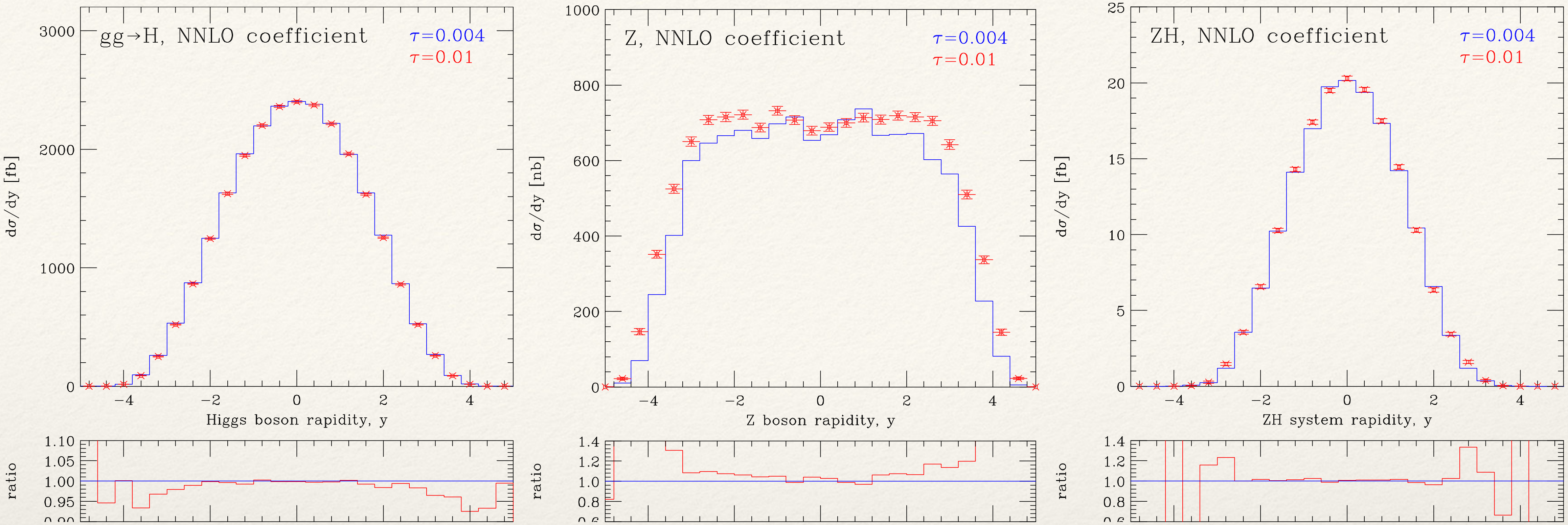
The dependence on tau can be formulated as follows

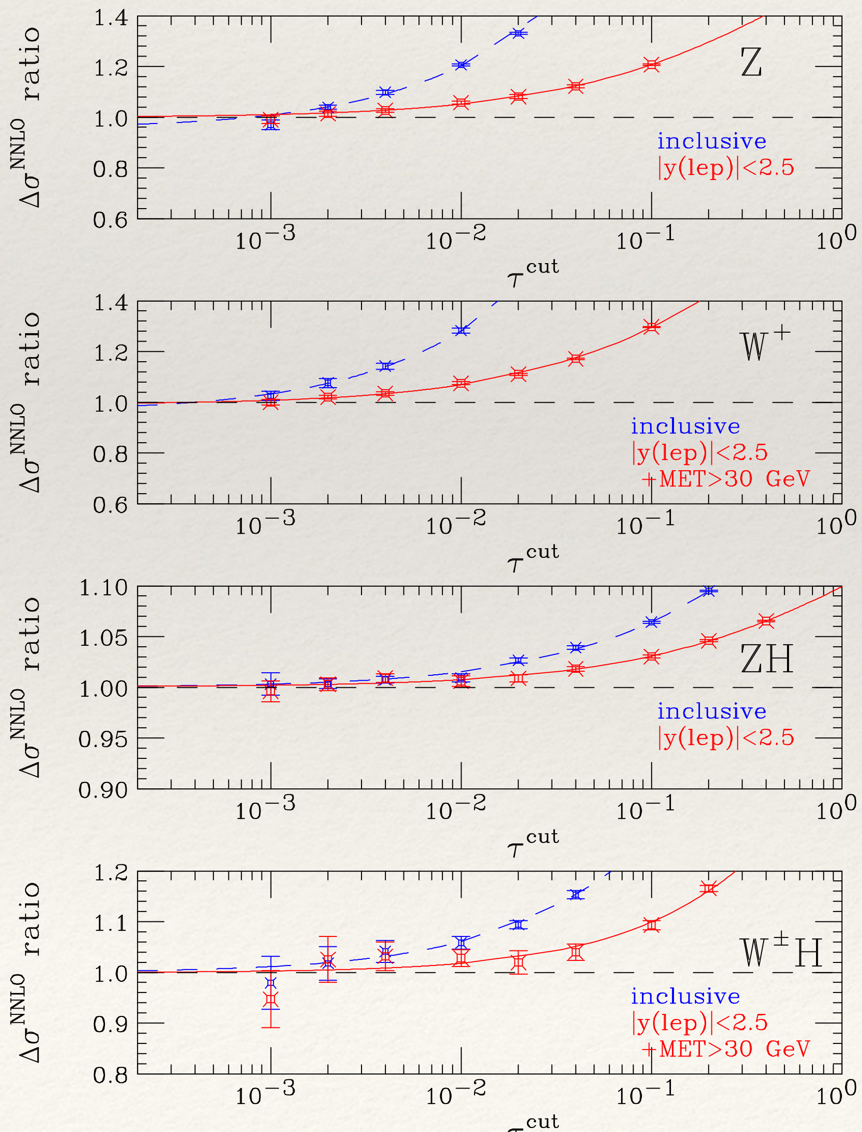
$$\Delta\sigma^{NNLO}_{\text{jettiness}}(\tau^{\text{cut}}) = \Delta\sigma^{NNLO} + c_3 \left(\frac{\tau^{\text{cut}}}{Q}\right) \log\left(\frac{\tau^{\text{cut}}}{Q}\right)^3 + c_2 \left(\frac{\tau^{\text{cut}}}{Q}\right) \log\left(\frac{\tau^{\text{cut}}}{Q}\right)^2 ,$$

Note, if desired a tau->0 extrapolation can be made (even at reasonably large tau).

Note also the change in scale from the NLO comparisons.

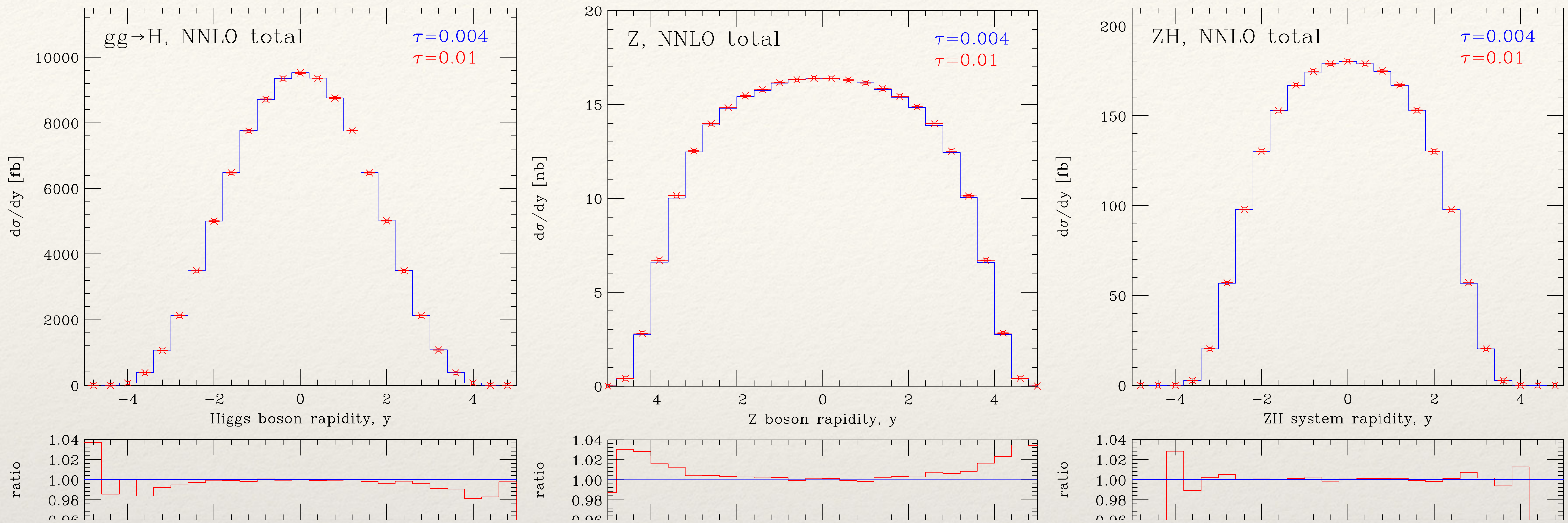
O(10%) corrections are more prevalent at NNLO for larger tau





The issues at large rapidity are also more pronounced at NNLO, leading to substantial tau dependence at larger rapidity (especially for DY)

Cuts on the rapidity of final state particles (relevant for phenomenology) improve the situation substantially.



Process		$\Delta\sigma^{NNLO}$		σ^{NNLO}	
		10% accuracy	2% accuracy	1% accuracy	0.2% accuracy
$gg \to H$	inclusive	1	0.02	0.03	0.002
Z	inclusive	0.005	0.001	0.01	0.002
	lep. cuts	0.03	0.002	0.07	0.005
W^+	inclusive	0.003	0.0008	0.005	0.001
	lep. cuts	0.02	0.002	0.03	0.003
ZH	inclusive	0.2	0.01	0.3	0.02
	lep. cuts	1	0.05	0.8	0.04
$W^\pm H$	inclusive	0.01	0.002	0.2	0.01
	lep. cuts	0.1	0.01	0.8	0.08

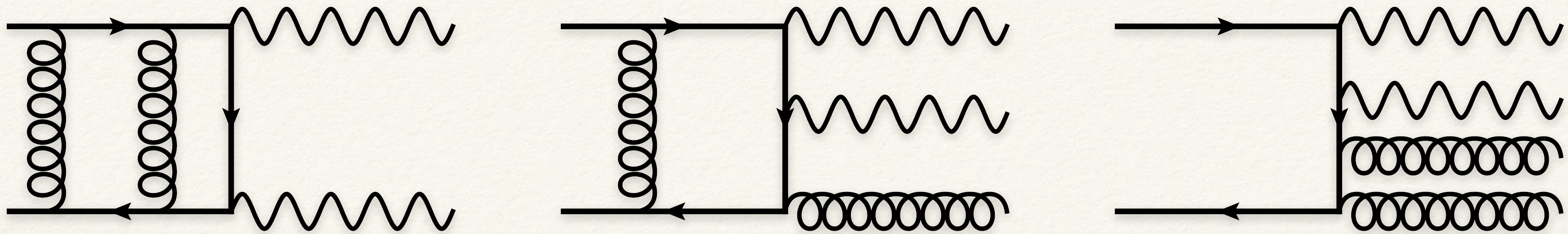
When combined with the NLO (computed with dipoles) the total tau dependence is suppressed.

This method is acceptable for phenomenological applications.

Recipe for values of tau_cut to achieve a given accuracy

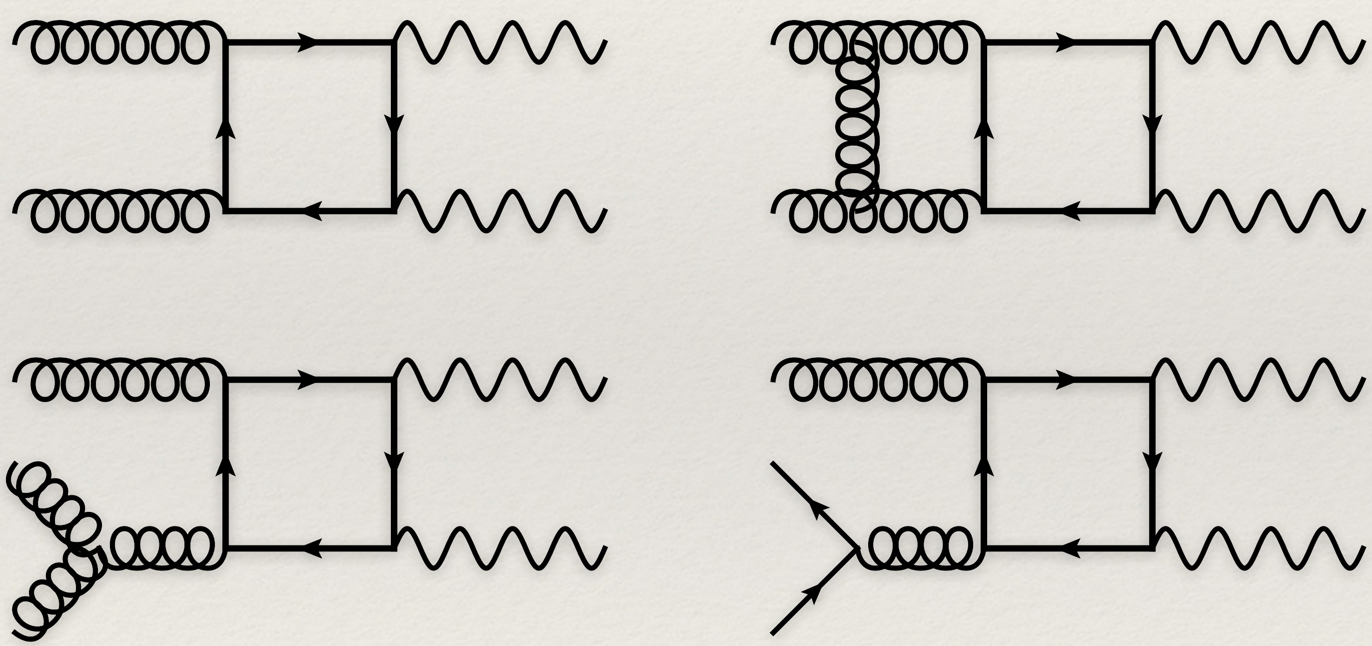
$\gamma\gamma$@NNLO

Campbell, Williams 14



(Anastasiou, Glover, Tejeda-Yeomans 02)

$$\sigma^{\mathrm{NNLO}} + \Delta\sigma^{\mathrm{N3LO}}_{gg,n_F}$$



Aside from the regular $q\bar{q}$ NNLO topologies, there are interesting effects from gg initiated pieces too.

gg@NLO was calculated first by (Bern, De Freitas Dixon 01), (Bern, Dixon, Schmidt 02)

One other (pure NNLO) calculation in the literature,

(S. Catani, L. Cieri, D. de Florian, G. Ferrera and M. Grazzini 1110.2375)

$\gamma\gamma$-cross section comparison

2γNNLO

σ[fb]	LO	NLO	NNLO
$\mu_F = \mu_R = m_{\gamma\gamma}/2$	5045 ± 1	26581 ± 23	45588 ± 97
$\mu_F = \mu_R = m_{\gamma\gamma}$	5712 ± 2	26402 ± 25	43315 ± 54
$\mu_F = \mu_R = 2m_{\gamma\gamma}$	6319 ± 2	26045 ± 24	41794 ± 77

Catani et al , 1110.2375

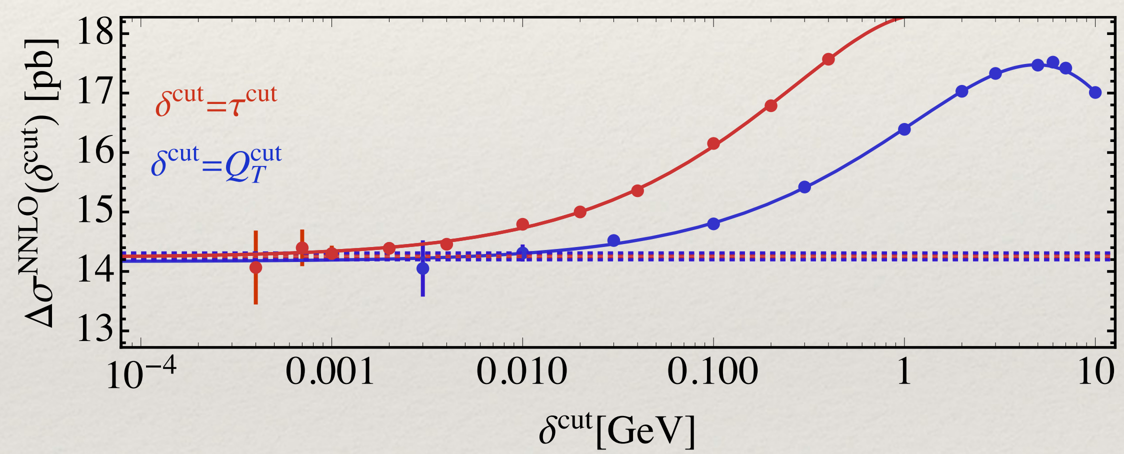
MCFM

σ[fb]	LO	NLO	NNLO
$\mu_F = \mu_R = m_{\gamma\gamma}/2$	5043 ± 1	26578 ± 13	42685 ± 35
$\mu_F = \mu_R = m_{\gamma\gamma}$	5710 ± 1	26444 ± 12	40453 ± 30
$\mu_F = \mu_R = 2m_{\gamma\gamma}$	6315 ± 2	26110 ± 13	38842 ± 27

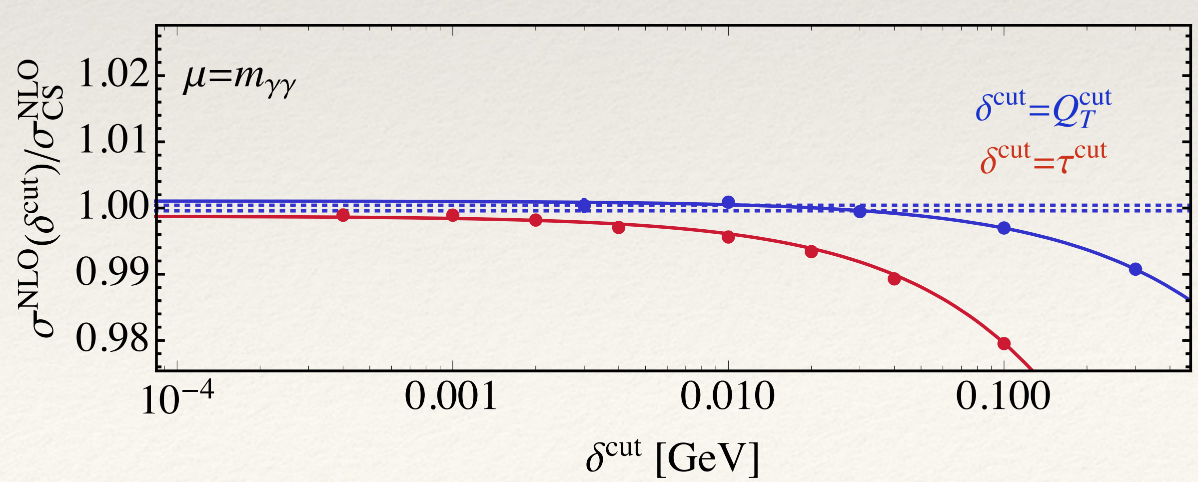
Campbell et al ,1603.02663

We disagree with the existing published result (PRL.108.072001 and 1110.2375) by around 8% (~ LO/2). We have been informed by Catani et al. that they had discovered a bug in the numerical program used to produce the numbers in their PRL paper, but that they had neither issued an erratum nor corrected the paper on the ArXiV

NNLO coefficient

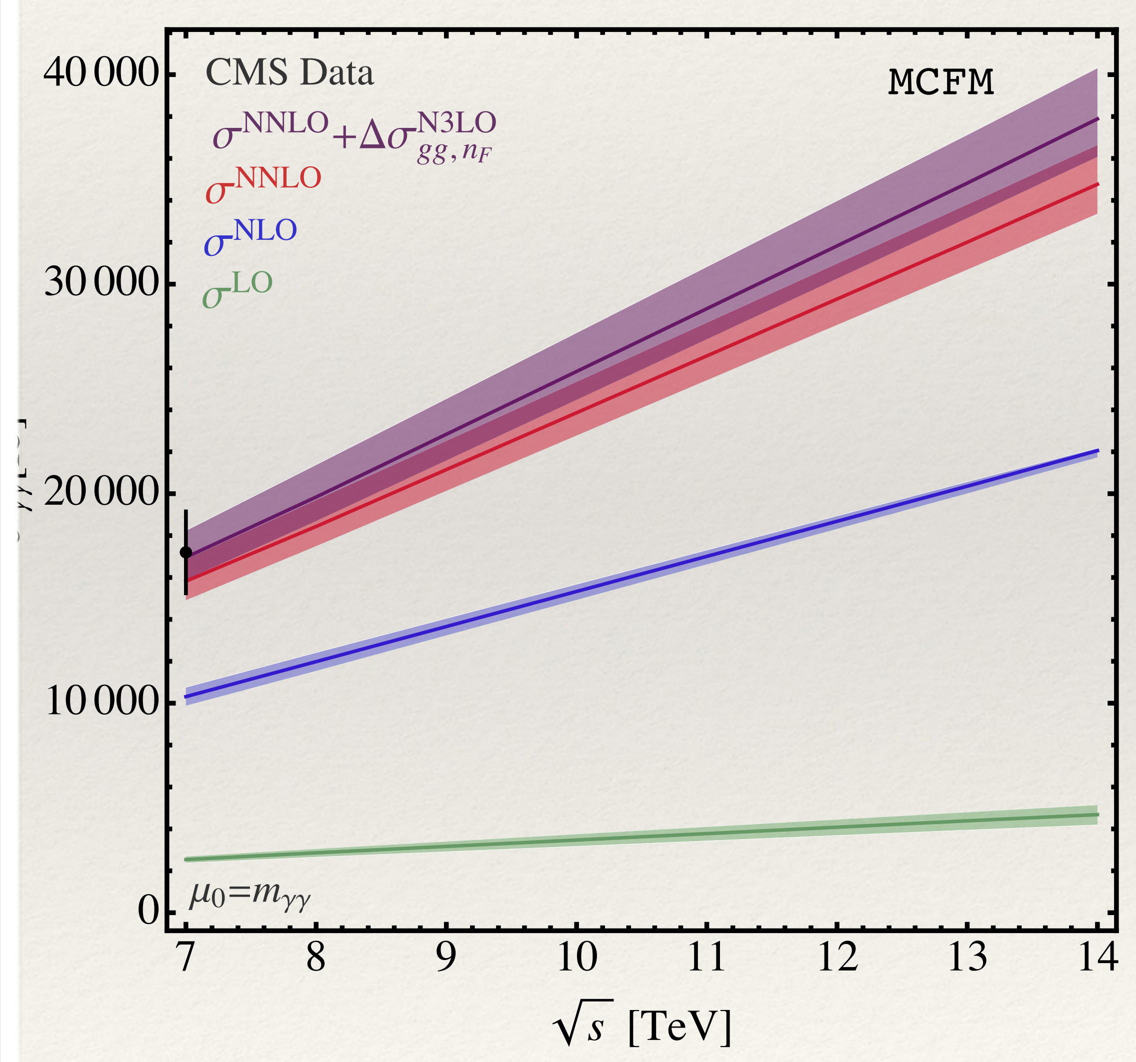


NLO Cross section

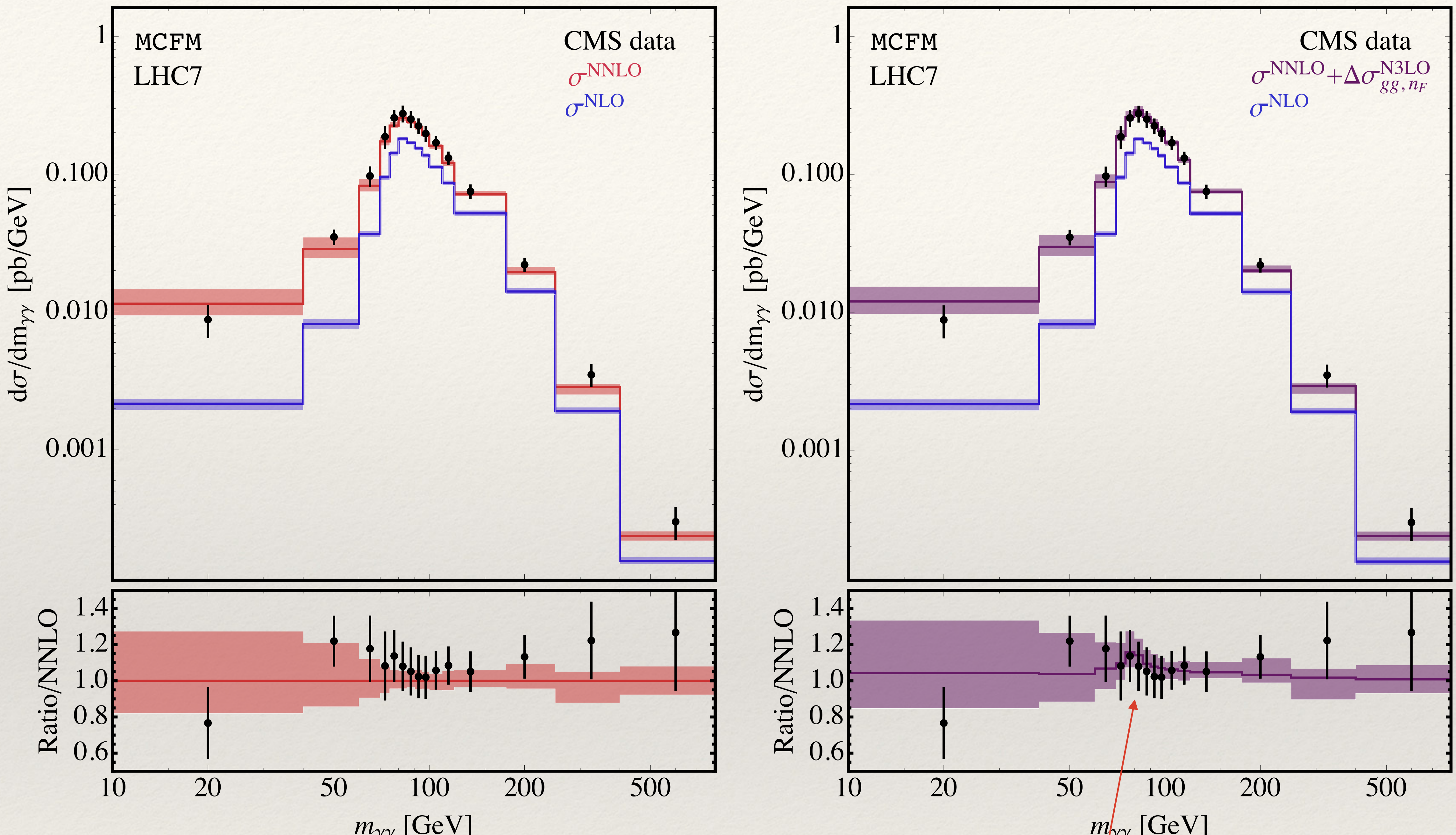


Cross sections

- It is interesting to compare NNLO with NNLO+gg@NLO
- At 7 TeV, given the experimental error, it is hard to distinguish which is better.
- At 13 TeV, predictions of NNLO and NNLO+gg@NLO are distinct
- It would be interesting to see which is the better prediction.



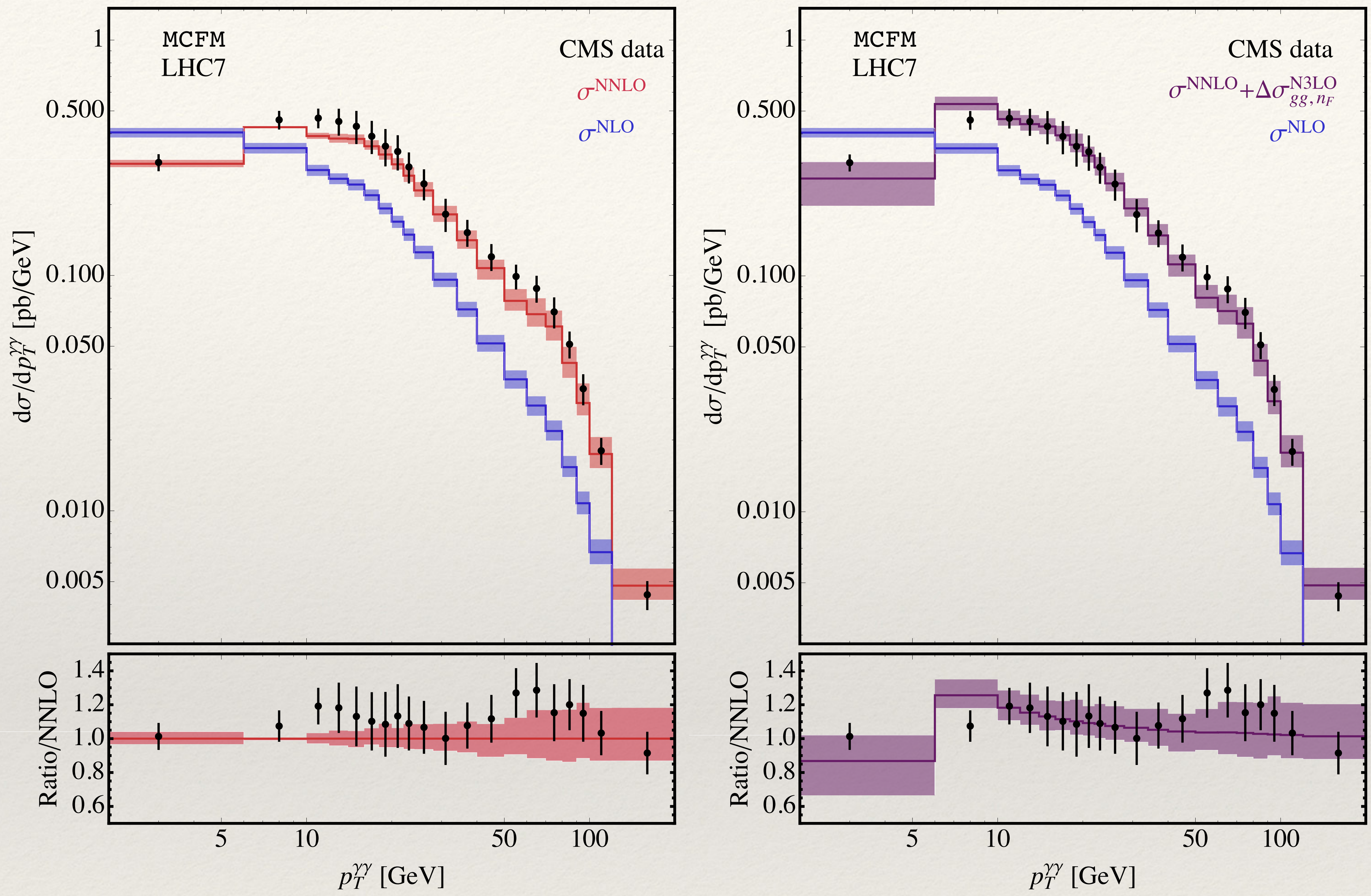
Differential predictions: Invariant masses



Out of the box NNLO does a very nice job of describing CMS 7 TeV Data

Looks like adding in additional gluon pieces helps in this region

Differential Predictions $p_T^{\gamma\gamma}$

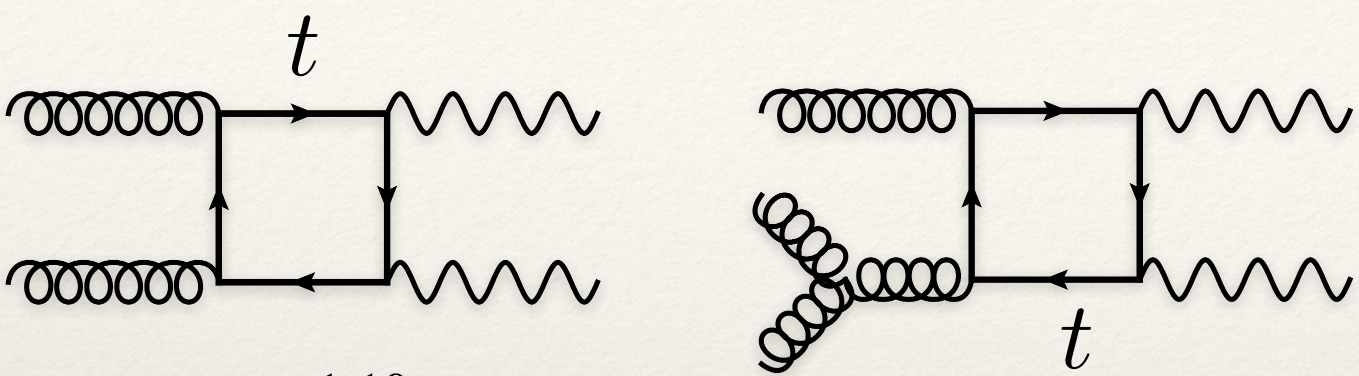


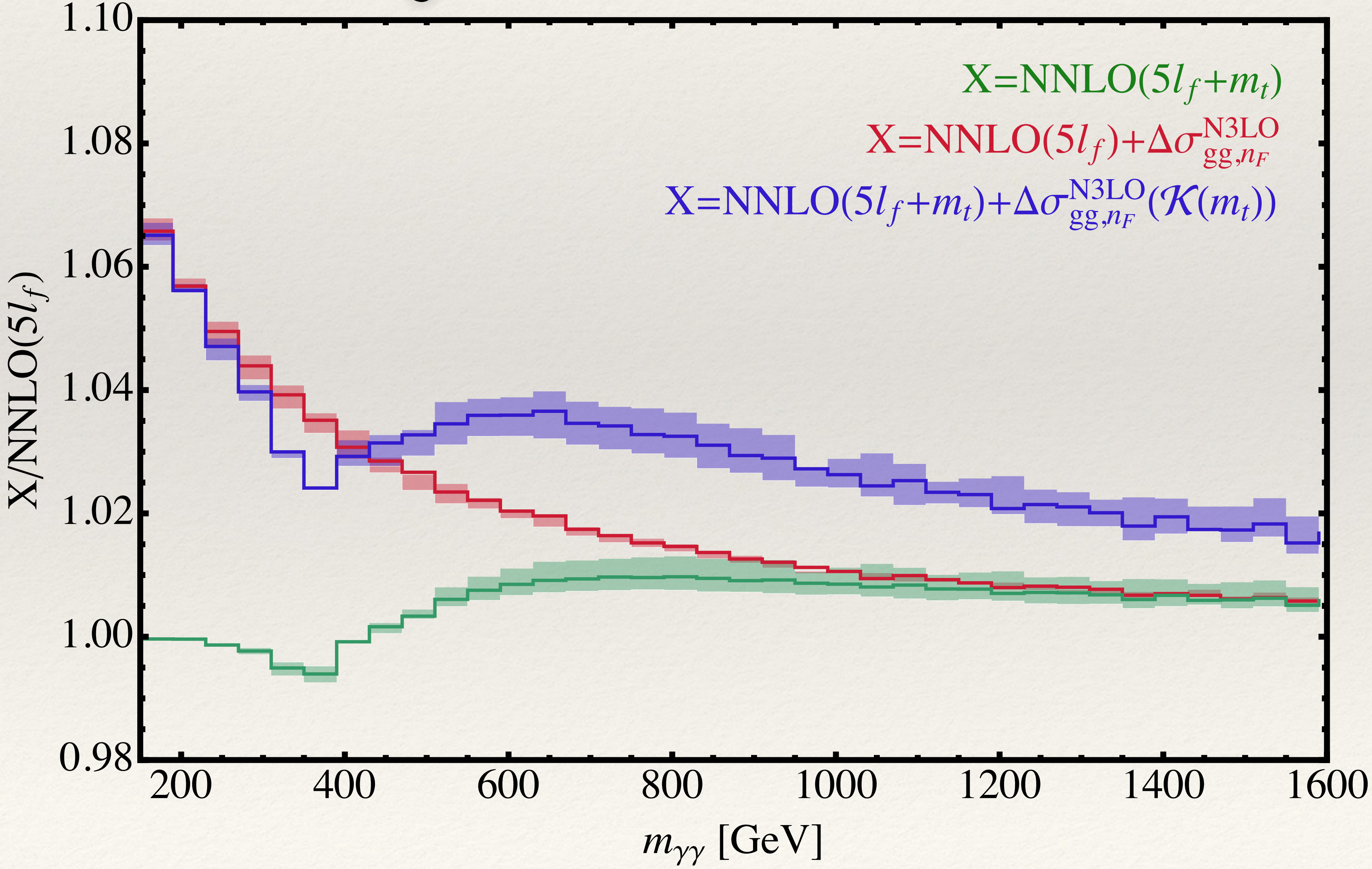
NNLO works well, (even though it is not an NNLO observable)

Additional gg pieces help at higher pt, but not really in the soft region

Predictions at high invariant mass

Bump hunting in the diphoton system assume a smooth function which can be fitted to the data.

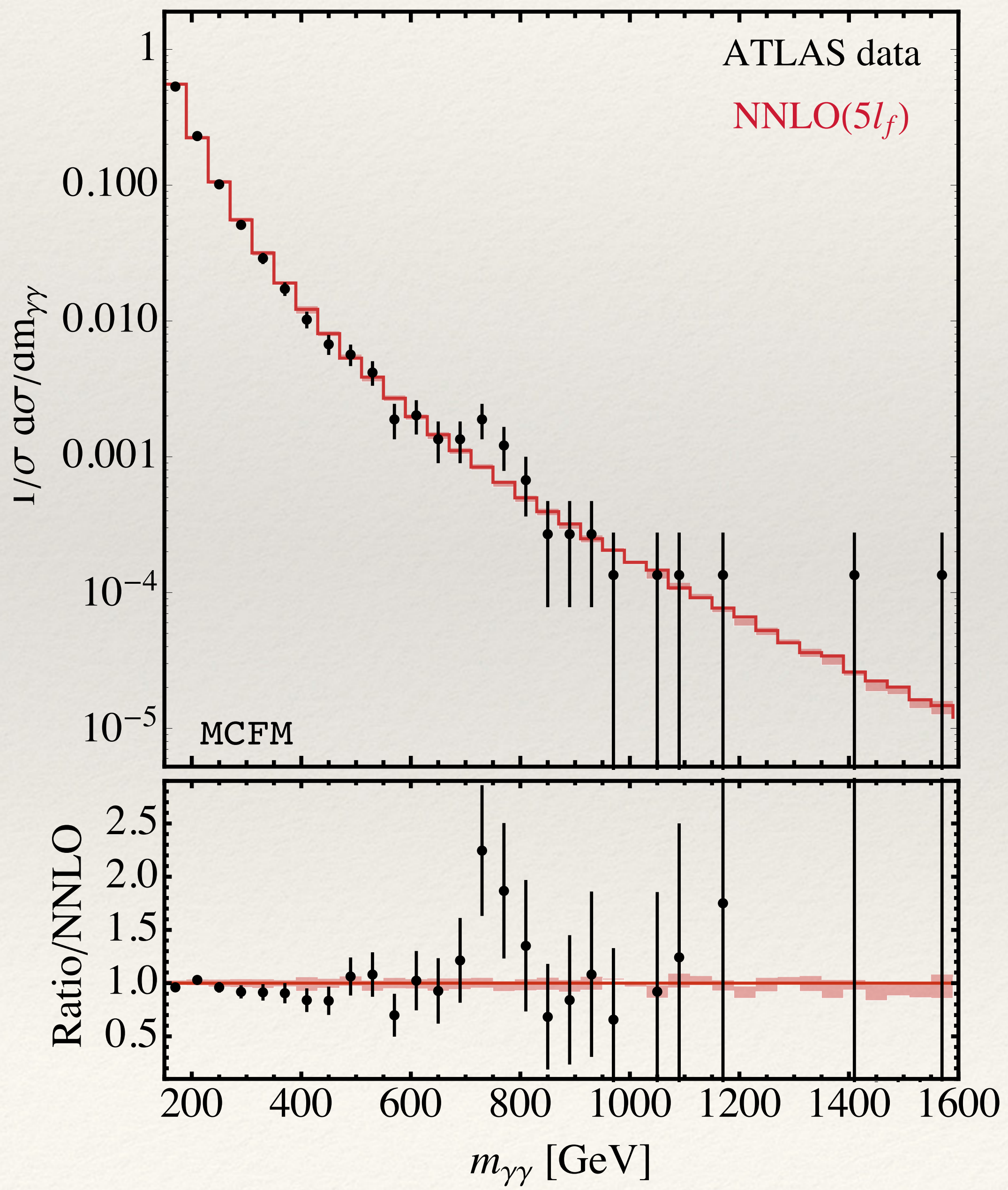


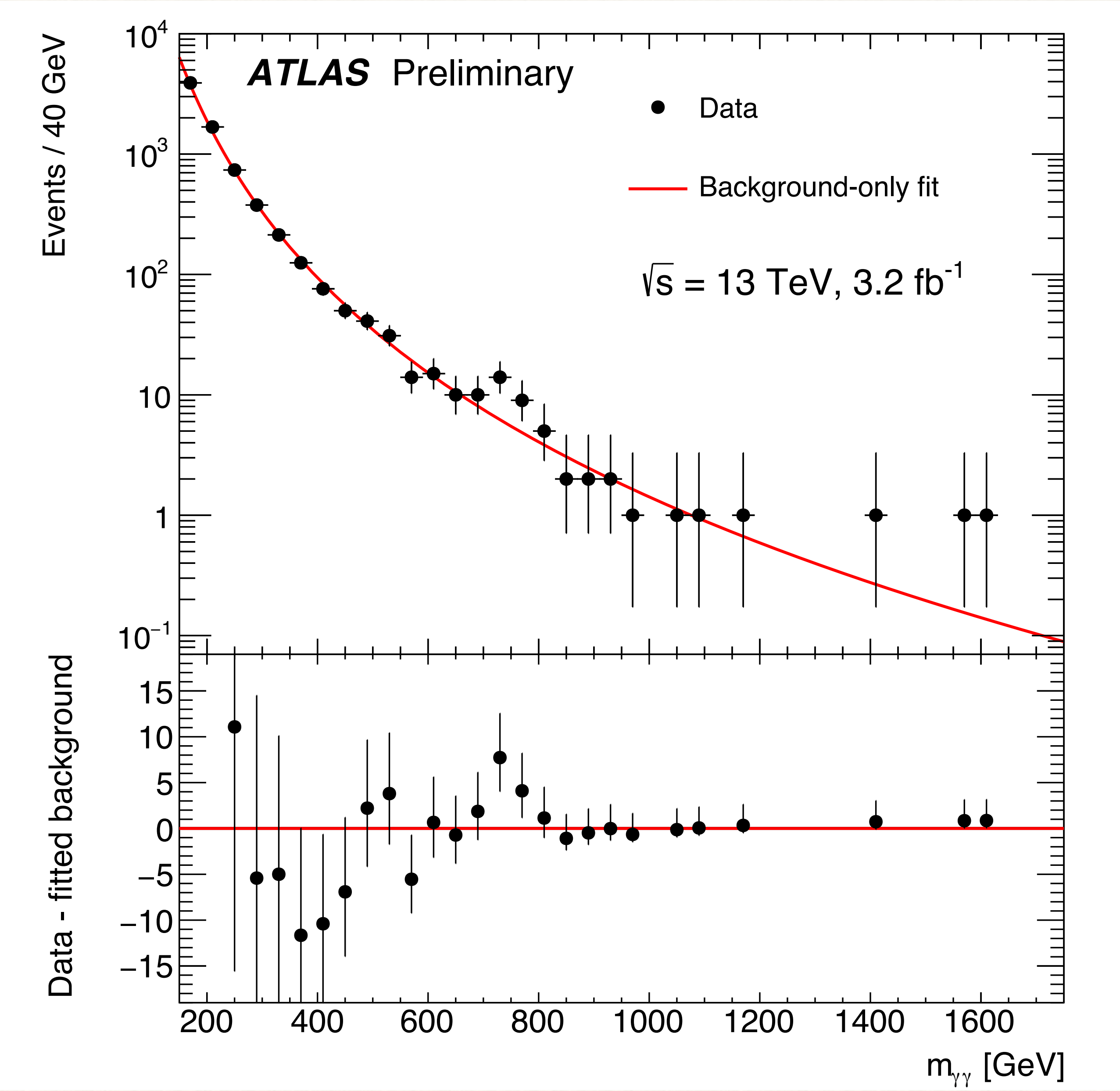


Diphoton invariant mass

A natural concern is that the fit, while good in the region of lots of data, may not correctly describe tails with limited data.

Can check with a first principles calculation of the shape of the SM prediction and compare the shape to the data.





MCFM 8.0

Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello and Williams (in prep),

Lots of work by many people upgrading MCFM to NNLO version, including MPI on top of OMP (Campbell, Giele, Ellis 14) version. We will release code very soon.

Process	v8.0 ($\sim$ weeks)	v8.x ($\sim$ months)	Calculation in MCFM framework
H	X		
W/Z	X		
HW/HZ	X		
$\gamma\gamma$	X		
$V\gamma$		X	
VV		X	
$Z+j$			[1]
$W+j$			[2]
$H+j$			[3]

[1] Boughezal et al, 1512.01291

[2] Boughezal et al,1504.02131

[3] Boughezal et al, 1505. 03893

Conclusions

- We have validated the jettiness slicing method by reproducing known results for colour singlet final states.
- A new result for $\gamma\gamma$ production corrects the published result in the literature.
- The observed $\gamma\gamma$ mass distribution is well predicted by a theoretical calculation.
- MCFM-8.0 will have a suite of NNLO processes.