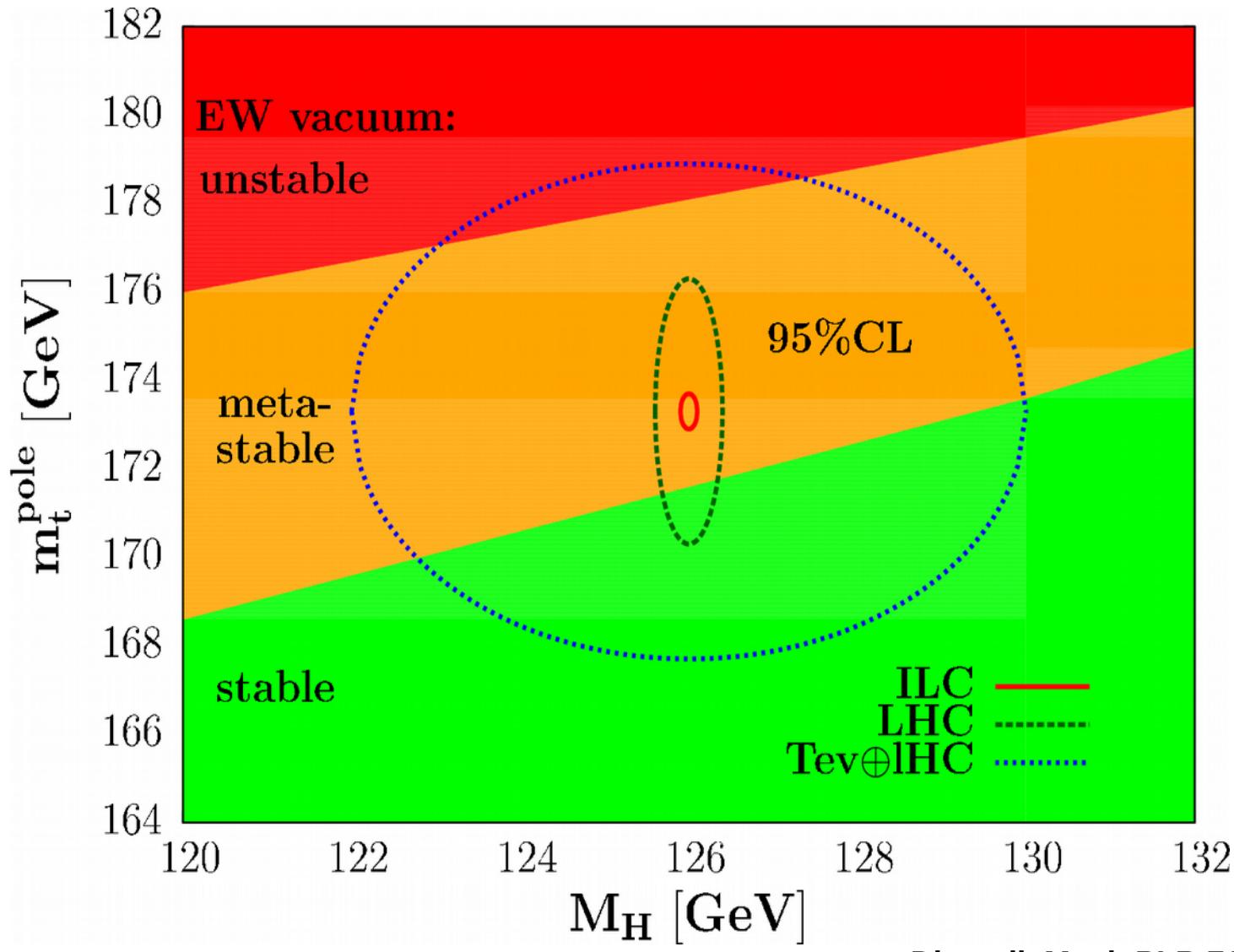


# PDF, $\alpha_s$ , and quark masses from global fits

S.Alekhin (*Univ. of Hamburg & IHEP Protvino*)  
*(in collaboration with J.Blümlein, S.Moch, and R.Plačakyté)*



sa, Djouadi, Moch PLB 716, 214 (2012)

Vacuum stability is quite sensitive to the t-quark mass

# The ABMP16 ingredients

DATA:

- DIS NC/CC inclusive (HERA I+II added, no deuteron data included)
- DIS NC charm production (HERA)
- DIS CC charm production (HERA, NOMAD, CHORUS, NuTeV/CCFR)
- fixed-target DY sa, et al. [hep-ph/1404.6469](#)
- LHC DY distributions (ATLAS, CMS, LHCb)
- t-quark data from the LHC and Tevatron

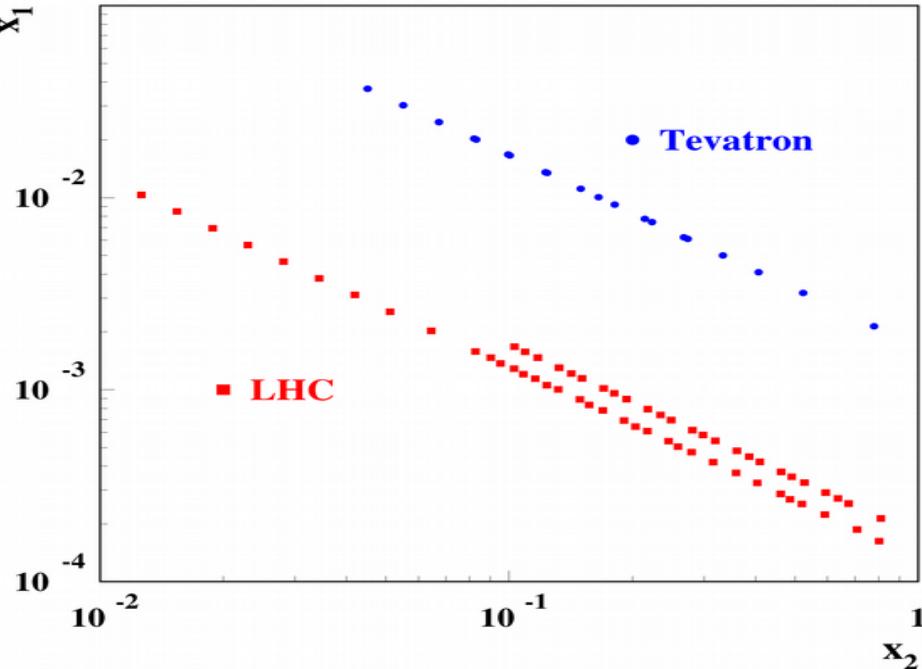
QCD:

- NNLO evolution
- NNLO massless DIS and DY coefficient functions
- NLO+ massive DIS coefficient functions (**FFN scheme**)
  - NLO + NNLO threshold corrections for NC
  - NNLO CC at  $Q \gg m_c$
  - running mass
- NNLO exclusive DY (FEWZ 3.1)
- NNLO inclusive ttbar production ( pole / running mass )
- Relaxed form of (d-bar-ubar) at small  $x$  sa, Blümlein, Moch, Plačakytė [hep-ph/1508.07923](#)

Power corrections in DIS:

- target mass effects
- dynamical twist-4 terms

# Collider W&Z data used in the fit



In the forward region  $x_2 \gg x_1$

$$\sigma(W^+) \sim u(x_2) \bar{d}(x_1)$$

$$\sigma(W^-) \sim d(x_2) \bar{u}(x_1)$$

$$\sigma(Z) \sim Q_u^2 u(x_2) \bar{u}(x_1) + Q_d^2 d(x_2) \bar{d}(x_1)$$

$$\sigma(\text{DIS}) \sim q_u^2 u(x_2) + q_d^2 d(x_2)$$

*Forward W&Z production probes small/large x and is complementary to the DIS → constraint on the quark iso-spin asymmetry*

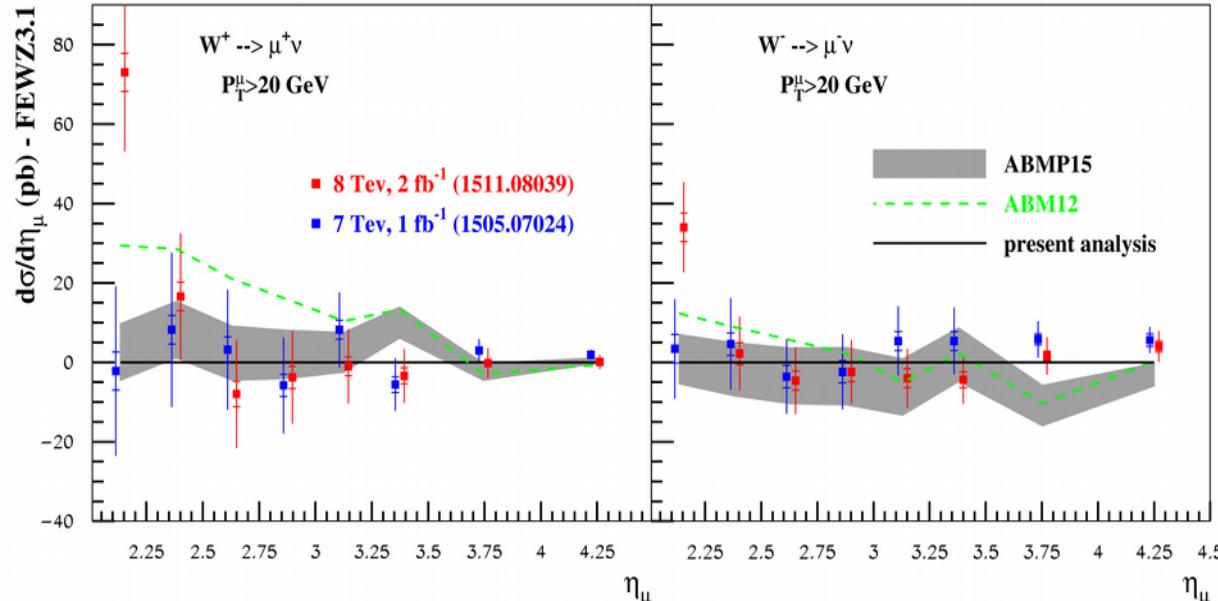
Experiment	ATLAS	CMS		D0		LHCb		
$\sqrt{s}$ (TeV)	7	7	8	1.96		7	8	8
Final states	$W^+ \rightarrow l^+ \nu$	$W^+ \rightarrow \mu^+ \nu$	$W^+ \rightarrow \mu^+ \nu$	$W^+ \rightarrow \mu^+ \nu$	$W^+ \rightarrow e^+ \nu$	$W^+ \rightarrow \mu^+ \nu$	$Z \rightarrow e^+ e^-$	$W^+ \rightarrow \mu^+ \nu$
	$W^- \rightarrow l^- \nu$	$W^- \rightarrow \mu^- \nu$	$W^- \rightarrow \mu^- \nu$	$W^- \rightarrow \mu^- \nu$	$W^- \rightarrow e^- \nu$	$W^- \rightarrow \mu^- \nu$		$W^- \rightarrow \mu^- \nu$
	$Z \rightarrow l^+ l^-$					$Z \rightarrow \mu^+ \mu^-$		$Z \rightarrow \mu^+ \mu^-$
Cut on the lepton $P_T$	$P_T^l > 20 \text{ GeV}$	$P_T^\mu > 25 \text{ GeV}$	$P_T^\mu > 25 \text{ GeV}$	$P_T^\mu > 25 \text{ GeV}$	$P_T^e > 25 \text{ GeV}$	$P_T^\mu > 20 \text{ GeV}$	$P_T^e > 20 \text{ GeV}$	$P_T^e > 20 \text{ GeV}$
<i>NDP</i>		30	11	22	10	13	31	17
$\chi^2$	ABMP16	30.0	22.0	16.8	18.2	19.6	45.4	21.5
	CJ15	–	–	–	20	29	–	–
	CT14	42	– <sup>a</sup>	–	–	34.7	–	–
	JR14	–	–	–	–	–	–	–
	HERAFitter	–	–	–	13	19	–	–
	MMHT14	39	–	–	21	–	–	–
	NNPDF3.0	35.4	18.9	–	–	–	–	–

<sup>a</sup>Statistically less significant data with the cut of  $P_T^\mu > 35 \text{ GeV}$  are used.

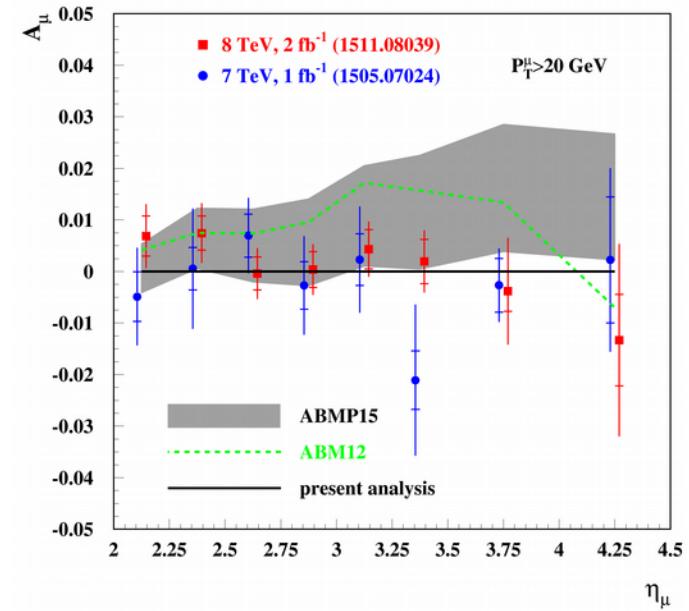
*Obsolete/superseded/low-accuracy Tevatron and LHC data are not used*

# Most recent DY inputs

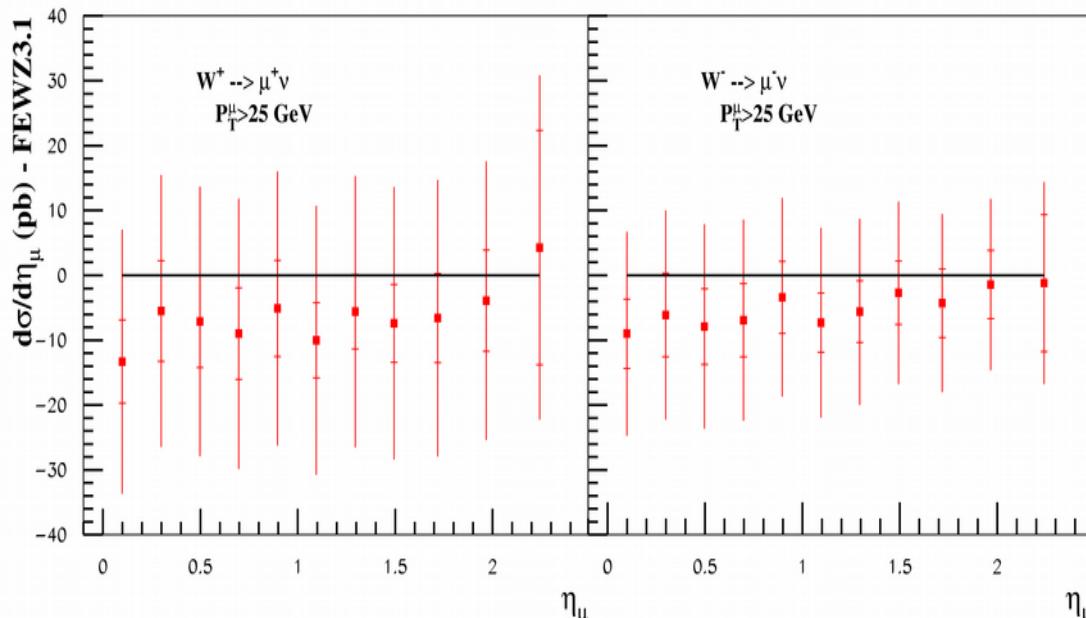
LHCb



LHCb



CMS (8 TeV,  $18.8 \text{ fb}^{-1}$ ) 1603.01803



A filtering of the LHCb data has been performed:

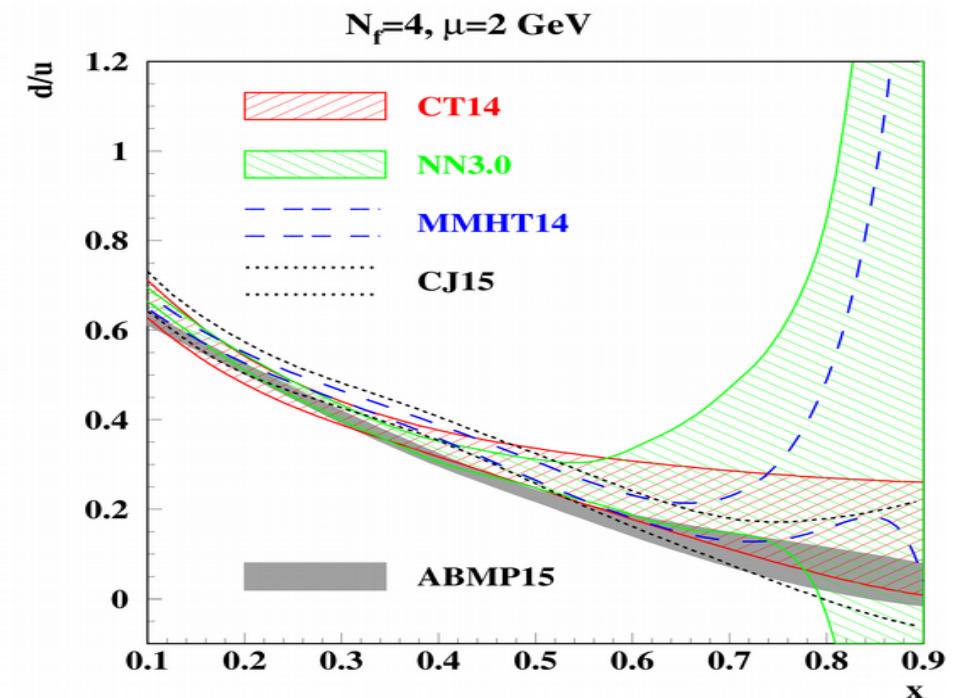
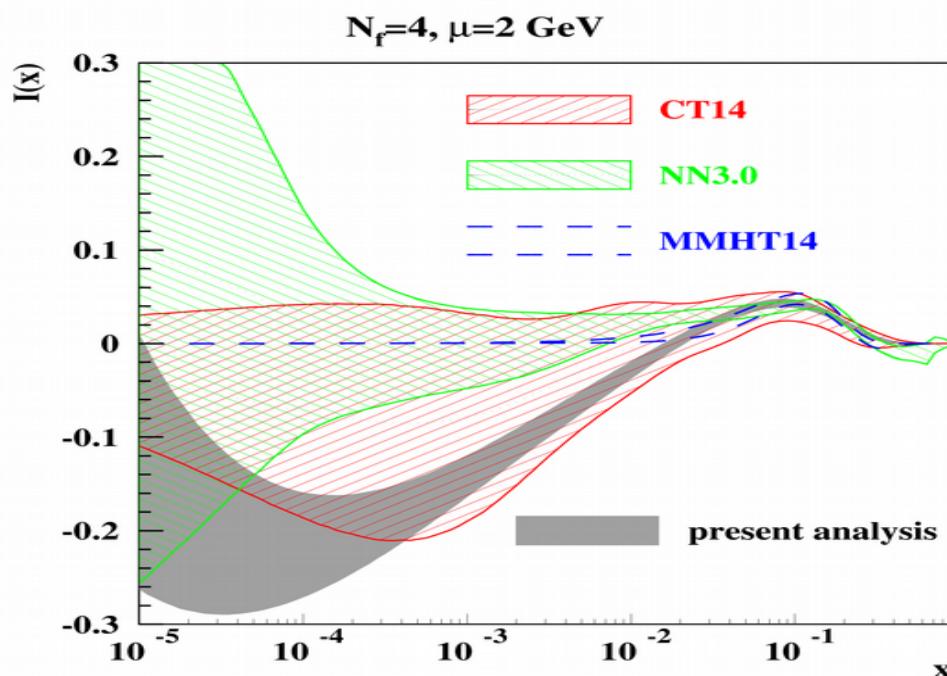
- a bump at 7 TeV and  $\eta=3.275$   
(not confirmed by the LHCb data at 8 TeV)
- and excess at 8 TeV and  $\eta=2.125$   
(not confirmed by the CMS data at 8 TeV)

The CMS data at 8 TeV are much smoother than the ones at 7 TeV:

$$\chi^2 = 17/22 \text{ versus } 22/11$$

cf. earlier data in sa, Blümlein, Moch, Plačakytė, hep-ph/1508.07923

# Impact of the forward Drell-Yan data



sa, Blümlein, Moch, Plačakytė, hep-ph/1508.07923

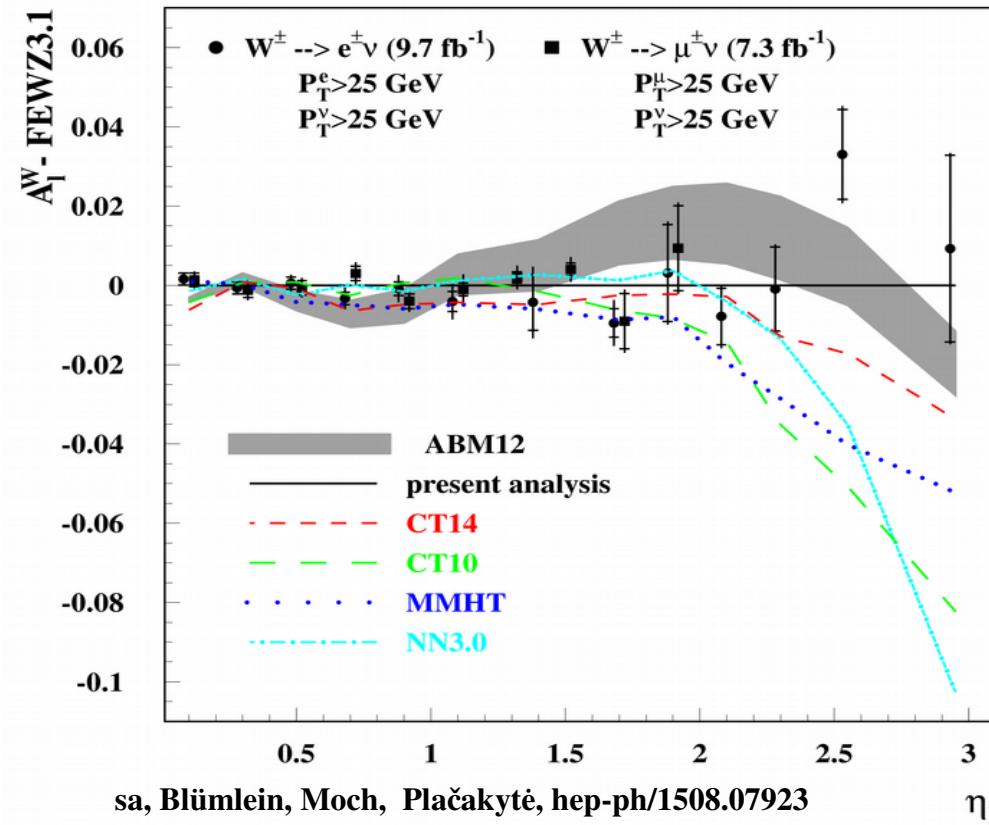
Accardi, et al. hep-ph/1603.08906

- Relaxed form of the sea iso-spin asymmetry  $I(x)$  at small  $x$ ; Regge-like behaviour is recovered only at  $x \sim 10^{-6}$ ; at large  $x$  it is still defined by the phase-space constraint
- Good constraint on the  $d/u$  ratio w/o deuteron data → independent extraction of the deuteron corrections Accardi, Brady, Melnitchouk, Owens, Sato hep-ph/1602.03154; talks by Accardi and Petti at DIS2016
- Big spread between different PDF sets, up to factor of 30 at large  $x$  → PDF4LHC averaging is misleading in this part

# DY at large rapidity

**D0 (1.96 TeV)**

**LHCb (7 TeV, 1  $\text{fb}^{-1}$ )**



sa, Blümlein, Moch, Plačakytė, hep-ph/1508.07923

- The data can be evidently used for consolidation of the PDFs, however, unification of the theoretical accuracy is also needed

ABM

Interpolation of accurate  
NNLO grid (a la FASTNLO)

CT

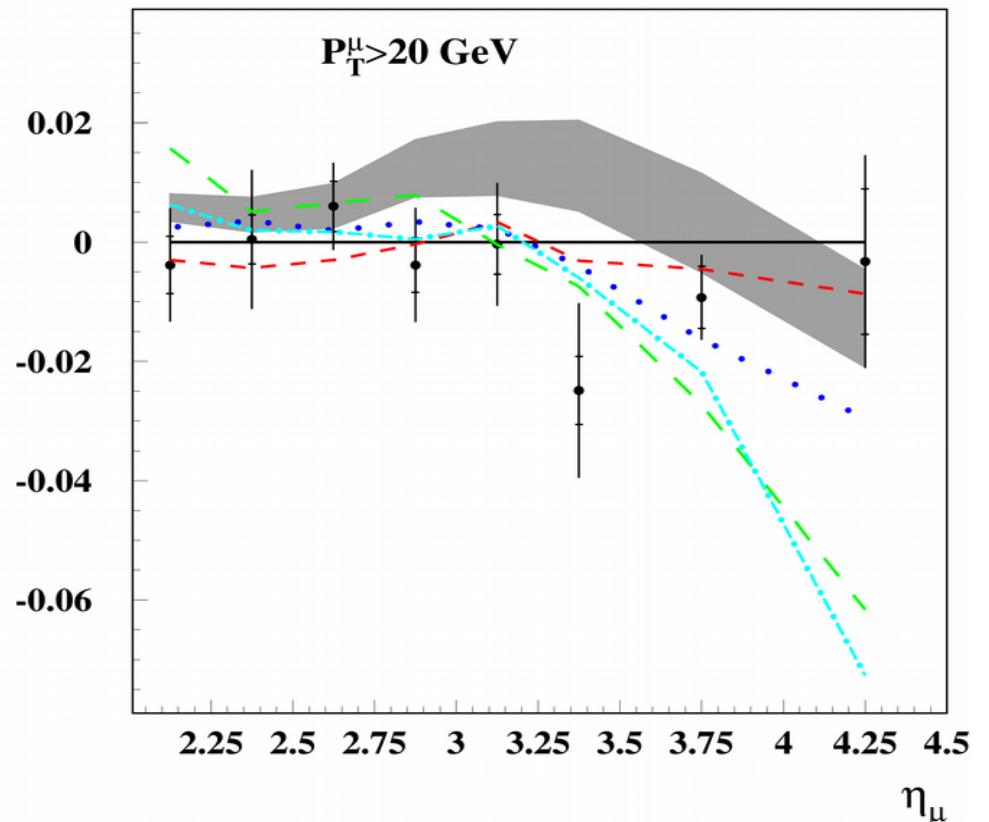
NNLL (ResBos)

MMHT

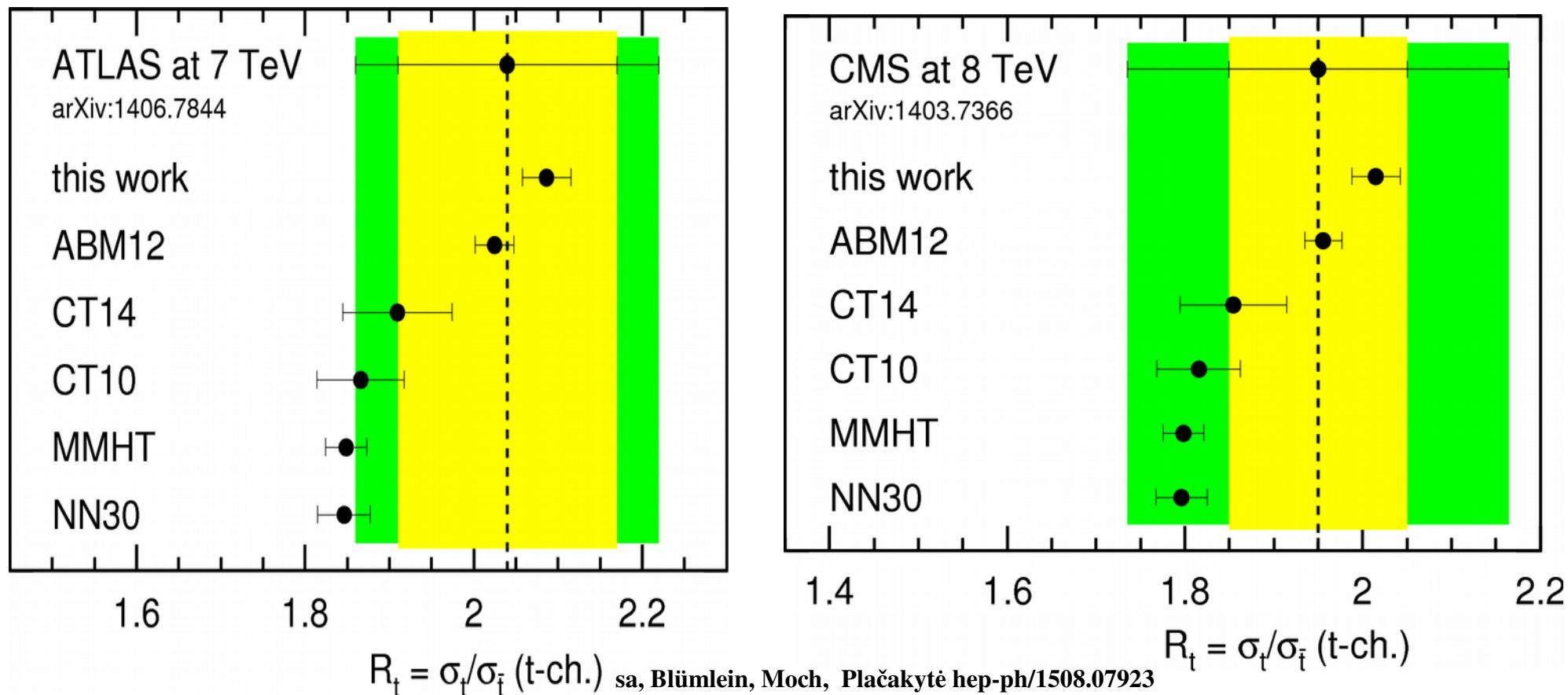
NLO +  
NNLO K-factor

NNPDF

NLO +  
NNLO C-factors  
(y-dependent  
K-factors)



# Implication for(of) the single-top production



- ATLAS and CMS data on the ratio t/tbar are in a good agreement
- The predictions driven by the froward DY data are in a good agreement with the single-top data (N.B.: ABM12 is based on the deuteron data → consistent deuteron correction was used talk by Petti at DIS2016 )

*Single-top production discriminate available PDF sets and can serve as a standard candle process*

# Heavy-quark electro-production in the FFNS

- Only 3 light flavors appear in the initial state
- The dominant mechanism is photon-gluon fusion
- The coefficient functions are known up to the NLO

Witten NPB 104, 445 (1976)

Laenen, Riemersma, Smith, van Neerven NPB 392, 162 (1995)

- Involved high-order calculations:

- NNLO terms due to threshold resummation

Laenen, Moch PRD 59, 034027 (1999)

Lo Presti, Kawamura, Moch, Vogt [hep-ph 1008.0951]

Kawamura, Lo Presti, Moch, Vogt NPB 864, 399 (2012)

- limited set of the NNLO Mellin moments

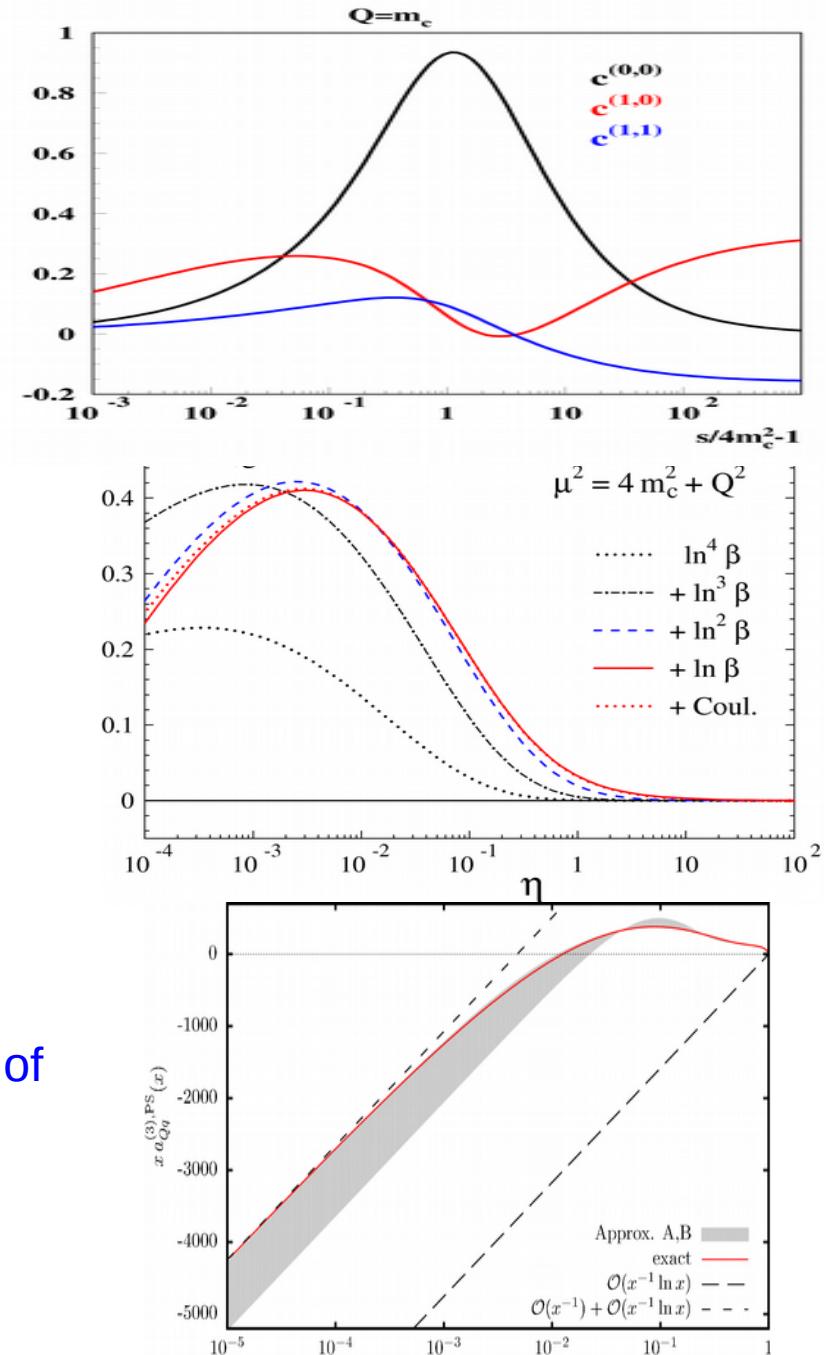
Ablinger et al. NPB 844, 26 (2011)

Bierenbaum, Blümlein, Klein NPB 829, 417 (2009)

- At large  $Q$  the leading-order coefficient  $\rightarrow \ln(Q/m_\eta)$   
and may be quite big despite the suppression by factor of  
 $\alpha_s$  and should be resummed

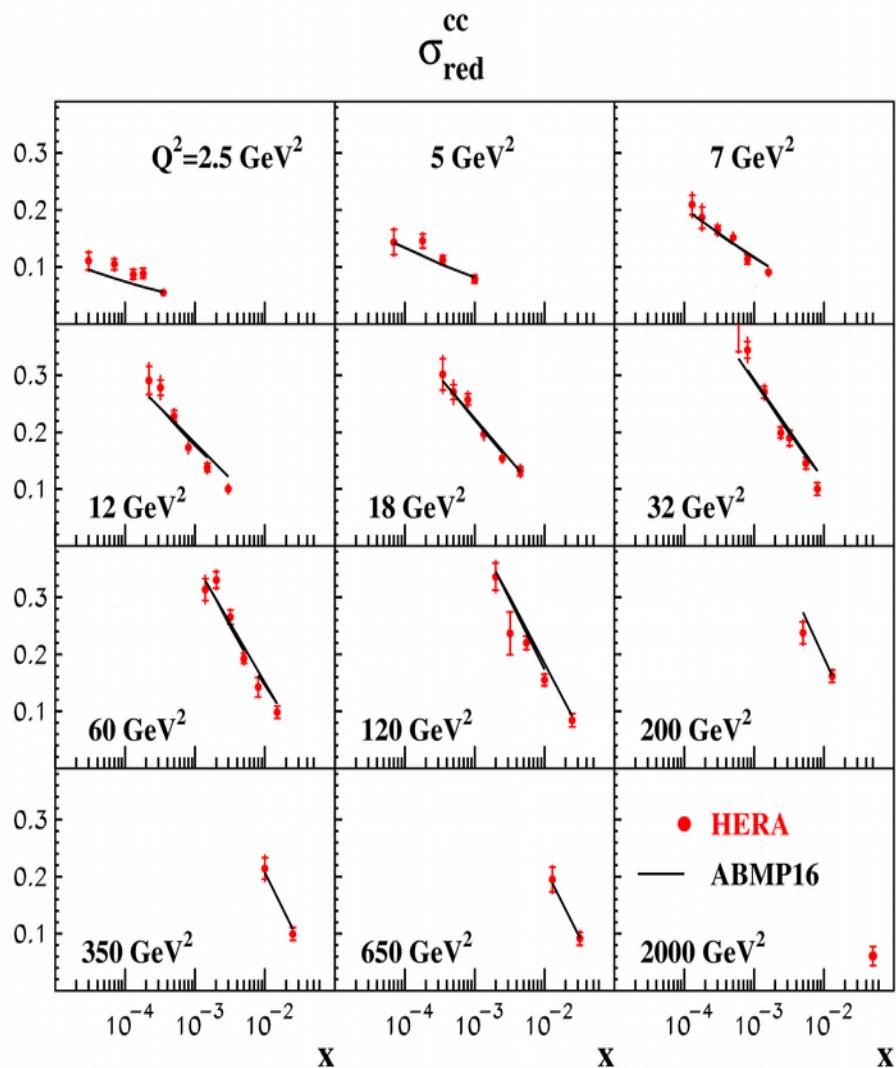
Shifman, Vainstein, Zakharov NPB 136, 157 (1978)

- a motivation to derive the VFN scheme matched to  
the FFNS (ACOT..., RT..., FONLL....)



# HERA charm data and $m_c(m_c)$

H1/ZEUS PLB 718, 550 (2012)



$$m_c(m_c) = 1.275 \pm 0.025 \text{ GeV} \quad \text{PDG2016}$$

$$m_c(m_c) = 1.246 \pm 0.023 \text{ (h.o.) GeV} \quad \text{NNLO}$$

Kiyo, Mishima, Sumino hep-ph/1510.07072

- Approximate NNLO massive Wilson coefficients (combination of the threshold corrections, high-energy limit, and the NNLO massive OMEs)

Kawamura, Lo Presti, Moch, Vogt NPB 864, 399 (2012)

- Running-mass definition of  $m_c$   
 $X^2/\text{NDP}=61/52$   
 $m_c(m_c) = 1.250 \pm 0.020 \text{ (exp.) GeV} \quad \text{ABMP16}$   
 $m_c(m_c) = 1.24 \pm 0.03 \text{ (exp.) GeV} \quad \text{ABM12}$

*Good agreement with the e+e- determinations → the FFN scheme nicely works for the existing data*

- RT optimal  
 $X^2/\text{NDP}=82/52$   
 $m_c(\text{pole})=1.25 \text{ GeV} \quad \text{NNLO}$
- FONLL  
 $X^2/\text{NDP}=60/47$   
 $m_c(\text{pole})=1.275 \text{ GeV} \quad \text{NNLO}$
- S-ACOT-χ  
 $X^2/\text{NDP}=59/47$   
 $m_c(\text{pole})=1.3 \text{ GeV} \quad \text{NNLO}$
- NNPDF3.0 JHEP 1504, 040 (2015)  
CT14 hep-ph 1506.07443

PDF sets	$m_c$ [GeV]	$m_c$ renorm. scheme	theory method ( $F_2^c$ scheme)	theory accuracy for heavy quark DIS Wilson coeff.	$\chi^2/\text{NDP}$ for HERA data [127] with xFitter [128, 129]		
ABM12 [2] <sup>a</sup>	$1.24^{+0.05}_{-0.03}$	$\overline{\text{MS}}$ $m_c(m_c)$	FFNS ( $n_f = 3$ )	NNLO <sub>approx</sub>	65/52	66/52	
CJ15 [1]	1.3	$m_c^{\text{pole}}$	SACOT [122]	NLO	117/52	117/52	
CT14 [3] <sup>b</sup>							
(NLO)	1.3	$m_c^{\text{pole}}$	SACOT( $\chi$ ) [123]	NLO	51/47	70/47	
(NNLO)	1.3	$m_c^{\text{pole}}$	SACOT( $\chi$ ) [123]	NLO	64/47	130/47	
HERAPDF2.0 [4]							
(NLO)	1.47	$m_c^{\text{pole}}$	RT optimal [125]	NLO	67/52	67/52	
(NNLO)	1.43	$m_c^{\text{pole}}$	RT optimal [125]	NLO	62/52	62/52	
JR14 [5] <sup>c</sup>	1.3	$\overline{\text{MS}}$ $m_c(m_c)$	FFNS ( $n_f = 3$ )	NNLO <sub>approx</sub>	62/52	62/52	
MMHT14 [6]							
(NLO)	1.4	$m_c^{\text{pole}}$	RT optimal [125]	NLO	72/52	78/52	
(NNLO)	1.4	$m_c^{\text{pole}}$	RT optimal [125]	NLO	71/52	83/52	
NNPDF3.0 [7]							
(NLO)	1.275	$m_c^{\text{pole}}$	FONLL-B [124]	NLO	58/52	60/52	
(NNLO)	1.275	$m_c^{\text{pole}}$	FONLL-C [124]	NLO	67/52	69/52	
PDF4LHC15 [8] <sup>d</sup>	–	–	FONLL-B [124]	–	58/52	64/52	
	–	–	RT optimal [125]	–	71/52	75/52	
	–	–	SACOT( $\chi$ ) [123]	–	51/47	76/47	

No advantage of the GMVFN schemes: the VFN  $\chi^2$  values are systematically bigger than the FFN ones

Accardi, et al. hep-ph/1603.08906

# Quark mass renormalization

## Pole mass

- Based on (unphysical) concept of heavy-quark being a free parton

$$\not{p} - m_q - \Sigma(p, m_q) \Big|_{p^2 = m_q^2}$$

- heavy-quark self-energy  $\Sigma(p, m_q)$  receives contributions from regions of all loop momenta – also from momenta of  $\mathcal{O}(\Lambda_{QCD})$
- Renormalon ambiguity in definition of pole mass of  $\mathcal{O}(\Lambda_{QCD})$   
Bigi, Shifman, Uraltsev, Vainshtein '94; Beneke, Braun '94; Smith, Willenbrock '97

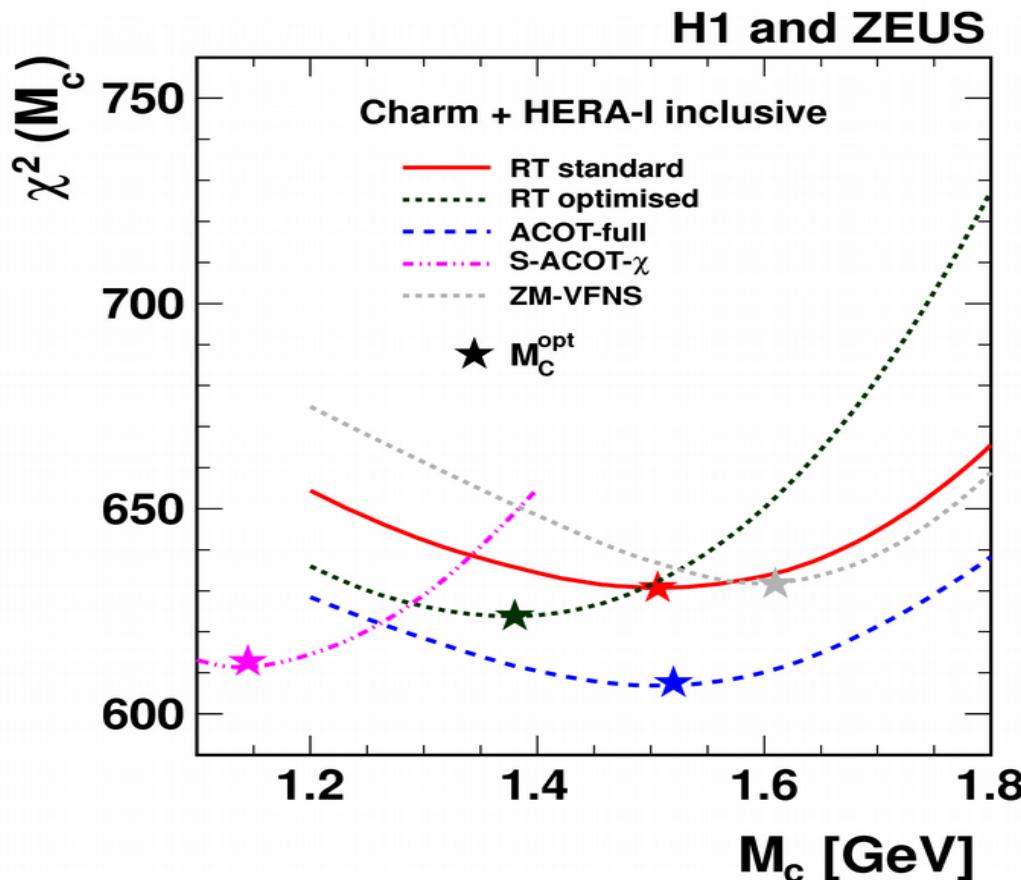
## $\overline{MS}$ mass

- Free of infrared renormalon ambiguity
- Conversion between  $m_{\text{pole}}$  and  $\overline{MS}$  mass  $m(\mu_R)$  in perturbation theory known to four loops in QCD Marquard, Smirnov, Smirnov, Steinhauser '15
  - does not converge in case of charm quark

$$\begin{aligned} m_c(m_c) &= 1.27 \text{ GeV} \longrightarrow m_c^{\text{pole}} = 1.47 \text{ GeV} \text{ (one loop)} \\ &\longrightarrow m_c^{\text{pole}} = 1.67 \text{ GeV} \text{ (two loops)} \\ &\longrightarrow m_c^{\text{pole}} = 1.93 \text{ GeV} \text{ (three loops)} \\ &\longrightarrow m_c^{\text{pole}} = 2.39 \text{ GeV} \text{ (four loops)} \end{aligned}$$

# c-quark mass in the CMVFN schemes

The values of pole mass  $m_c$  used by different groups and preferred by the PDF fits are systematically lower than the PDG value



H1/ZEUS PLB 718, 550 (2012)

$m_c(m_c) = 1.19 + 0.08 - 0.15 \text{ GeV}$  ACOT....

Gao, Guzzi, Nadolsky EPJC 73, 2541 (2013)

Wide spread of the  $m_c$  obtained in different version of the GMVFN schemes → quantitative illustration of the GMVFNS uncertainties

# *Charm quark mass and the Higgs cross section*

## *MMHT*

- “Tuning” Charm mass  $m_c$  parameter effects the Higgs cross section
  - linear rise in  $\sigma(H) = 40.5 \dots 42.6 \text{ pb}$  for  $m_c = 1.15 \dots 1.55 \text{ GeV}$  with MMHT14 PDFs Martin, Motylinski, Harland-Lang, Thorne ‘15

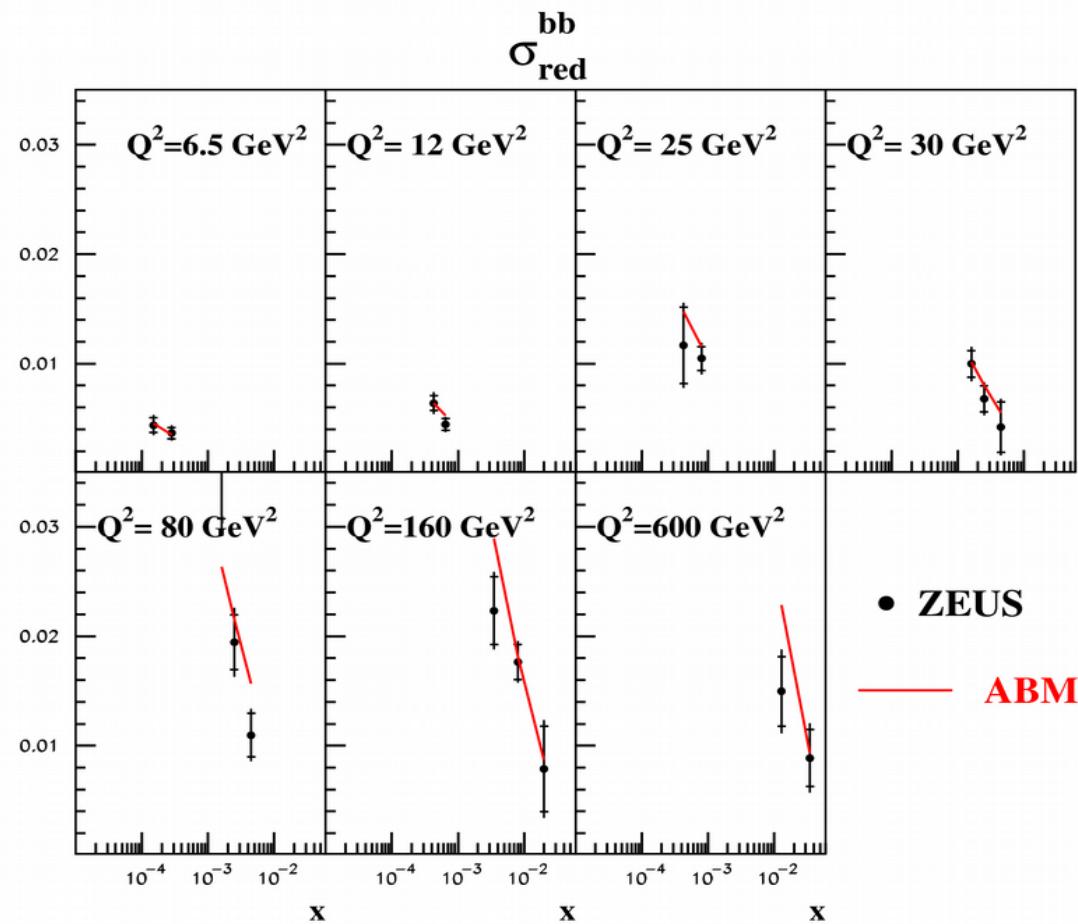
$m_c^{\text{pole}}$ [GeV]	$\alpha_s(M_Z)$ (best fit)	$\chi^2/\text{NDP}$ (HERA data on $\sigma^{c\bar{c}}$ )	$\sigma(H)^{\text{NNLO}}$ [pb] best fit $\alpha_s(M_Z)$	$\sigma(H)^{\text{NNLO}}$ [pb] $\alpha_s(M_Z) = 0.118$
1.15	0.1164	78/52	40.48	(42.05)
1.2	0.1166	76/52	40.74	(42.11)
1.25	0.1167	75/52	40.89	(42.17)
1.3	0.1169	76/52	41.16	(42.25)
1.35	0.1171	78/52	41.41	(42.30)
1.4	0.1172	82/52	41.56	(42.36)
1.45	0.1173	88/52	41.75	(42.45)
1.5	0.1173	96/52	41.81	(42.51)
1.55	0.1175	105/52	42.08	(42.58)

*A spread of 41.0 ..... 42.3 pb was obtained by R.Thorne with  $\alpha_s$  varied; the same trend is observed for MSTW08 and NNPDF 2.3*

Accardi, et al. hep-ph/1603.08906

# ZEUS bottom data and $m_b(m_b)$

ZEUS hep-ex/1405.6915



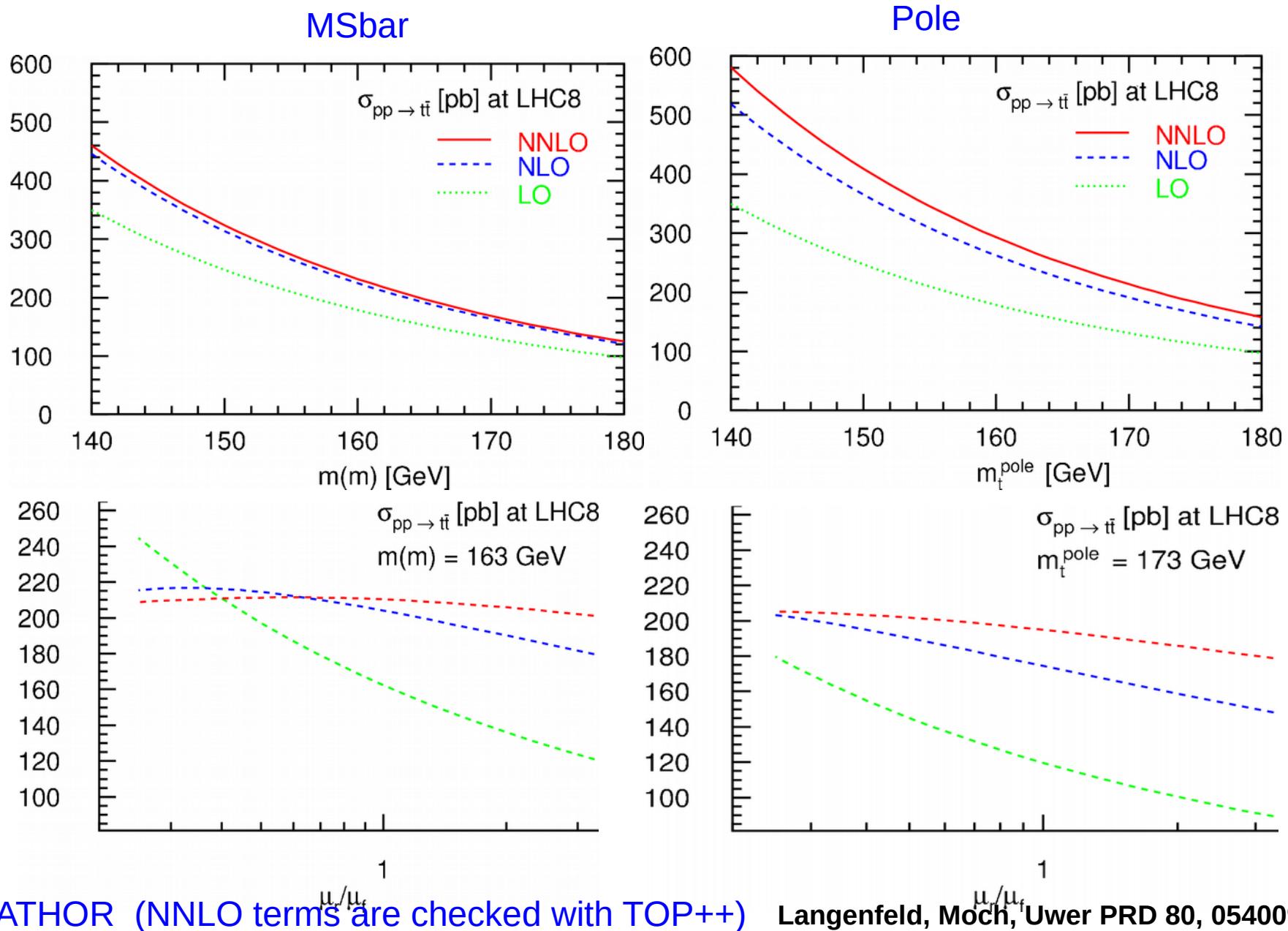
$$\chi^2/\text{NDP}=16 / 17$$

$$m_b(m_b) = 3.91 \pm 0.14 (\text{exp.}) \text{ GeV} \quad \text{ABMP16}$$

$$m_b(m_b) = 4.07 \pm 0.17 (\text{exp.}) \text{ GeV}$$

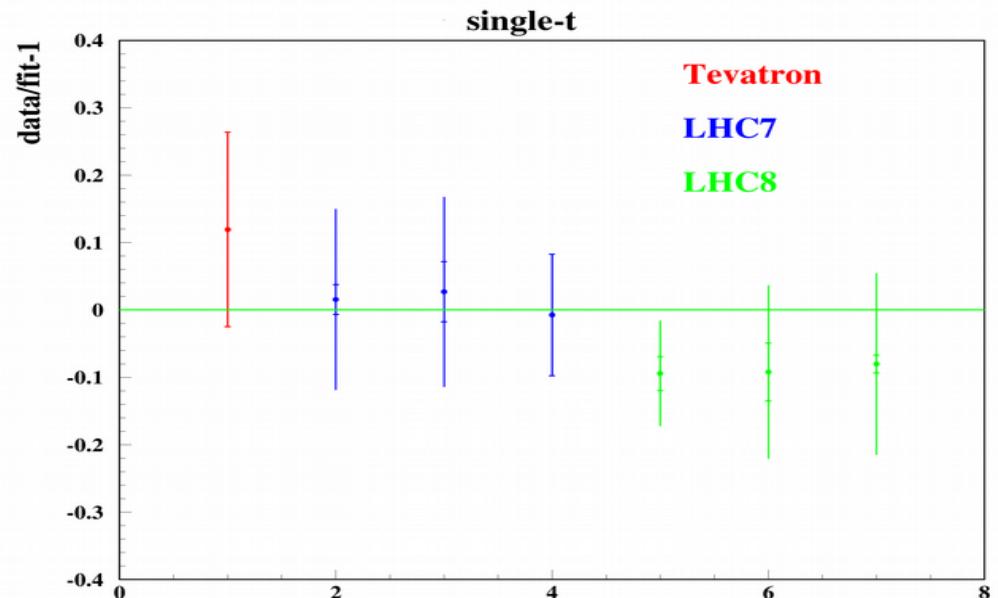
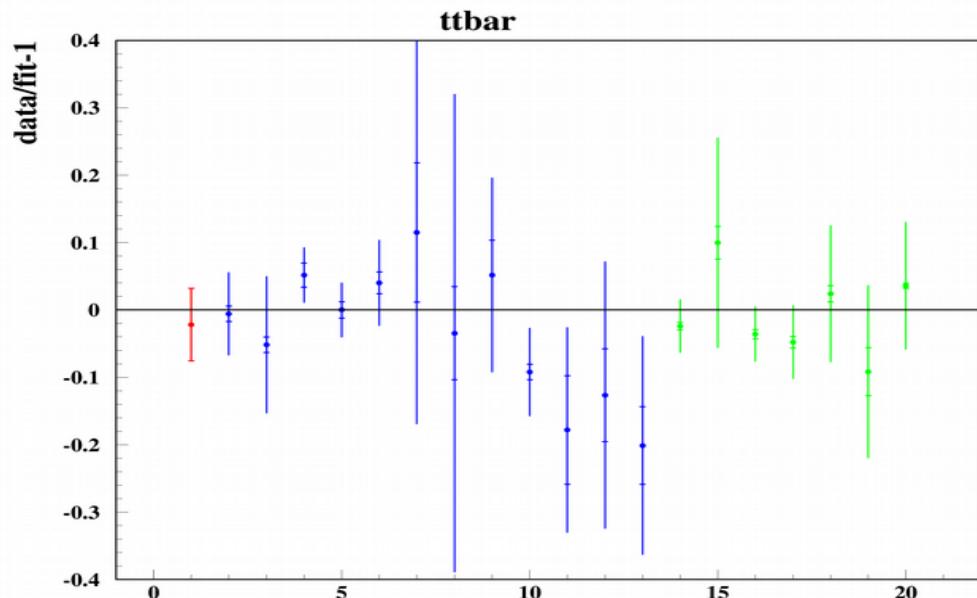
ZEUS JHEP 1409, 127 (2014)

# ttbar production with pole and Msbar mass



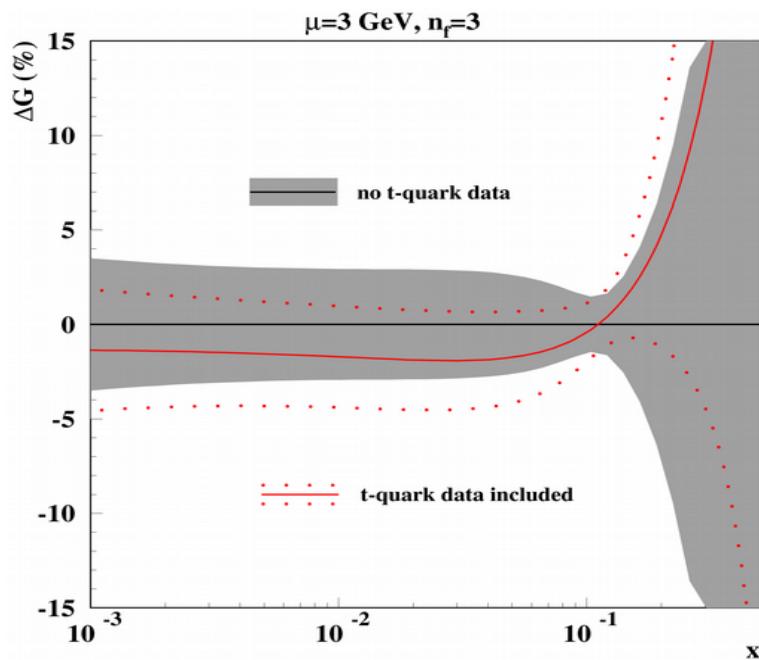
*Running mass definition provides nice perturbative stability*

# t-quark data from the LHC and Tevatron



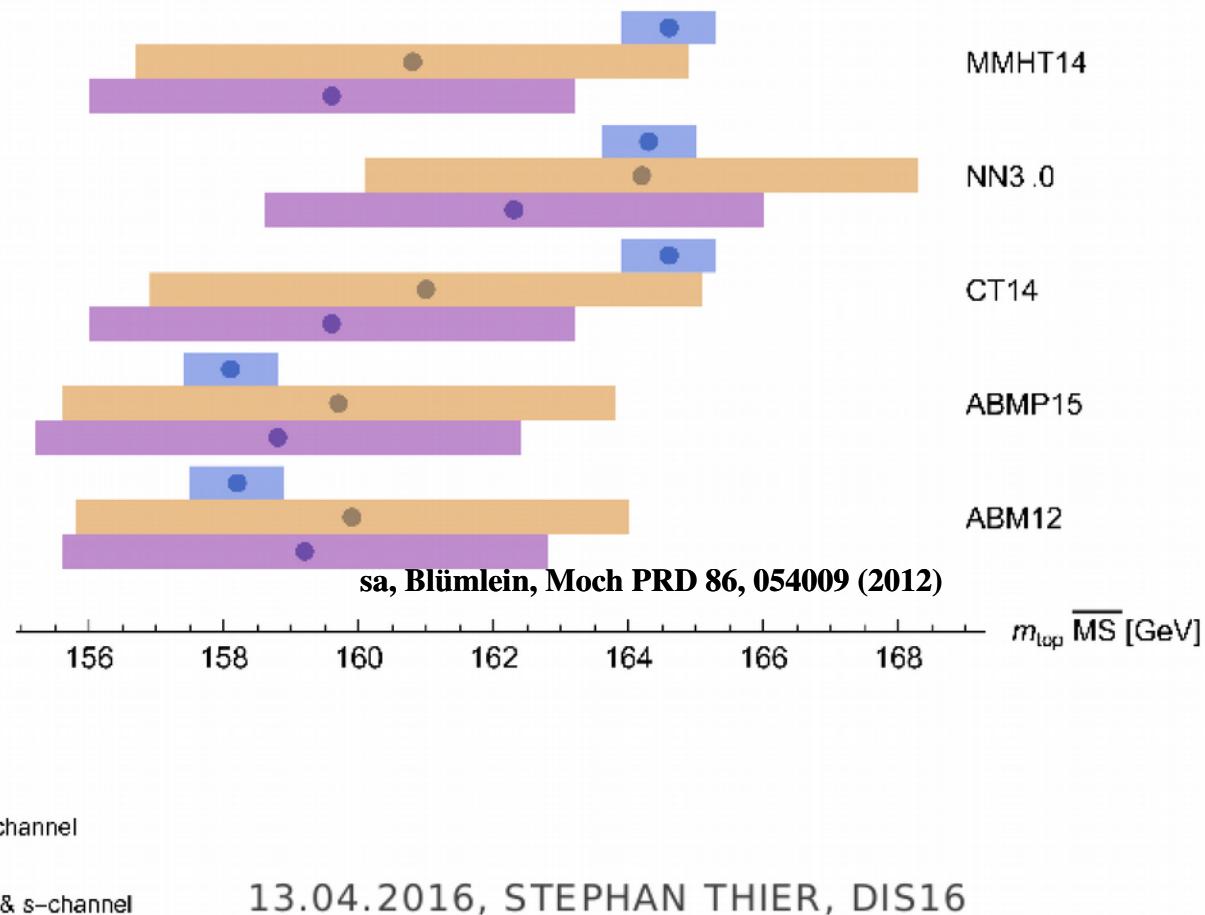
Running mass definition → better perturbative stability

sa, Blümlein, Moch PRD 86, 054009 (2012)



- $m_t(m_t) = 160.9 \pm 1.2(\text{exp.}) \text{ GeV}$  NNLO
- $\alpha_s(M_z) = 0.1145(9) \rightarrow 0.1149(9)$  NNLO
- moderate change in the large-x gluon distribution

# $\overline{\text{MS}}$ mass fit: comparison



T-channel (tq): NNLO

Brucherseifer, Caola, Melnikov PLB 736, 58 (2014)

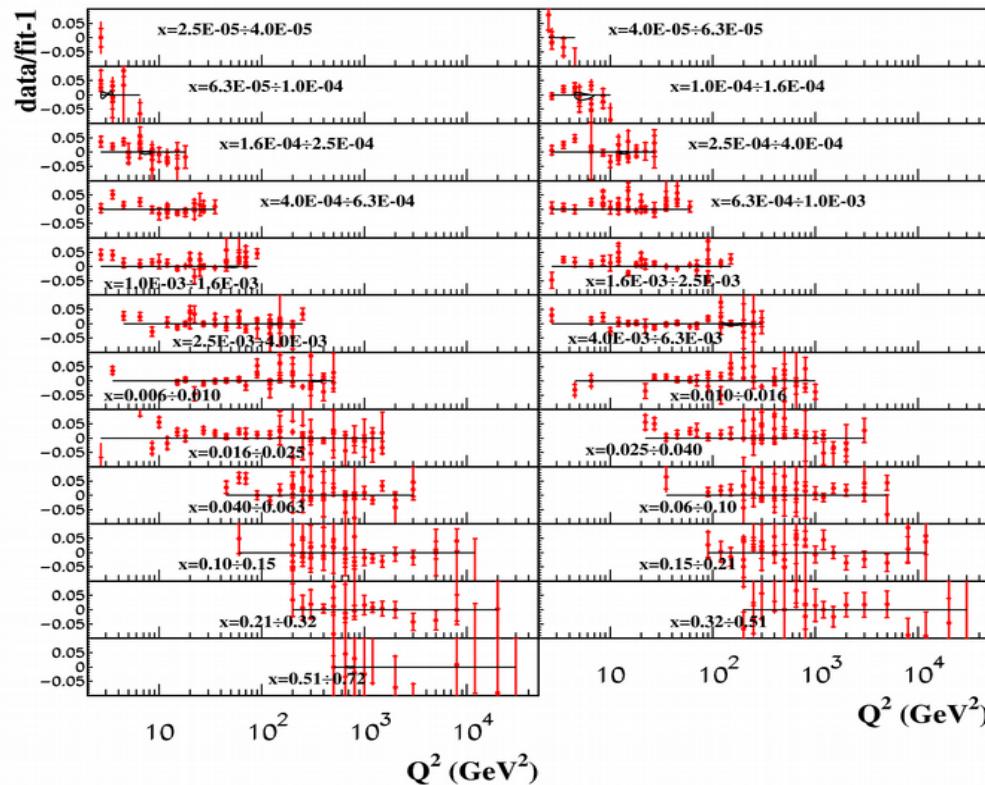
S-channel (tb): NLO + NNLO(threshold)

sa, Moch, Thier in preparation

# Inclusive HERA I+II data

H1 and ZEUS hep-ex/1506.06042

**HERA I+II ( $e^+ p$ )**



$Q^2(\text{HERA})$

$>2.5 \text{ GeV}^2$

$>5 \text{ GeV}^2$

$>10 \text{ GeV}^2$

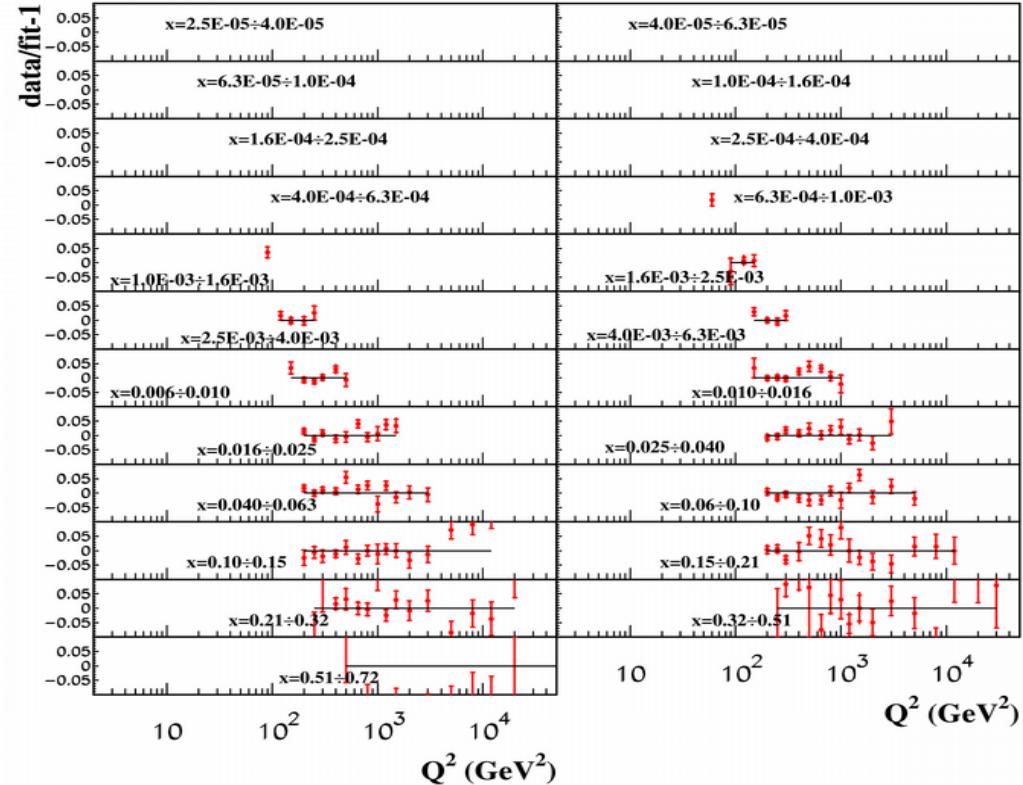
$\chi^2/\text{NDP}(\text{HERA})$

$1505/1168=1.29$

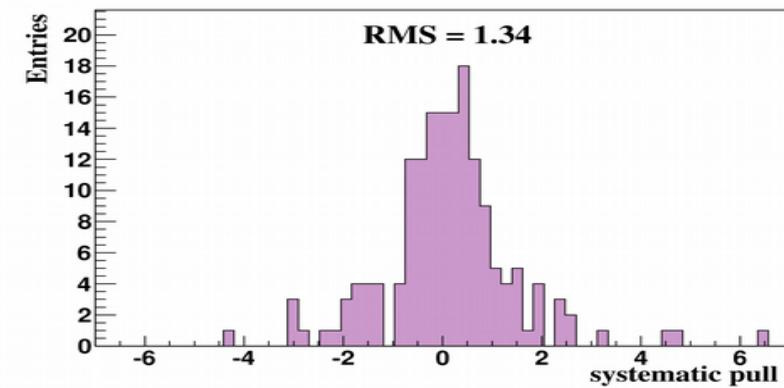
$1350/1092=1.24$

$1225/1007=1.22$

**HERA I+II ( $e^- p$ )**

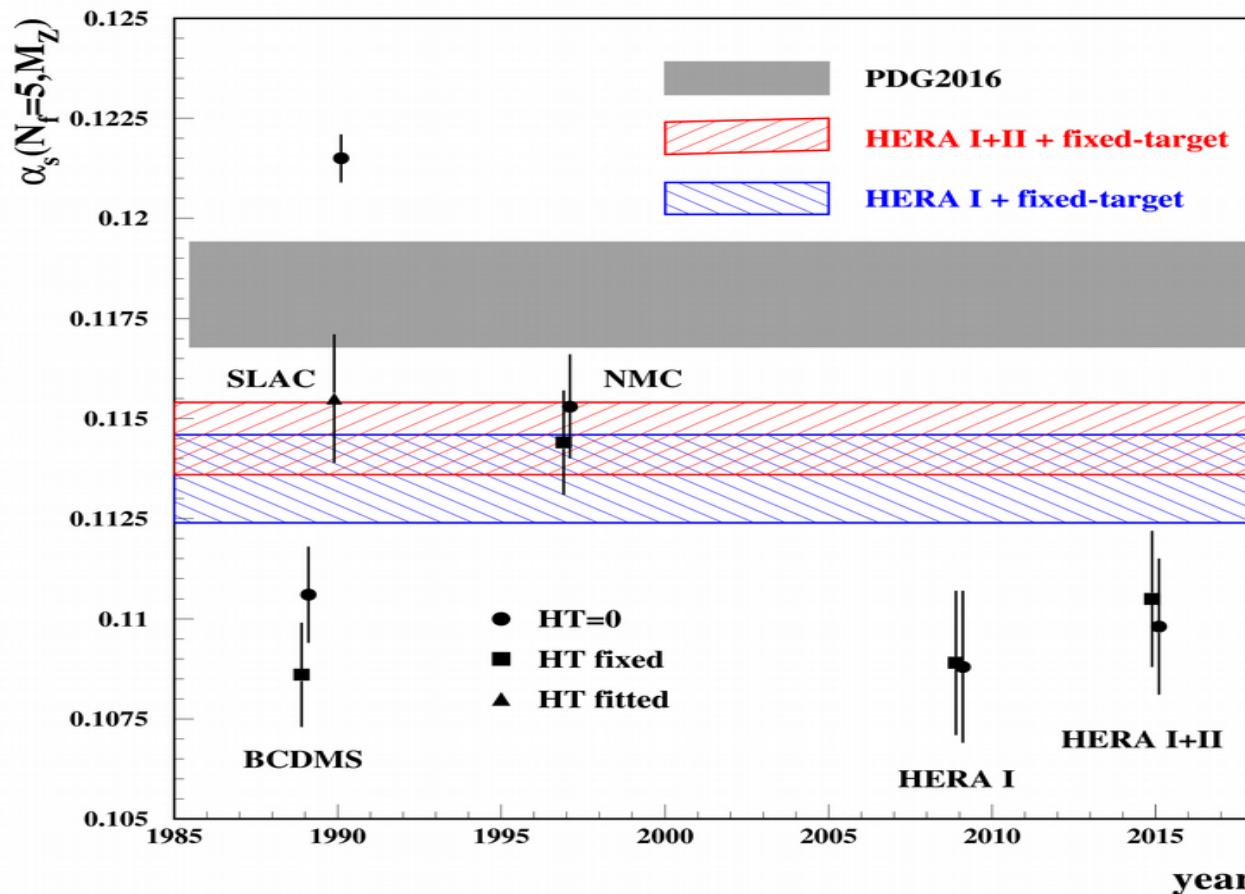


**H1 and ZEUS**



The value of  $\chi^2/\text{NDP}$  is bigger than 1, however still comparable to the pull distribution width

# $\alpha_s$ updated



- Combination of the DY data (disentangle PDFs) and the DIS ones (constrain  $\alpha_s$ )
- the value of  $\alpha_s$  is still lower than the PDG one: pulled up by the SLAC and NMC data; pulled down by the BCDMS and HERA ones
- only SLAC determination overlap with the PDG band provided the high-twist terms are taken into account

# Summary

- The FFN scheme provides a nice description of the existing DIS data with a consistent determination of the heavy-quark masses

$$m_c(m_c) = 1.250 \pm 0.020 \text{ GeV}$$

$$m_b(m_b) = 3.91 \pm 0.14 \text{ GeV}$$

In contrast to the GMVFN schemes suffering from the uncertainties due to missing NNLO corrections to the OMEs and requiring tuning of  $m_c(\text{pole}) \sim 1.3 \text{ GeV}$

- The value of  $m_t(m_t) = 160.9 \pm 1.2 \text{ GeV}$  from the ttbar and single-top data at the LHC and Tevatron
- Updated value of

$$\alpha_s(M_z) = 0.1145(9) \quad \text{DIS}$$

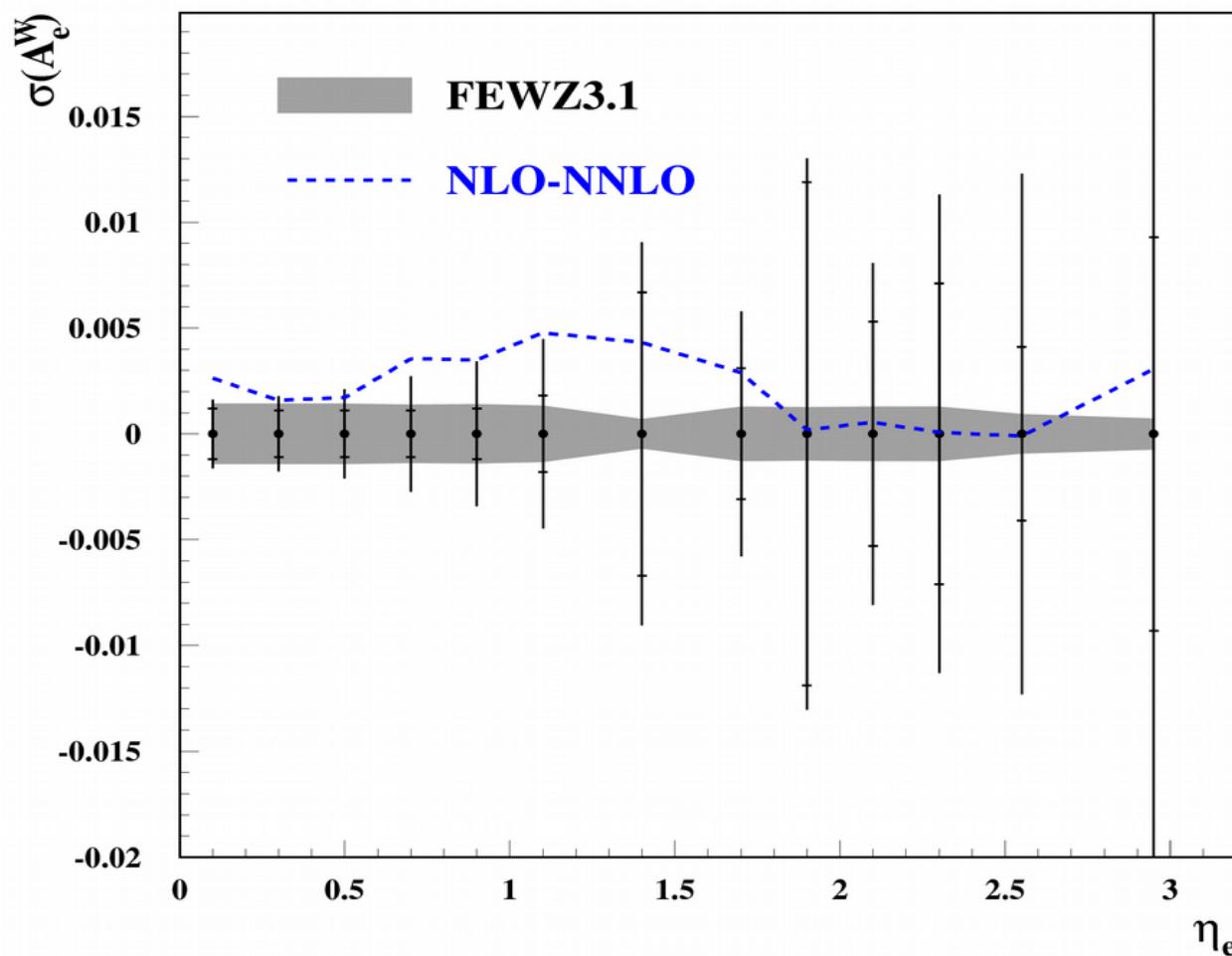
$$\alpha_s(M_z) = 0.1149(9) \quad \text{DIS+t-quark}$$

- Updated PDFs (bulk of DY data, HERA I+II, t-quark included, deuteron excluded):
  - reduced theoretical uncertainties in PDFs, in particular in d/u at large x; the small-x iso-spin sea asymmetry is relaxed and turns negative at  $x \sim 10^{-3}$  with onset of the Regge-like asymptotics at  $x < 10^{-5}$ ; moderate increase in the large-x gluon distribution due to impact of the ttbar data

# EXTRAS

# Computation accuracy

D0(1.96 TeV, 9.7 fb<sup>-1</sup>)



- Accuracy of O(1 ppm) is required to meet uncertainties in the experimental data → O( $10^4$  h) of running FEWZ 3.1 in NNLO
- An interpolation grid a la FASTNLO is used

# NNLO DY corrections in the fit

The existing NNLO codes (DYNNLO, FEWZ) are quite time-consuming → fast tools are employed (FASTNLO, Applgrid,.....)

- the corrections for certain basis of PDFs are stored in the grid
- the fitted PDFs are expanded over the basis
- the NNLO c.s. in the PDF fit is calculated as a combination of expansion coefficients with the pre-prepared grids

The general PDF basis is not necessary since the PDFs are already constrained by the data, which do not require involved computations → *use as a PDF basis the eigenvalue PDF sets obtained in the earlier version of the fit*

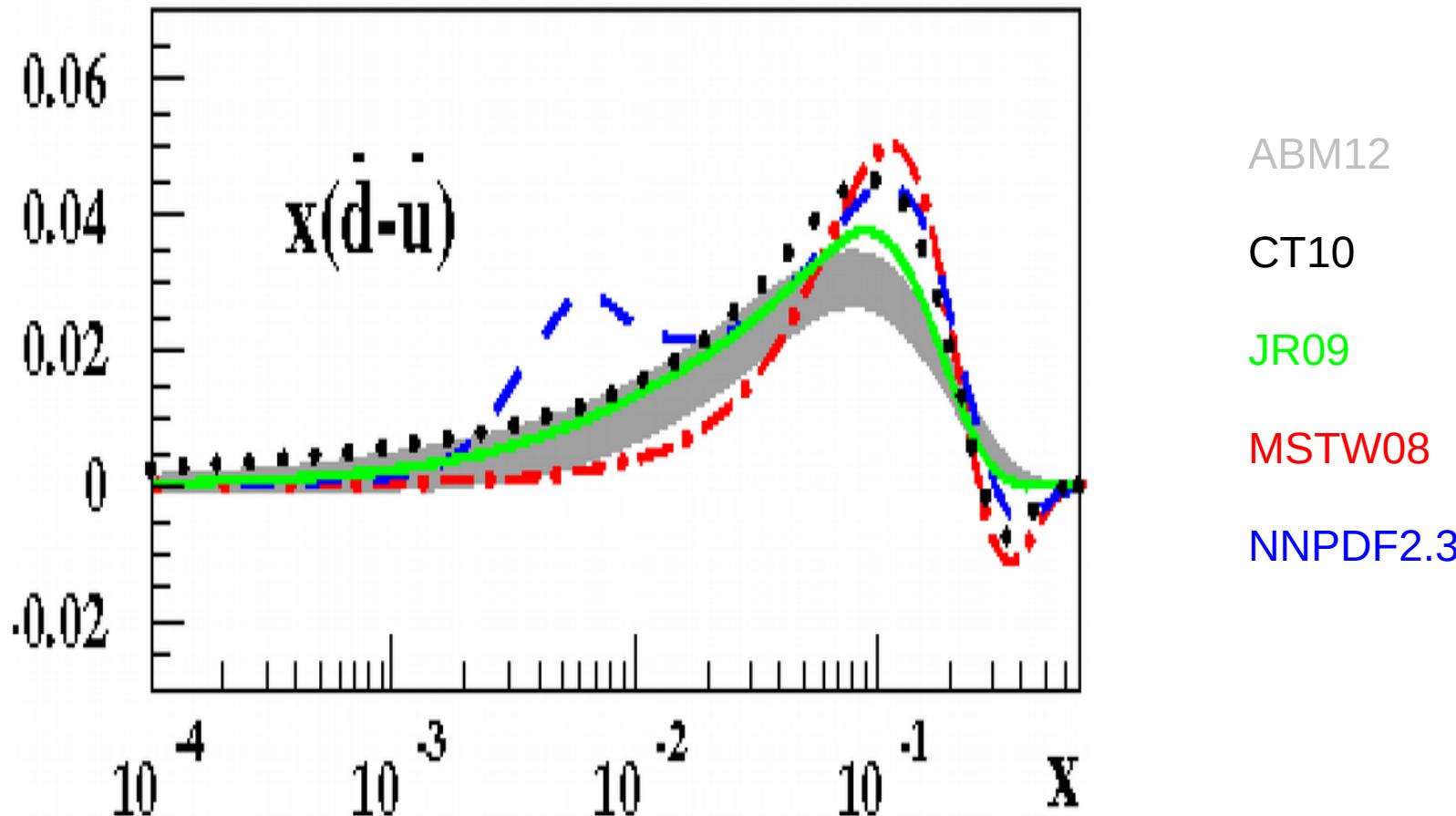
$\mathbf{P}_0 \pm \Delta\mathbf{P}_0$  – vector of PDF parameters with errors obtained in the earlier fit

$\mathbf{E}$  – error matrix

$\mathbf{P}$  – current value of the PDF parameters in the fit

- store the DY NNLO c.s. for all PDF sets defined by the eigenvectors of  $\mathbf{E}$
- the variation of the fitted PDF parameters ( $\mathbf{P} - \mathbf{P}_0$ ) is transformed into this eigenvector basis
- the NNLO c.s. in the PDF fit is calculated as a combination of transformed ( $\mathbf{P} - \mathbf{P}_0$ ) with the stored eigenvector values

# Sea quark iso-spin asymmetry



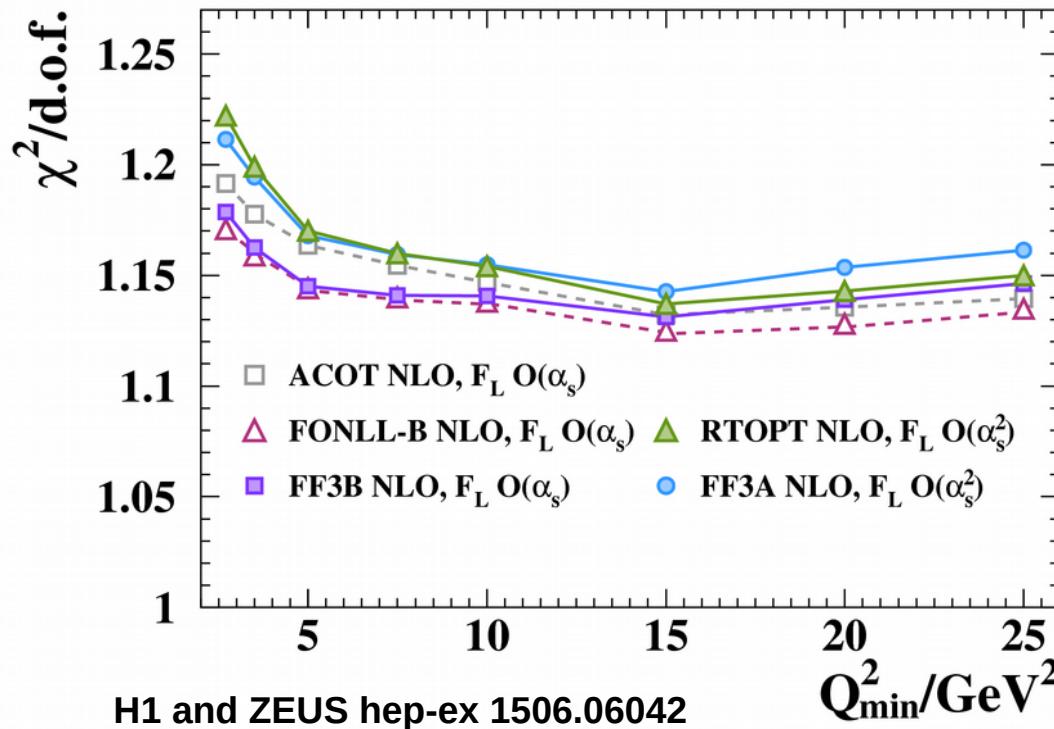
sa, Blümlein, Moch PRD 89, 054028 (2014)

- At  $x \sim 0.1$  the sea quark iso-spin asymmetry is controlled by the fixed-target DY data (E-866), weak constraint from the DIS (NMC)
- At  $x < 0.01$  Regge-like constraint like  $x^{(a-1)}$ , with a close to the meson trajectory intercept; the “unbiased” NNPDF fit follows the same trend

*Onset of the Regge asymptotics is out of control*

# Factorization scheme benchmarking

## H1 and ZEUS



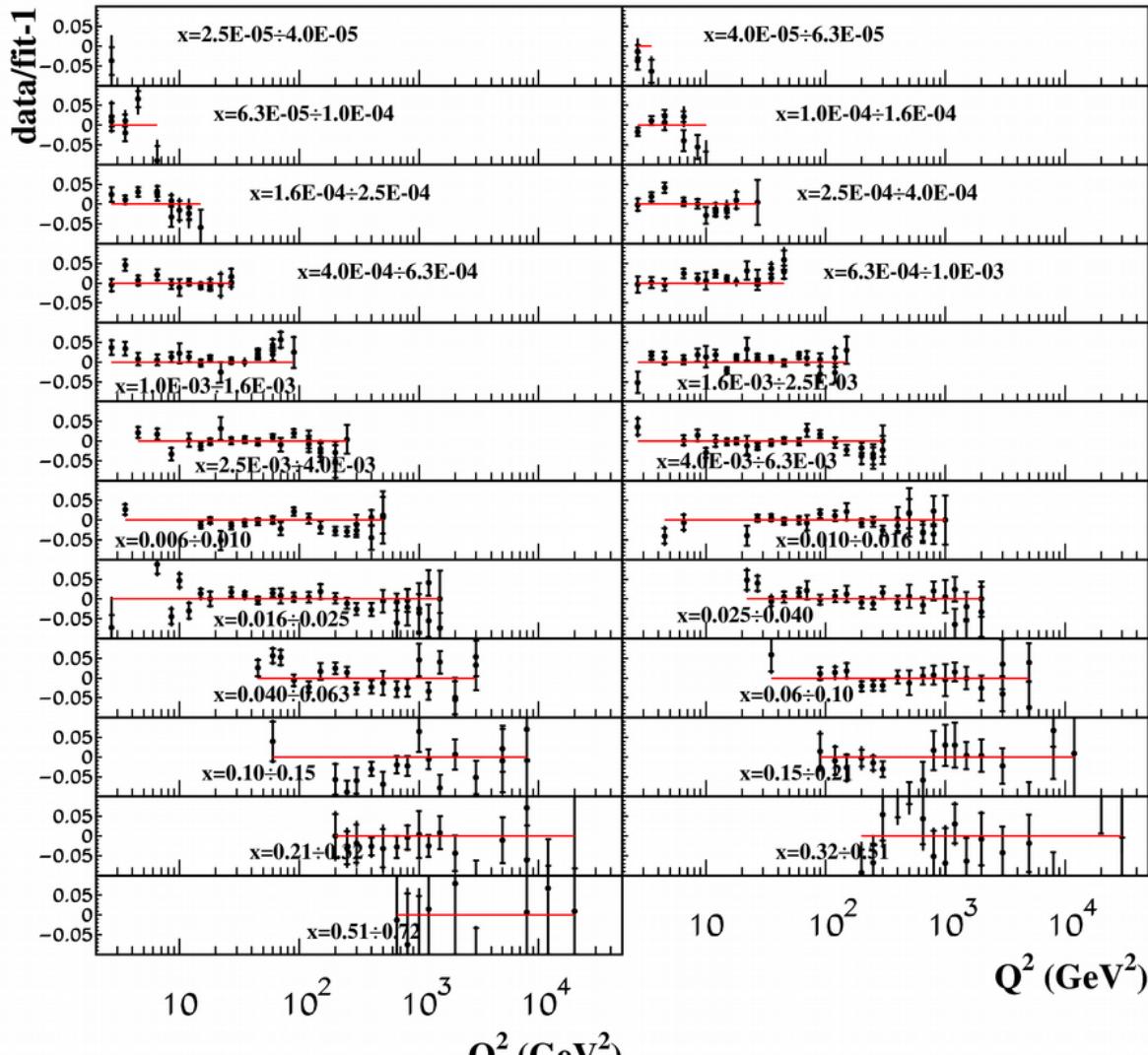
- Data allow to discriminate factorization schemes
- FFN scheme works very well in case of correct setting (running mass definition and correct value of  $m_c$ ) → no traces of big logs due to resummation

$x_{\min}$	$x_{\max}$	$Q^2_{\min}$ (GeV)	$Q^2_{\max}$ (GeV)	$\Delta\chi^2$ (DIS)	$N_{\text{dat}}^{\text{DIS}}$	$\Delta\chi^2$ (HERA-I)	$N_{\text{hera-I}}^{\text{dat}}$
$4 \cdot 10^{-5}$	1	3	$10^6$	72.2	2936	77.1	592
$4 \cdot 10^{-5}$	0.1	3	$10^6$	87.1	1055	67.8	405
$4 \cdot 10^{-5}$	0.01	3	$10^6$	40.9	422	17.8	202
$4 \cdot 10^{-5}$	1	10	$10^6$	53.6	2109	76.4	537
$4 \cdot 10^{-5}$	1	100	$10^6$	91.4	620	97.7	412
$4 \cdot 10^{-5}$	0.1	10	$10^6$	84.9	583	67.4	350
$4 \cdot 10^{-5}$	0.1	100	$10^6$	87.7	321	87.1	227

We conclude that the FFN fit is actually based on a less precise theory, in that it does not include full resummation of the contribution of heavy quarks to perturbative PDF evolution, and thus provides a less accurate description of the data

# Statistical check of big-log impact in ABM12 fit

HERA-I e<sup>+</sup>p

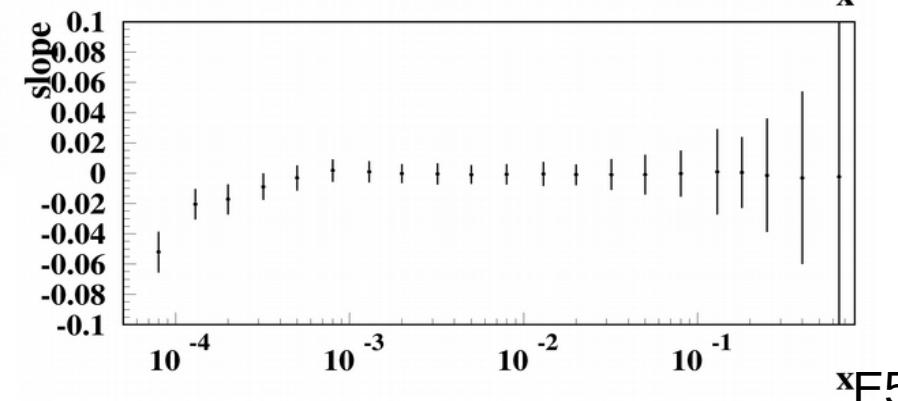
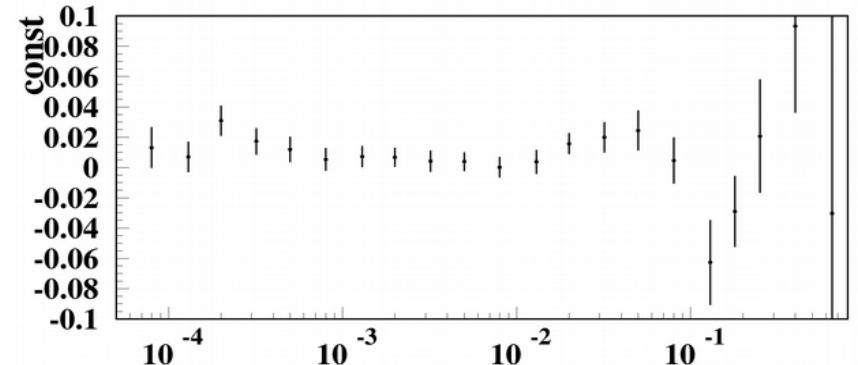


$$\text{pulls} = \text{const} + \text{slope} * \log(Q^2/Q^2_0)$$

*No traces of big logs*

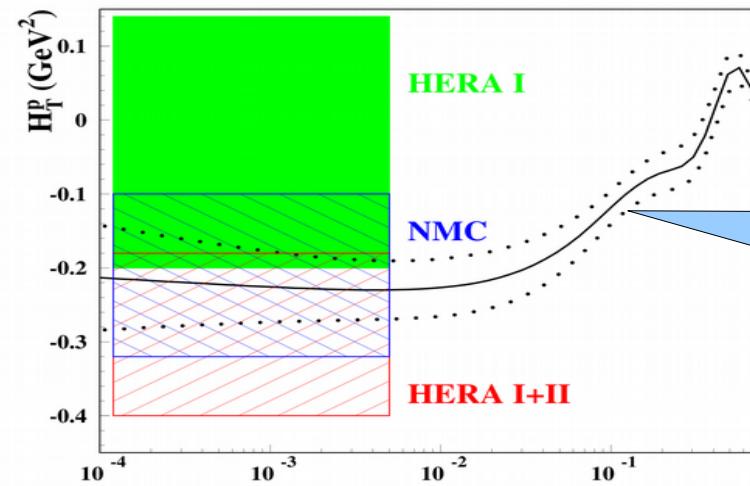
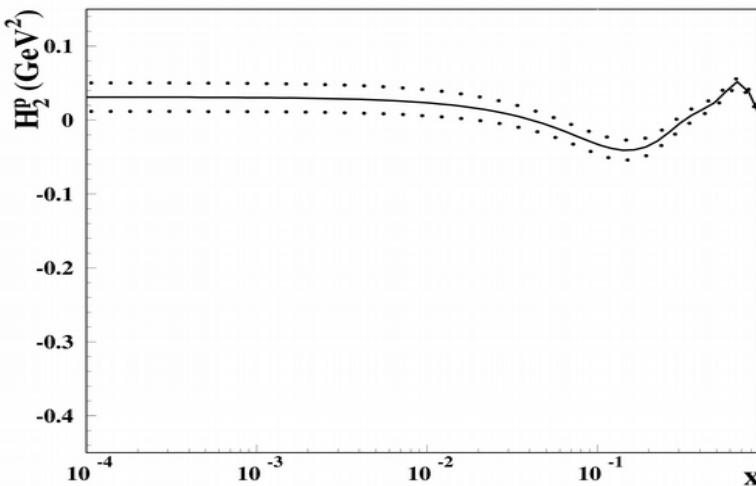
(cf. Extras and sa, Blümlein, Moch hep-ph/1307.7258)

$Q^2_{\min}$ (GeV <sup>2</sup> )	$\chi^2/\text{NDP}$
10	366 / 324
100	193 / 201
1000	95 / 83



# High twists at small x

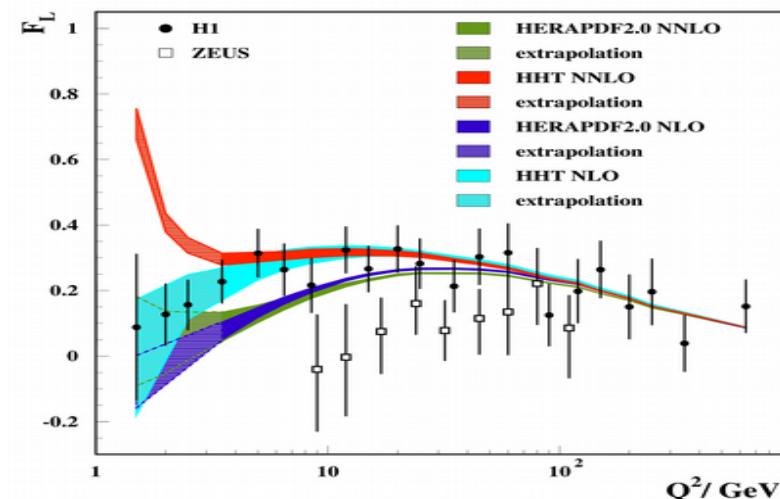
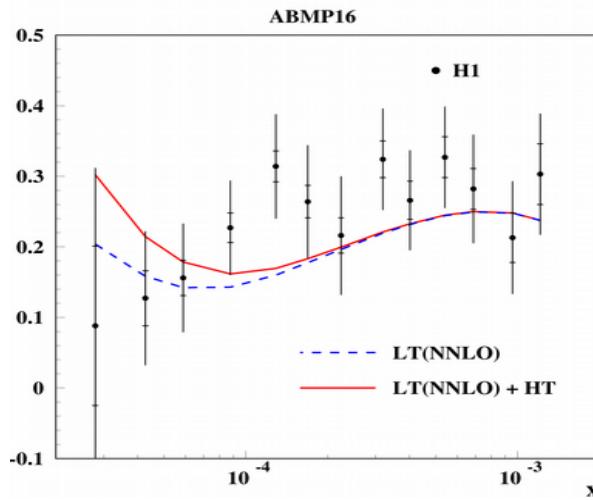
$$F_{2,L} = F_{2,L}(\text{leading twist}) + H_{2,L}(x)/Q^2 \quad H(x) = x^h P(x)$$



sa, Blümlein, Moch  
PRD 86, 054009 (2012)

- $H_T(x)$  continues a trend observed at larger  $x$ ;  $H_2(x)$  is comparable to 0 at small  $x$
- $h_T = 0.05 \pm 0.07 \rightarrow$  slow vanishing at  $x \rightarrow 0$
- $\Delta\chi^2 \sim -40$

Harland-Lang, Martin, Motylinski, Thorne hep-ph/1601.03413



No dramatic increase of  $F_L$  at small  $x$

Abt, et al. hep-ex/1604.02299