# Higgs boson pair production in gluon fusion at NLO with full top-quark mass dependence

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Loops and Legs, Leipzig

MAX-PLANCK-GESELLSCHAFT

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arXiv:1604.06447

In collaboration with:
S. Borowka, N. Greiner, G. Heinrich, S. Jones, J. Schlenk, U. Schubert, T. Zirke

# Motivation

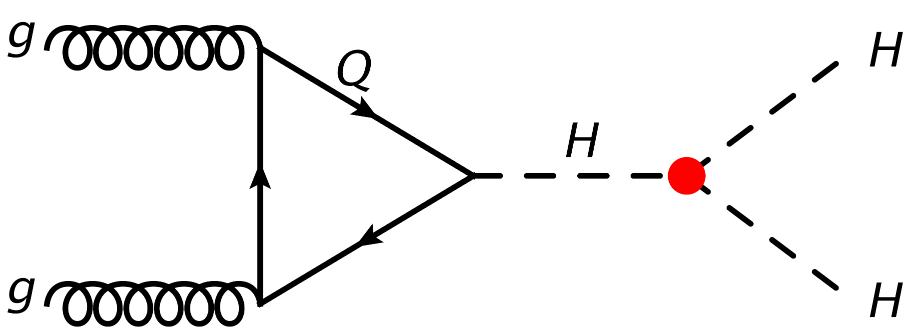
Higgs Lagrangian:

$$\mathcal{L} \supset -V(\Phi), \quad V(\Phi) = \frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4$$

↓ EW symmetry breaking

$$\frac{m_H^2}{2}H^2 + \frac{m_H^2}{2v}H^3 + \frac{m_H^2}{8v^2}H^4$$

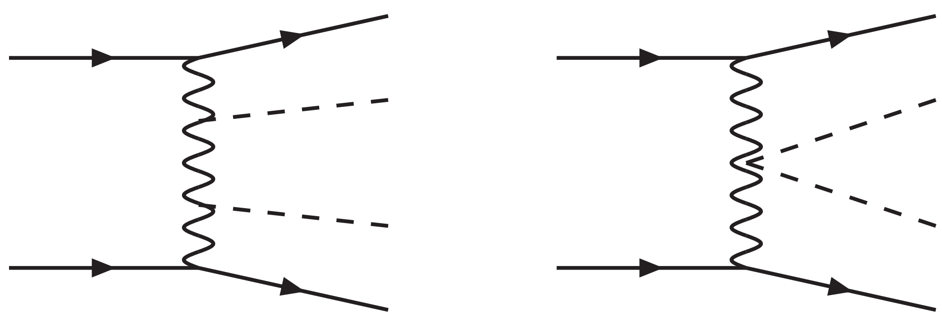
triple-Higgs coupling    measured in Higgs pair production



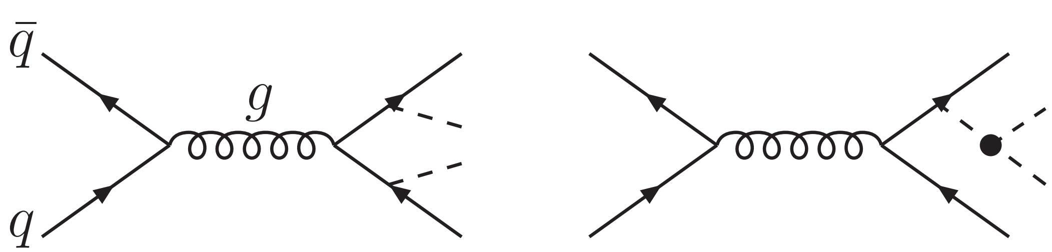
# Higgs Pair Production channels

- gluon fusion

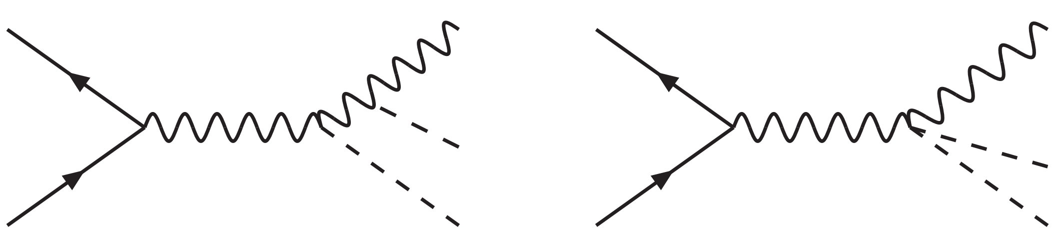

- vector boson fusion

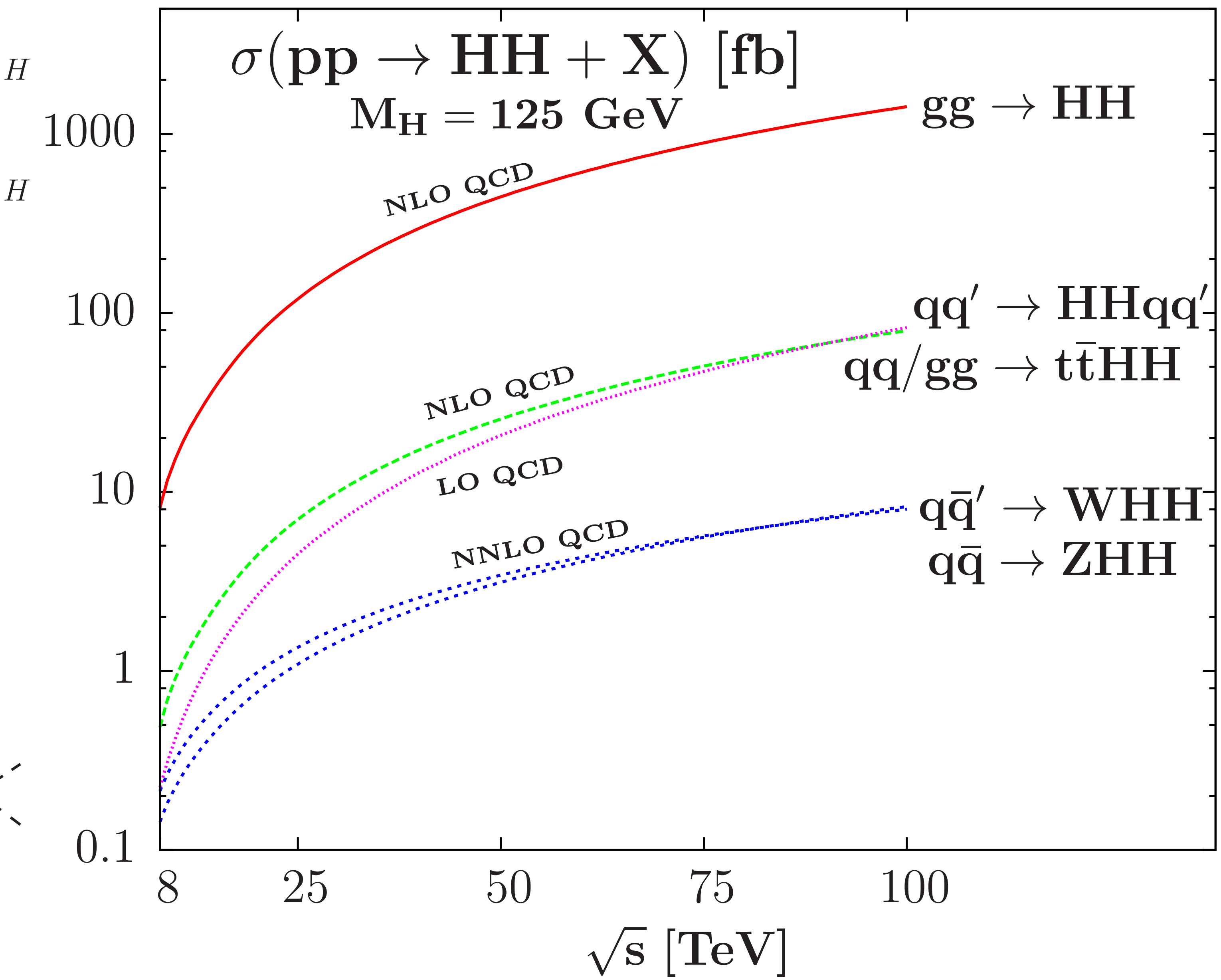


- top-quark associated



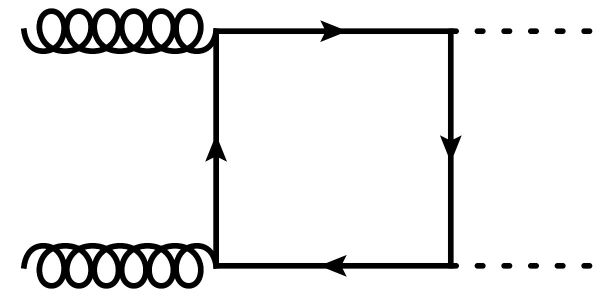

- Higgs strahlung






Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira `12

# gg→HH known results

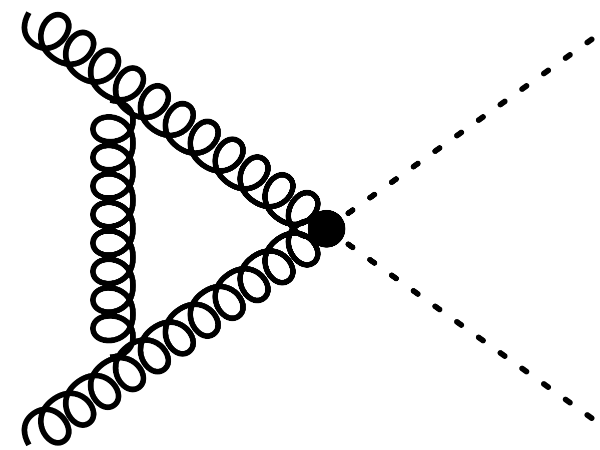


1. LO, including full $m_T$ dependence

   Glover, van der Bij `88

2. NLO ( $m_t \to \infty$ limit)

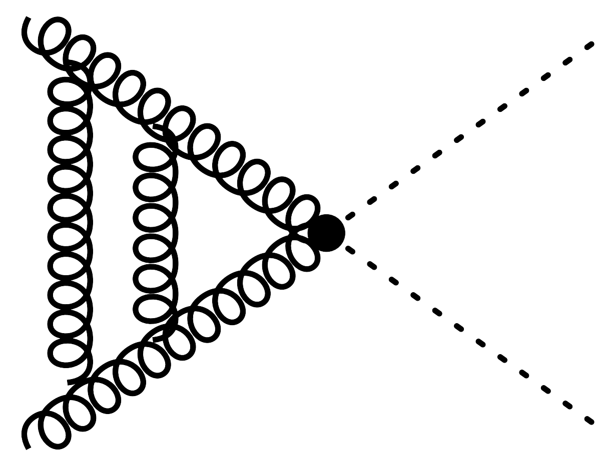
   Plehn, Spira, Zerwas `96, `98; Dawson, Dittmaier, Spira `98



   - including full $m_T$ dependence in real radiation
     Maltoni, Vryonidou, Zaro `14
   - including $1/m_T$ expansion
     Grigo, Hoff, Melnikov, Steinhauser `13; Grigo, Hoff, Steinhauser `15
     Degrassi, Giardino, Gröber `16

3. NNLO ( $m_t \to \infty$ limit)

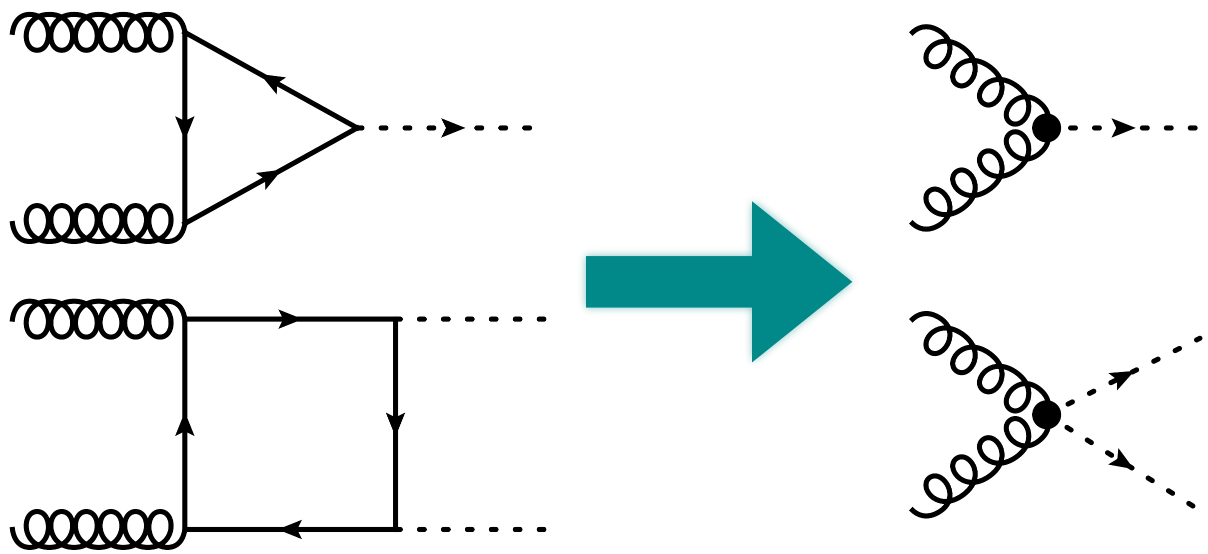
   De Florian, Mazzitelli `13



   - including all matching coefficients
     Grigo, Melnikov, Steinhauser `14
   - including $1/m_T$ expansion
     Grigo, Hoff, Steinhauser `15
   - NNLL soft gluon resummation
     Shao, Li, Li, Wang `13; de Florian, Mazzitelli `15

# HEFT and approximated NLO results

- $m_T \to \infty$ limit (Higgs EFT)



HEFT valid for $\sqrt{s} \ll 2\, m_T$

Higgs pair production:

$$2\, m_H < \sqrt{s}$$

- Born improved NLO HEFT

$$d\sigma_{NLO} \approx d\sigma_{NLO}^{HEFT} = \frac{d\sigma_{NLO}(m_t \to \infty)}{d\sigma_{LO}(m_t \to \infty)}\, d\sigma_{LO}(m_t)$$

Spira et al. (HPAIR)

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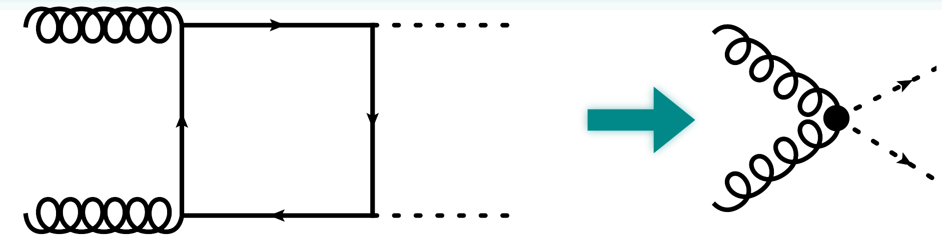

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**K ≈ 2**

Spira et al. (HPAIR)

- further improvements:

Maltoni Vryonidou, Zaro `14

$$d\sigma_{NLO}^{V,HEFT}$$

$$d\sigma_{NLO}^{R}(m_t)$$

**-10%**

Grigo, Hoff, Melnikov, Steinhauser `13

$$\sigma_{exp} = \sum_{n}^{6} c_n \rho^n, \quad \rho = \frac{m_H^2}{m_t^2}$$

$$\sigma^{NLO} = \sigma_{exp}^{NLO} \cdot \frac{\sigma^{LO}}{\sigma_{exp}^{LO}}$$

**+10%**

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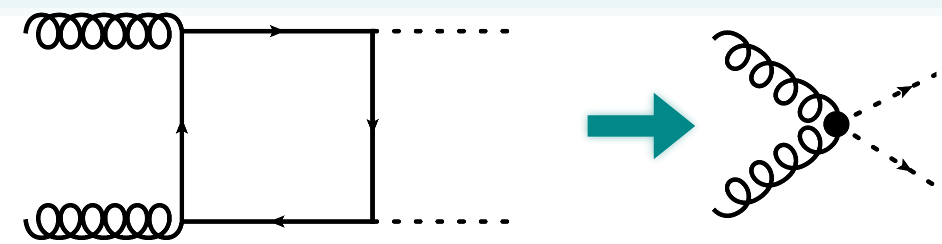

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**±10%**

$$\sigma^{NLO} = \int \mathrm{d}Q^2 \frac{\mathrm{d}\sigma_{exp}^{NLO}}{\mathrm{d}Q^2} \cdot \frac{\mathrm{d}\sigma^{LO}/\mathrm{d}Q^2}{\mathrm{d}\sigma_{exp}^{LO}/\mathrm{d}Q^2}$$

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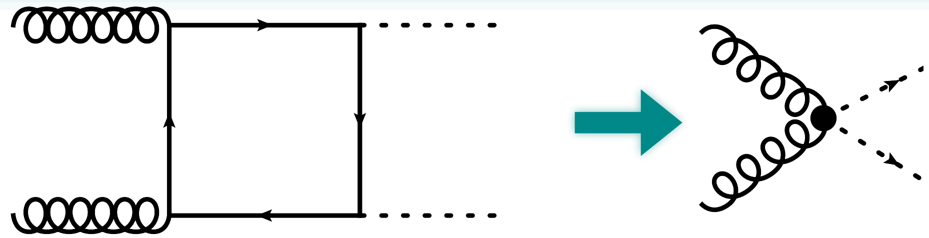

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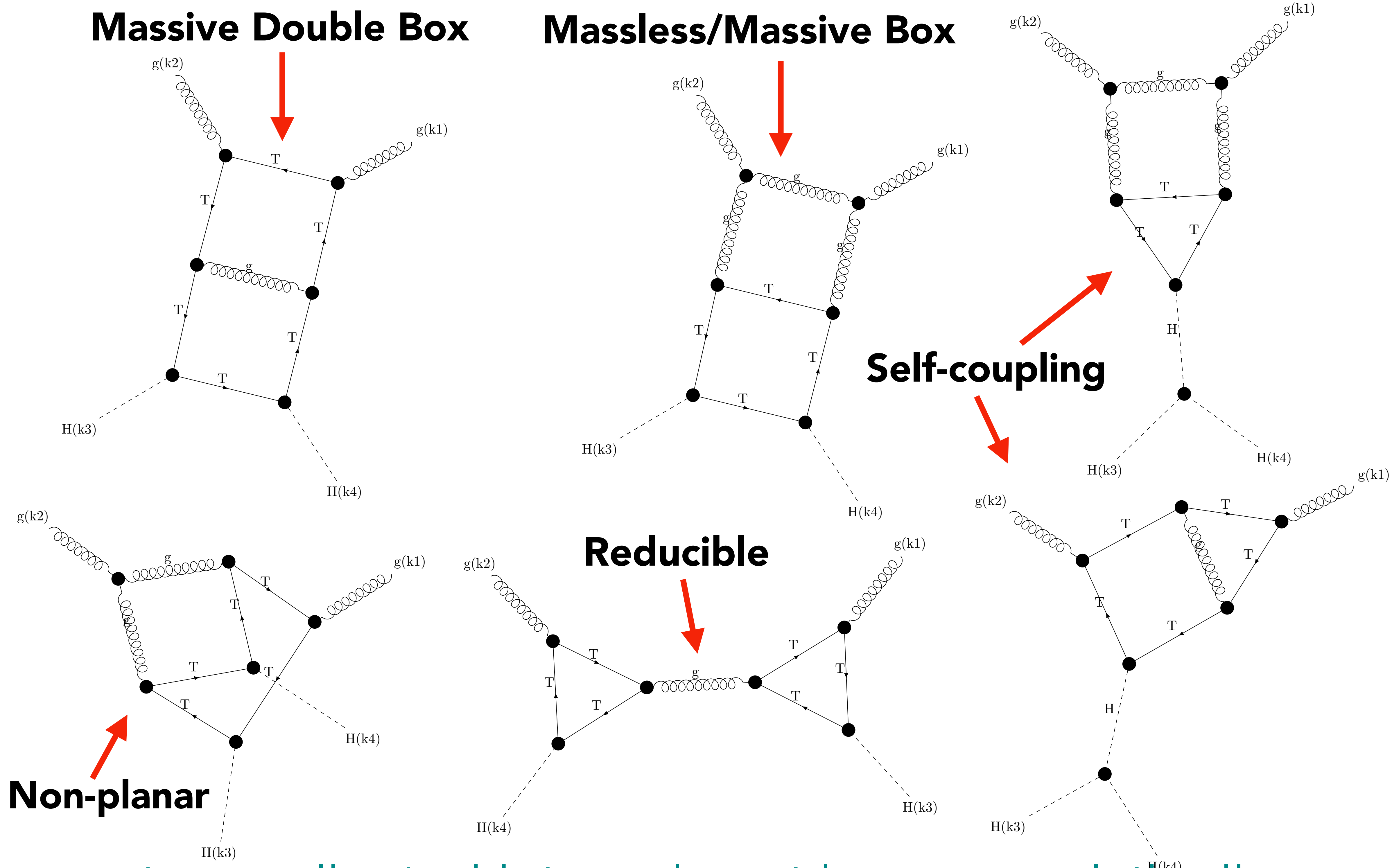
**±10%**

mass effects largest uncertainty

→calculation with full mass dependence needed

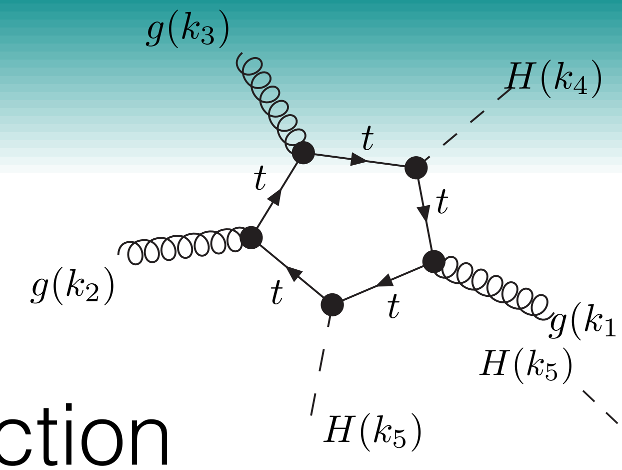
# Two Loop Diagrams

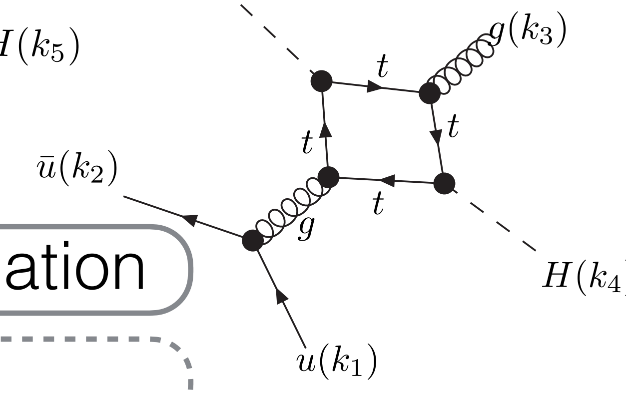



most complicated integrals not known analytically
→ numeric calculation using SecDec

# Tools

- LO and Real Radiation
  - Gosam
    Cullen, van Deurzen, Greiner, Heinrich, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Schlenk, von Soden-Fraunhofen, Tramontano
  - dipole subtraction
    Catani Seymour




- Virtual Corrections — GoSam-2Loop

this calculation

future?

reduction ⟷ amplitude generation ⟷ loop integrals

**reduction**

- Reduze
  von Manteuffel, Studerus `12
- FIRE
  Smirnov, Smirnov `13
- LiteRed
  Lee `13

- integrand reduction
  Mastrolia, Ossola, Peraro, Schubert

→talks by Mastrolia, Ossola

**amplitude generation**

GoSam-2Loop

GoSam-1L collaboration + Jahn, Jones, MK, Zirke

using QGRAF (Nogueira `93)
and FORM (Vermaseren et al. `12 )

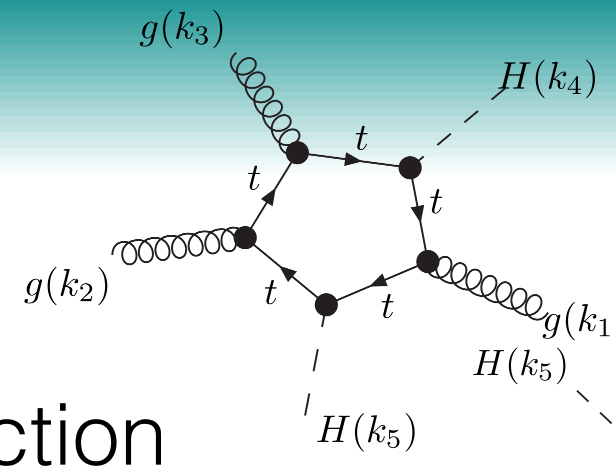
HH: 2nd implementation
using QGRAF, Reduze, Mathematica

**loop integrals**

- SecDec
  Borowka, Heinrich, Jahn, Jones, MK, Schlenk, Zirke

- analytic results
  Mastrolia, Schubert

# Tools

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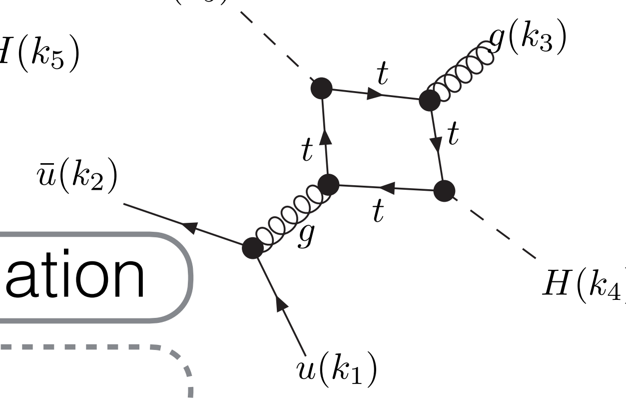


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- analytic results
  Mastrolia, Schubert

more details: Talk by Stephen Jones

# Two Loop Amplitude

- tensor structure Glover, van der Bij `88

$$\mathcal{M} = \epsilon_\mu(p_1,n_1)\epsilon_\nu(p_2,n_2)\,\mathcal{M}^{\mu\nu}$$

$$\mathcal{M}^{\mu\nu} = A_1(s,t,m_H^2,m_t^2,D)\,T_1^{\mu\nu} + A_2(s,t,m_H^2,m_t^2,D)\,T_2^{\mu\nu}$$

with

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\nu\, p_2^\mu}{p_1\cdot p_2}$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{1}{p_T^2\,(p_1\cdot p_2)}\left\{m_H^2 p_1^\nu\, p_2^\mu - 2\,(p_1\cdot p_3)\,p_3^\nu\, p_2^\mu - 2\,(p_2\cdot p_3)\,p_3^\nu\, p_1^\mu + 2\,(p_1\cdot p_2)\,p_3^\nu\, p_3^\mu\right\}$$

$$\mathcal{M}^{++} = \mathcal{M}^{--} = -A_1$$
$$\mathcal{M}^{+-} = \mathcal{M}^{-+} = -A_2$$

triangle diagrams gg→H→HH only contribute to $A_1$

- projectors

construct $P_i^{\mu\nu} = \sum_j c_{ij} T_j^{\mu\nu}$ such that

$$P_1^{\mu\nu}\mathcal{M}_{\mu\nu} = A_1(s,t,m_H^2,m_t^2,D)$$
$$P_2^{\mu\nu}\mathcal{M}_{\mu\nu} = A_2(s,t,m_H^2,m_t^2,D)$$

# Integral Reduction

Reduction to master integrals using Reduze

- integral families with 9 propagators: 5(3) planar(non-planar) families
- full dependence on $s,\ t,\ m_t^2,\ m_H^2$ challenging
  ➡ simplification: fix $m_t = 173\,\mathrm{GeV},\ m_H = 125\,\mathrm{GeV}$
- (mostly) finite basis
  von Manteuffel, Panzer, Schabinger
- non-planar sectors still unreduced

| Integrals | 1-loop | 2-loop |
|---|---|---|
| **Direct** | 63 | 9865 |
| **+ Symmetries** | 21 | 1601 |
| **+ IBPs** | 8 | ~260-270<br>currently: 327 |

Non-planar integrals    rewrite inverse prop. → scalar products

$$\int \mathrm{d}^d p_1 \mathrm{d}^d p_2 \frac{(p_1+k_1)^2}{p_1^2} f(p_i,k_i) = \int \mathrm{d}^d p_1 \mathrm{d}^d p_2 \left(1 + \frac{k_1^2}{p_1^2} + \frac{2\,p_1\cdot k_1}{p_1^2}\right) f(p_i,k_i)$$

rank-2    rank-1

up to 4 inverse propagators → up to rank-4 tensors

# Amplitude — Loop Integrals

## SecDec

- sector decomposition of loop integrals Binoth, Heinrich
- contour deformation Nagy, Soper

→ numerical integration possible

interface

## Amplitude & numerical integration

- using Quasi-Monte-Carlo (QMC) integration
  $\mathcal{O}(n^{-1})$ scaling of integration error
- split each integral into sectors
- dynamically set n for each integral, minimizing

$$T = \sum_{\text{integral } i} t_i + \lambda \left( \sigma^2 - \sum_i \sigma_i^2 \right) \qquad \sigma_i = c_i \cdot t_i^{-e}$$

$\sigma_i =$ error estimate (including coefficients in amplitude)

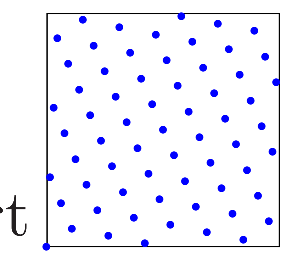
$\lambda =$ Lagrange multiplier $\qquad \sigma =$ precision goal

- avoid reevaluation of integrals for different orders in $\varepsilon$ and form factors
- parallelization on gpu

### QMC rank-1 lattice rule

$$I = \int \mathrm{d}\vec{x} f(\vec{x}) \approx I_k = \frac{1}{n} \sum_{i=1}^{n} f(\vec{x}_{i,k})$$

$$\vec{x}_{i,k} = \left\{ \frac{i \cdot \vec{g}}{n} + \vec{\Delta}_k \right\}$$



$\{\ldots\} =$ fractional part

$\vec{g} =$ generating vector

$\vec{\Delta}_k =$ randomized shift

$m$ different estimates $I_1 \ldots I_m$

$\rightarrow$ error estimate

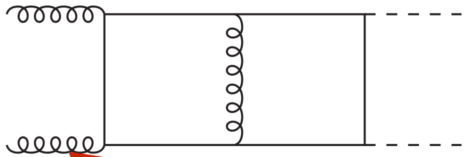
Review: Dick, Kuo, Sloan

# Amplitude Evaluation — Example

$$\sqrt{s} = 327.25\,\text{GeV},\ \sqrt{-t} = 170.05\,\text{GeV},\ M^2 = s/4$$

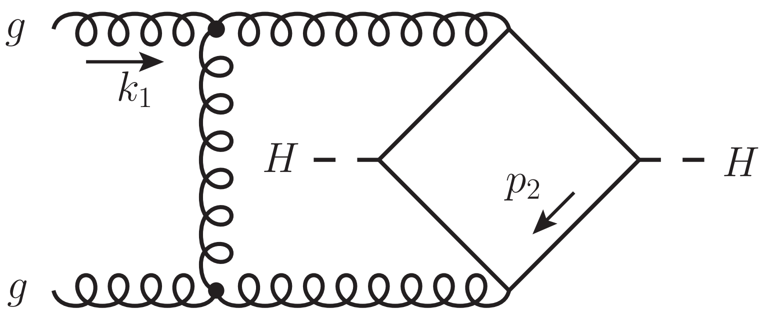
contributing integrals:

| integral | value | error | time [s] |
|---|---|---|---|
| ... | | | |
| F1_011111110_ord0 | (0.484, 4.96e-05) | (4.40e-05, 4.23e-05) | 11.8459 |
| ... | | | |
| N3_111111100_k1p2k2p2_ord0 | (0.0929, -0.224) | (6.32e-05, 5.93e-05) | 235.412 |
| N3_111111100_1_ord0 | (-0.0282, 0.179) | (8.01e-05, 9.18e-05) | 265.896 |
| N3_111111100_k1p2k1p2_ord0 | (0.0245, 0.0888) | (5.06e-05, 5.31e-05) | 282.794 |
| N3_111111100_k1p2_ord0 | (-0.00692, -0.108) | (3.05e-05, 3.05e-05) | 433.342 |



$\approx 700$ integrals

$$I(s,t,m_t^2,m_h^2) = -\left(\frac{\mu^2}{M^2}\right)^{2\varepsilon} \Gamma(3+2\epsilon) M^{-4} \left(\frac{A_{-2}}{\epsilon^2} + \frac{A_{-1}}{\epsilon^1} + A_0 + \mathcal{O}(\epsilon)\right)$$



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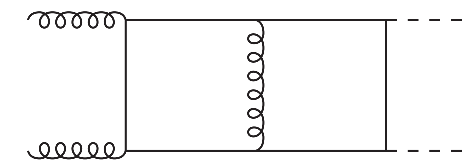

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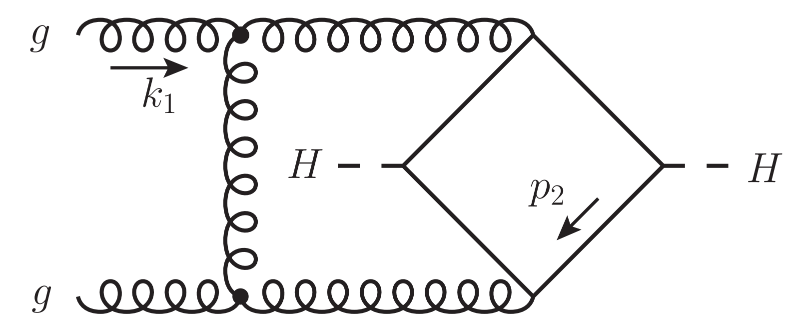


sector decomposition

| sector | integral value | error | time [s] | #points |
|---|---|---|---|---|
| 5 | (-1.34e-03, 2.00e-07) | (2.38e-07, 2.69e-07) | 0.255 | 1310420 |
| 6 | (-1.58e-03, -9.23e-05) | (7.44e-07, 5.34e-07) | 0.266 | 1310420 |
| ... | | | | |
| 41 | (0.179, -0.856) | (1.10e-05, 1.22e-05) | 29.484 | 79952820 |
| 42 | (0.359, -1.308) | (1.40e-06, 1.58e-06) | 80.24 | 211436900 |
| 44 | (0.0752, -1.185) | (5.44e-07, 6.76e-07) | 99.301 | 282904860 |

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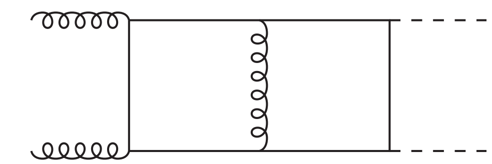

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|---|---|---|---|
| … | | | |
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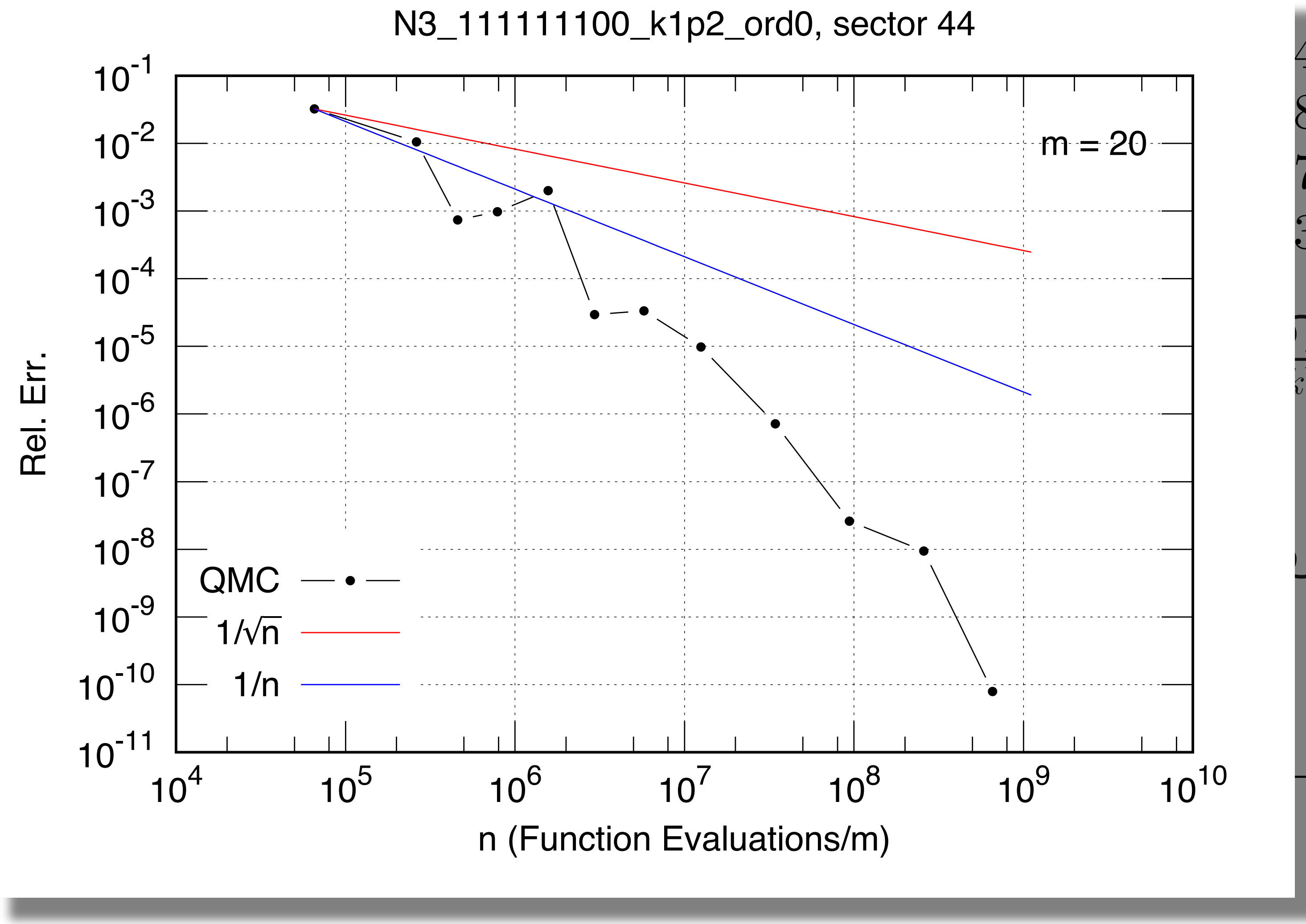
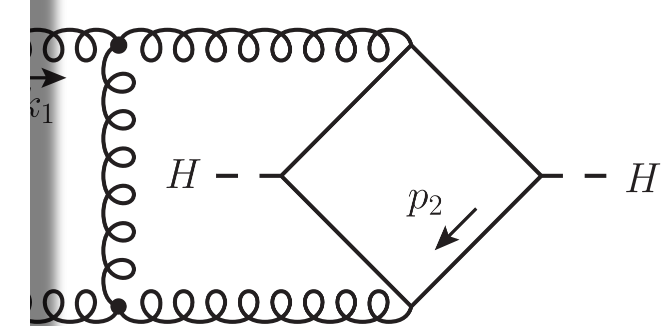
| … | | | |
| N3_111111100_k1p2 | | | 412 |
| N3_111111100_1_or | | | 896 |
| N3_111111100_k1p2 | | | 794 |
| N3_111111100_k1p2 | | | 342 |

≈ 700 integrals

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| sector | in | | | | |
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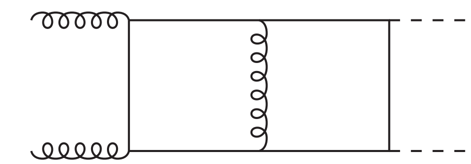

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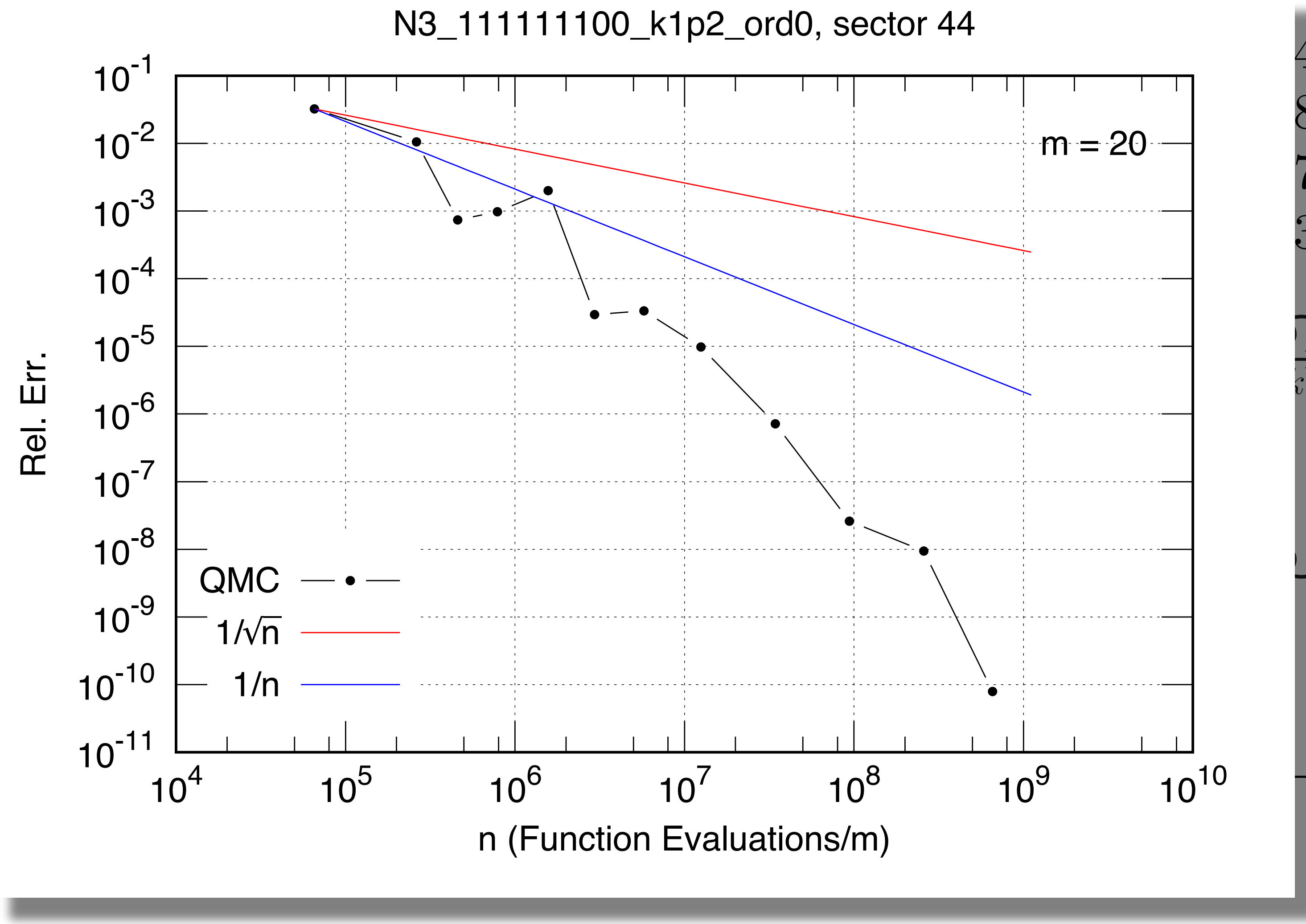
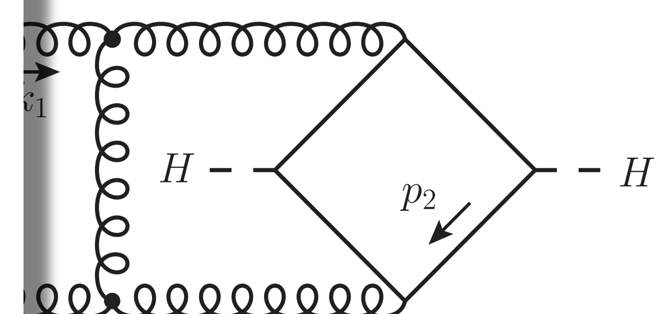
≈ 700 integrals

$I(s,t,m_t^2,m_h^2) = - ($




| sector | in | | | |
|---|---|---|---|---|
| 5 | (-1.34e-0 | | | |
| 6 | (-1.58e-03 | | | |
| ... | | | | |
| 41 | (0.179, -0.856) | (1.10e-05, 1.22e-05) | 29.484 | 79952820 |
| 42 | (0.359, -1.308) | (1.40e-06, 1.58e-06) | 80.24 | 211436900 |
| 44 | (0.0752, -1.185) | (5.44e-07, 6.76e-07) | 99.301 | 282904860 |

**amplitude result:**

**gpu time: 2 h**

**accuracy**
**$F_1$ ($F_2$): 0.2% (11%)**

# Phase-space points

- Importance sampling:

  LO calculation → unweighted events → sampling points virtual amplitude } $\sigma^V$ with 2.5% accuracy using 665 phase-space points

- Accuracy goal:
  - 3% for form factor $F_1$
  - 5-20% for form factor $F_2$ (depending on $F_2/F_1$)

- Run time: (gpu time)
  - 80 min - 2 d (≙wall-clock limit)
  - median: 2h

- one point at $s/m_t^2 = 4.01$ excluded (huge integration error) (stable results for points at $s/m_t^2 = 3.98$ and $s/m_t^2 = 4.05$ )

# Checks

## Real Emission / Subtraction Terms

- Independence of dipole-cut $\alpha$ parameter Nagy `03
- Agreement with Maltoni, Vryonidou, Zaro `14

## Virtual Corrections

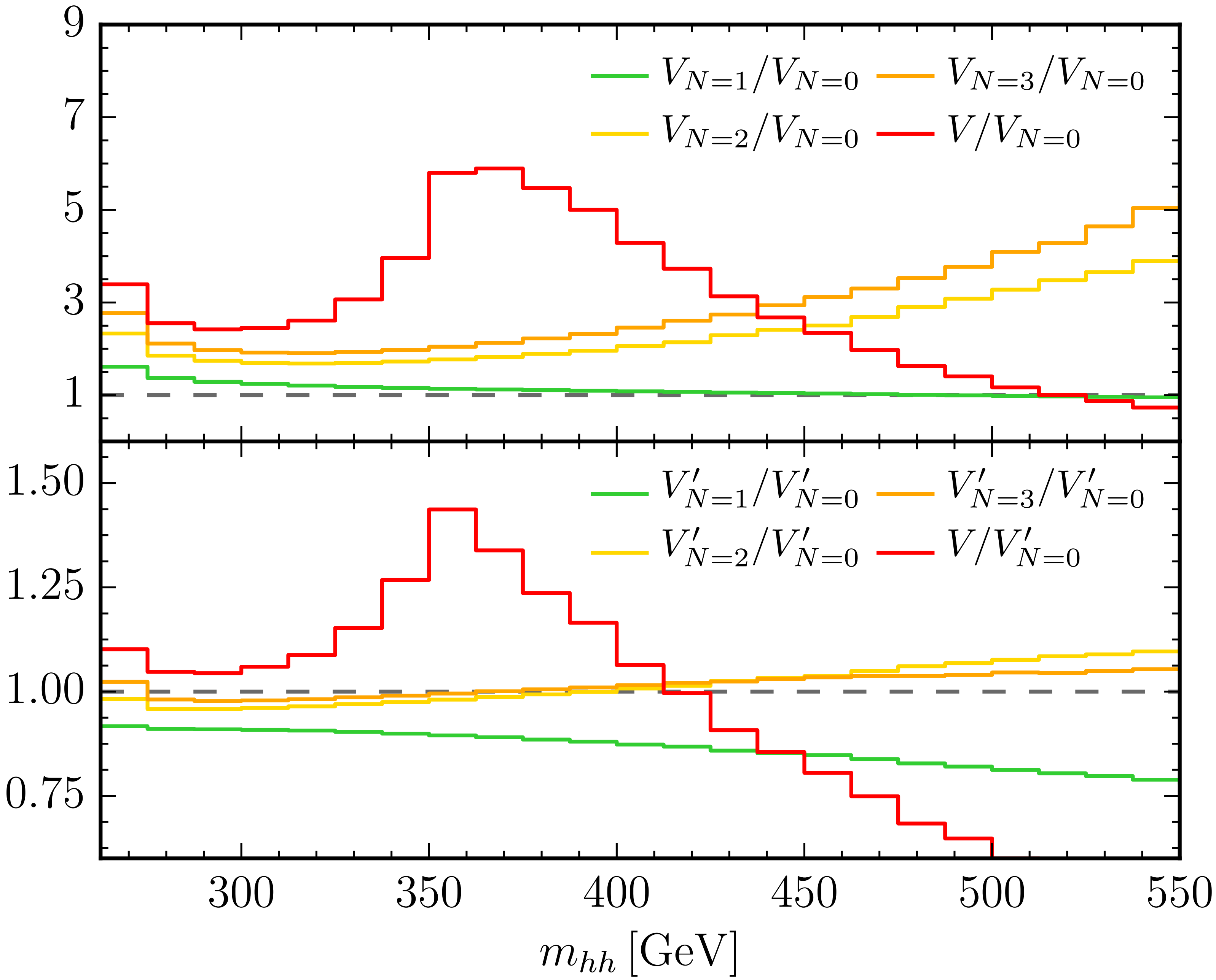
- Two calculations of amplitude up to reduction
- Amplitude result invariant under $t \leftrightarrow u$
- Pole cancellation

Harlander,Liebler, Mantler

- Agreement of contributions gg → H → HH with SusHi
- Convergence of $1/m_T$ expansion towards full result

# Results - Amplitude

comparison with HEFT and expansion in $1/m_t$



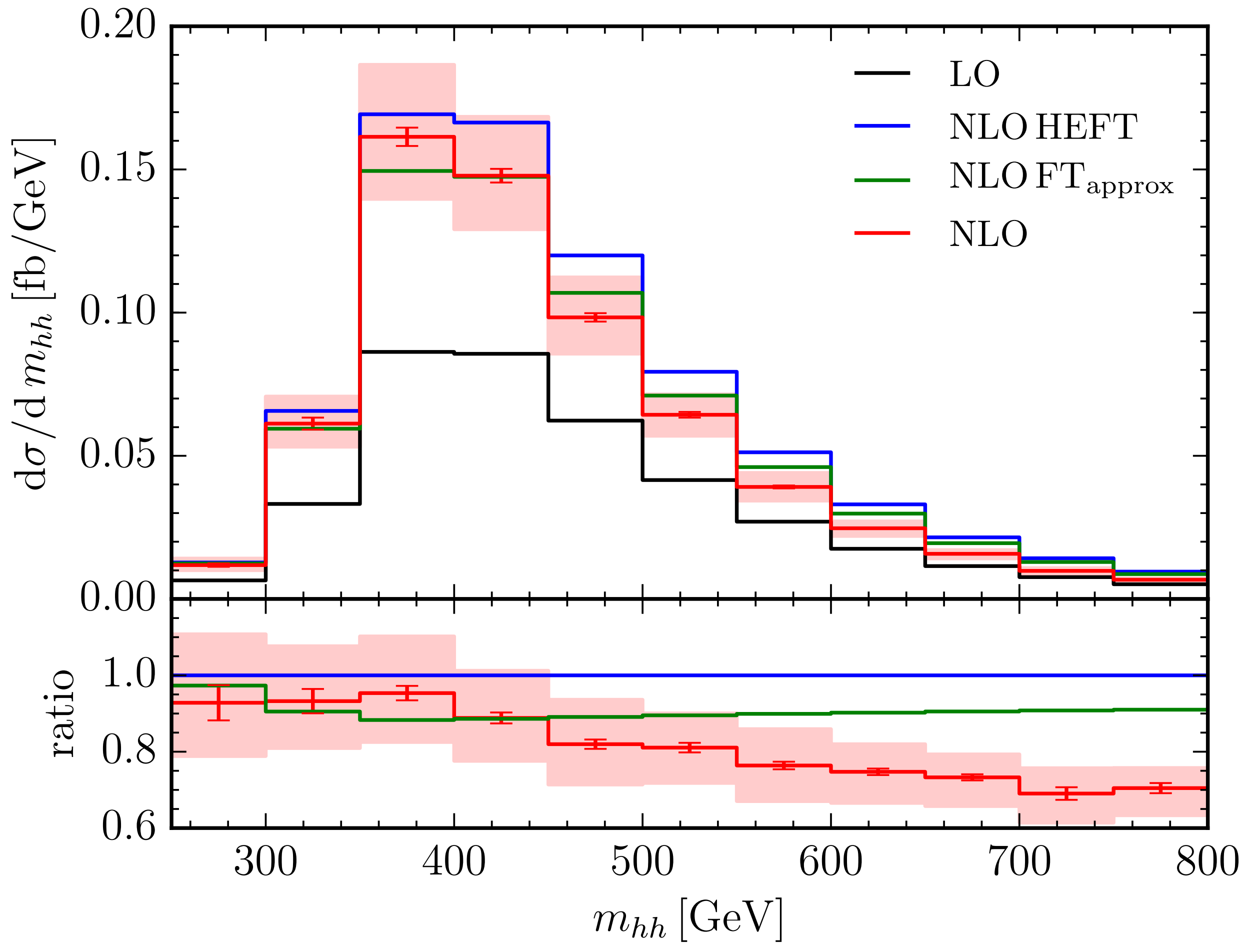

$$V_N = \left(d\hat{\sigma}^{\text{virt}}_{\text{exp},N} + d\hat{\sigma}^{\text{LO}}_{\text{exp},N}(\epsilon) \otimes I\right) \frac{d\hat{\sigma}^{\text{LO}}(\epsilon)}{d\hat{\sigma}^{\text{LO}}_{\text{exp},N}(\epsilon)}$$

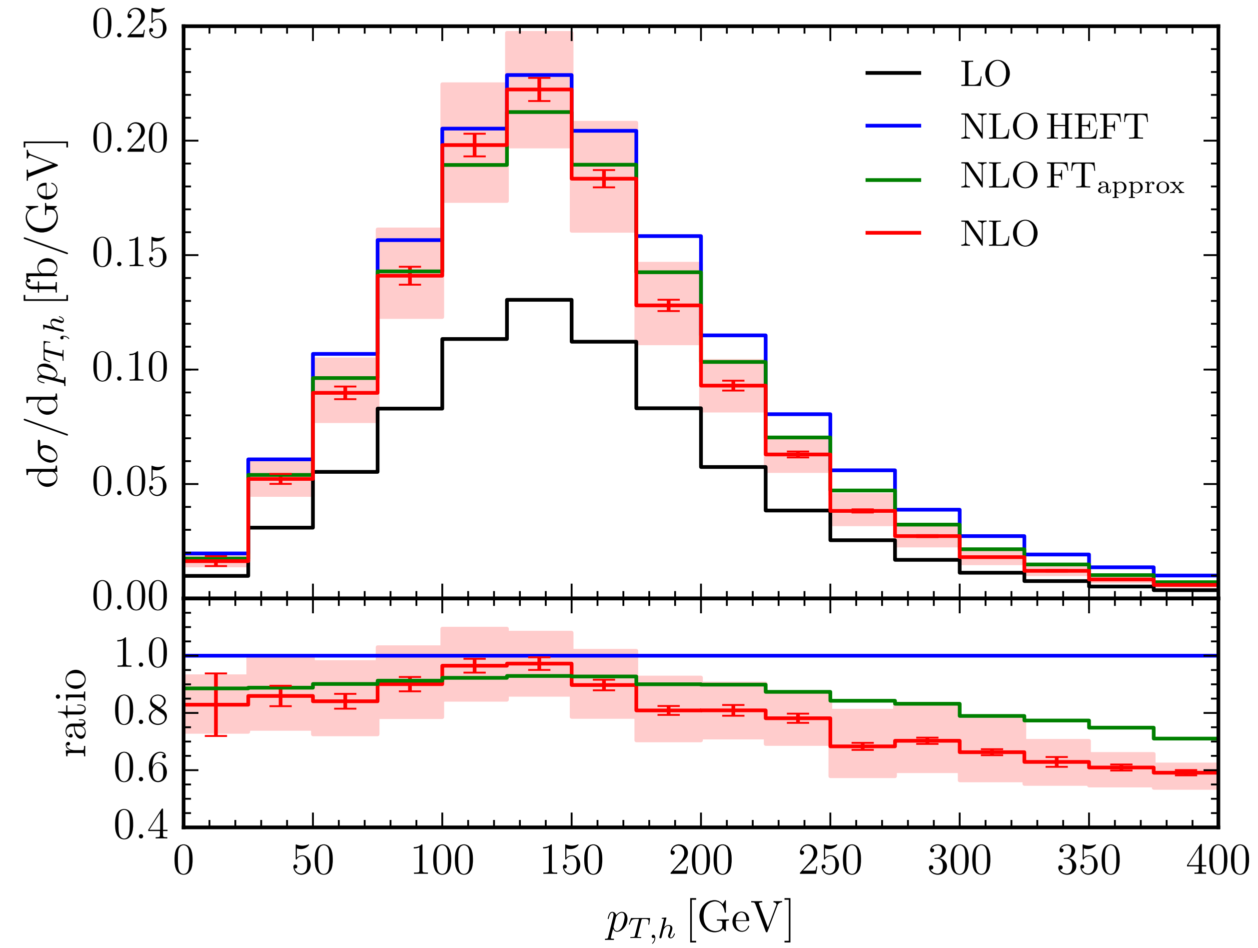
$$d\hat{\sigma}_{\text{exp},N} = \sum_{\rho=0}^{N} d\hat{\sigma}^{(\rho)} \left(\frac{\Lambda}{m_t}\right)^{2\rho}$$

$$\Lambda \in \left\{\sqrt{\hat{s}}, \sqrt{\hat{t}}, \sqrt{\hat{u}}, m_h\right\}$$

$$V'_N = V_N \cdot \frac{B}{B_N}$$

# Results - Cross Section




$$\sigma^{NLO} = 32.80^{+13\%}_{-12\%}\,\text{fb} \pm 0.4\%\,(\text{stat.}) \pm 0.1\%\,(\text{int.})$$

Born-improved HEFT: $\sigma^{NLO}_{HEFT} = 38.32^{+18\%}_{-15\%}\,\text{fb}$

LO: $\sigma^{LO} = 19.85^{+28\%}_{-21\%}\,\text{fb}$

# Summary

## Higgs pair production at NLO

- full $m_t$ dependence
- reduce cross section by 14% relative to Born-improved HEFT

→ relevant contribution to cross section

- numerical integration of loop integrals using SecDec
  - new interface to amplitude code
  - dynamically adjust #sampling points
  - Quasi Monte Carlo

- First step towards automated 2-loop calculations using GoSam-2L

  more details: → talk by Stephen Jones

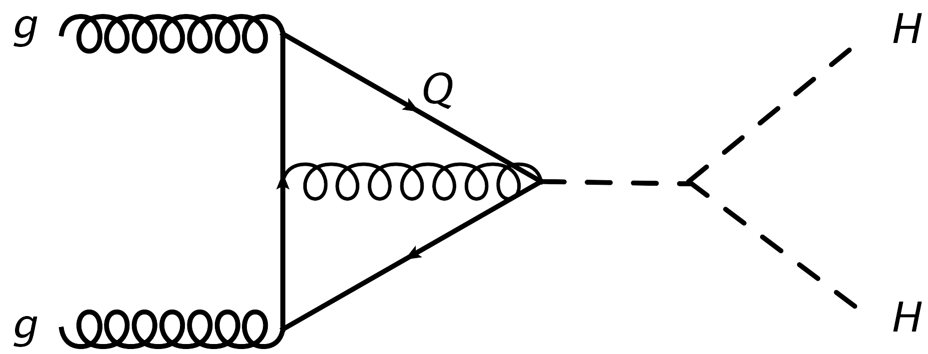
# Backup

# Analytically known integrals

- master integrals

  known analytically

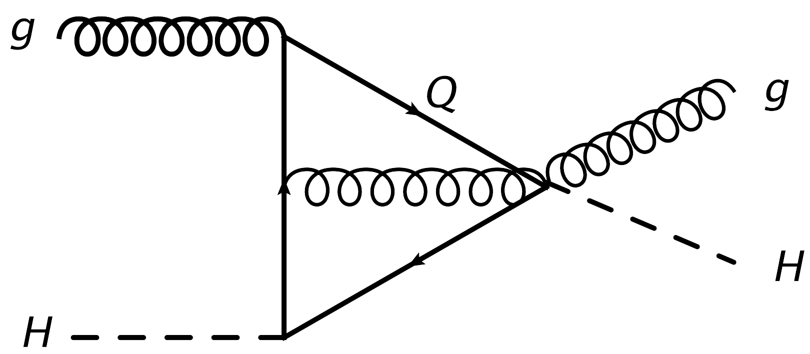
3-point, 1 off-shell leg




Spira, Djouadi et al. `93, `95
Bonciani, Mastrolia `03, `04
Anastasiou, Beerli et al. `06

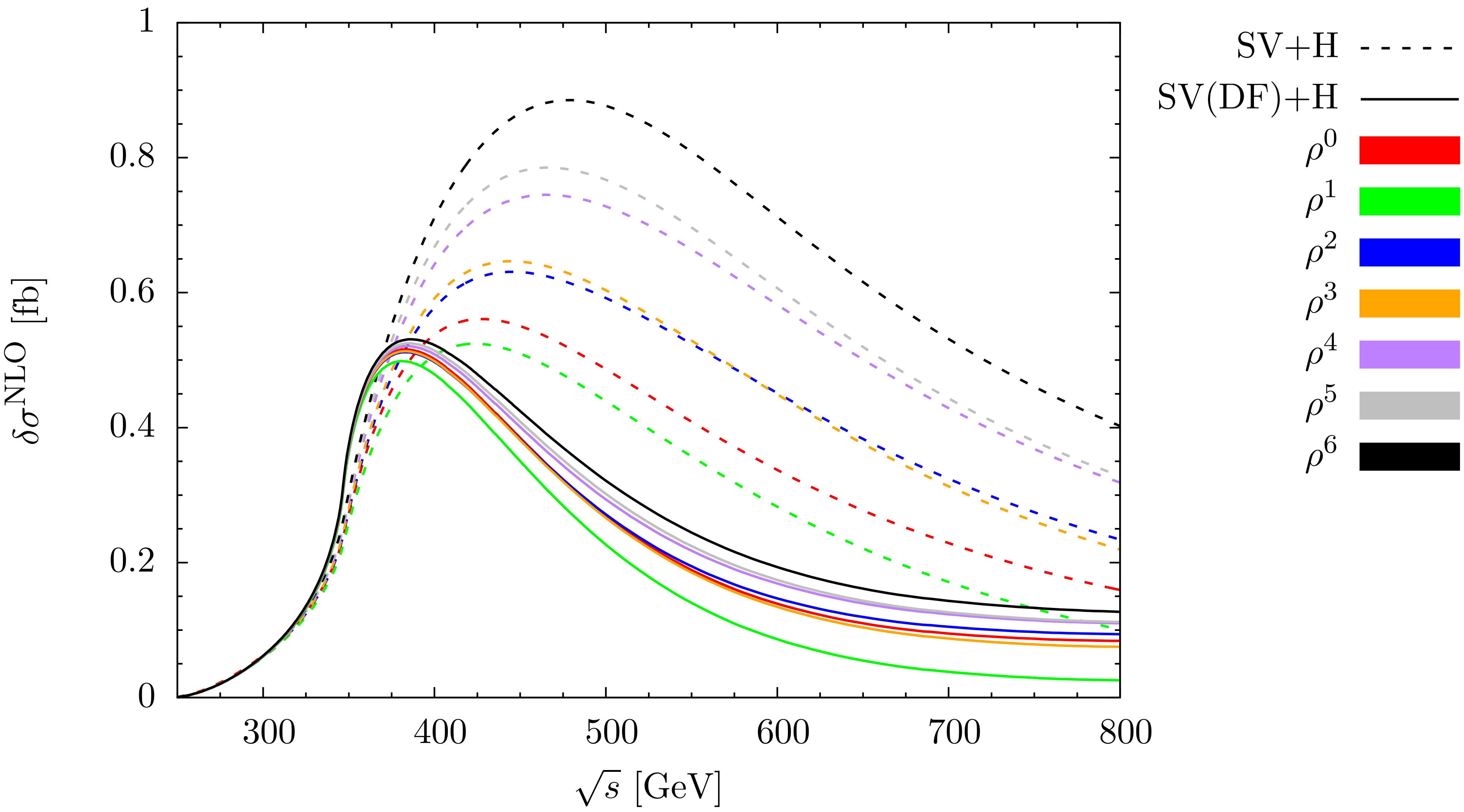
→HPLs

3-point, 2 off-shell leg

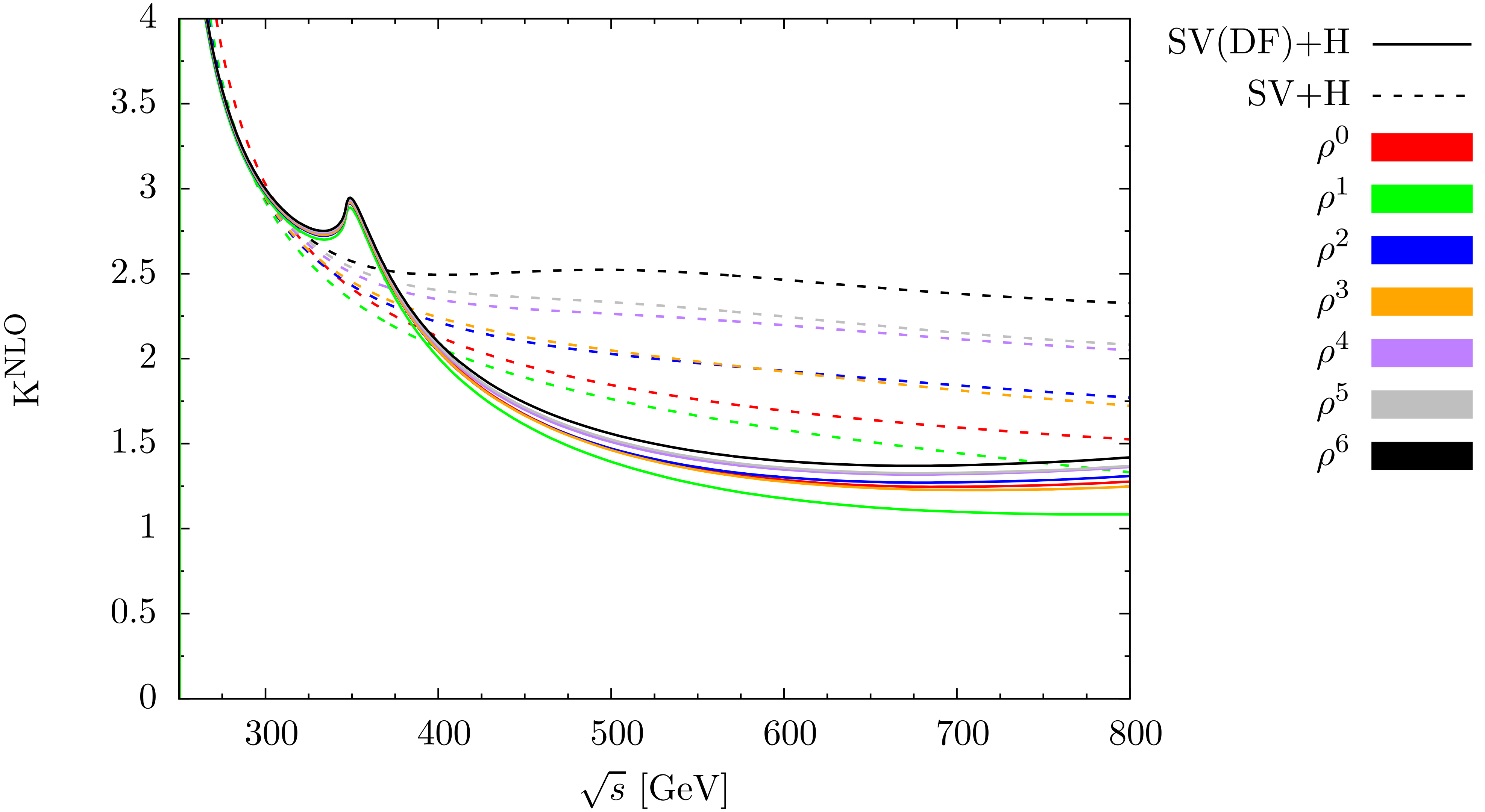



Gehrmann, Guns, Kara `15

→ generalized HPLs, 12 letters

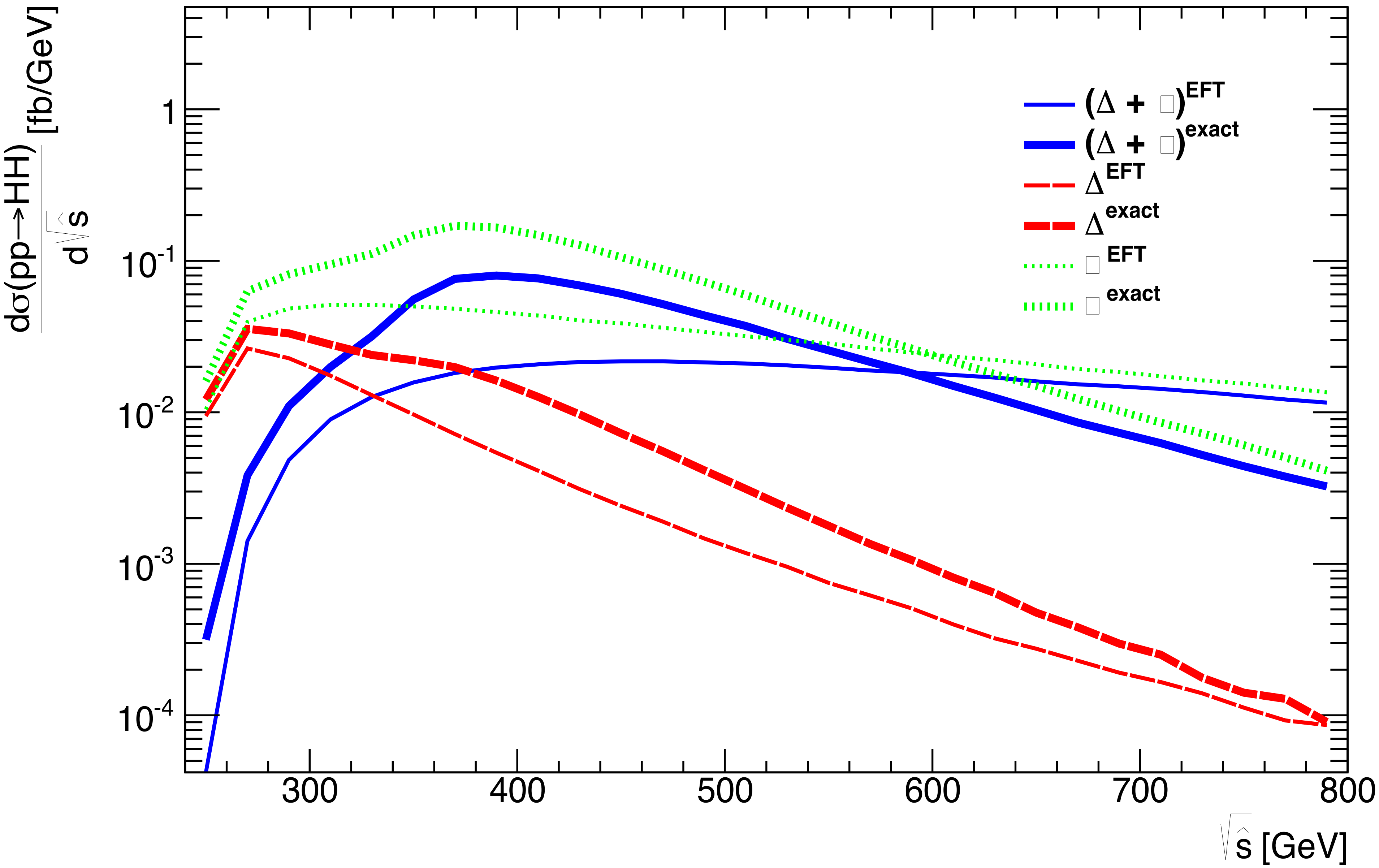



Grigo, Hoff, Steinhauser `15



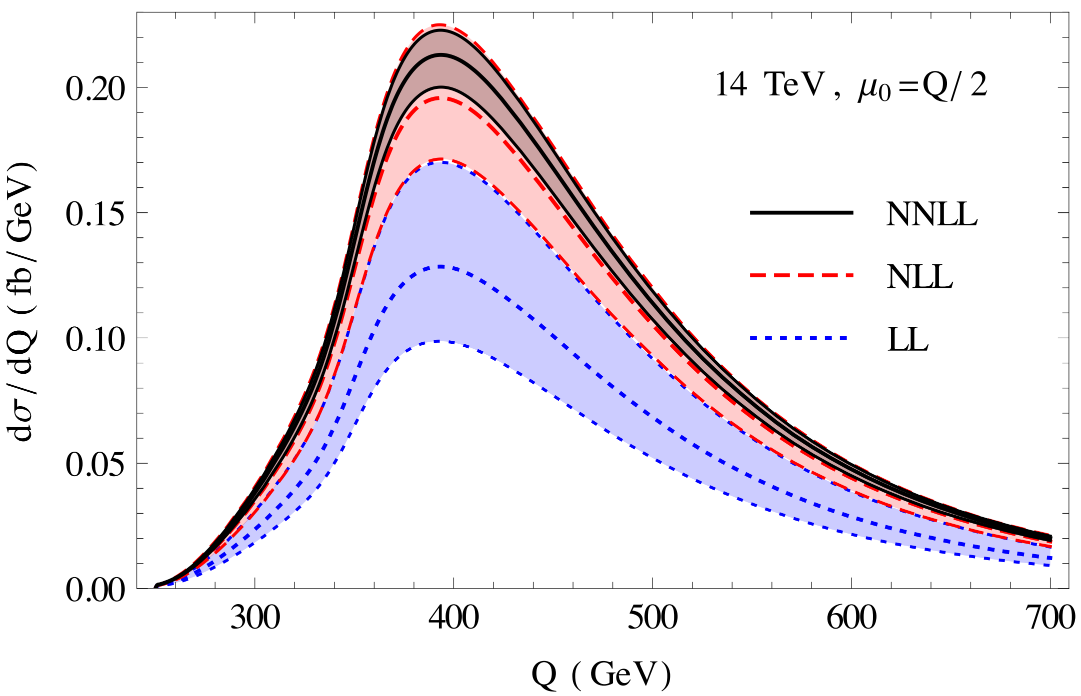

Grigo, Hoff, Steinhauser `15

# Differential Cross Section

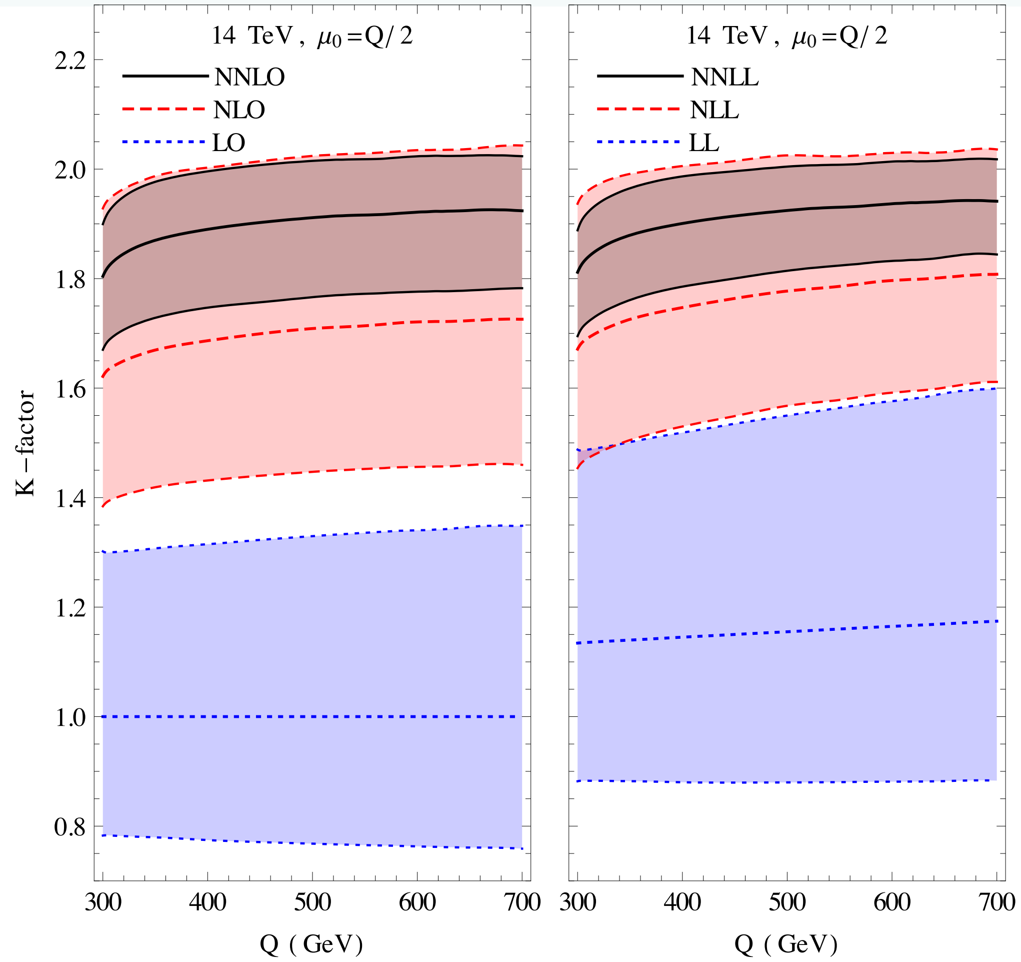



Slawinska, van den Wollenberg,
van Eijk, Bentvelsen `14

# NNLO and NNLL results




de Florian, Mazzitelli `15




| $\mu_0 = Q$ | NNLO (fb) | scale unc. (%) | NNLL (fb) | scale unc. (%) | PDF unc. (%) | PDF+$\alpha_S$ unc. (%) |
|---|---|---|---|---|---|---|
| 8 TeV | 9.92 | $+9.3-10$ | 10.8 | $+5.4-5.9$ | $+5.6-6.0$ | $+9.3-9.2$ |
| 13 TeV | 34.3 | $+8.3-8.9$ | 36.8 | $+5.1-6.0$ | $+4.0-4.3$ | $+7.7-7.5$ |
| 14 TeV | 40.9 | $+8.2-8.8$ | 43.7 | $+5.1-6.0$ | $+3.8-4.0$ | $+7.5-7.3$ |
| 33 TeV | 247 | $+7.1-7.4$ | 259 | $+5.0-6.1$ | $+2.2-2.8$ | $+6.1-6.1$ |
| 100 TeV | 1660 | $+6.8-7.1$ | 1723 | $+5.2-6.1$ | $+2.1-3.0$ | $+5.7-5.8$ |
| $\mu_0 = Q/2$ | NNLO (fb) | scale unc. (%) | NNLL (fb) | scale unc. (%) | PDF unc. (%) | PDF+$\alpha_S$ unc. (%) |
| 8 TeV | 10.8 | $+5.7-8.5$ | 11.0 | $+4.0-5.6$ | $+5.8-6.1$ | $+9.6-9.3$ |
| 13 TeV | 37.2 | $+5.5-7.6$ | 37.4 | $+4.2-5.8$ | $+4.1-4.3$ | $+7.8-7.6$ |
| 14 TeV | 44.2 | $+5.5-7.6$ | 44.5 | $+4.2-5.9$ | $+3.9-4.1$ | $+7.6-7.4$ |
| 33 TeV | 264 | $+5.3-6.6$ | 265 | $+4.6-6.1$ | $+2.4-2.7$ | $+6.3-6.1$ |
| 100 TeV | 1760 | $+5.3-6.7$ | 1762 | $+4.9-6.4$ | $+2.2-3.1$ | $+6.2-7.0$ |