Towards a numerical unitarity approach for two-loop amplitudes in QCD

based on: [arXiv 1510.05626, HI]

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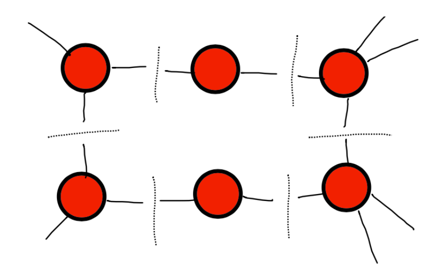
Content

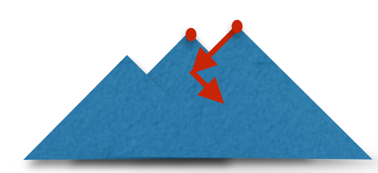
Physics Motivation

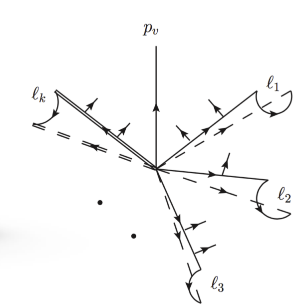
Loop Computations



Surface-Terms

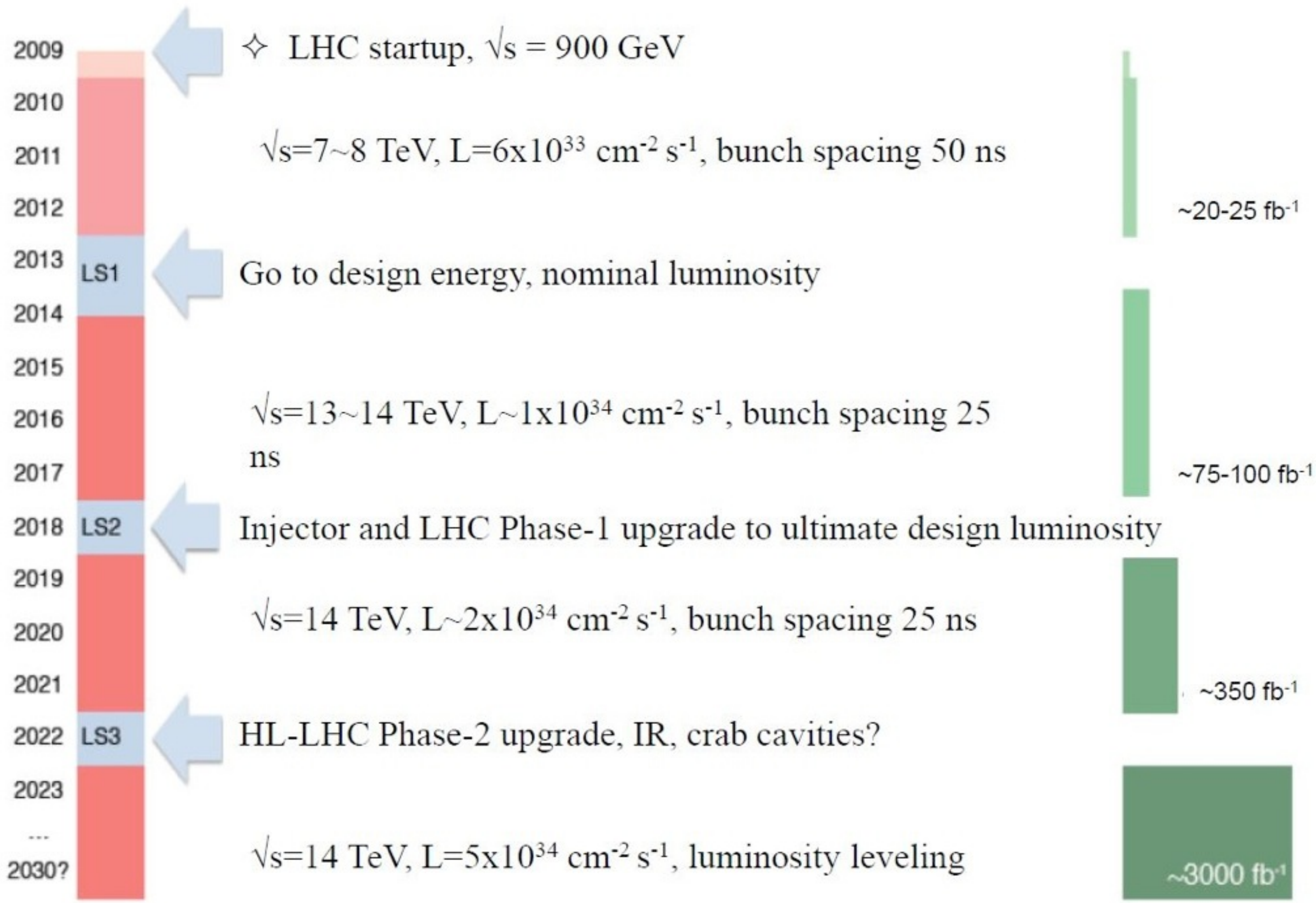
Formal Structures



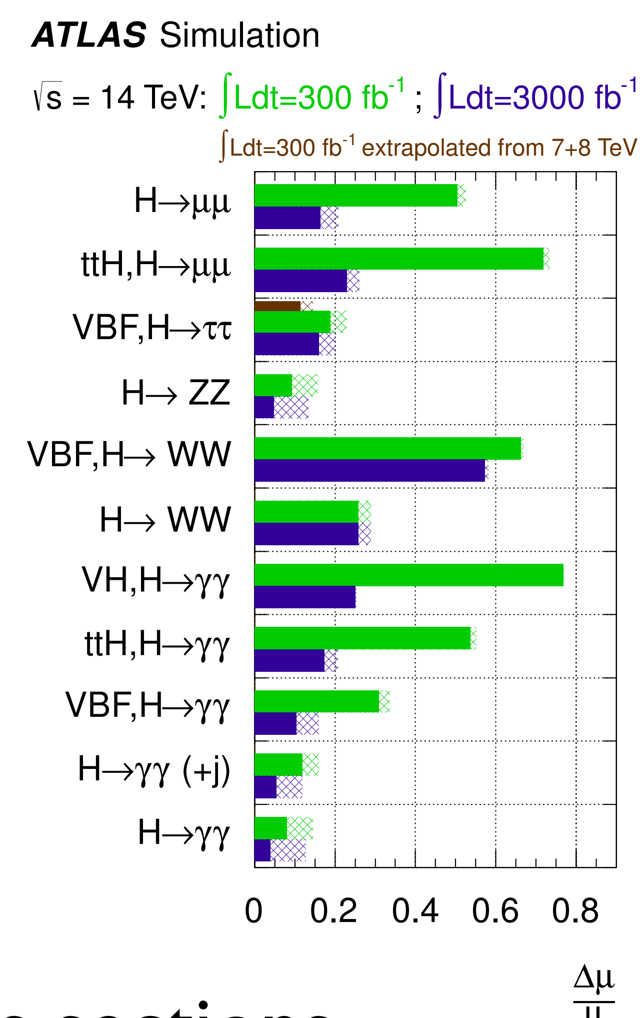


LHC Timeline

Taken from Rolf Heuer in CERN General Meeting January 2013.



(LS = Long Shutdown)



Potential: new physics & percent-level cross sections.

Theory Input

- Precision theory predictions at the few percent level (input parameters, backgrounds, predictions)

- Broad range of predictions for indirect detection & characterisation of New Physics

- Go deep by inclusion of multiple final states (new channels, resolve kinematic dependence of interactions)

- Complete including of higher-order effects (electroweak, final-state description)

Complex dynamic of proton collisions:

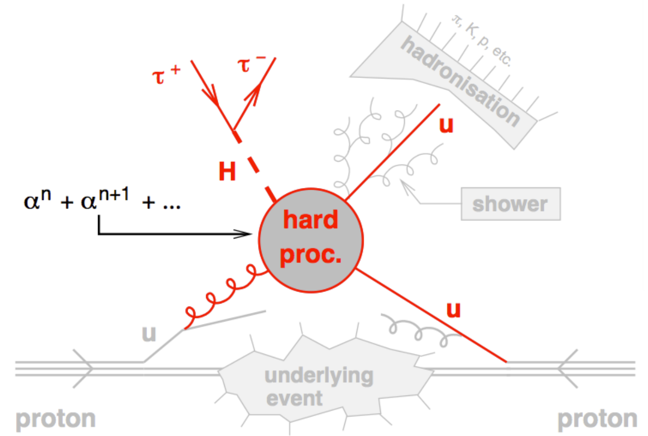


diagram from G. Salam

Precise predictions are a multilayered problem:

$$
\begin{aligned}
\mathrm{d}\hat{\sigma}_{ij,NNLO} &= \int_{\mathrm{d}\Phi_{n+2}} \left[\mathrm{d}\hat{\sigma}^{RR}_{ij,NNLO} - \mathrm{d}\hat{\sigma}^{S}_{ij,NNLO}\right] \\
&+ \int_{\mathrm{d}\Phi_{n+1}} \left[\mathrm{d}\hat{\sigma}^{RV}_{ij,NNLO} - \mathrm{d}\hat{\sigma}^{T}_{ij,NNLO}\right] \\
&+ \int_{\mathrm{d}\Phi_{n}} \left[\mathrm{d}\hat{\sigma}^{VV}_{ij,NNLO} - \mathrm{d}\hat{\sigma}^{U}_{ij,NNLO}\right]
\end{aligned}
$$

[Antenna subtraction; Gehrmann-De Ridder, Gehrmann, Glover; Kosower]

Current frontier: NNLO computations in QFT

NLO Progress

Driving ideas & methods:

- Unitarity and numerical unitarity methods [Bern, Dixon, Kosowers; Britto Cachazo Feng; Ossola, Papadopoulos, Pittau; Ellis, Giele, Kunszt, Melnikov; Forde; Badger + many others]

- Integral reduction algorithms [e.g. COLLIER; Denner, Dittmaier, Hofer; GOLEM; Binoth, Guillet, Pilon, Heinrich, Schubert]

- Refined techniques; single cuts, spinor helicity, color tricks, recursions [e.g. Berends,Giele; Britto, Cachazo, Feng,Witten; many contribution from amplitudes field]

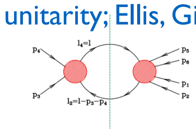
Mature tools; e.g.:

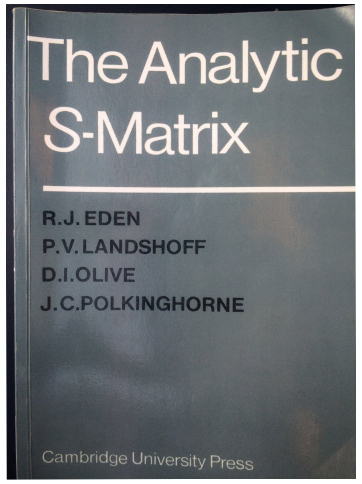
- Sherpa, Munich, Madgraph

- BlackHat, GoSam, Hawk, MCFM, Madgraph, NJet, OpenLoops, Recola, Rocket, Prospino, Prophecy4f,…

Ideas:

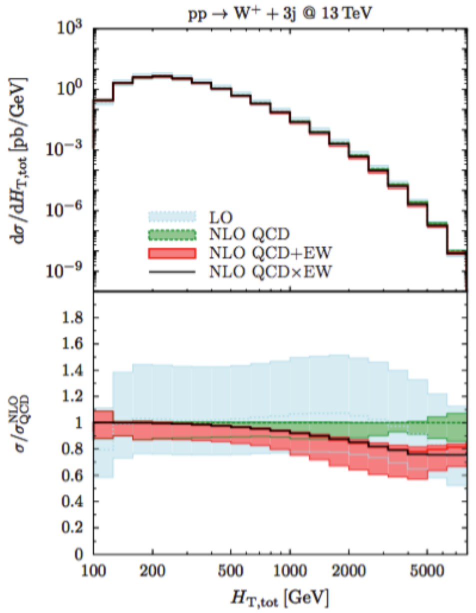
[Numerical unitarity; Ellis, Giele Kunszt '07]



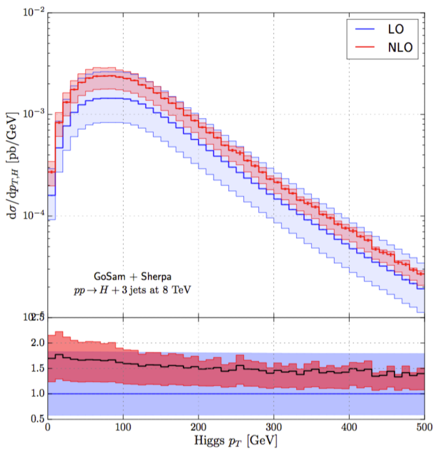


Some state-of-the-art predictions:

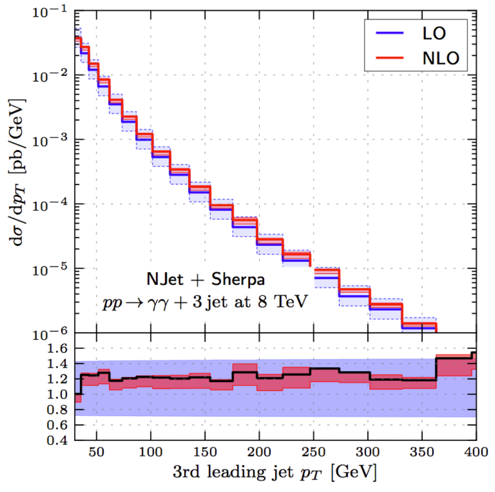
[OpenLoops; Kallweit, Lindert, Maierhofer, Pozzorini, Schonherr, '14]



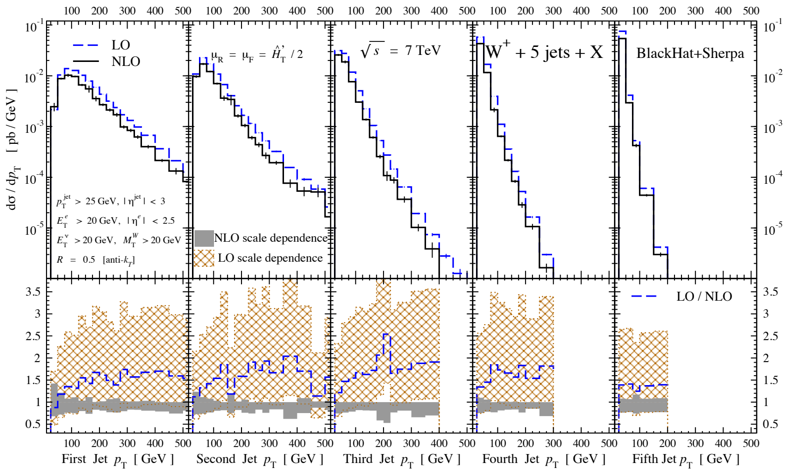
[GoSam; Cullen, van Deurzen, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Tramontano,'13]



[NJet; Badger, Biedermann, Guffanti, Uwer, Yundin, '14]



[BlackHat; Bern, Dixon, Febres Cordero, HI, Kosower, Maitre, '13]



NNLO Progress (2 to 2)

Recent highlights for proton collisions:

[see talk by K. Ellis]

- di-photon: [Catani, Cieri, di Florian, Ferrera, Grazzini 11; Campbell, Ellis, Li, Williams 16]
- dijet: [Currie, Gehrmann-De Ridder, Gehrmann, Glover, Pires, Wells 14]
- W+1-jet: [Boughezal, Focke, Liu, Petriello 15]
- Z+1-jet: [Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan 15, 16; Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello 15]
- H+1-jet: [Chen, Gehrmann, Glove, Jaquier 14; Caola, Melnikov, Schulze 15; Boughezal, Focke, Giele, Liu, Periello 15]
- tt: [Czakon, Fiedler, Heymes, Mitov 15, 16]
- WW: [Gehrmann, Grazzini, Kallweit, Maierhöfer, v.Mannteuffel, Pozzerini, Rathlev, Tancredi 14; Caola, Melnikov, Röntsch, Tancredi 15]
- ZZ: [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, v.Mannteuffel, Pozzerini, Rathlev, Tancredi, Weihs 14; Grazzini, Kallweit, Rathlev 15; Caola, Melnikov, Röntsch, Tancredi 15]
- ZH: [Ferrera, Grazzini, Tramontano 14; Campbell, Ellis, Williams 16]
- Z/Wphoton: [Grazzini, Kallweit, Rathlev, Torre 14, 16]

A frontier:

- 3jets : amplitudes [Badger, Frellesvig, Zhang 15; Gehrmann, Henn, Lo Presti 15]

Three/more final states? Masses? Jets as probes of interactions?
Complexity bottleneck?

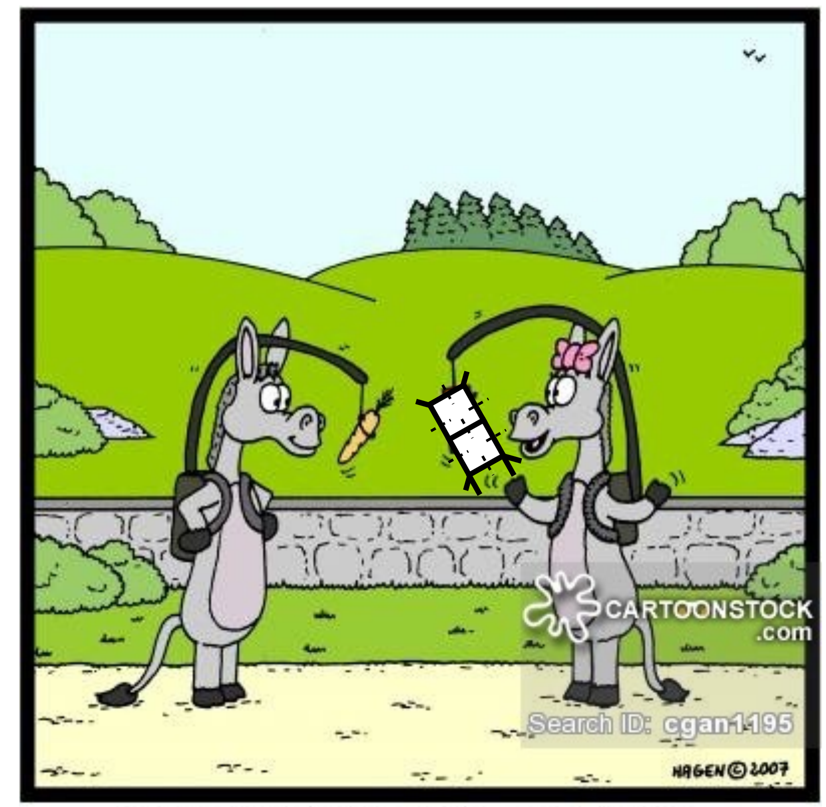
Unitarity Approach @ Two-loops

Status:

- Selected analytic computations of QCD amplitudes [see talk by S. Badger]
- Many formal computation in gravity & N=4 SYM [amplitudes field]

Discussed here:

- Integral coefficients
- Extend one-loop ideas for multi-loop amplitudes
- Numerical algorithm for QFT



Carrots just didn't get me going anymore, so I switched to NNLO instead...

Applications:

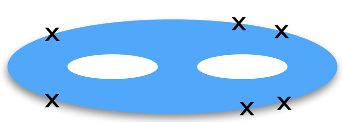
- QCD amplitudes for NNLO, integral-reduction tools [see talk by K. Larsen]
- Perturbative quantum-gravity theories

Loop Computations

Loop Computations

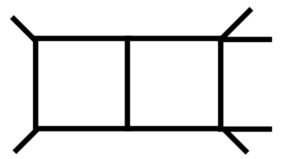
Feynman amplitude:

$$A(p_i) = \int [d^{(nD)}\ell] \; \tilde{A}(\ell, p_i)$$



Universal ansatz:

$$A(p_i) = \sum_{\text{integral basis}} c_j(p_i)\, \mathcal{I}_j(p_i)$$



$$\int [d^{(nD)}\ell] \; \frac{m_j(\ell, p_i)}{\rho^1 \cdots \rho^N}$$

Challenges: number of diagrams, integral reduction, integrals, algebra, numerical stability

Is there a viable process-independent approach to obtain coefficients?

Classical - Approach

Isolate tensor integrals:

$$\int [d^{(nD)}\ell]\ \tilde{A}(\ell, p_i) \longrightarrow \sum_{\text{tensor topologies}} T^{k}_{\mu_1\cdots\mu_m} \mathcal{I}_k^{\mu_1\cdots\mu_m}$$

$$\int [d^{(nD)}\ell]\ \frac{\ell^{\mu_1}\cdots\tilde{\ell}^{\mu_m}}{\rho^1\cdots\rho^N}$$

Integral reduction:

$$\sum_{\substack{\text{k in tensor integrals} \\ \text{j in integral basis}}} T_{k,\mu_1\cdots\mu_m} C_{k,j}^{\mu_1\cdots\mu_m}\ \mathcal{I}_j$$

2-loop methods: Tensor reduction [Tarasov 96; Anastasiou, Glover, Oleari 99], Integration-by-parts identities [Tkachov, Chetyrkin 81], Lorentz invariance identities [Gehrmann, Remiddi 99], Laporta algorithm [Laporta 01]

Programs in use: Reduze [Mannteuffel, Studerus], AIR [Anastasiou, Lazopoulos], FIRE [Smirnov, Smirnov], LiteRed [Lee], SecDec [Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke]; COLLIER [Denner, Dittmaier, Hofer], GOLEM [Binoth, Guillet, Pilon, Heinrich, Schubert]

Unitarity Approach

Exploit analytic structure and use cutting rules:

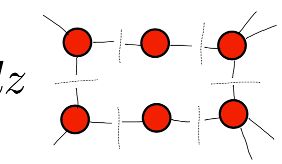
$$\int_{\mathcal{C}} [\mathrm{dLIPS}]\ \tilde{A}_1(\ell, p_i) \times \cdots \times \tilde{A}_m(\ell, p_i)$$

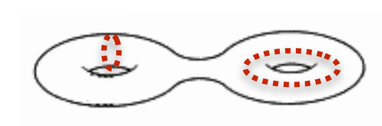
$$= \sum_{\text{integrals with cuts}} c_j(p_i) \int_{\mathcal{C}} [\mathrm{dLIPS}] \frac{m_j(\ell, p_i)}{(\text{uncut propagator terms})}$$

Properties:

- Fine-grained set of equations
- Organise equations: from maximal to minimal number of cut propagators
- Remaining integration
- Analytic approach for very compact results

Multi-loop pioneers [Bern, Dixon, Kosower, Dunbar 94; Bern, Dixon, Dunbar, Perelstein, Rozowsky 98; Bern, Dixon, Kosower 00]

$\oint_{\mathcal{C}} dz$ 

$\mathcal{C}$: 

Recently: duality between master integrals and contours [Kosower, Larsen 11; Caron-Huot, Larsen 12; Georgoudis, Zhang 15; Sogaard, Zhang 14; HI 15]

Unitarity Approach - Integrands

[see talks by G. Ossola, P.P. Mastrolia and S. Badger]

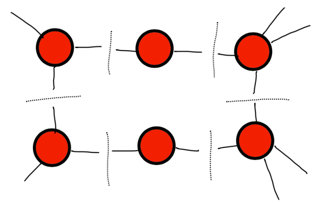
Use integrand basis and remove integration:

$$\tilde{A}(\ell, p_i) = \sum_{\text{j in integrand basis}} c_j(p_i) \frac{m_j(\ell, p_i)}{\rho^1 \cdots \rho^N}$$

Classification of integrands [Badger, Frellesvig, Zhang 12; Mastrolia, Mirabella, Ossola, Peraro 12]

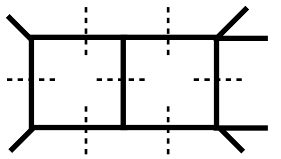
Factorisation in loop momenta [Ellis, Giele, Kunszt 07]:

$$\lim_{\{\rho^i\}\to 0} \tilde{A}(\ell, p_i) \to \tilde{A}_1(\ell, p_i) \times \cdots \times \tilde{A}_m(\ell, p_i) \frac{1}{\text{(large propagator terms)}}$$



Algebraic equations using tree-level data:

$$\tilde{A}_1(\ell, p_i) \times \cdots \times \tilde{A}_m(\ell, p_i) = \sum_{\text{j in large integrands}} c_j(p_i)\, m_j(\ell, p_i) + \text{previously computed topologies}$$



Properties:

- Universal and numerical
- But: additional integral reduction required

Numerical Unitarity Approach

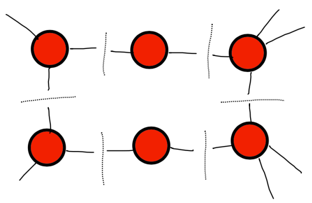
Integrand basis as direct sum of vanishing integrals and master integrals:

$$\tilde{A}(\ell, p_i) = \sum_{\text{j in master integrands}} c_j(p_i) \frac{m_j(\ell, p_i)}{\rho^1 \cdots \rho^N} + \sum_{\text{j in surface terms}} \hat{c}_j(p_i) \frac{\hat{m}_j(\ell, p_i)}{\rho^1 \cdots \rho^N}$$

@ one-loop [Ossola Papadopoulos, Pittau 07; Ellis, Giele Kunszt 07; Giele Kunszt, Melnikov 08]

@ two-loop [HI15]

Properties:



- Unitarity cuts give fine set of equations for coefficients
- Integral reduction implicit: just drop surface terms
- Process & multiplicity independent numerical approach

How to construct vanishing integrals?

- Use particular integration-by-parts identities [HI 15]

IBPs

Vanishing Integrals

Surface Terms

Integration-by-parts identities:

[Tkachov, Chetyrkin 81]

loop momenta: $(\ell, \tilde{\ell})$

$$0 = \int [d^{((n=2)D)}\ell] \left[\partial_\mu \left(\frac{u^\mu}{\rho^1 \cdots \rho^N} \right) + \tilde{\partial}_\nu \left(\frac{\tilde{u}^\nu}{\rho^1 \cdots \rho^N} \right) \right]$$

Doubled propagators entangle unitarity equations:

$$\partial_\mu \left(\frac{u^\mu}{\rho^i(\ell)} \right) = \frac{1}{\rho^i(\ell)} \partial_\mu u^\mu - \frac{1}{(\rho^i(\ell))^2} u^\mu \partial_\mu \rho^i(\ell)$$

IBP-generating vectors:

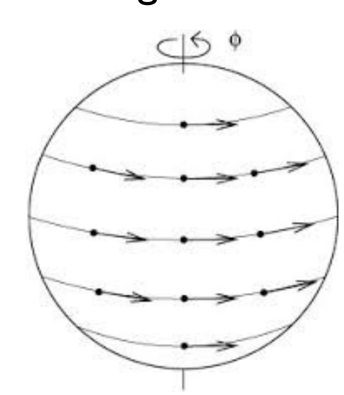
[Gluza, Kajda, Kosower 10]

$$(u^\mu \partial_\mu + \tilde{u}^\nu \tilde{\partial}_\nu) \rho^i(\ell, \tilde{\ell}) = f^i(\ell, \tilde{\ell}) \rho^i(\ell, \tilde{\ell}) \qquad \text{for all} \quad \rho^i(\ell, \tilde{\ell})$$

Geometric interpretation [Zhang 14; HI 15]:

- Unitarity-cut surface: defined by $\rho^i(\ell, \tilde{\ell}) = 0$
- Solutions are tangent vectors to unitarity cut surface
- Intrinsic structure of unitarity cut

Unitarity-cut surface and IBP-generating vector field:



Natural Coordinates (1)

Need to solve

$$(u^\mu \partial_\mu + \tilde{u}^\nu \tilde{\partial}_\nu)\rho^i(\ell,\tilde{\ell}) = f^i(\ell,\tilde{\ell})\rho^i(\ell,\tilde{\ell}) \qquad \text{for all} \quad \rho^i(\ell,\tilde{\ell})$$

solutions with computer algebra [Gluza, Kajda, Kosower 10; Schabinger 11];
explicit solutions [HI 15];
recently [Larsen, Zhang 15]

Coordinates [e.g. Cutkosky 60]

General coordinate transformation [HI 15]

inverse propagators

$$(\ell,\tilde{\ell}) \quad \rightarrow \quad (\rho^i, \alpha^a)$$

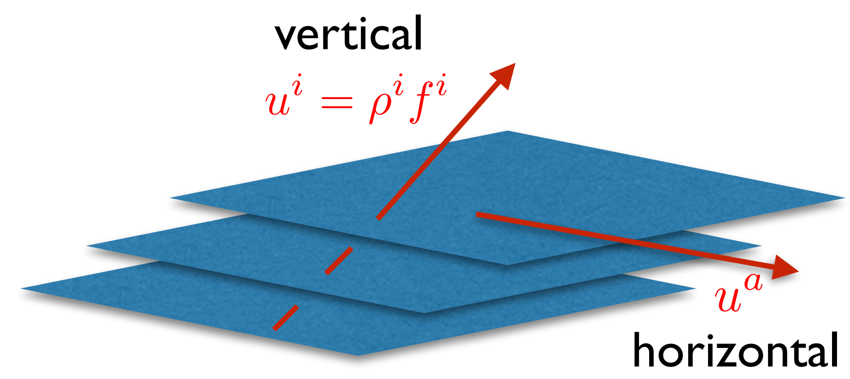
auxiliary coordinates

$$\left(u^j \frac{\partial}{\partial \rho^j} + u^a \frac{\partial}{\partial \alpha^a}\right)\rho^i = u^j \delta^i_j = f^i \rho^i \quad \Longrightarrow \quad (u^i, u^a) = (f^i \rho^i, u^a)$$

Properties:

- Simple solution
- But: want polynomial vector fields for polynomial surface terms; careful treatment of algebraic structure — no problem

… continued



foliation of momentum space
in ρ^i = const. slices

Classification of IBP-generating vectors:

- Horizontal:

$$(u^i, u^a) = (0, u^a) \longrightarrow \text{relations within integral topology}$$

- Vertical:

$$(u^i, u^a) = (f^i \rho^i, 0) \longrightarrow \text{relations between distinct integral topologies}$$

- Mixed:

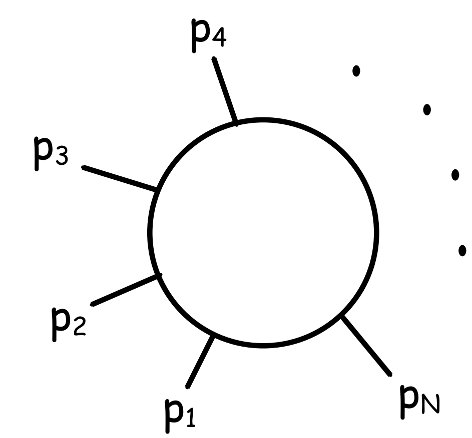
$$(u^i, u^a) = (f^i \rho^i, u^a) \longrightarrow \text{massless integrals, D-dependent relations}$$

Role of intrinsic vector component u^a :

- Local translations/rotations
- Conformal transformations (massless integrals)

IBP-generating vectors (I)

one NV basis: n_a



van Neerven Vermaseren basis:

- External momenta

$$\{p_1, p_2, \dots p_{N-1}\}$$

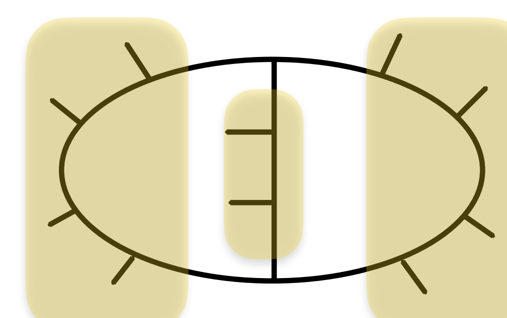
- Dual basis

$$v^{i\mu} = (G^{-1})^{ij} p_j^\mu, \quad \text{with} \quad G_{ij} = p_i^\mu p_{j\mu}$$
$$\to \quad v^{i\mu} p_{j\mu} = (G^{-1}G)^i_j = \delta^i_j$$
$$\to \quad v^{i\mu} \quad \text{in span of} \quad p_i^\mu$$

- Transverse space

$$n_a, \quad \text{with} \quad n_a^\mu p_{i\mu} = 0$$

three NV bases:



n_a $\hat{n}_a$ $\tilde{n}_a$

IBP-generating vectors (2)

One-loop integrals:

- Inverse propagators

$$\ell^2, (\ell - p_1)^2 - m_1^2, (\ell - p_1 - p_2)^2 - m_2^2, \ldots$$

- IBP-generating vectors are rotation generators

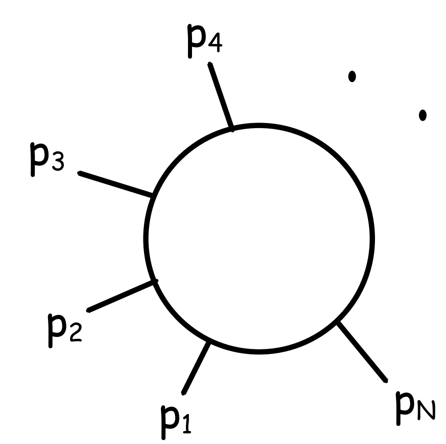
$$u^{\nu}_{[ab]}\partial_\nu = n^{\mu}_{[a|}\ell_\mu n^{\nu}_{|b]}\partial_\nu$$

- Check

$$u^{\nu}_{[ab]}\partial_\nu(\ell^2) = 2\, n^{\mu}_{[a|}\ell_\mu n^{\nu}_{|b]}\ell_\nu = 0 \quad \text{(anti-symmetry)}$$

$$u^{\nu}_{[ab]}\partial_\nu(\ell^\sigma p_i^\sigma) = 2\, n^{\mu}_{[a|}\ell_\mu n^{\nu}_{|b]}p_{i\nu} = 0 \quad \text{(transversality)}$$

one NV basis: n_a



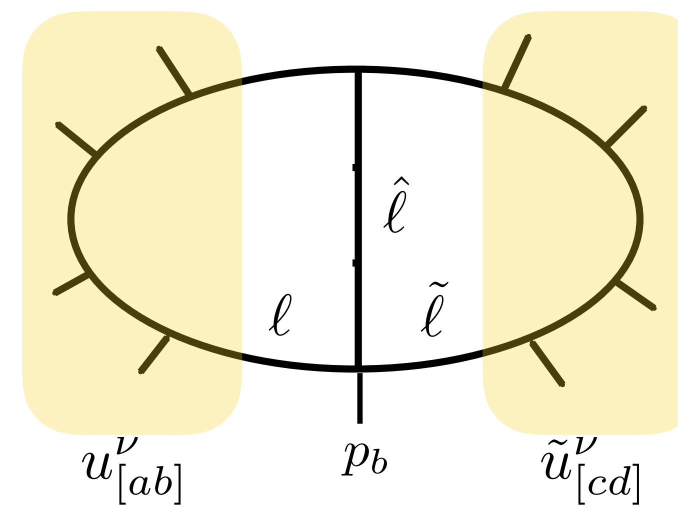
Remark: complete for massive integrals

IBP-generating vectors (3)

Two-loop integrals: planar, massive

Combine vectors of sub loops:

- Type (a) $u^{\nu}_{[abc]}\partial_\nu = n^{\kappa}_{[a|}\hat{\ell}_\kappa n^{\mu}_{|b|}\ell_\mu n^{\nu}_{|c]}\partial_\nu$
- Type (b) $n^{\mu}_{[a|}\ell_\mu n^{\nu}_{|b]}\partial_\nu + n^{\mu}_{[a|}\tilde{\ell}_\mu n^{\nu}_{|b]}\tilde{\partial}_\nu \qquad \text{for} \quad n_a = \tilde{n}_a$
- Type (c) $\tilde{u}^{\nu}_{[ab]}\tilde{\partial}_\nu(\hat{\ell}^2)u^{\mu}_{[cd]}\partial_\mu - u^{\mu}_{[cd]}\partial_\mu(\hat{\ell}^2)\tilde{u}^{\nu}_{[ab]}\tilde{\partial}_\nu$



Properties:

- Integral topology determines types and number of vectors
- Multiplication with loop momentum polynomial give complete set of IBP-generating vectors
- Relations from type (b) known as `spurious terms' [Badger, Frellesvig, Zhang 12; Mastrolia, Mirabella, Peraro Ossola 12]

[see talk by P.P. Mastrolia]

Surface terms

Properties of IBP-generating vectors:

- Horizontal: $[u^\mu \partial_\mu + \tilde{u}^\nu \tilde{\partial}_\nu] \rho^i = 0$
- Divergence free (observation): $\partial_\mu u^\mu + \tilde{\partial}_\nu \tilde{u}^\nu = 0$

Surface terms:

$$\left[\partial_\mu \left(\frac{g\, u^\mu}{\rho^1 \cdots \rho^N}\right) + \tilde{\partial}_\nu \left(\frac{g\, \tilde{u}^\nu}{\rho^1 \cdots \rho^N}\right)\right] = \frac{[u^\mu \partial_\mu + \tilde{u}^\nu \tilde{\partial}_\nu] g}{\rho^1 \cdots \rho^N}$$

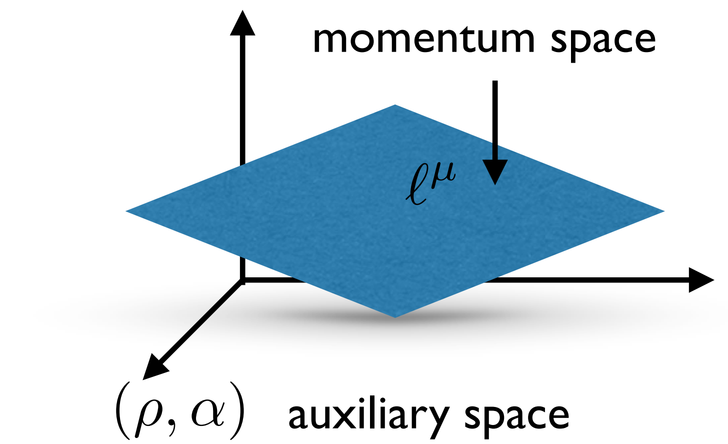
Algorithm:

- Input: numerator polynomials
- Directional derivatives
- Remove linear dependent terms

Reproduces known results @ one-loop level.

Construction of IBP-vectors

Natural Coordinates (2)



Coordinates applied @ one-loop by [Ellis, Giele Kunszt 07]

Loop momentum parametrization:

$$\ell = v^i r_i + \alpha^a n^a$$

- Linear equations: $(p_i, \ell) = -\frac{1}{2}\left((\rho^i + m_i^2 - q_i^2) - (\rho^{i-1} + m_{i-1}^2 - q_{i-1}^2)\right) = r_i$
- Quadratic equations (implicit):

$$c(\rho, \alpha) := (\ell^2 - m_0^2) - \rho^0 = \sum_a (\alpha^a)^2 + (G^{-1})^{ij} r_i r_j - \rho^0 - m_0^2 = 0$$

- Each strand gives a quadratic expression:

@ 1-loops: $c(\rho, \alpha)$

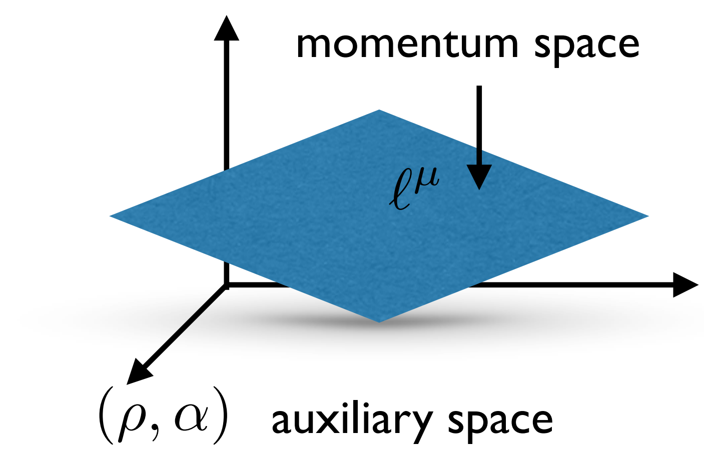
@ 2-loops: $c(\rho, \alpha)$, $\tilde{c}(\tilde{\rho}, \tilde{\alpha})$ and $\hat{c}(\rho, \alpha, \tilde{\rho}, \tilde{\alpha}) = \left(\ell + \tilde{\ell} + p_b\right)^2 - \hat{\rho}^0 - \widehat{m}_0^2$

(many useful different coordinate choices [Larsen, Zhang '15])

Natural Coordinates (3)

Numerators give polynomials

$$t^{\mu_1\cdots\mu_k}\ell_{\mu_1}\cdots\ell_{\mu_k} \quad \sim \quad (\rho,\alpha)-\text{polynomials}$$



- Auxiliary space as coordinate space
- Momentum space as algebraic variety from $\{c,\tilde{c},\hat{c}\}=0$

Vector fields:

- Map polynomials to polynomials — polynomial components
- Tangent to momentum space $(u^a\partial_a + u^k\partial_k)\,\{c,\tilde{c},\hat{c}\}=0$

syzygy equations

for

$\{c,\tilde{c},\hat{c}\}=0$

equations give rotation generators as solutions

One-loop Examples

One-loop relation:

$$c(\rho,\alpha,\mu) := (\ell^2 - m_0^2) - \rho^0 = \sum_a (\alpha^a)^2 + \sum_i (\mu^i)^2 + (G^{-1})^{ij} r_i r_j - \rho^0 - m_0^2 = 0$$

Rotation:

$$u_{[ab]} \ = \ \alpha^a \frac{\partial}{\partial \alpha^b} - \alpha^b \frac{\partial}{\partial \alpha^a}$$

Scaling:

$$u_s = f_s\, \alpha^a \frac{\partial}{\partial \alpha^a} + \sum_i f_s^i \rho^i \frac{\partial}{\partial \rho^i} \qquad \text{for} \qquad C(0) = 0$$

D-dim rotation:

$$u_a \ = \ \alpha^a \mu^b \partial_{\mu^b} - (\mu^b \mu^b) \partial_{\alpha^a}$$

Completeness of IBP-vectors

middle rank
holomorphic form

Cutting loop integral:

$$\int \frac{t}{\rho^1 \cdots \rho^N} J[d\alpha d\rho] \quad \xrightarrow{\text{cut}} \int t\, J[d\alpha]$$

exact form

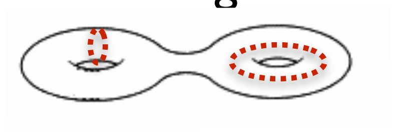
IBP-relations give exact forms on shell:

$$\int \left[\left(\sum_i \frac{\rho^i \partial_i (f^i\, J)}{\rho^1 \cdots \rho^N} \right) + \left(\frac{\partial_a (u^a\, J)}{\rho^1 \cdots \rho^N} \right) \right] [d\alpha d\rho] \xrightarrow{\text{cut}} \int \partial_a (u^a\, J)[d\alpha]$$

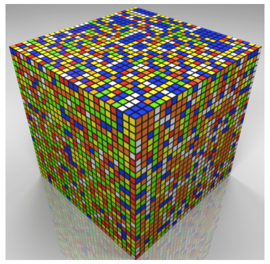
Master integrands:

- (holomorphic forms) modulo (exact forms) ~ (cohomology of unitarity cut variety)

a topological count gives
master integrands



Completeness of generators:

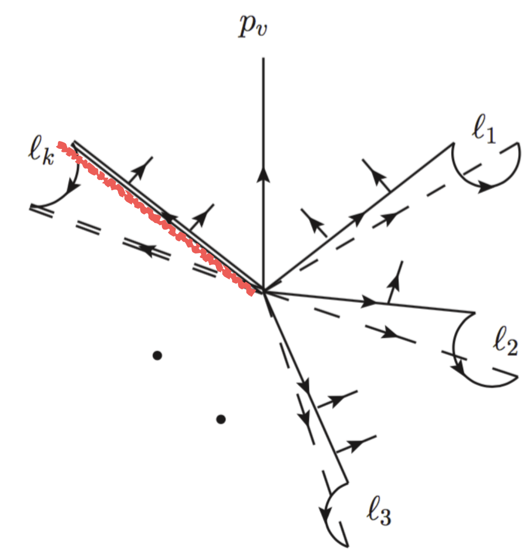


- compare number of (exact forms) vs. (on-shell IBP-relations) — OK!

Formal structures

IBP-generating vectors:

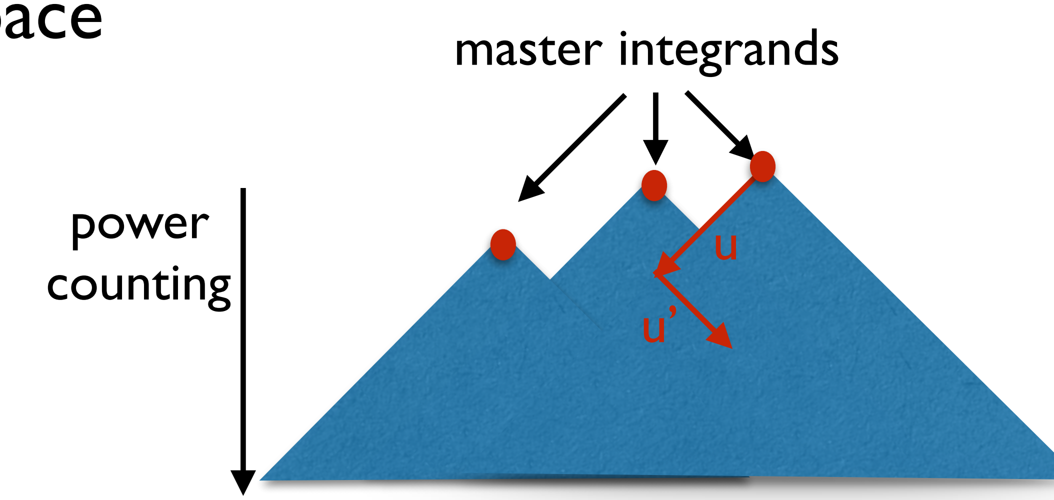
- Rotation/scaling generators for each rung, consistent with momentum conservation at vertices.
- Scaling vectors and off-shell extension



Lie algebra structure: $[u_a, u_b] = f_{ab}^c(\ell, p_i) u_c$

- Representation theory
- Can we understanding of unitarity-cut phase-space from algebra?

Numerator polynomials:



Purely vertical IBP-generators, cycles, intersection numbers …

Conclusions

Methods for numerical unitarity approach presented

- Numerical, universal approach

Next steps to setup:

- Special cases (massless topologies, D-dim)
- Explicit proof-of-principle computation

Structures:

- Unitarity-cut phase spaces
- Coordinates adapted to integrals
- Classification of IBP vectors

… continued

Open questions:

- Representation theory of current algebra
- Further lessons from natural coordinates (integrals, diff. equations)
- Topology of unitarity-cut phase-spaces
- Efficiency of numerical approach
- Doubled propagators (cut less approach).
- Analytic realisations [see K. Larsen's talk]

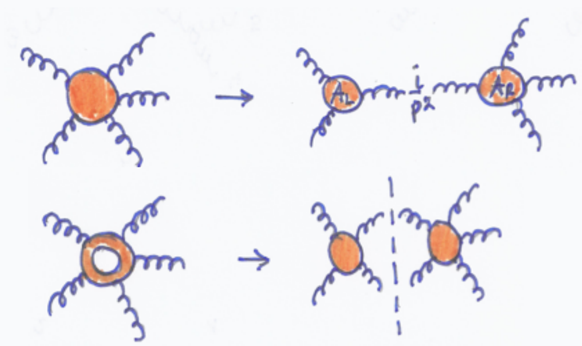
Thanks!

History of Unitarity Method

Origins in bootstrap program for strong interactions in 60s:

[Chew, Mandelstam; Eden, Landshoff, Olive, Polkinghorne; Veneziano; Virasoro, Shapiro; …]

- Reconstruct amplitudes directly from analytic properties:
  - Poles explained by factorization.
  - Branch cuts from optical theorem.



Replaced in 70s by rise of field theory (QCD) and Feynman rules.
Return of analyticity within perturbative field theory:

[Bern, Dixon, Dunbar, Kosover]

- Cutting rules, on-shell and unitarity methods.
- Adding to field theory methods.

Reduce Reuse Recycle

Numerical unitarity methods are algorithm to construct complicated loop amplitudes from simpler tree amplitudes:

Many contributors: [Badger, Berger, Bern, Bjerrum-Bohr, Brandhumber, Britto, Cachazo, Dixon, Dunbar, Ellis, Febres-Cordero, Feng, Forde, Giele, Harmeren, Kosower, Kunszt, Mastrolia, Melnikov, Ossola, Papadopoulos, Pittau, Schwinn, Spence, Travaglini, Weinzierl, Witten]

- Built from analytic inspiration, N=4 SYM, etc

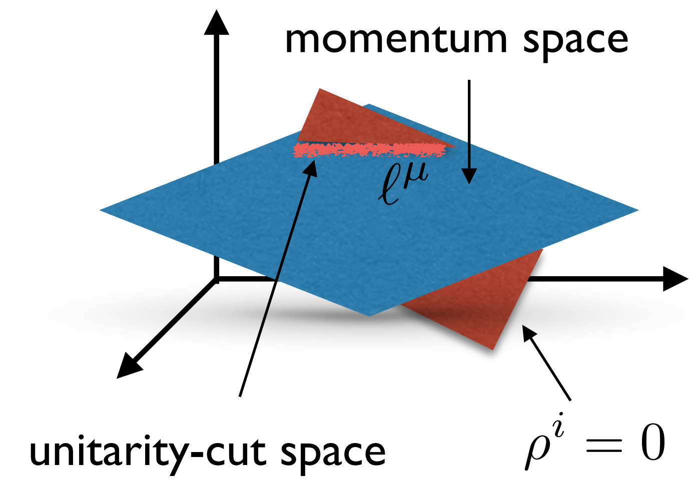
Numerator Basis

Numerators and Unitarity cuts:

- Numerators modulo lower topologies = modulo inverse props
- Function algebra on on-shell varieties

$$\rho^i \sim 0, \{c, \tilde{c}, \hat{c}\} = 0$$

- Remove redundant surface terms on-shell
- Finally consider the linear independent surface terms off shell.



unitarity cut as algebraic variety