CSR & Beam Dynamics Simulations

CSR & Beam Dynamics Simulations

Peter Kuske, HZB

3rd ARD ST3 Workshop
15. - 17.07.2015

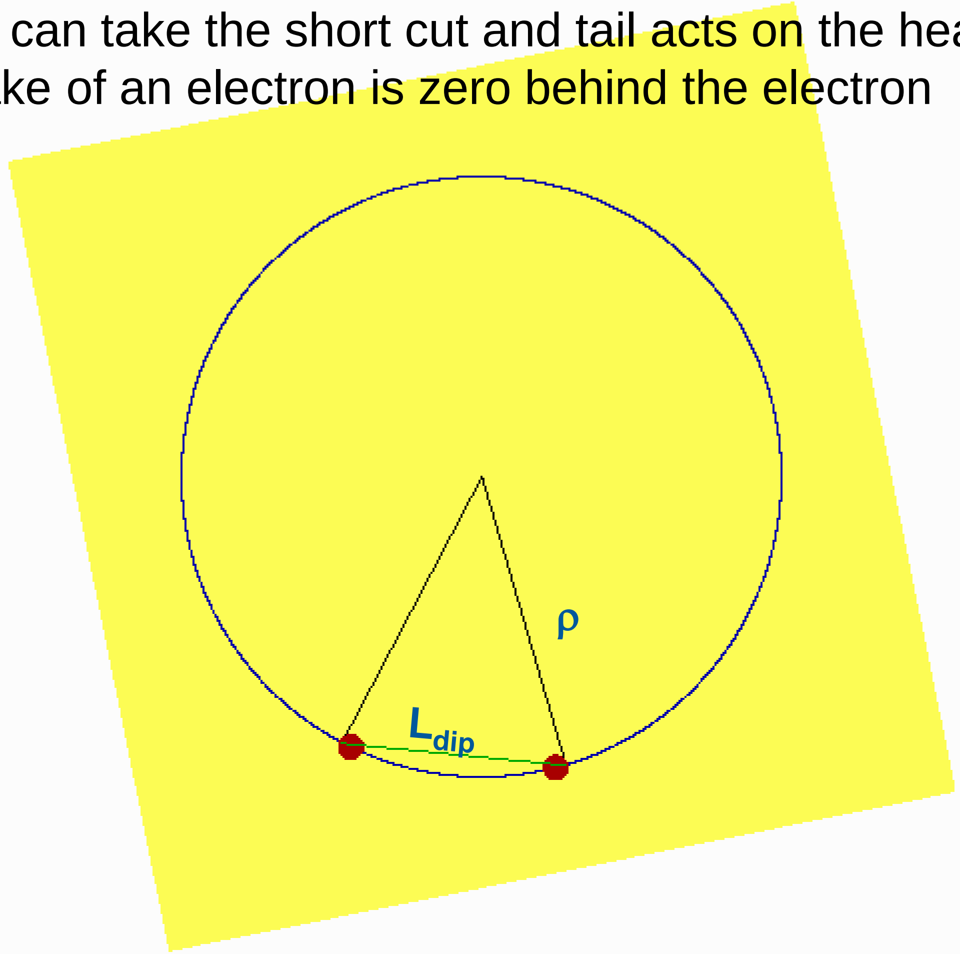
Content of the talk

- Introduction/Motivation
- Model of shielded CSR-interaction
- Some Details of the Simulations
- Selected Results
- Summary

Model of Shielded CSR

Bunch moves clock wise on a circular path

Photons (green) can take the short cut and tail acts on the head – in free space wake of an electron is zero behind the electron



Radiation from the tail can overtake the Bunch and interact with the head if:

$$L_{dip}^3 > 24 \cdot \rho^2 \cdot \sigma$$

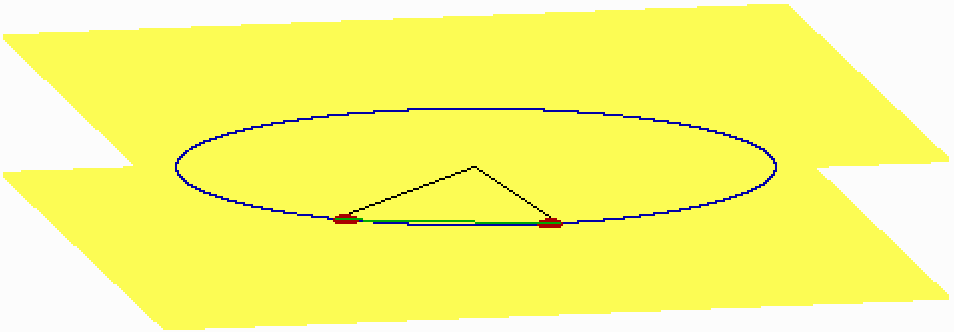
BESSY II:

$\sigma/c < 4$ps

Model of Shielded CSR

In reality bunches move inside of a metallic vacuum chamber

Analytical expression for the wake of an electron moving midway between two perfectly conducting infinite parallel plates



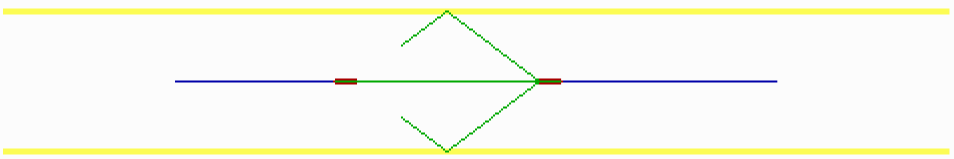
J. B. Murphy, et al., Part. Acc. 1997, Vol. 57, pp 9-64

Model of Shielded CSR

In reality bunches move inside of a metallic vacuum chamber

Analytical expression for the wake of an electron moving midway between two perfectly conducting infinite parallel plates

Photons emitted towards the walls can be back reflected and the electron acts in the forward and backward direction

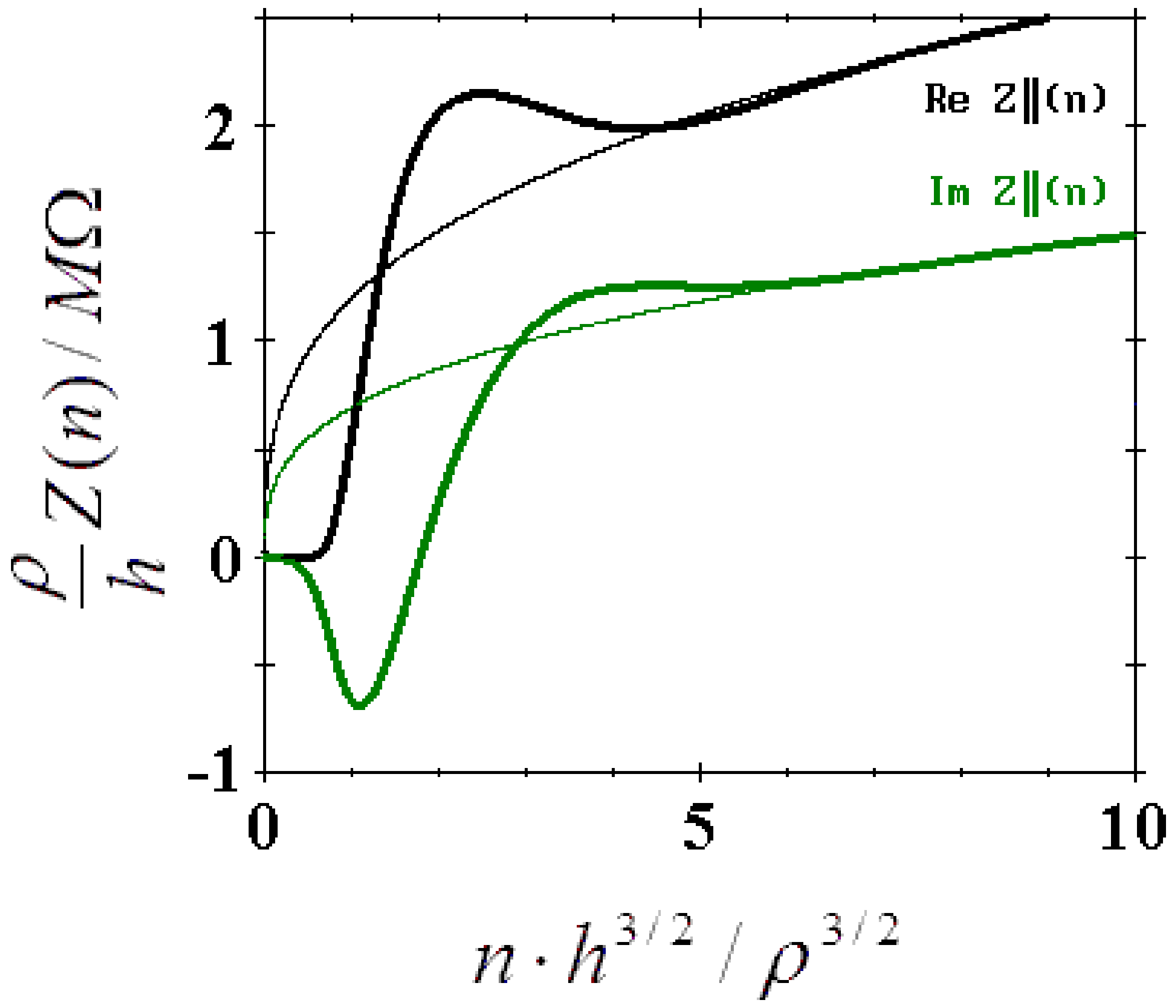


J. B. Murphy, et al., Part. Acc. 1997, Vol. 57, pp 9-64

Shielded CSR Impedance

2h = plate separation

ρ = bending radius



"Broad band resonator with low Q":

$$F_{res} = c\sqrt{\pi / 24}\rho^{1/2} h^{-3/2}$$

ANKA: F_{res} ~ 127 GHz

BESSY II: F_{res} ~ 100 GHz

MLS: F_{res} ~ 44 GHz

R.L. Warnock, PAC'91,
PAC1991_1824,
http://www.JACoW.org

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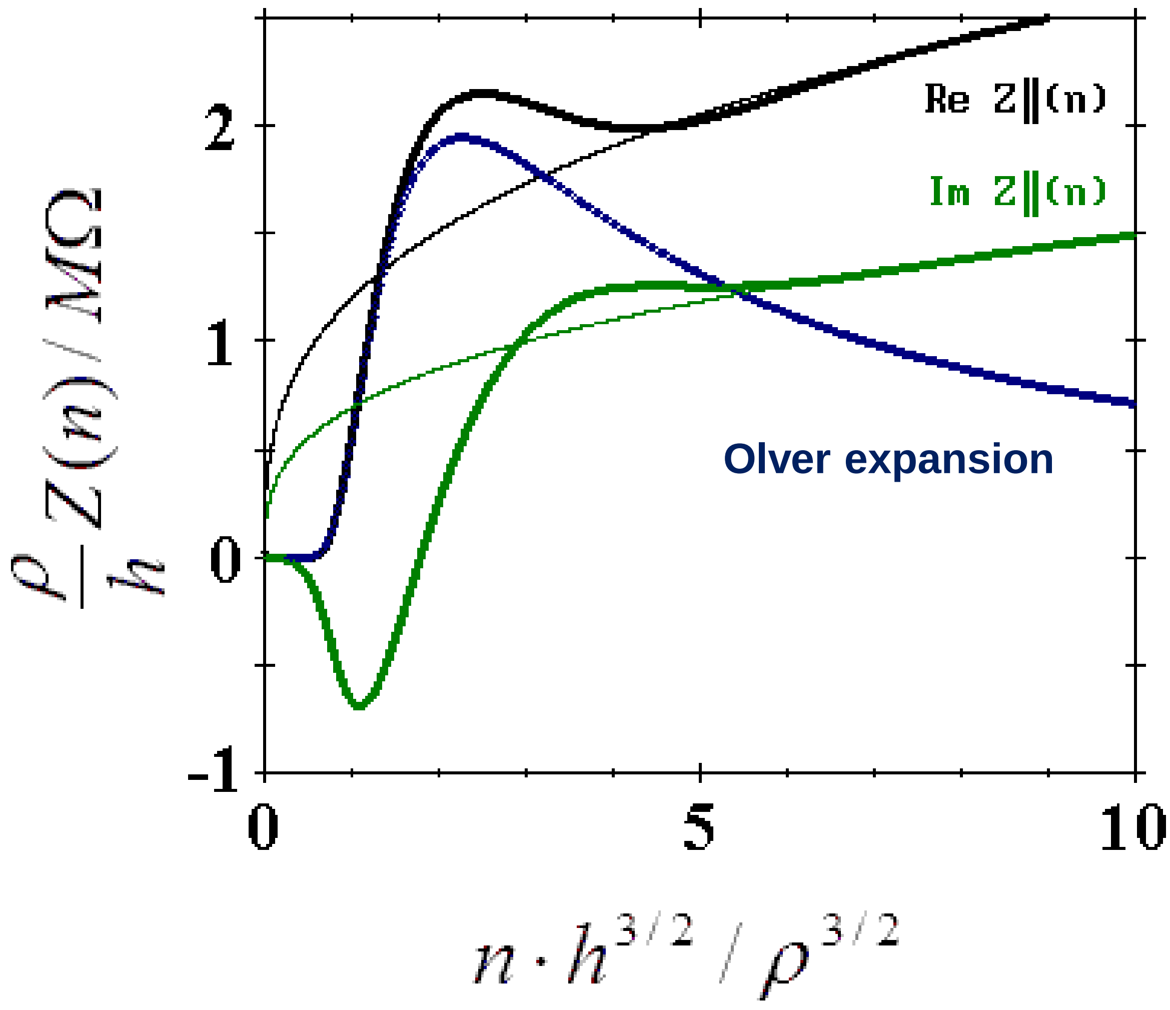

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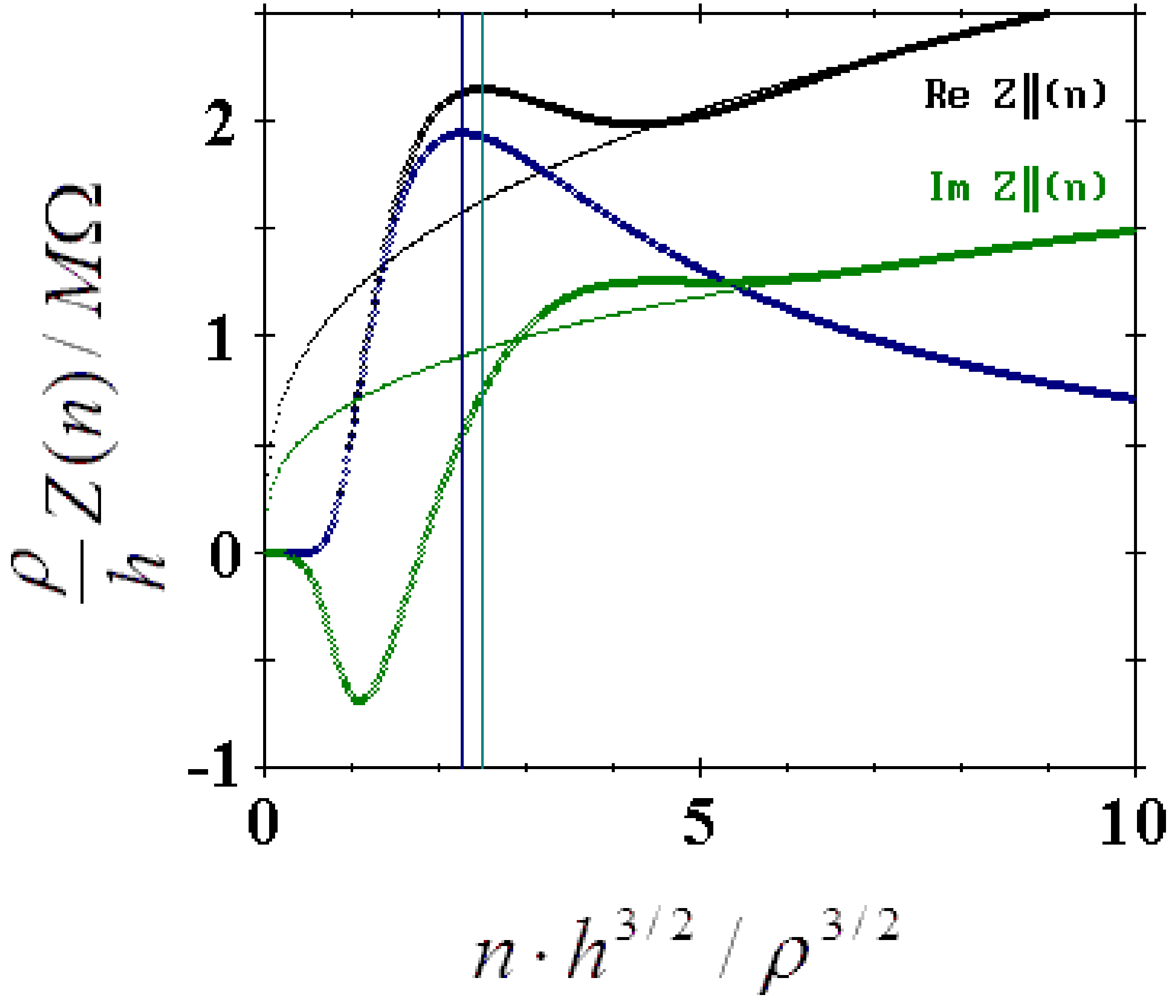

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MLS: F_{res} ~ 44 GHz

~10% higher

R.L. Warnock, PAC'91, PAC1991_1824, http://www.JACoW.org

this frequency and the bunch length determine the CSR-burst rate

Observations @ ANKA by V. Judin, et al.

TUPPP010 Proceedings of IPAC2012, New Orleans, Louisiana, USA

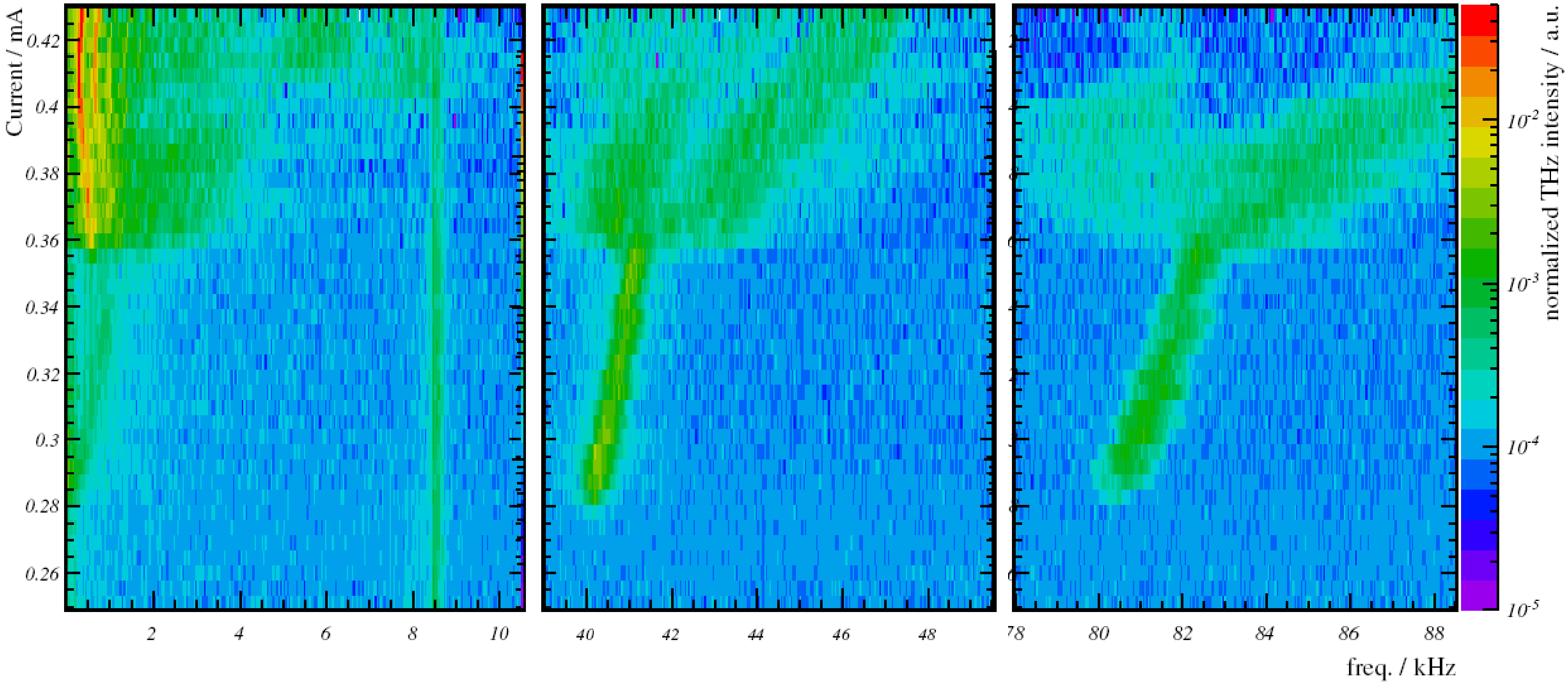
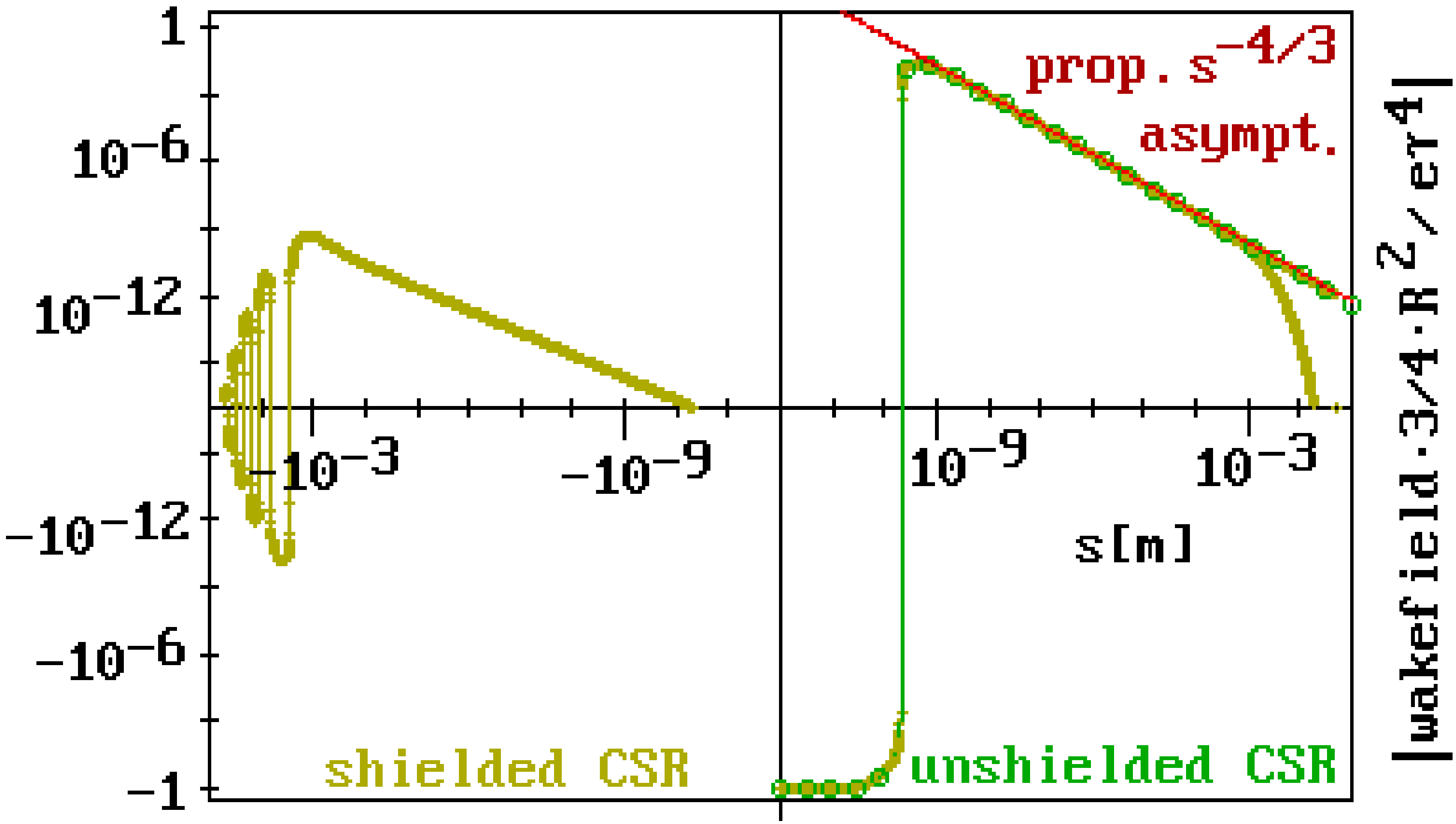


Figure 1: Instability spectrum of low frequency bursting is shown on the left diagram. Bursting occurs at currents > 0.28 mA and changes mode at 0.36 mA. The main instability can be found at about 400-600 Hz and drifts with the current. Its harmonics are also clearly observable. The $1f_s$ frequency line can be seen at 8.5 kHz. The middle and left figure show typical bursting patterns with onset at the same thresholds.

Parallel Plate Shielded CSR-Wake

J. B. Murphy, et al., Part. Acc. 1997, Vol. 57, pp 9-64



tail head

Haissinski Equation

Hamiltonian for the harmonic longitudinal motion – plus potential well distortion:

$$H = \frac{p^2}{2} + \frac{q^2}{2} + \frac{\alpha \cdot e}{E_0 T_0} \int_0^q V_W(q')dq'$$

current distribution, I(τ), with $\int I d\tau = N \cdot e =$ **total charge**

V_w, the induced voltage is given by folding the wake function per unit charge, W(q-q‘), with the current distribution:

$$V_W(q) = \int_{-\infty}^{+\infty} W(q-q')I(q')dq'$$

As long as the momentum distribution remains Gaussian there is only potential well distortion and the distorted current distribution can be calculated by the Haissinski equation:

$$I(t) = K \exp\left(-\frac{t^2}{2\sigma_0^2} - \frac{1}{\dot{V}_{rf}\sigma_0^2} \int_{-\infty}^{\tau} V_{ind}(\tau')d\tau'\right)$$

With $\int I(t)dt = 1$

see for example: R.D. Ruth in ‚Frontiers of Particel Beams; Observations, Diagnosis and Correction‘ in Lecture Notes in Physics 343, p. 247 ff, Springer Verlag Berlin Heidelberg 1989

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Calculation of Induced Voltage

$$V_W(q) = \int_{-\infty}^{+\infty} W(q-q')I(q')dq'$$

$$w(q) = w_0(q) + w_1(q)$$

free space ← w_0 | **shielding** ← w_1

$$w_0(q) = -\frac{4\pi}{3^{4/3}} H(q) \frac{\rho^{1/3}}{(q\sigma_{z0})^{4/3}}.$$

$$w_1(q) = -\rho^{1/3}\left(\frac{\Pi}{\sigma_{z0}}\right)^{4/3} G(\Pi q)$$

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **13**, 104402 (2010)

Threshold studies of the microwave instability in electron storage rings

K. L. F. Bane, Y. Cai, and G. Stupakov

SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA

(Received 12 July 2010; published 7 October 2010)

$$\Pi = \sigma_{z0}\rho^{1/2}/h^{3/2}$$

shielding factor

$$G(\zeta) = 8\pi \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2} \frac{Y_k(\zeta)[3 - Y_k(\zeta)]}{[1 + Y_k(\zeta)]^3}$$

where Y_k is a root of the equation

$$Y_k - \frac{3\zeta}{k^{3/2}} Y_k^{1/4} - 3 = 0$$

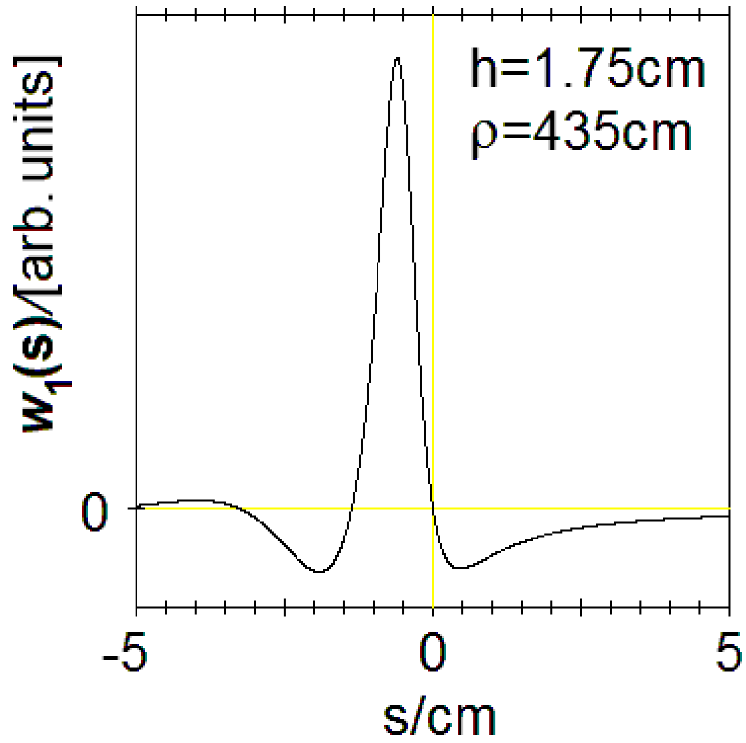
$$v_{\text{ind}}(q) = \int s(q')\lambda'(q-q')dq'$$

$$s(q) = \int_{-\infty}^{q} w(q')dq'$$ **step response function**

λ' **derivative of charge distribution**

Calculation of Induced Voltage, My Way

```
SUB Wparplates0 (W1, s)
'
'    wake function between parallel plates
'
h = .0175
radius = 4.35
Z0 = 480 * ATN(1)
delta = h / radius
eps = .000004
x = s / 2 / radius / delta ^ (1.5)
Ykstart = 3 ^ (1 / 4)
G2 = 0
FOR kk = 1 TO 30
     xx = x / kk ^ (1.5): Yk = .0001
     IF ABS(xx) < .000000000000001# THEN Yk = Ykstart: GOTO 1371
11351 F = Yk ^ 3 / 6 - 1 / 2 / Yk - xx
     dF = 3 * Yk ^ 2 + 1 / 2 / Yk ^ 2
     Yk = Yk - F / dF
     IF ABS(F) > eps THEN GOTO 11351
1371  G2 = G2 + (-1) ^ (kk + 1) / kk ^ 2 * 8 * Yk ^ 4 * (3 - Yk ^ 4) / (1 + Yk ^ 4) ^ 3
NEXT kk
W1 = -1 / 4 * G2 / delta ^ 2 / radius * Z0
END SUB
```



Calculation of Induced Voltage

$$V_W(q) = \int_{-\infty}^{+\infty} W(q-q')I(q')dq'$$

$$w(q) = w_0(q) + w_1(q)$$

free space

shielding

$$w_0(q) = -\frac{4\pi}{3^{4/3}} H(q) \frac{\rho^{1/3}}{(q\sigma_{z0})^{4/3}}.$$

$$w_1(q) = -\rho^{1/3}\left(\frac{\Pi}{\sigma_{z0}}\right)^{4/3} G(\Pi q)$$

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$$Y_k - \frac{3\zeta}{k^{3/2}} Y_k^{1/4} - 3 = 0$$

$$v_{\text{ind}}(q) = \int s(q')\lambda'(q-q')dq'$$

$$s(q) = \int_{-\infty}^{q} w(q')dq'$$ step response function

λ' derivative of charge distribution

Calculation of Induced Voltage, My Way

Solution of the Vlasow-Fokker-Planck equations, which produces very smooth charge distributions I apply a second integration by parts:

$$V_{ind} \approx 0.5 \int ds Z_0 3^{2/3} \rho^{1/3} s^{2/3} \lambda''(s)$$

Multi-particle tracking creates very noisy distributions (2^20 particles, bin width σ/10 and smaller) use Taylor expansion of λ(q) and direct integration – singularity as example:

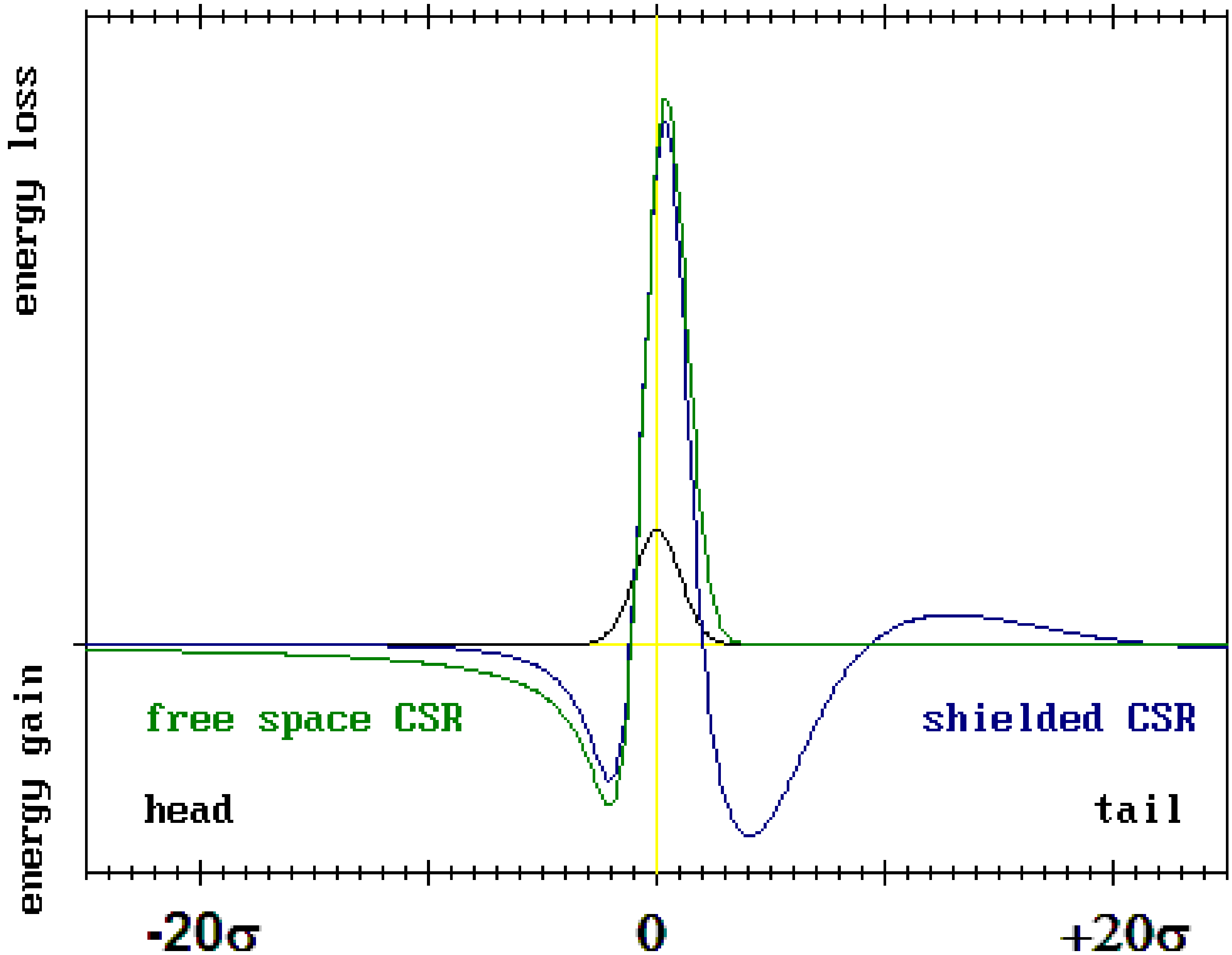
$$\Delta V_{ind} \approx \int_0^{\Delta q=\sigma/10} w_0(q-q')\lambda(q')dq' \approx \int_0^{\Delta q} w_0(q-q')\left[\lambda(0)+\lambda'(0)q'+\ldots\right]dq'$$

$$\int_0^{\Delta q} w_0(q-q')dq' = -\int_{\Delta q}^{\infty} w_0(q-q')dq'$$

Integrated wake functions:

$$w_a(q) = \int_q^{q+\Delta q} w_0(q-q')dq' \quad w_b(q) = \int_q^{q+\Delta q} w_0(q-q')q'dq'$$

Calculation of Induced Voltage, My Way



Shielded CSR-Wake: Theoretical Results

TUPD078 Proceedings of IPAC'10, Kyoto, Japan

COMPARISON OF SIMULATION CODES FOR MICROWAVE INSTABILITY IN BUNCHED BEAMS *

K.L.F. Bane, Y. Cai, G. Stupakov, SLAC National Accelerator Laboratory, Stanford, CA 94309, USA

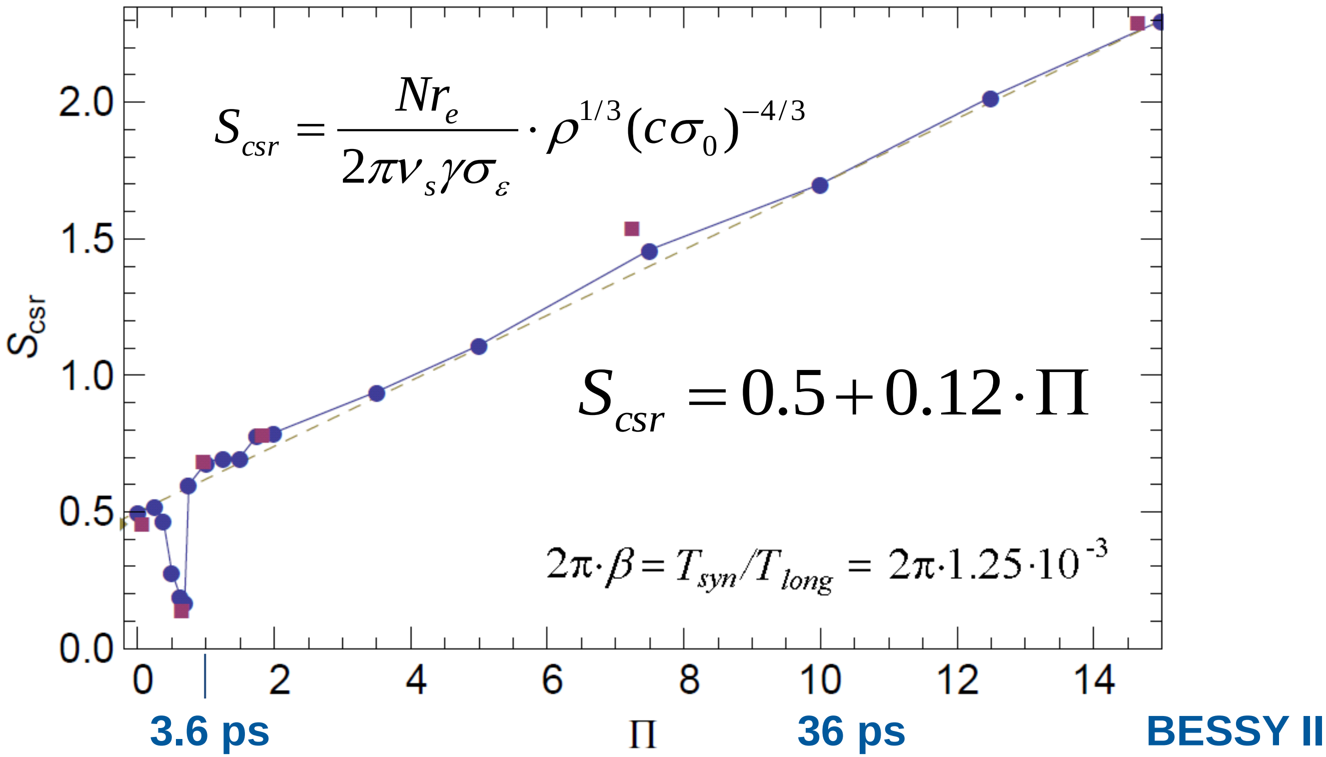
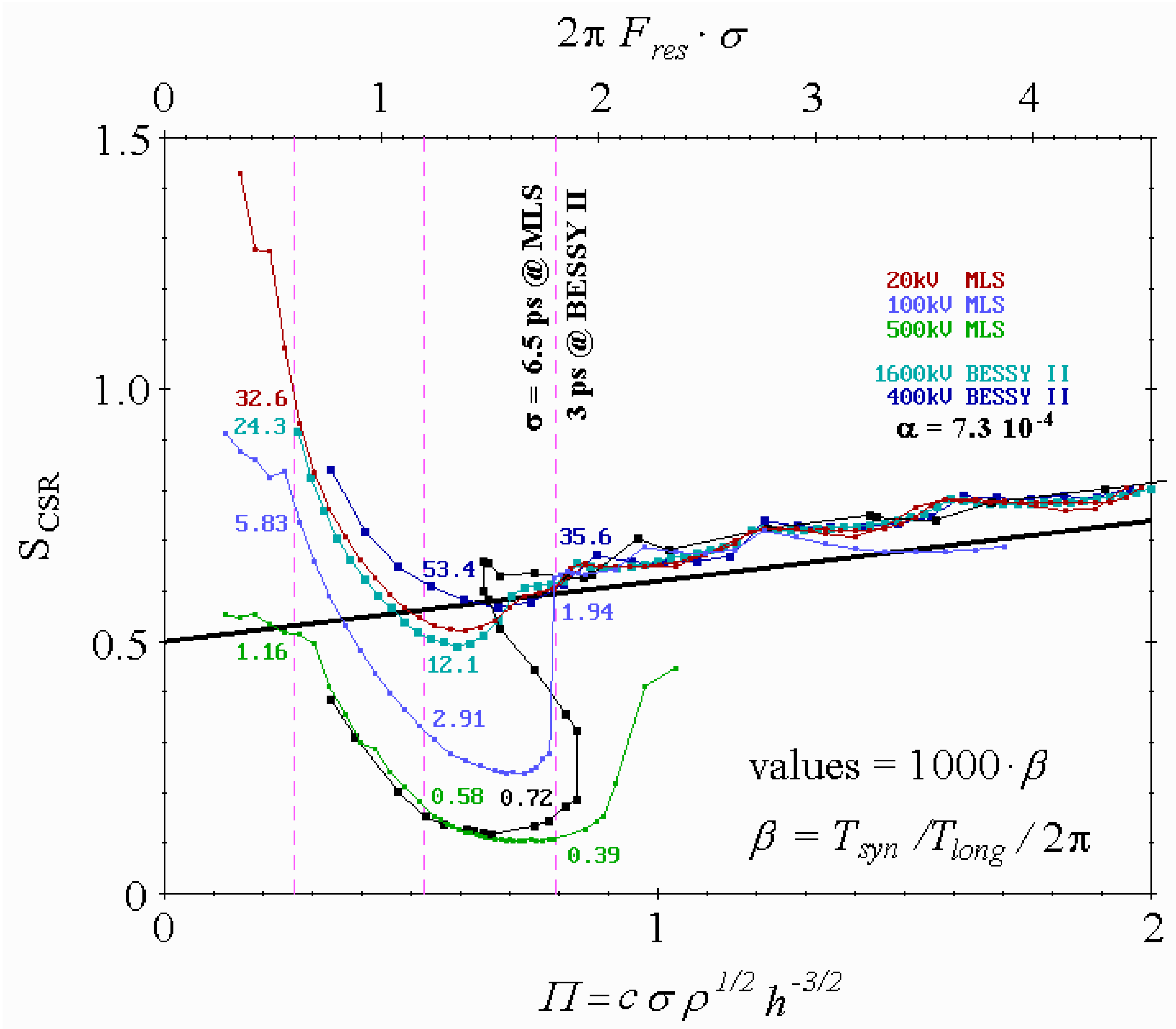


Figure 2: For the CSR wake, threshold value of $S_{\rm csr}$ *vs.* shielding parameter, $\Pi = \rho^{1/2}\sigma_{z0}/h^{3/2}$. Symbols give results of the VFP solver (blue) and the LV code (red).

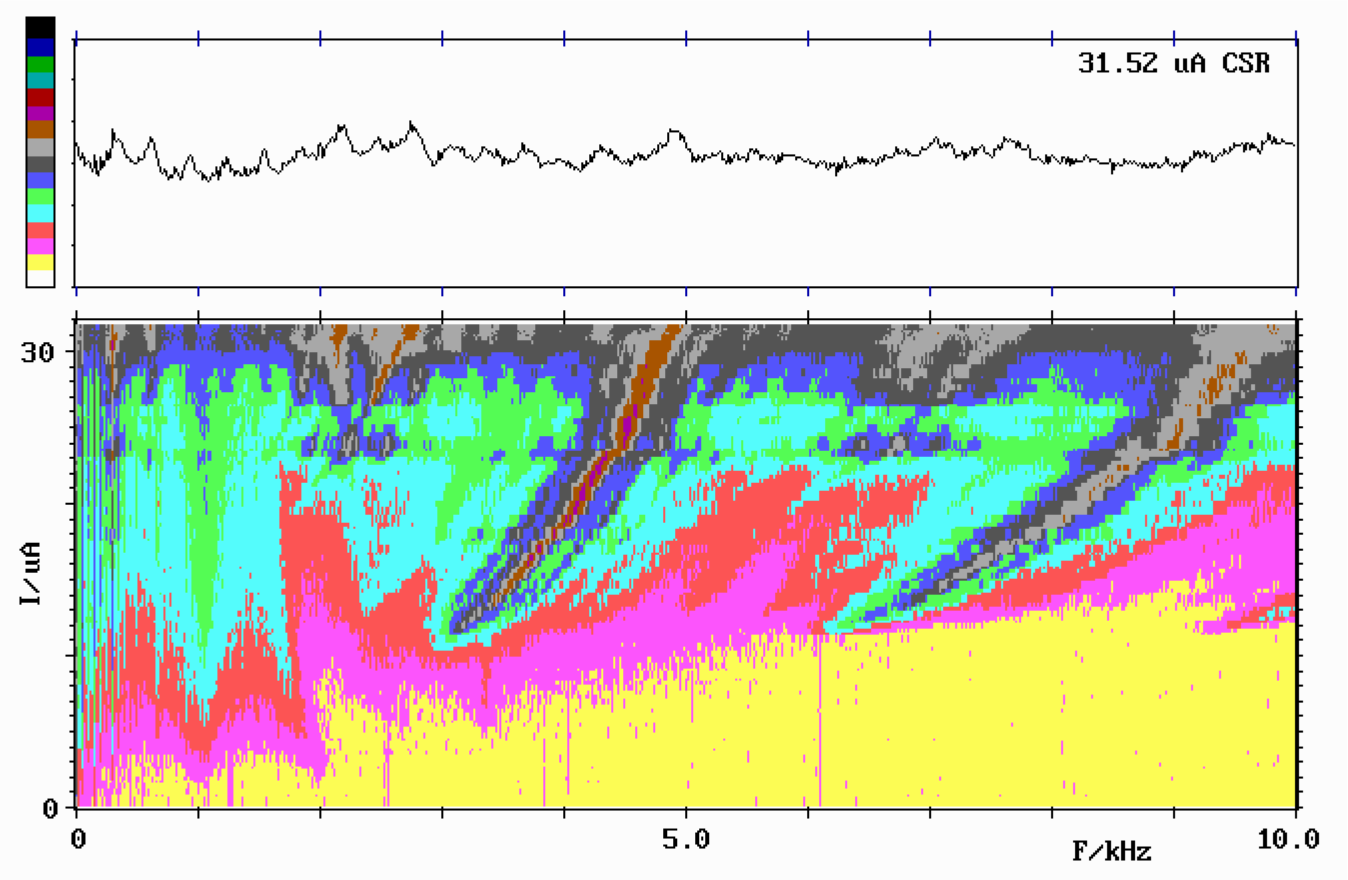
Shielded CSR-Wake: My Theoretical Results

$$S_{csr} = \frac{N r_e}{2\pi \nu_s \gamma \sigma_\varepsilon} \cdot \rho^{1/3} (c\sigma)^{-4/3}$$



CSR-Threshold Current Measurement

BESSY II, F_{syn_o}=1 kHz, σ_o~1.5 ps



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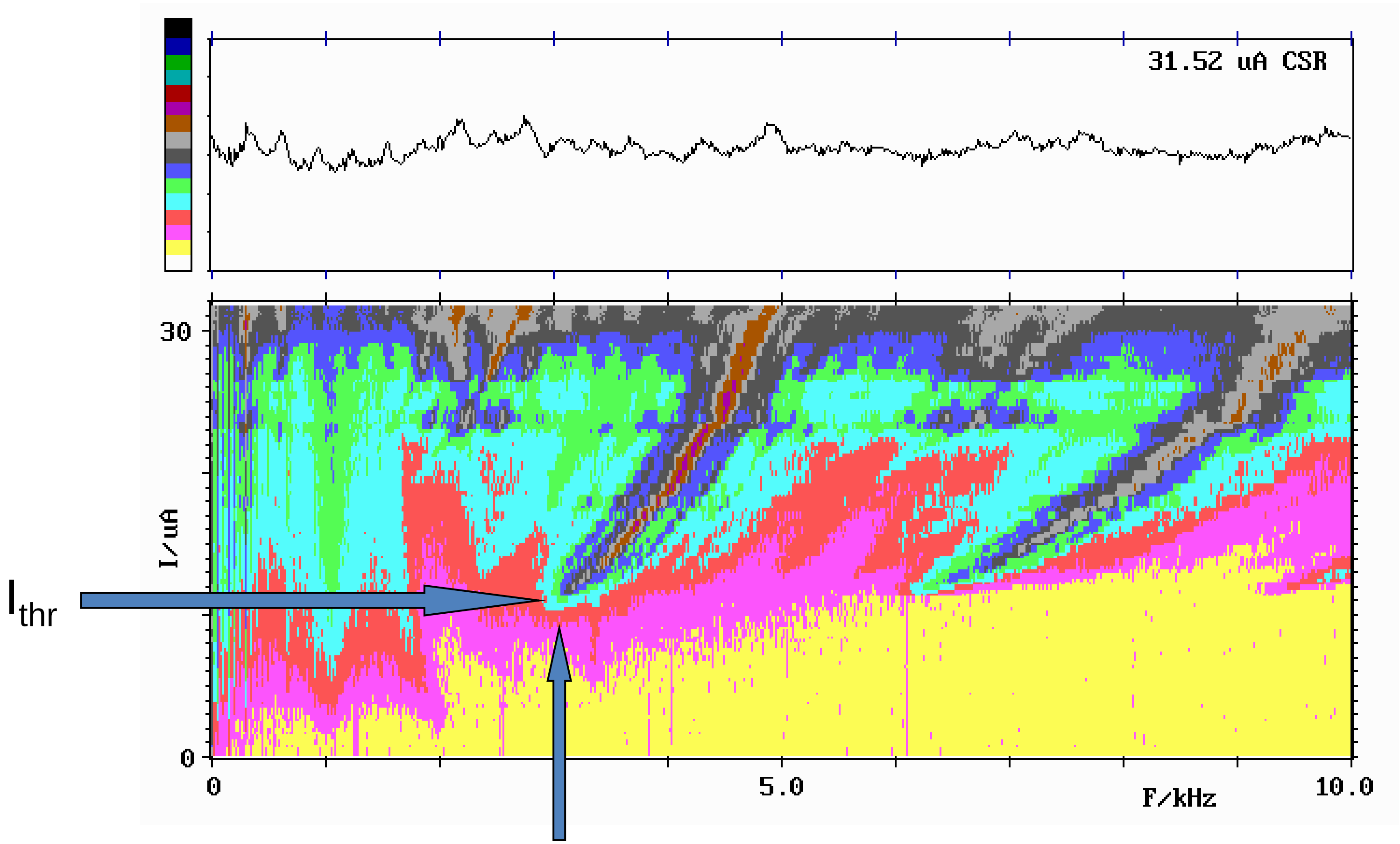
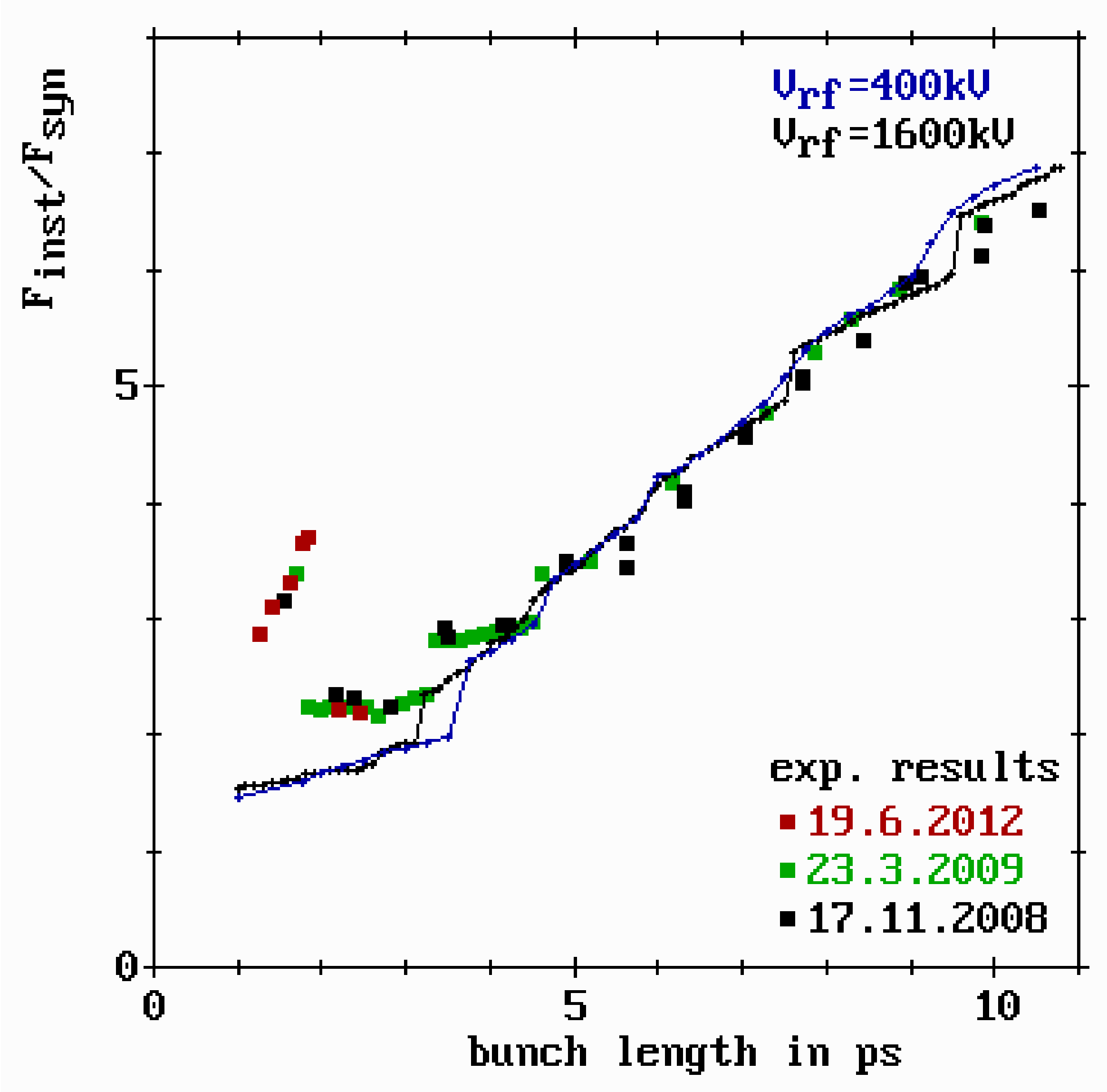


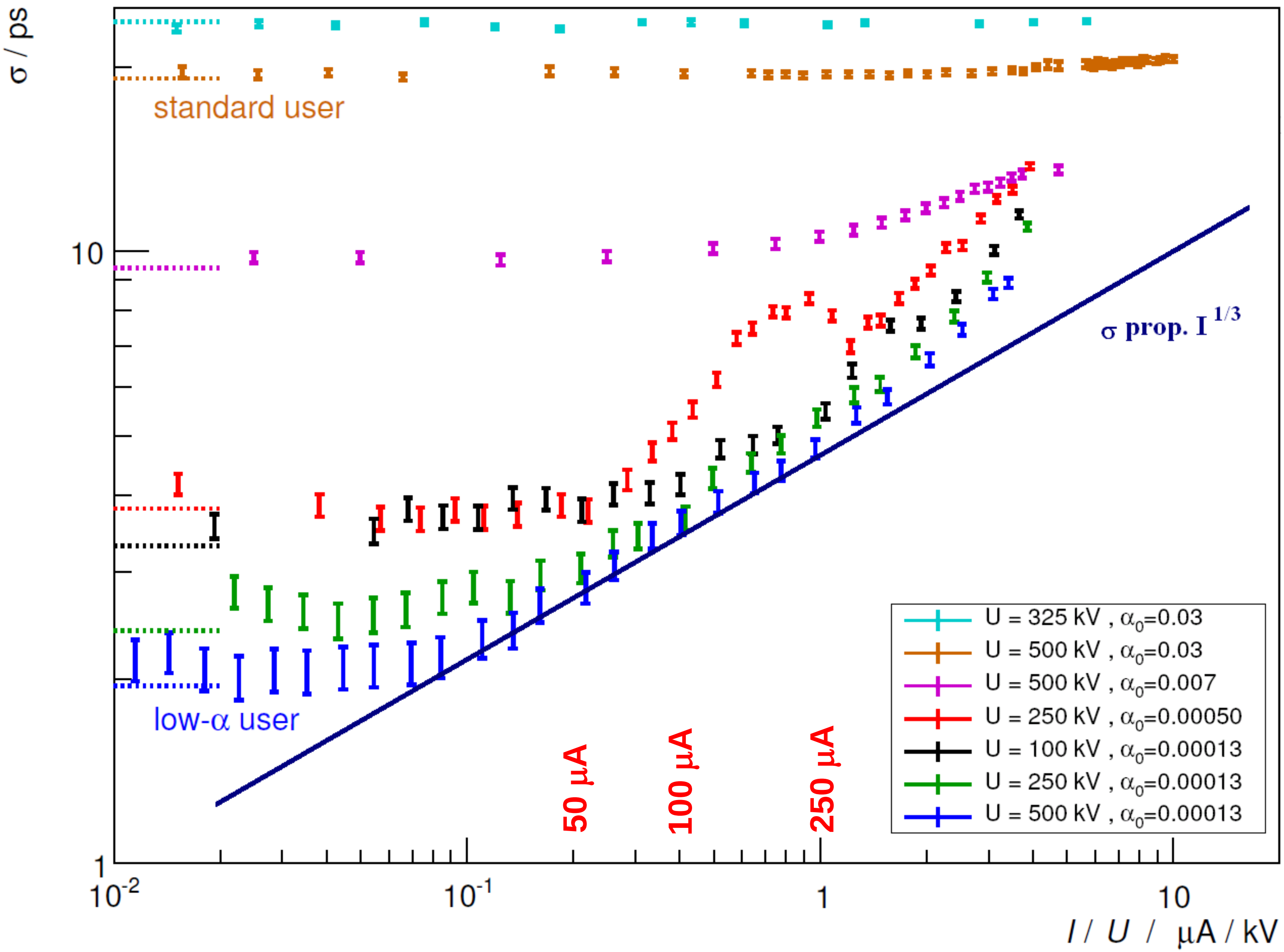
F_{inst}/F_{syn} ~ 3.1 instability mode number

First Unstable Modes BESSY II



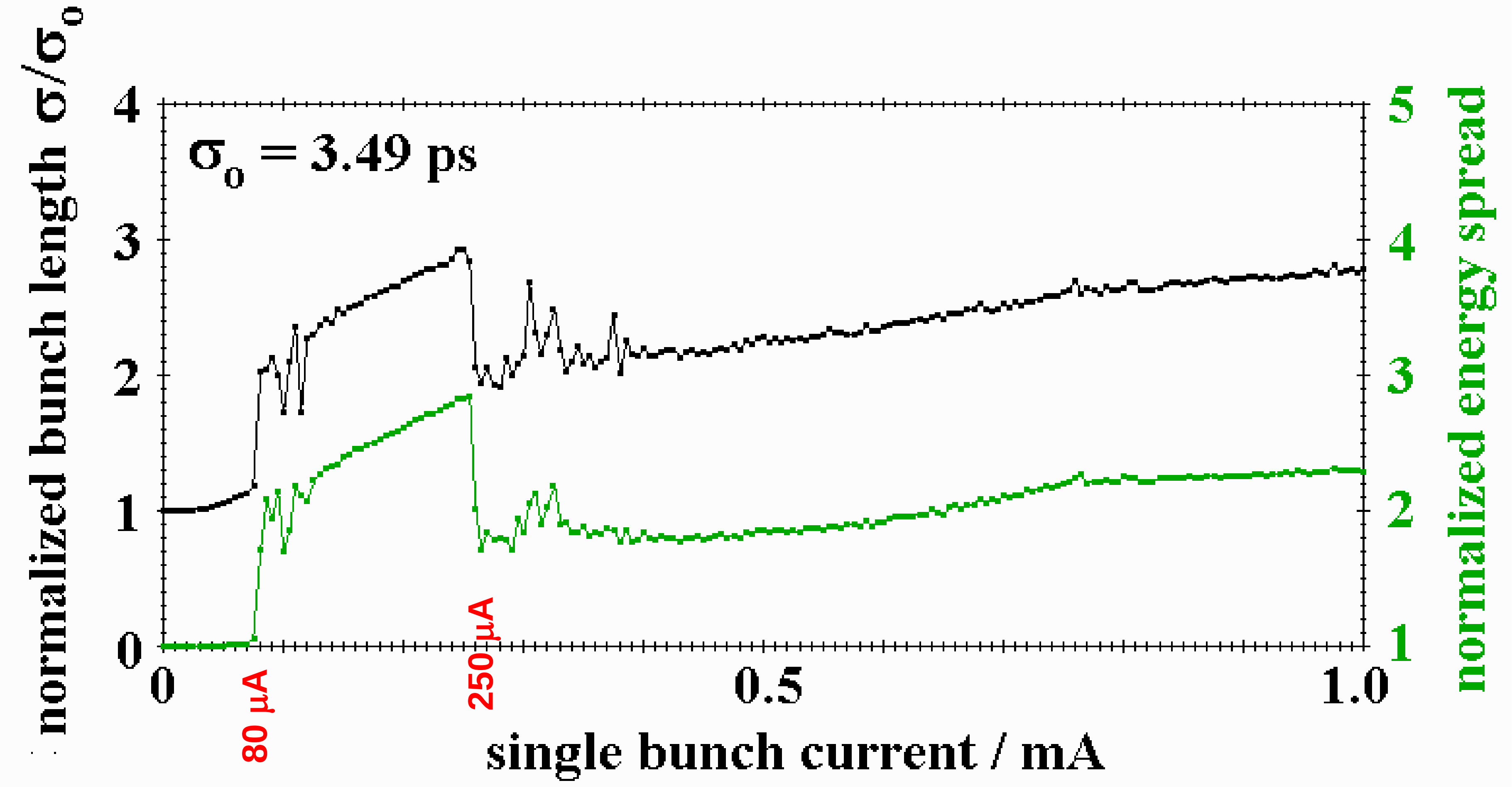
Slope agrees with resonance F_{res}~100 GHz

MLS – Bunch Length Measurements (M. Ries, PhD Thesis)

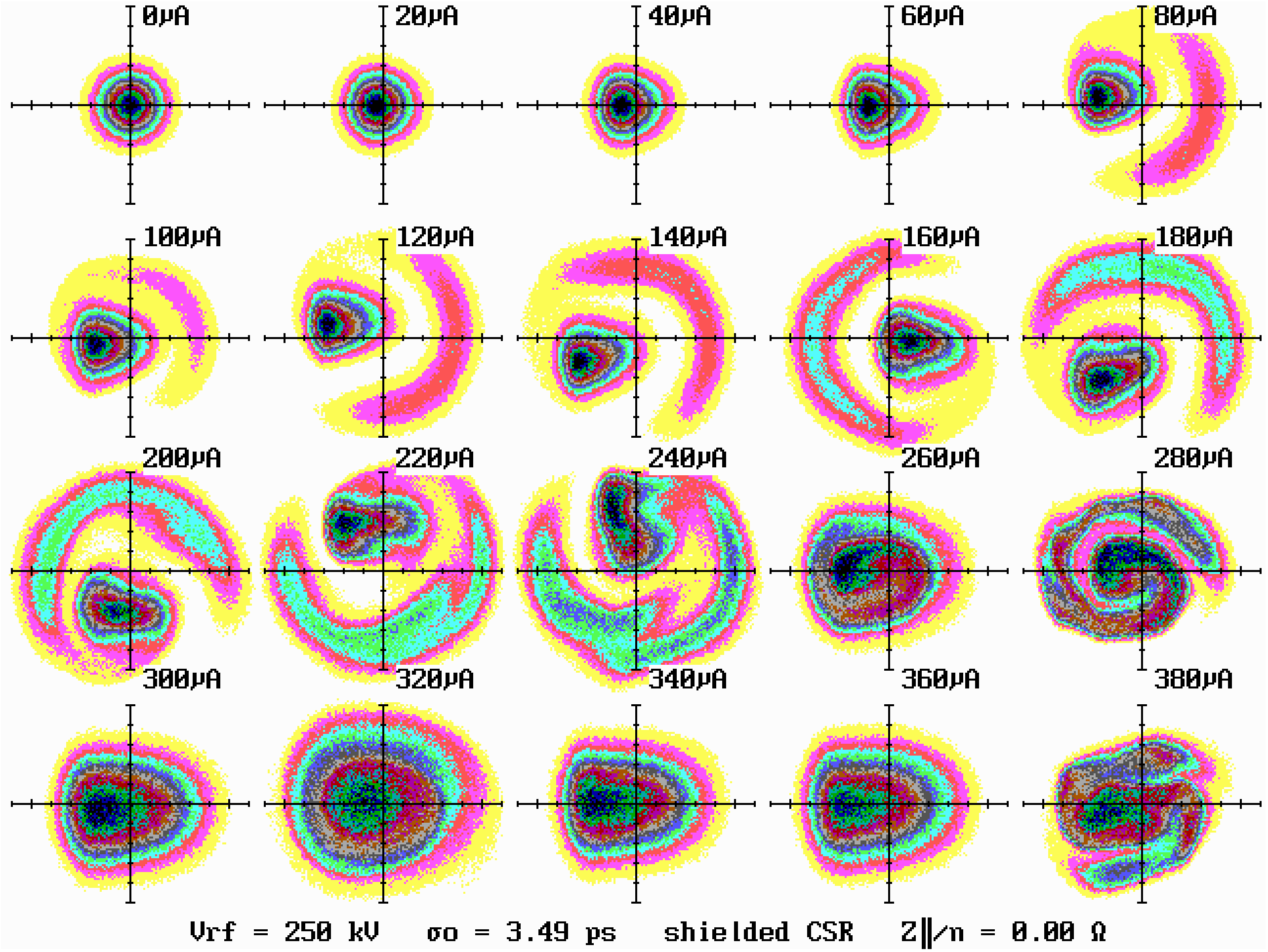


Simulated Bunch Length – Current Relation

MLS-Parameters: V_{rf} = 250 kV, α =5·10^{-4}, shielded CSR-interaction, no other impedance contributions



Simulated Longitudinal Phase Space Distributions



Like ‚Binary Star' Instability' – M. D'yachkov, R. Baartman, WEP130G, EPAC'96

Summary

Numerical solutions of the Vlasow-Fokker-Planck equation and simulations with multi particle tracking deliver identical results.

Predictions with simple shielded CSR model are in surprisingly good agreement with observed features of threshold currents and bunch length – not only at BESSY II and MLS.

Shielded CSR-interaction + inductive impedance seems to be an appropriate model for estimating the longitudinal single bunch threshold current for BESSY VSR.

A better model for the geometric and resistive impedance of the BESSY vacuum chamber would reduce uncertainties further.

Thank you for your attention!